

# GENERALIZED SINGLE VALUED NEUTROSOPHIC GRAPHS OF FIRST TYPE

Said Broumi, Assia Bakali , Mohamed Talea, Florentin Smarandache, Ali Hassan

Laboratory of Information processing, Faculty of Science Ben M'Sik, University Hassan II, B.P 7955, Sidi Othman, Casablanca, Morocco.

Ecole Royale Navale, Boulevard Sour Jdid, B.P 16303 Casablanca, Morocco.

Department of Mathematics, University of New Mexico, 705 Gurley Avenue, Gallup, NM 87301, USA.

Department of Mathematics, University of Punjab (New campus), Lahore (Pakistan)

**Abstract**—Motivated by the notion of generalized fuzzy graphs proposed by Samanta et al.[40] and the notion of single valued neutrosophic graphs proposed by Broumi et al.[30]. In this paper, we define the concept of generalized single valued neutrosophic graphs of first type ( GSVNG-1) and presented a matrix representation for it and studied few properties of this new concept. The concept of GSVNG is a generalization of generalized fuzzy graphs (GFG1).

**Keywords:** Single valued neutrosophic graph; Generalized single valued neutrosophic graphs first type; Matrix representation

# I.INTRODUCTION

In 1998, Smarandache [8] grounded the concept of proposed the neutrosophic set theory (NS) from philosophical point of view by incorporating the degree of indeterminacy or neutrality as independent component to deal with problems involving imprecise, indeterminate and inconsistent information. The concept of neutrosophic set theory is a generalization of the theory of fuzzy sets [17], intuitionistic fuzzy sets [14, 15], interval-valued fuzzy sets [13] and interval-valued intuitionistic fuzzy sets [16]. The concept of neutrosophic set is characterized by a truth-membership degree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval  $]^{-0, 1^{+}}$ . Therefore, if their range is restrained within the real standard unit interval  $[0, 1]$ , Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in  $]^{-0, 1^{+}}$ . The single valued neutrosophic set was introduced for the first time by Smarandache in his book [8]. The single valued neutrosophic sets as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Later on, Wang et al. [12] studied some properties related to single valued neutrosophic sets. The concept of neutrosophic sets and its extensions such as single valued neutrosophic sets, interval neutrosophic sets, simplified neutrosophic sets and so on have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine and economic and can be found in [44].

Graphs are the most powerful and handfull tool used in representing information involving relationship between objects and concepts. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from  $[0, 1]$ . The most complete trends on fuzzy graphs is [2]. Later on Atanassov [3] defined intuitionistic fuzzy graphs (IFGs) using five types of Cartesian products. The concept fuzzy graphs, intuitionistic fuzzy graphs and their extensions such interval valued fuzzy graphs, bipolar fuzzy graph, bipolar intuitionistic fuzzy graphs, interval valued intuitionistic fuzzy graphs, hesitancy fuzzy graphs, vague graphs and so on, have been studied deeply in over hundred papers. All these types of graphs have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. In 2016, Samanta et al [40] proposed a new concept called the generalized fuzzy graphs (GFG) and studied some major properties such as completeness and regularity with proved results. The authors classified the GFG into two type. The first type is called generalized fuzzy graphs of first type (GFG1). The second is called generalized fuzzy graphs of second type 2 (GFG2). Each type of GFG are represented by matrices similar to fuzzy graphs. In this paper, the authors claim that fuzzy graphs defined by several researches are limited to represent for some systems such as social network.

When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy intuitionistic fuzzy, bipolar fuzzy, vague and interval valued fuzzy graphs. So, for this purpose, Smarandache [12] proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Many book on neutrosophic graphs based on literal indeterminacy (I) was completed by Smarandache and Kandasamy [43]. Later on, Smarandache [5, 6] gave another definition for neutrosophic graph theory using the neutrosophic truth-values (T, I, F) without and constructed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on Smarandache [10] proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripolar/ multipolar graph. In a short period of time, few authors have focused deeply on the study of neutrosophic vertex-edge graphs and explored diverse types of different neutrosophic graphs. Broumi et al.[29] combined the concept of single valued neutrosophic sets and graph theory, and introduced certain types of single valued neutrosophic graphs (SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph and investigate some of their properties with proofs and examples. Later on, Broumi et al.[30] also introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, Broumi et al.[31] proved a necessary and sufficient condition for a single valued neutrosophic graph to be a single valued neutrosophic graph.

