Generalizations in Extenics of the Location Value and Dependent Function from A Single Finite Interval to *2D, 3D*, and *n-D* Spaces

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Abstract

- Qiao-Xing Li and Xing-Sen Li have defined in *2011* the Location Value of a Point and the Dependent Function of a Point on a single finite or infinite interval in the paper: *The Method to Construct Elementary Dependent Function on Single Interval.*
- In this paper we extend their definitions from one dimension (*1D*) to *2D, 3D*, and in general *n-D* spaces.
- Several examples are given in *2D* and *3D* spaces.

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Short Introduction to Extenics

- *Extenics* is a science initiated by Professor Cai Wen in *1983*. It is at the intersection of mathematics, philosophy, and engineering. Extenics solves contradictory problems. It is based on modeling and remodeling, on transforming and retransforming until getting a reasonable solution to apparently an unreasonable problem.
- Extenics solves unconventional and non-traditional problems and finding ingenious, perspicacious and novelty solutions.
- Extenics helps in solving problems in hard conditions, incomplete conditions, conflicting conditions. Where mathematics doesn’t work, i.e. for inconsistent problems where mathematics says that there is no solution, Extenics does work because it can obtain a solution.
- Everything is dynamic; we have dynamic structure, dynamic classification, and dynamic change.

Short Introduction to Extenics (2)

- In Extenics a problem may have more solutions, some of them even contradictory with each other, but all of them can be valid solutions.
- The five basic transformations are: substitution, increasing/decreasing, expansion/contraction, decomposition, and duplication.

What Extenics Studies:

- the antithetic properties of the matter: physical part (real) and non-physical part (imaginary), soft and hard parts of the matter, negative and positive parts of the matter;
- unfeasible problems are transformed to feasible problems;
- false propositions are transformed in true propositions;
- wrong inference is transformed into correct inference;
- non-conformity transformed to conformity;
- in business non–customers are transformed to customers;
- there are qualitative and quantitative transformations;
- transformation of matter-element, transformation of affair-element, transformation of relation-element;
- transformation of the characteristics;
- one considers transformation of a single part too (not of the whole);

What Extenics Studies (2)

- Extenics deals with unconventional problems which are transformed into conventional;
- inconsistent problems which are transformed into consistent;
- also one determines the composability and conductivity of transformations;
- Extenics finds rules and procedures of solving contradictory problems;
- gets structures and patterns to deal with contradictions;
- gets new methods of solving contradictions;
- reduces the degree of inconsistency of the problems;
- moves from divergent to less-divergent problems.

Location Value of a Point on a Single Finite Interval (1D-Space)

- Suppose *S* = <*a*, *b*> is a finite interval. By the notation <*a*, *b*> one understands any type of interval: open *(a, b)*, closed *[a, b]*, or semi-open/semi-closed *(a, b]* and *[a, b)*.
- For any real point $x_0 \in R$, Qiao-Xing Li and Xing-Sen Li have considered:

$$D(x_0, S) = a - b$$

as the location value of point $P(x_0)$ on the single finite interval <*a*, *b*>.

Thus $D(x_0, S) = D(P, S) = a - b < 0$ *since* $a < b$.

As we can see, *a*-*b* is the negative distance between the frontiers of the single finite interval *S* in the *1D*-space.

Dependent Function of a Point on a Single Finite Interval (1D-Space)

- Afterwards, the above authors defined for any real point *P(x_0)*, with $x_0 \in S$, the elementary dependent function on the single interval *S* in the following way:

$$k(x_0) = \frac{\rho(x_0, S)}{D(x_0, S)}$$

where $\rho(x_0, S)$ is the extension distance between point x_0 and the finite interval *X* in the *1D*-space.

We can re-write the above formula as:

$$k(P) = \frac{\rho(P, S)}{D(P, S)}$$

Attraction Point

Let *S* be a given set in the universe of discourse *U*, and the optimal point $O \in S$.

Then each point $P(x_1, x_2, \ldots, x_n)$ from the universe of discourse tends towards, or is attracted by, the optimal point *O*, because the optimal point *O* is an ideal of each other point.

