International Workshop on Information Fusion

Information fusion with belief functions: A DSmT perspective

Jean Dezert, Ph.D.

jean.dezert@onera.fr

http://www.onera.fr/fr/staff/jean-dezert

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Outline

1 - Belief functions and Dempster-Shafer Theory (DST)

2 - Introduction to Dezert-Smarandache Theory (DSmT)

3 - Some applications of DSmT

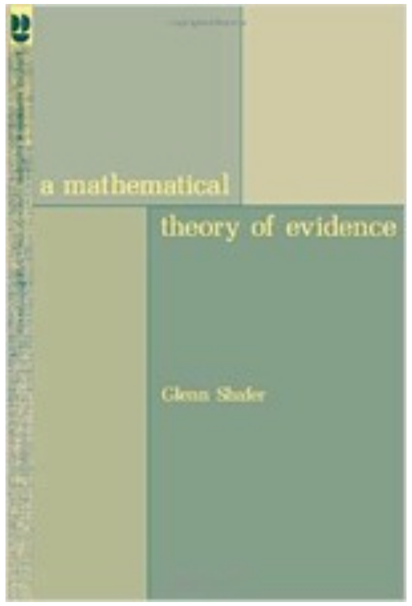
Part 1

Belief functions and Dempster-Shafer Theory

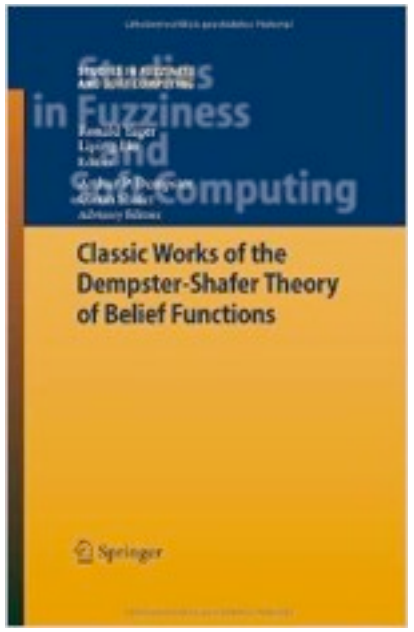
Belief functions and DST

Main references on Dempster-Shafer Theory (DST)

http://www.glennshafer.com/books/amte.html



G. Shafer, A mathematical theory of evidence, Princeton Univ., 1976.



R. Yager, L. Liu, Classic Works of the Dempster-Shafer Theory of Belief Functions, Springer, 2008.

Belief functions and DST

Limitations of probabilities

They do not account for partial/incomplete knowledge.

They deal generally with information drawn from generic knowledge based either on population of items, laws of physics, common sense, ...

They capture only one aspect of the uncertainty
(the randomness, i.e. the variability through repeated measurements).

They can’t distinguish between uncertainty due to variability, and uncertainty due to the incompleteness/lack of knowledge (epistemic uncertainty).

Variability is related with precisely observed random observations

Incompleteness/non specificity is related with missing/partial information

Belief functions and DST

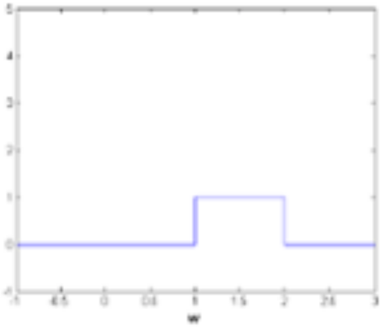
Limitation of uniform prior pdf to model the full ignorance

Consider a random variable W taking its value w in [1,2], and the random variable V=1/W which obviously takes its value v=1/w in [0.5,1].

To model ignorance of value of W, it is usually assumed **uniform prior pdf**.

$$W \sim u([1,2)]) \Leftrightarrow P(W \leq w) = \begin{cases} 0 & \text{if} \quad w < 1 \\ w - 1 & \text{if} \quad 1 \leq w \leq 2 \\ 1 & \text{if} \quad w > 2 \end{cases}$$