When description of the object or their relations or both is indeterminate and inconsistent, it cannot be handled by fuzzy intuitionistic fuzzy, bipolar fuzzy, vague and interval valued fuzzy graphs. So, for this purpose, Smarandache [12] proposed the concept of neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. Many book on neutrosophic graphs based on literal indeterminacy (I) was completed by Smarandache and Kandasamy [43]. Later on, Smarandache [5, 6] gave another definition for neutrosophic graph theory using the neutrosophic truth-values (T, I, F) without and constructed three structures of neutrosophic graphs: neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on Smarandache [10] proposed new version of neutrosophic graphs such as neutrosophic offgraph, neutrosophic bipolar/tripolar/ multipolar graph. In a short period of time, few authors have focused deeply on the study of neutrosophic vertex-edge graphs and explored diverse types of different neutrosophic graphs. Broumi et al.[29] combined the concept of single valued neutrosophic sets and graph theory, and introduced certain types of single valued neutrosophic graphs (SVNG) such as strong single valued neutrosophic graph, constant single valued neutrosophic graph, complete single valued neutrosophic graph and investigate some of their properties with proofs and examples. Later on, Broumi et al.[30] also introduced neighborhood degree of a vertex and closed neighborhood degree of vertex in single valued neutrosophic graph as a generalization of neighborhood degree of a vertex and closed neighborhood degree of vertex in fuzzy graph and intuitionistic fuzzy graph. In addition, Broumi et al.[31] proved a necessary and sufficient condition for a single valued neutrosophic graph to be a single valued neutrosophic graph.

The same authors [36] defined the concept of bipolar single neutrosophic graphs as the generalization of bipolar fuzzy graphs, N-graphs, intuitionistic fuzzy graph, single valued neutrosophic graphs and bipolar intuitionistic fuzzy graphs. In addition, the same authors [38] introduce different types of bipolar single valued neutrosophic graphs such as bipolar single valued neutrosophic graphs, complete bipolar single valued neutrosophic graphs, regular bipolar single valued neutrosophic graphs and investigate some of their related properties. In [32, 33], the authors defined the concept of interval valued neutrosophic graphs, the concept of strong interval valued neutrosophic graph, and studied some operations. Nasir et al. [25, 26] proposed a new type of graph called neutrosophic soft graphs and established a link between graphs and neutrosophic soft sets. The authors also, defined some basic operations of neutrosophic soft graphs such as union, intersection and complement. Recently, Akram et al. [18] introduce the notion of single-valued neutrosophic graphs in a different way and presented some fundamental operations on single-valued neutrosophic graphs. And explored some interesting properties of single-valued neutrosophic graphs by level graphs. Akram et al. [23] introduced the concept of single-valued neutrosophic graph structures and present certain operations of single-valued neutrosophic graph structures. Further, the authors discussed some novel applications of single-valued neutrosophic graph structures in decision-making problems Akram et al. [19, 20, 21] portrayed the concept of single valued neutrosophic hypergraphs, single valued neutrosophic planar graphs, neutrosophic soft graphs and intuitionistic neutrosophic soft graphs.

Malik and Hassan [22] defined the concept of single valued neutrosophic trees and studied some of their properties. Later on, Ali Hassan et Malik [1] introduced some classes of bipolar single valued neutrosophic graphs and studied some of their properties. P. K. Singh[27] has discussed adequate analysis of uncertainty and vagueness in medical data set using the properties of three-way fuzzy concept lattice and neutrosophic graph introduced by Broumi et al [29]. This study provided a precise representation of medical diagnoses problems using the vertices and edges of neutrosophic graph. Further to refine the knowledge three-way fuzzy concepts generation and their hierarchical order visualization in the concept lattice is proposed using neutrosophic graph and component-wise Godel resituated lattice. One application of the proposed method is also discussed to analyze the multi-criteria decision making process

Ashraf et al.[41], proposed some novels concepts of edge regular, partially edge regular and full edge regular single valued neutrosophic graphs and investigated some of their properties. Also the authors, introduced the notion of single valued neutrosophic digraphs (SVNDGs) and presented an application of SVNDG in multi-attribute decision making. Fathhi et al.[42] computed the dissimilarity between two neutrosophic graphs based on the concept of Hausdorff distance. In 2017, Mehra and Singh [40] introduced the concept of single valued neutrosophic Signed graphs and examined the properties of this concept with examples.



Similar to the fuzzy graphs, which have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects. Also, the single valued neutrosophic graphs presented in the literature [18, 25, 39] have a common property, that edge membership value is less than the minimum of its end vertex values. Whereas the edge indeterminacy-membership value is less than the maximum of its end vertex values or is greater than the maximum of its end vertex values. And the edge non-membership value is less than the minimum of its end vertex values or is greater than the maximum of its end vertex values.

In [9], Smarandache has discussed the removal of the edge degree restriction of single valued neutrosophic graphs with the following example: we can consider the most general case of single-valued neutrosophic graphs, i.e. when the neutrosophic truth-values of the vertices are independent from the neutrosophic truth values of the edges. This is just from our everyday life, because: If John ( $u$ ) and George ( $v$ ) are two individuals (vertices) in a given set association ( $A$ ), and  $u$  and  $v$  belong each of them in a specific neutrosophic degree respectively  $(t_u, i_u, f_u)$  and  $(t_v, i_v, f_v)$  to the set  $A$ , then the edge  $uv$  (meaning the relationship between  $u$  and  $v$ ) is not necessarily dependent on the degree of appurtenance of  $u$  and  $v$  to  $A$ .