There could be one or more linearly or non-linearly trajectories (curves) that the same point *P* may converge on towards *O*.

Let's call all such points' trajectories as the **Network of Attraction Curves (NAC)**.

Generalized Location Value of Point *P* on the Single Finite Set *S* in *n-D* Space

- $D_{n\text{-}D}(x_0, S)$, is the classical geometric distance (yet taken with a negative sign in front of it) between the set frontiers, distance taken on the line (or in general taken on the curve or geodesic) passing through the optimal point *O* and the given point *P.*
- If there are many distinct curves passing through both *O* and *P* in the Network of Attraction Curves, then one takes that curve for which one gets the maximum geometric distance (and one assigns a negative sign in front of this distance).
- We can also denote it as $D_{n\text{-}D}(P, S)$.

1D Extension-Distance Principle

- Prof. Cai Wen's 1983 definition of Extension Distance $\rho(x_0,S)$ in *1D* satisfies the following principle:
- $\rho(x_0,S)$ = *the classical geometric distance between the point* x_0 *and the closest extremity point of the interval <a, b> to it (going in the direction that connects* x_0 *with the optimal point), distance*

taken as negative if $x_0 \in Int(<a, b>)$*,*

as positive if $x_0 \in Ext(<a, b>)$*,*

and as zero if $x_0 \in Fr(<a, b>)$*;*

where Int(<a, b>) = interior of the set <a, b>,

Ext(<a, b>) = exterior of the set <a, b>,

and Fr(<a, b>) = frontier of the set <a, b>.

n-D Extension-Distance

$$\rho_{nD}(P,S)=\begin{cases}-\max\limits_{c\in NAC}\ \ d\limits_{P'\in Fr(S)}(P,P';c), & P\neq O, P\in c(OP');\\ \max\limits_{c\in NAC}\ d\limits_{P'\in Fr(S)}(P,P';c), & P\neq O, P'\in c(OP);\\ -\max\limits_{c\in NAC, M\in Fr(S), M\in c(O)} d(P,M;c), & P=O.\end{cases}$$

where $\rho_{nD}(P,S)$ means the extension distance as measured along the curve *c* in the *n-D* space;

- *O* is the optimal point (or non-linearly attraction point);
- the points are attracting by the optimal point *O* on trajectories described by an injective curve *c*;
- *d(P,P';c)* means the non-linearly *n-D*-distance between two points *P* and *P'* along the curve *c*, or the arclength of the curve *c* between the points *P* and *P'*;

n-D Extension Distance (2)