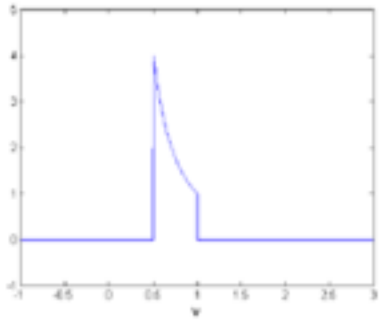
$$p_W(w) = \frac{\partial}{\partial w} P(W \leq w) = \begin{cases} 0 & \text{if} \quad w \notin [1,2] \\ 1 & \text{if} \quad w \in [1,2] \end{cases}$$



By doing so, however we get **Non-uniform prior** pdf for V=1/W.

$$P(V \leq v) = P(\frac{1}{W} \leq v) = P(W \geq \frac{1}{v}) = 1 - P(W < \frac{1}{v})$$
$$= \begin{cases} 1 & \text{if} \quad \frac{1}{v} < 1 \\ 2 - \frac{1}{v} & \text{if} \quad \frac{1}{v} \in [1,2] \\ 0 & \text{if} \quad \frac{1}{v} > 2 \end{cases}$$

$$p_V(v) = \frac{\partial}{\partial v} P(V \leq v) = \begin{cases} 0 & \text{if} \quad v \notin [\frac{1}{2}, 1] \\ \frac{1}{v^2} & \text{if} \quad v \in [\frac{1}{2}, 1] \end{cases}$$



which is not satisfactory because, we are a priori fully ignorant on the true value of W as well as of 1/W !!! So the choice of uniform pdf **does not model properly** our prior full ignorance of values w and v.

Belief functions and DST

Paradigm shift with Belief Functions (BF)

Beliefs often are related with singular event and are **not necessarily related with statistical data** and generic knowledge, but with singular evidence. BF are well adapted for modeling partial knowledge.

Frame of discernment (FoD) $\Theta = \{\theta_i, i = 1, \ldots, n\}$

Shafer's model Close world assumption with exclusivity of elements

Power-set $\mathcal{P}(\Theta) \triangleq 2^{\Theta}$

Any subset A of the FoD corresponds to the proposition

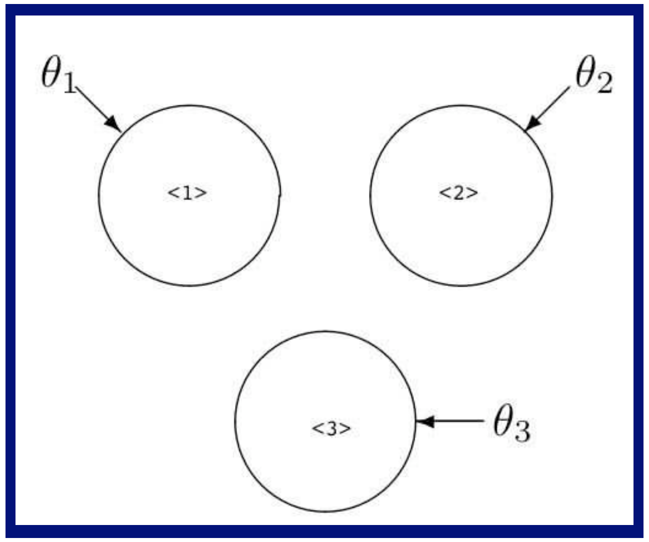
$$\mathcal{P}_{\theta}(A) \triangleq \textit{The true value of } \theta \textit{ is in a subset } A \textit{ of } \Theta.$$

There is equivalence between operators on sets and logical operators

Belief functions and DST

Example

$$\Theta = \{\theta_1, \theta_2, \theta_3\} \Rightarrow$$



$$2^\Theta = \{\overset{\text{Impossibility}}{\emptyset}, \theta_1, \theta_2, \theta_3, \overbrace{\theta_1 \cup \theta_2, \theta_1 \cup \theta_3, \theta_2 \cup \theta_3}^{\text{partial ignorances}}, \overbrace{\theta_1 \cup \theta_2 \cup \theta_3}^{\text{full ignorance}}\}$$

$$|2^\Theta| = 2^3 = 8$$

Belief functions and DST

Basic belief assignment (BBA) $m(.) : 2^\Theta \to [0,1]$

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Theta} m(A) = 1$$

Focal element A: iff m(A)>0

Vacuous BBA $\forall A \neq \Theta, m_v(A) = 0 \text{ and } m_v(\Theta) = 1$

Credibility

$$\text{Bel}(A) = \sum_{B \in 2^\Theta, B \subseteq A} m(B)$$

Total mass of subsets implying A

Plausibility

$$\text{Pl}(A) = \sum_{B \in 2^\Theta, B \cap A \neq \emptyset} m(B)$$

Total mass of subsets intersecting A

In general, $0 \leq \text{Bel}(A) \leq \text{Pl}(A) \leq 1$

Bayesian BBA Focal elements are singletons Bel(A)=Pl(A)=P(A)