Motivated by the Samanta's work, and the question proposed by Smarandache in his book [9]. The main objective of this paper is to extend the concept of generalized fuzzy graph of first type to single valued neutrosophic graphs first type (GSVNG1) to model systems having an indeterminate information and introduced a matrix representation of GSVNG1.

Malik and Hassan [22] defined the concept of single valued neutrosophic trees and studied some of their properties. Later on, Ali Hassan et Malik [1] introduced some classes of bipolar single valued neutrosophic graphs and studied some of their properties. P. K. Singh[27] has discussed adequate analysis of uncertainty and vagueness in medical data set using the properties of three-way fuzzy concept lattice and neutrosophic graph introduced by Broumi et al [29]. This study provided a precise representation of medical diagnoses problems using the vertices and edges of neutrosophic graph. Further to refine the knowledge three-way fuzzy concepts generation and their hierarchical order visualization in the concept lattice is proposed using neutrosophic graph and component-wise Godel resituated lattice. One application of the proposed method is also discussed to analyze the multi-criteria decision making process

Ashraf et al.[41], proposed some novels concepts of edge regular, partially edge regular and full edge regular single valued neutrosophic graphs and investigated some of their properties. Also the authors, introduced the notion of single valued neutrosophic digraphs (SVNDGs) and presented an application of SVNDG in multi-attribute decision making. Fathhi et al.[42] computed the dissimilarity between two neutrosophic graphs based on the concept of Hausdorff distance. In 2017, Mehra and Singh [40] introduced the concept of single valued neutrosophic Signed graphs and examined the properties of this concept with examples.

The remainder of this paper is organized as follows. In Section 2, we review some basic concepts about neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graph and generalized fuzzy graphs. In Section 3, the concept of generalized single valued neutrosophic graphs type 1 is proposed with an illustrative example. In Section 4 a representation matrix of generalized single valued neutrosophic graphs type 1 is introduced. Finally, Section 5 outlines the conclusion of this paper and suggests several directions for future research.

## 2. Preliminaries

In this section, we mainly recall some notions related to neutrosophic sets, single valued neutrosophic sets, single valued neutrosophic graphs and generalized fuzzy graphs relevant to the present work. See especially [7,12, 29, 30, 39] for further details and background.

**Definition 2.1** [7]. Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ ; then the neutrosophic set  $A$  (NS  $A$ ) is an object having the form  $A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \}$ , where the functions  $T, I, F: X \rightarrow ]-0,1+[$  define respectively the truth-membership function, indeterminacy-membership function, and falsity-membership function of the element  $x \in X$  to the set  $A$  with the condition:

$$-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (1)$$

The functions  $T_A(x), I_A(x)$  and  $F_A(x)$  are real standard or nonstandard subsets of  $]-0,1+[$ .

Since it is difficult to apply NSs to practical problems, Smarandache [8] introduced the concept of a SVNS, which is an instance of a NS and can be used in real scientific and engineering applications.

**Definition 2.2 [12].** Let  $X$  be a space of points (objects) with generic elements in  $X$  denoted by  $x$ . A single valued neutrosophic set  $A$  (SVNS  $A$ ) is characterized by truth-membership function  $T_A(x)$ , an indeterminacy-membership function  $I_A(x)$ , and a falsity-membership function  $F_A(x)$ . For each point  $x$  in  $X$ ,  $T_A(x), I_A(x), F_A(x) \in [0, 1]$ . A SVNS  $A$  can be written as

$$A = \{ \langle x: T_A(x), I_A(x), F_A(x) \rangle, x \in X \} \quad (2)$$

**Definition 2.3-[29, 30].** A single valued neutrosophic graph (SVN-graph) with underlying set  $V$  is defined to be a pair  $G = (A, B)$  where

1. The functions  $T_A : V \rightarrow [0, 1]$ ,  $I_A : V \rightarrow [0, 1]$  and  $F_A : V \rightarrow [0, 1]$  denote the degree of truth-membership, degree of indeterminacy-membership and falsity-membership of the element  $v_i \in V$ , respectively, and

$$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3 \text{ for all } v_i \in V. \quad (3)$$

2. The Functions  $T_B : E \subseteq V \times V \rightarrow [0, 1]$ ,  $I_B : E \subseteq V \times V \rightarrow [0, 1]$  and  $F_B : E \subseteq V \times V \rightarrow [0, 1]$  are defined by

$$T_B(v_i, v_j) \leq \min [T_A(v_i), T_A(v_j)], \quad (4)$$

$$I_B(v_i, v_j) \geq \max [I_A(v_i), I_A(v_j)] \text{ and} \quad (5)$$

$$F_B(v_i, v_j) \geq \max [F_A(v_i), F_A(v_j)] \quad (6)$$

denotes the degree of truth-membership, indeterminacy-membership and falsity-membership of the edge  $(v_i, v_j) \in E$  respectively, where

$$0 \leq T_B(v_i, v_j) + I_B(v_i, v_j) + F_B(v_i, v_j) \leq 3 \text{ for all } (v_i, v_j) \in E \text{ (i, j = 1, 2, \dots, n)} \quad (7)$$

$A$  is the single valued neutrosophic vertex set of  $V$ ,  $B$  is the single valued neutrosophic edge set of  $E$ , respectively.