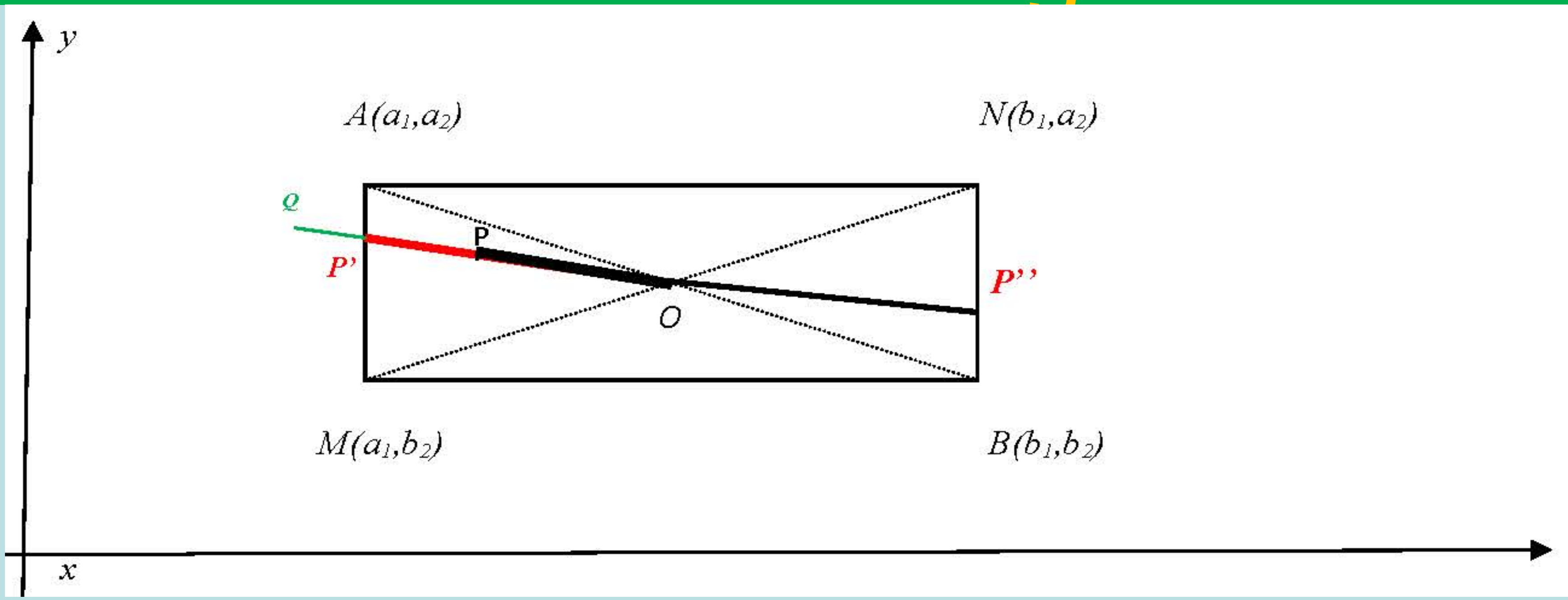
- *Fr(S)* means the frontier of set *S*;
- and c(*OP')* means the curve segment between the points *O* and *P'* (the extremity points *O* and *P'* included), therefore *P*∈*c(OP')* means that *P* lies on the curve *c* in between the points *O* and *P'*.
- For *P* coinciding with *O*, one defined the distance between the optimal point *O* and the set *S* as the negatively maximum curvilinear distance (to be in concordance with the *1D*-definition).
- In the same way, if there are many curves, *c* in the Network of Attraction Curves, passing through both *O* and *P,* then one chooses that curve which maximizes the geometric distance.
- We do these maximizations in order to be consistent with the case when the point *P* coincides with the optimal point *O*.

Generalized Dependent Function of Point *P* on the Single Finite Set *S* in *n-D*-Space

- is the geometric distance between point *P* and the closest frontier on the line (or in general on the curve/geodesic c that connects *P* with the optimal point *O)* in the same side of the optimal point, divided by the distance [taken along the line (or in general on the curve/geodesic c that connects P with the optimal point *O*)] between the set frontiers.
- If there are more curves passing through *P* and *O*, then one takes that curve which maximizes the value of $k_{nD}(P)$.
- Its formula is:

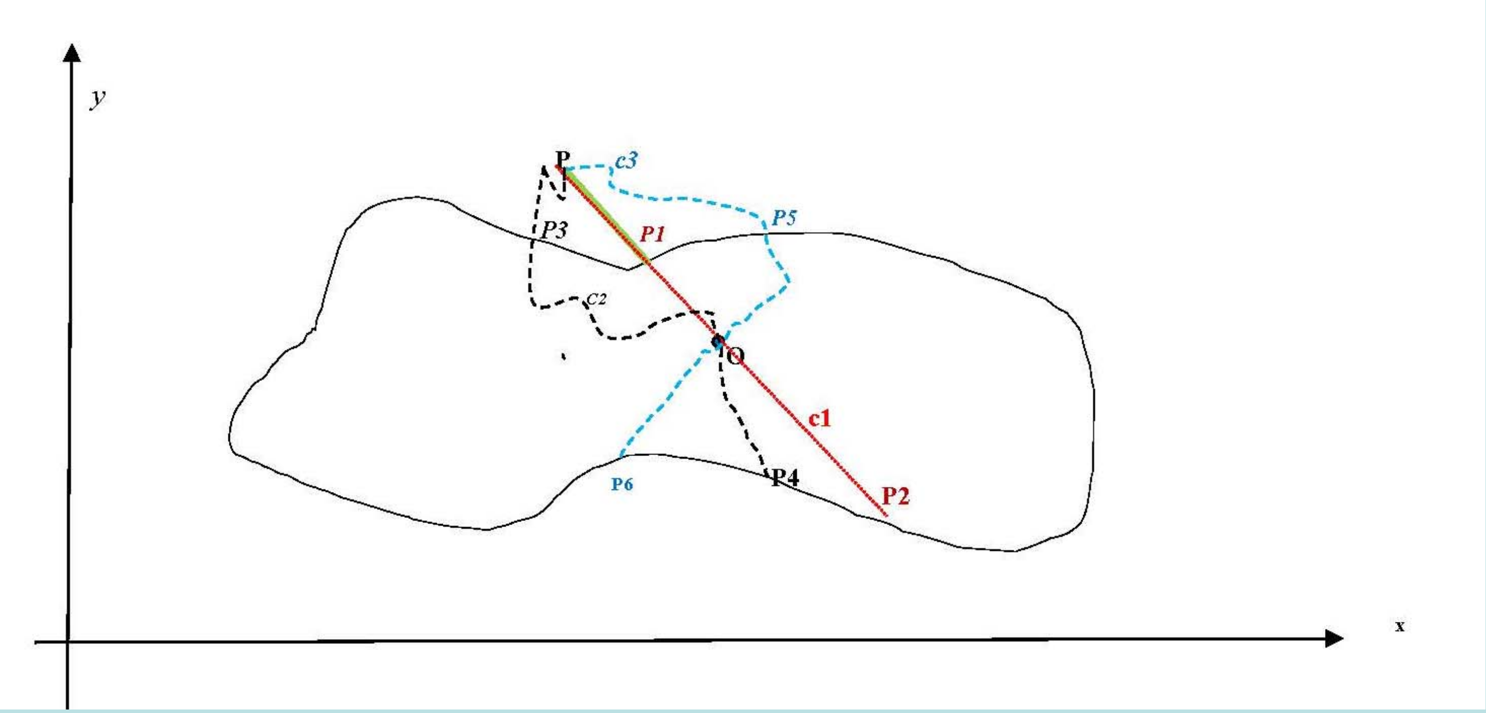
$$k_{nD}(P) = \max_{c \in NAC} \frac{\rho_{nD}(P,S;c)}{D_{nD}(P,S;c)}$$

2D-Example for Finite Single Set with One Attracting Curve



- **Location Value of the Point P on the Single Finite 2D-Set (rectangle AMBN) is: the geometrical distance |P'P''|.**
- **Dependent Function of a Point on the Single Finite 2D-Set (rectangle AMBN) is:**
- **K(P) = + |PP'| / |P'P''|, where P = interior point;**
- **k(P') = 0, where P' = frontier point;**
- **k(Q) = - |QP'| / |P'P''|, where Q = exterior point**