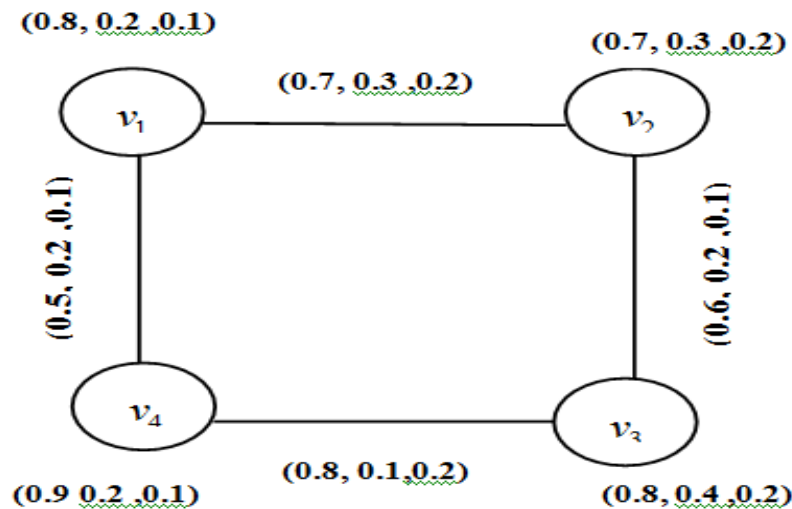


Fig.1. Single valued neutrosophic graph

**Definition 2.4 [40].** Let  $V$  be a non-void set. Two functions are considered as follows:

$\rho: V \rightarrow [0, 1]$  and  $\omega: V \times V \rightarrow [0, 1]$ . suppose  $A = \{(\rho(x), \rho(y)) \mid \omega(x, y) > 0\}$ , where  $\omega_T > 0$  for all set  $A$

The triad  $(V, \rho, \omega)$  is defined to be generalized fuzzy graph of first type (GFG1) if there is function

$\alpha: A \rightarrow [0, 1]$  such that  $\omega(x, y) = \alpha((\rho(x), \rho(y)))$  Where  $x, y \in V$ .

The  $\rho(x)$ ,  $x \in V$  are the membership of the vertex  $x$  and  $\omega(x, y)$ ,  $x, y \in V$  are the membership, values of the edge  $(x, y)$ .

### III. Generalized Single Valued Neutrosophic Graph of First Type

In this section, based on the generalized fuzzy graphs first type proposed by Samanta et al .[39], the definition of generalized single valued neutrosophic graphs of first type is defined as follow:

**Definition 3.1.** Let  $V$  be a non-void set. Two functions are considered as follows:

$$\rho = (\rho_T, \rho_I, \rho_F): V \rightarrow [0, 1]^3 \text{ and}$$

$$\omega = (\omega_T, \omega_I, \omega_F): V \times V \rightarrow [0, 1]^3. \text{ We suppose}$$

$$A = \{(\rho_T(x), \rho_T(y)) \mid \omega_T(x, y) \geq 0\}, \quad B = \{(\rho_I(x), \rho_I(y)) \mid \omega_I(x, y) \geq 0\}, \quad C = \{(\rho_F(x), \rho_F(y)) \mid \omega_F(x, y) \geq 0\},$$

We have considered  $\omega_T, \omega_I$  and  $\omega_F \geq 0$  for all set  $A, B, C$ , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad  $(V, \rho, \omega)$  is defined to be generalized single valued neutrosophic graph of first type (GSVNG1) if there are functions

$$\alpha: A \rightarrow [0, 1], \quad \beta: B \rightarrow [0, 1] \text{ and } \delta: C \rightarrow [0, 1] \text{ such that}$$

$$\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y))), \quad \omega_I(x, y) = \beta((\rho_I(x), \rho_I(y))), \quad \omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$$

Where  $x, y \in V$ .

Here  $\rho(x) = (\rho_T(x), \rho_I(x), \rho_F(x)), x \in V$  are the membership, indeterminacy and non-membership of the vertex  $x$  and  $\omega(x, y) = (\omega_T(x, y), \omega_I(x, y), \omega_F(x, y)), x, y \in V$  are the membership, indeterminacy and non-membership values of the edge  $(x, y)$ .



**Example 3.2 :** Let the vertex set be  $V=\{x, y, z, t\}$  and edge set be  $E=\{(x, y),(x, z),(x, t),(y, t)\}$

Table 1: membership, indeterminacy and non-membership of the vertex set.

	x	y	z	t
$\rho_T$	0.5	0.9	0.3	0.8
$\rho_I$	0.3	0.2	0.1	0.5
$\rho_F$	0.1	0.6	0.8	0.4

Let us consider functions  $\alpha(m, n)=m \vee n = \beta(m, n)=\delta(m, n)$

Here,  $A=\{(0.5, 0.9), (0.5, 0.3), (0.5, 0.8), (0.8, 0.8)\}$

$B=\{(0.3, 0.2), (0.3, 0.1), (0.3, 0.5), (0.2, 0.5)\}$

$C=\{(0.1, 0.6), (0.1, 0.8), (0.1, 0.4), (0.6, 0.4)\}$ . Then

Table 2: membership, indeterminacy and non-membership of the edge set.

$\omega$	$(x, y)$	$(x, z)$	$(x, t)$	$(y, t)$
$\omega_T$	0.9	0.5	0.8	0.9
$\omega_I$	0.3	0.3	0.5	0.5
$\omega_F$	0.6	0.8	0.4	0.6

The corresponding generalized single valued neutrosophic graph is shown in Fig.2

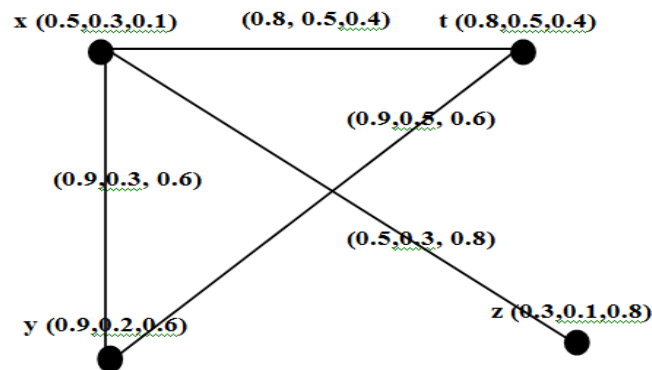


Fig 2. GSVNG of first type.

The easier way to represent any graph is to use the matrix representation. The adjacency matrices, incident matrices are the widely matrices used. In the following section GSVNG1 is represented by adjacency matrix.

#### IV. Matrix Representation of Generalized Single Valued Neutrosophic Graph of First Type

Because membership, indeterminacy and non-membership of the vertices are considered independents. In this section, we adopted the representation matrix of generalized fuzzy graphs defined in [39].

The generalized single valued neutrosophic graph (GSVNG1) has one property that edge membership values (T, I, F) depends on the membership values (T, I, F) of adjacent vertices. Suppose  $\xi=(V, \rho, \omega)$  is a GSVNG1 where vertex set  $V=\{v_1, v_2, \dots, v_n\}$ . The functions

$\alpha :A \rightarrow (0, 1]$  is taken such that  $\omega_T(x, y) = \alpha((\rho_T(x), \rho_T(y)))$  Where  $x, y \in V$  and  $A = \{(\rho_T(x), \rho_T(y)) \mid \omega_T(x, y) \geq 0\}$ ,

$\beta :B \rightarrow (0, 1]$  is taken such that  $\omega_I(x, y) = \beta((\rho_I(x), \rho_I(y)))$  Where  $x, y \in V$  and  $B = \{(\rho_I(x), \rho_I(y)) \mid \omega_I(x, y) \geq 0\}$ , and

$\delta :C \rightarrow (0, 1]$  is taken such that  $\omega_F(x, y) = \delta((\rho_F(x), \rho_F(y)))$  Where  $x, y \in V$  and  $C = \{(\rho_F(x), \rho_F(y)) \mid \omega_F(x, y) \geq 0\}$ . The GSVNG1 can be represented by  $(n+1) \times (n+1)$  matrix  $M_{G_1}^{T,I,F} = [a^{T,I,F}(i,j)]$  as follows:

The membership (T), indeterminacy-membership (I) and the non-membership (F) values of the vertices are provided in the first row and first column. The  $(i+1, j+1)$ -th entry are the membership (T), indeterminacy-membership (I) and the non-membership (F) values of the edge  $(x_i, x_j)$ ,  $i, j=1, \dots, n$  if  $i \neq j$ .

The  $(i, i)$ -th entry is  $\rho(x_i) = (\rho_T(x_i), \rho_I(x_i), \rho_F(x_i))$ , where  $i=1, 2, \dots, n$ . The membership (T), indeterminacy-membership (I) and the non-membership (F) values of the edge can be computed easily using the functions  $\alpha, \beta$  and  $\delta$  which are in  $(1,1)$ -position of the matrix. The matrix representation of GSVNG1, denoted by  $M_{G_1}^{T,I,F}$ , can be written as three matrix representation  $M_{G_1}^T, M_{G_1}^I$  and  $M_{G_1}^F$ .