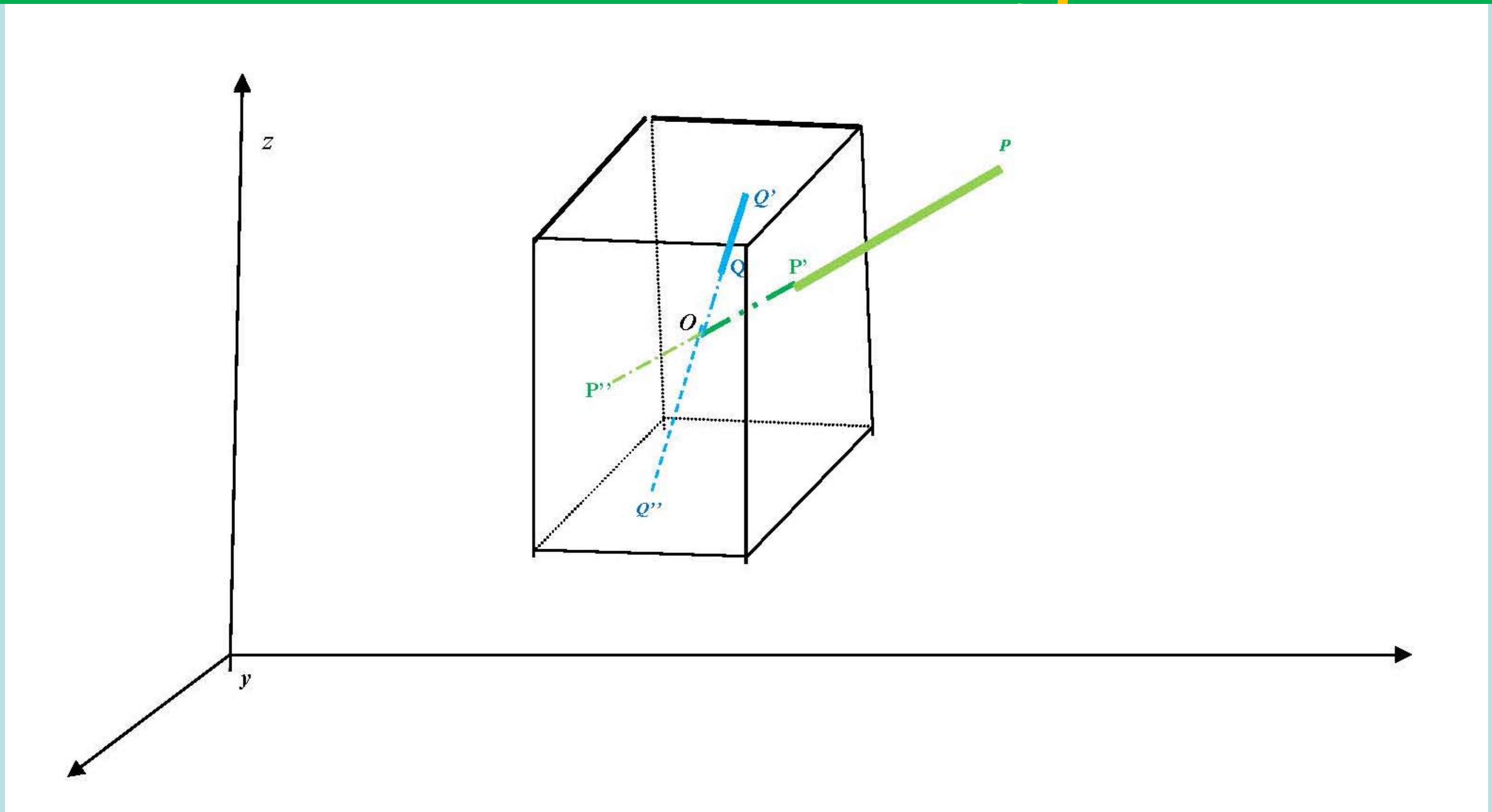
2D-Example for Finite Single Set with Many Attracting Curves



- **The dependent function of point P is:**

$$k(P) = -\max\{\frac{c1(PP1)}{c1(P1P2)} = \frac{|PP1|}{|P1P2|}, \frac{c2(PP3)}{|c2(P3P4)}, \frac{c3(PP5)}{c3(P5P6)}\}$$

3D-Example for Finite Single Set With One Attracting Curve



The 3D-dependent function calculated for points P, Q, and Q' is: $k(P) = -\frac{|PP'|}{|P''P'|}, k(Q) = +\frac{|QQ'|}{|Q''Q'|}, k(P') = k(Q') = 0$

Conclusions

- **The Location Value of a Point and the Dependent Function of a Point on a Single Finite or Infinite Set were generalized from *1D* to *n-D* Space with respect to a given Optimal Point (or Attraction Point);**
- **Examples for 2D- and 3D-spaces, using one or many attracting curves towards the attracting point, were presented;**

- **Research to be done in the future:**

To extend The Location Value of a Point and the Dependent Function of a Point on a Single Finite or Infinite Set to *n-D* Space without necessarily an attracting point.

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