The  $M_{G_1}^T$  can be represented as follows:

Table3. Matrix representation of T-GSVNG1

$\alpha$	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$\rho_T(v_1)$	$\alpha(\rho_T(v_1), \rho_T(v_2))$	$\alpha(\rho_T(v_1), \rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2), \rho_T(v_1))$	$\rho_T(v_2)$	$\alpha(\rho_T(v_2), \rho_T(v_2))$
...	....	...	...
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n), \rho_T(v_1))$	$\alpha(\rho_T(v_n), \rho_T(v_2))$	$\rho_T(v_n)$

The  $M_{G_1}^I$  can be represented as follows:

Table4. Matrix representation of I-GSVNG1

$\beta$	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$\rho_I(v_1)$	$\beta(\rho_I(v_1), \rho_I(v_2))$	$\beta(\rho_I(v_1), \rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2), \rho_I(v_1))$	$\rho_I(v_2)$	$\beta(\rho_I(v_2), \rho_I(v_2))$
...	....	...	...
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n), \rho_I(v_1))$	$\beta(\rho_T(v_n), \rho_I(v_2))$	$\rho_I(v_n)$

The  $M_{G_1}^I$  can be represented as follows

Table5. Matrix representation of F-GSVNG1

$\delta$	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$\rho_F(v_1)$	$\delta(\rho_F(v_1),\rho_F(v_2))$	$\delta(\rho_F(v_1),\rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2),\rho_F(v_1))$	$\rho_F(v_2)$	$\delta(\rho_F(v_2),\rho_F(v_2))$
...	....	...	...
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n),\rho_F(v_1))$	$\delta(\rho_F(v_n),\rho_F(v_2))$	$\rho_F(v_n)$

**Remark1:** If the indeterminacy-membership and non-membership values of vertices equals zero, the generalized single valued neutrosophic graphs type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Here the generalized single valued neutrosophic graph of first type (GSVNG1) can be represent by the matrix representation depicted in table 9.The matrix representation can be written as three matrices one containing the entries as T, I, F (see table 6, 7 and 8).

Table 6: Truth- matrix representation of GSVNG1

$\alpha = \max(x, y)$	$x(0.5)$	$y(0.9)$	$z(0.3)$	$t(0.8)$
$x(0.5)$	0.5	0.9	0.5	0.5
$y(0.9)$	0.9	0.9	0	0.9
$z(0.3)$	0.5	0	0.3	0
$t(0.8)$	0.8	0.8	0	0.8

Table 7: Indeterminacy- matrix representation of GSVNG1

$\beta = \max(x, y)$	$x(0.3)$	$y(0.2)$	$z(0.1)$	$t(0.5)$
$x(0.3)$	0.3	0.3	0.3	0.3
$y(0.2)$	0.3	0.2	0	0.5
$z(0.1)$	0.3	0	0.1	0
$t(0.5)$	0.5	0.5	0	0.5

Table 8: Falsity- matrix representation of GSVNG1

Table 8: Falsity- matrix representation of GSVNG1

$\delta = \max(x, y)$	x(0.1)	y(0.6)	z(0.8)	t(0.4)
x(0.1)	0.1	0.6	0.8	0.1
y(0.6)	0.6	0.6	0	0.6
z(0.8)	0.8	0	0.8	0
t(0.4)	0.4	0.6	0	0.4

The matrix representation of GSVNG1 can be represented as follows:

Table 9: Matrix representation of GSVNG1.

$(\alpha, \beta, \delta)$	x(0.5,0.3,0.1)	y(0.9,0.2,0.6)	z(0.3,0.1,0.8)	t(0.8,0.5,0.4)
x(0.5,0.3,0.1)	(0.5,0.3,0.1)	(0.9,0.3,0.6)	(0.5,0.3,0.8)	(0.5,0.3,0.1)
y(0.9,0.2,0.6)	(0.9,0.3,0.6)	(0.9,0.2,0.6)	(0,0,0)	(0.9,0.5,0.6)
z(0.3,0.1,0.8)	(0.5,0.3,0.8)	(0,0,0)	(0.3,0.1,0.8)	(0,0,0)
t(0.8,0.5,0.4)	(0.8,0.5,0.4)	(0.8,0.5,0.6)	(0,0,0)	(0.8,0.5,0.4)

**Theorem 1.** Let  $M_{G_1}^T$  be matrix representation of T-GSVNG1, then the degree of vertex  $D_T(x_k)$   
 $= \sum_{j=1, j \neq k}^n a^T(k+1, j+1), x_k \in V$  or  $D_T(x_p) = \sum_{i=1, i \neq p}^n a^T(i+1, p+1), x_p \in V$ .

**Proof:** I is similar as in theorem 1 of [39].

**Theorem 2.** Let  $M_{G_1}^I$  be matrix representation of I-GSVNG1, then the degree of vertex  $D_I(x_k)$   
 $= \sum_{j=1, j \neq k}^n a^I(k+1, j+1), x_k \in V$  or  $D_I(x_p) = \sum_{i=1, i \neq p}^n a^I(i+1, p+1), x_p \in V$ .

**Proof:** It is similar as in theorem 1 of [39].

**Theorem 3.** Let  $M_{G_1}^F$  be matrix representation of F-GSVNG1, then the degree of vertex  $D_F(x_k)$   
 $= \sum_{j=1, j \neq k}^n a^F(k+1, j+1), x_k \in V$  or  $D_F(x_p) = \sum_{i=1, i \neq p}^n a^F(i+1, p+1), x_p \in V$ .

**Proof:** It is similar as in theorem 1 of [39].

**Theorem 4.** Let  $M_{G_1}^{T,I,F}$  be matrix representation of GSVNG1, then the degree of vertex  $D(x_k)$

$= (D_T(x_k), D_I(x_k), D_F(x_k))$  where

$$D_T(x_k) = \sum_{j=1, j \neq k}^n a^T(k+1, j+1), x_k \in V.$$

$$D_I(x_k) = \sum_{j=1, j \neq k}^n a^I(k+1, j+1), x_k \in V.$$

$$D_F(x_k) = \sum_{j=1, j \neq k}^n a^F(k+1, j+1), x_k \in V$$

**Proof:** The proof is obvious.

## V. CONCLUSIONS

In this article, we present a new concept called generalized single valued neutrosophic graphs of first type and presented a matrix representation of it. In the future works, we plan to study the concept of completeness, the concept of regularity and to define the concept of generalized single valued neutrosophic graphs of type 2.



# REFERENCES

- [1] Hassan, A., Malik, M. A., *The Classes of bipolar single valued neutrosophic graphs*, TWMS Journal of Applied and Engineering Mathematics" (TWMS J. of Apl. & Eng. Math.), 2016accepted.
- [2] Shannon, A., Atanassov, K., *A First Step to a Theory of the Intuitionistic Fuzzy Graphs*, Proc. of the First Workshop on Fuzzy Based Expert Systems (D. akov, Ed.), Sofia, pp.59-61, 1994
- [3] Sharma, A.K., Padamwar, B.V., Dewangan, C.L., *Trends in fuzzy graphs*, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 2, Issue 9, pp. 4636-4640, 2013.
- [4] Dubois, D. and Prade, H., *Fuzzy Sets and Systems: Theory and Application*, Academic Press, New work, 1980.
- [5] Smarandache, F., *Refined Literal Indeterminacy and the Multiplication Law of Sub-Indeterminacies*," *Neutrosophic Sets and Systems*, Vol. 9, pp.58.63, 2015,
- [6] Smarandache, F., *Types of Neutrosophic Graphs and neutrosophic Algebraic Structures together with their Applications in Technology*," seminar, *Universitatea Transilvania din Brasov, Facultatea de Design de Produsi Mediu*, Brasov, Romania 06 June 2015.
- [7] Smarandache, F., *Neutrosophic set - a generalization of the intuitionistic fuzzy set*, Granular Computing, 2006 IEEE International Conference, p. 38 – 42, 2006.
- [8] Smarandache, F., *Neutrosophy. Neutrosophic Probability, Set, and Logic*, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 105 p., 1998
- [9] Smarandache, F., *Nidus idearum. Scilogs, III: Viva la Neutrosophia!*, Brussels, 2017, 134p. ISBN 978-1-59973-508-5
- [10] Smarandache, F., *Neutrosophic overset, neutrosophic underset, Neutrosophic offset. Similarly for Neutrosophic Over-/Under-/Off Logic, Probability, and Statistic*. Pons Editions, Brussels, 2016, 170p.
- [11] Smarandache, F.: *Symbolic Neutrosophic Theory* (Europanova asbl, Brussels, 195 p., Belgium 2015.
- [12] Wang, H. , Smarandache, F. , Zhang, Y. , and Sunderraman, R. , "Single valued Neutrosophic Sets," *Multisspace and Multistructure* 4, 2010, pp. 410-413.
- [13] Turksen, I., *Interval valued fuzzy sets based on normal forms*, *Fuzzy Sets and Systems*, vol. 20, pp. 191-210, 1986.
- [14] Atanassov, K., *Intuitionistic fuzzy sets*, *Fuzzy Sets and Systems*, vol. 20, pp. 87-96, 1986.
- [15] Atanassov, K., *Intuitionistic fuzzy sets: theory and applications*, *Physica*, New York, 1999.
- [16] Atanassov, K. and G. Gargov, *Interval valued intuitionistic fuzzy sets*, *Fuzzy Sets and Systems*, vol.31, pp. 343-349, 1989.
- [17] Zadeh, L., *Fuzzy sets*. *Inform and Control*, 8, pp.338-353, 1965.
- [18] Akram, M. and G. Shahzadi, *Operations on single-valued neutrosophic graphs*, *Journal of Uncertain System*, 11, pp.1-26, 2017.

- [19] Akram, M., *Single-Valued Neutrosophic Planar Graphs*, International Journal of Algebra and Statistics Volume 5: 2, pp.157-167, 2016.
- [20] Akram, M. and Shahzadi, S., *Neutrosophic soft graphs with application*, Journal of Intelligent and Fuzzy Systems, DOI:10.3233/JIFS-16090, pp.1-18, 2016.
- [21] Akram, M. and Shahzadi, S., *Representation of graphs using intuitionistic neutrosophic soft sets*, Journal of Mathematical Analysis, 7(6), 2016, pp.1-23.
- [22] Malik, M. A., Hassan, A., *Single valued neutrosophic trees*, TWMS Journal of Applied and Engineering Mathematics, TWMS J. of Apl. & Eng. Math., 2016, accepted.
- [23] Akram, M. and Muzzamal Sitara, *Novel applications of single-valued neutrosophic graph structures in decision making*, Journal of Applied Mathematics and Computing, pp. 1–32, 2016.
- [24] Sahin, M., Alkhazaleh, S. and Ulucay, V., *Neutrosophic soft expert sets*, Applied Mathematics 6(1), pp.116–127, 2015.
- [25] Shah N., *Some Studies in Neutrosophic Graphs*, Neutrosophic Sets and Systems, Vol. 12, pp.54-64, 2016.
- [26] Shah, N., and Hussain, A., *Neutrosophic Soft Graphs*, Neutrosophic Sets and Systems, Vol. 11, pp.31-44, 2016.
- [27] Singh, P K., *Three-way fuzzy concept lattice representation using neutrosophic set*, International Journal of Machine Learning and Cybernetics, pp 1–11, 2016.
- [28] Maji, P. K., *Neutrosophic soft set*, Annals of Fuzzy Mathematics and Informatics, 5/1, p157-168, 2013.
- [29] Broumi, S., Talea, M., Bakali, A., Smarandache, F., *Single Valued Neutrosophic Graphs*, Journal of New Theory, N 10, pp. 86-101, 2016,.
- [30] Broumi, S., Talea, M., Smarandache, F. and Bakali, A., *Single Valued Neutrosophic Graphs: Degree, Order and Size*, IEEE International Conference on Fuzzy Systems (FUZZ), 2016, pp. 2444-2451.
- [31] Broumi, S., Bakali, A., Talea, M, and Smarandache, F, *Isolated Single Valued Neutrosophic Graphs*. Neutrosophic Sets and Systems, Vol. 11, 2016, pp.74-78.
- [32] Broumi, S., Talea, M., Bakali, A., Smarandache, F., *Interval Valued Neutrosophic Graphs*, Critical Review, XII, p.5-33, 2016.
- [33] Broumi, S., F. Smarandache, M. Talea and A. Bakali, *Decision-Making Method Based On the Interval Valued Neutrosophic Graph*, IEEE International Conference, Future Technologie, pp44-50, 2016.
- [34] Broumi, S., Bakali, A., Talea, M., Smarandache, F. and Ali, M., *Shortest Path Problem under Bipolar Neutrosophic Setting*, Applied Mechanics and Materials. Vol. 859. pp. 59-66. 2016.

- [35] Broumi, S., Bakali, A., Talea, M., Smarandache, F. and Vladareanu, L., *Computation of Shortest Path Problem in a Network with SV-Trapezoidal Neutrosophic Numbers*, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, pp.417-422, 2016.
- [36] Broumi, S., Bakali, A., Talea, M., Smarandache, F. and Vladareanu, L., *Applying Dijkstra Algorithm for Solving Neutrosophic Shortest Path Problem*, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, November 30 - December 3, 2016, pp.412-416.
- [37] Broumi, S., Talea, M., Bakali, A., Smarandache, F., *On Bipolar Single Valued Neutrosophic Graphs*, Journal of New Theory, N11, pp.84-102, 2016.
- [38] Broumi, S., Smarandache, F., Talea, M. and Bakali, A., *An Introduction to Bipolar Single Valued Neutrosophic Graph Theory*. Applied Mechanics and Materials, vol.841, pp.184-191, 2016.
- [39] Samanta, S., Sarkar, B., Shin, D. and Pal, M., *Completeness and regularity of generalized fuzzy graphs*, SpringerPlus, 2016, DOI 10.1186/s40064-016-3558-6.
- [40] Mehra, S. and Singh, M., *Single valued neutrosophic signed graphs*, International Journal of computer Applications, Vol 157, N.9, pp 31., 2017
- [41] Ashraf, S., Naz, S., Rashmanlou, H., and Malik, M. A., *Regularity of graphs in single valued neutrosophic environment*,
- [42] Fathi, S., Elchawalby, H. and Salama, A. A., *A neutrosophic graph similarity measures*, chapter in book- New Trends in Neutrosophic Theory and Applications- Smarandache and Surpati Pramanik (Editors), pp. 223-230. 2016, ISBN 978-1-59973-498-9,
- [43] Vasantha Kandasamy, W. B., Ilanthenral, K. and Smarandache, F.: *Neutrosophic Graphs: A New Dimension to Graph Theory Kindle Edition, 2015.*
- [44] More information on <http://fs.gallup.unm.edu/NSS/>.

**THANKS**