Belief functions and DST

Discounting a source of evidence (Shafer's reliability discounting)

$$\begin{cases} m(A) \\ m(\Theta) \end{cases} \rightarrow \begin{cases} m'(A) = \alpha \cdot m(A) & \forall A \neq \Theta \\ m'(\Theta) = (1-\alpha) + \alpha \cdot m(\Theta) \end{cases}$$

$\alpha = 1$ means no discounting (full reliability of the source)

$\alpha = 0$ means total discounting (full unreliable/ignorant source)

To be used if one has a good estimation of the reliability factor of the source based on experiments and ground truth.

Other discounting techniques

- **Contextual discounting** [Denœux et al. 2005, 2006]
- **Importance discounting** [Smarandache, Dezert, Tacnet 2010]

Belief functions and DST

Combination of two distinct sources of evidence

Dempster's rule of combination $m_{DS}(\emptyset) \triangleq 0$

$$m_{DS}(X) = [m_1 \oplus m_2](X) = \frac{m_{12}(X)}{1 - K_{12}} \qquad \forall X \neq \emptyset \in 2^\Theta$$

$$\underbrace{m_{12}(X) \triangleq \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1) m_2(X_2)}_{\text{Conjunctive fusion}} \qquad \underbrace{K_{12} \triangleq m_{12}(\emptyset) = \sum_{\substack{X_1, X_2 \in 2^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1) m_2(X_2)}_{\text{Conflict level}}$$

DS rule = Normalized conjunctive rule

Belief functions and DST

Comments on DS rule of combination

- Properties: extension to $n > 2$ sources; associativity; commutativity; neutrality of vacuous bba $[m \oplus m_v](.) = m(.)$

- Conditioning: $m(.)$ combined with $m_Z(.)$ focused on Z (i.e. $m_Z(Z) = 1$) with DS rule yields $m(X|Z) = [m \oplus m_Z](X) = [m_Z \oplus m](X)$ and $Pl(X|Z) = Pl(X \cap Z)/Pl(Z)$ (similar to Conditioning rule for probas).

  Because of this, DS rule has often been interpreted as a **generalization** of Bayes rule.

- Drawbacks: Counter-intutitive and unexpected behaviors in some cases $\Rightarrow$ validity of DS rule has become very questionable over the years ... at least for highly conflicting cases.

Belief functions and DST

Drawbacks of DS rule of combination

DS rule is mathematically not defined when conlict is total (K=1).

DS rule doesn’t behave well not only in high conflicting case [Zadeh 1979], but even **in low** conflicting case [Dezert-Wang-Tchamova 2012]

DS rule is **not** a generalization of Bayes rule because it is incompatible with Bayes rule when the prior is not uniform, nor vacuous [Dezert-Tchamova-Han-Tacnet 2013].

Belief functions and DST

Zadeh's example (1979) **High conflict case** $\Theta = \{\theta_1, \theta_2, \theta_3\}$

Bayesian BBAs

$$m_1(\theta_1) = 1 - e_1 \qquad m_1(\theta_2) = 0 \qquad m_1(\theta_3) = e_1$$

$$m_2(\theta_1) = 0 \qquad m_2(\theta_2) = 1 - e_2 \qquad m_2(\theta_3) = e_2$$

$$k_{12} = (1 - e_1)(1 - e_2) + (1 - e_1)e_2 + e_1(1 - e_2) = 1 - e_1 e_2$$

$$\text{If } e_1 = 0.1 \text{ and } e_2 = 0.1, \text{ then } k_{12} = 1 - 0.01 = 0.99$$ **(high conf.)**

DS fusion

$$m(\theta_3) = \frac{e_1 e_2}{(1 - e_1) \cdot 0 + 0 \cdot (1 - e_2) + e_1 e_2} = 1$$

DS rule provides same result whatever the positive values of e_1 and e_2 are !!!

DS is not numerically robust to slight input changes.

Belief functions and DST

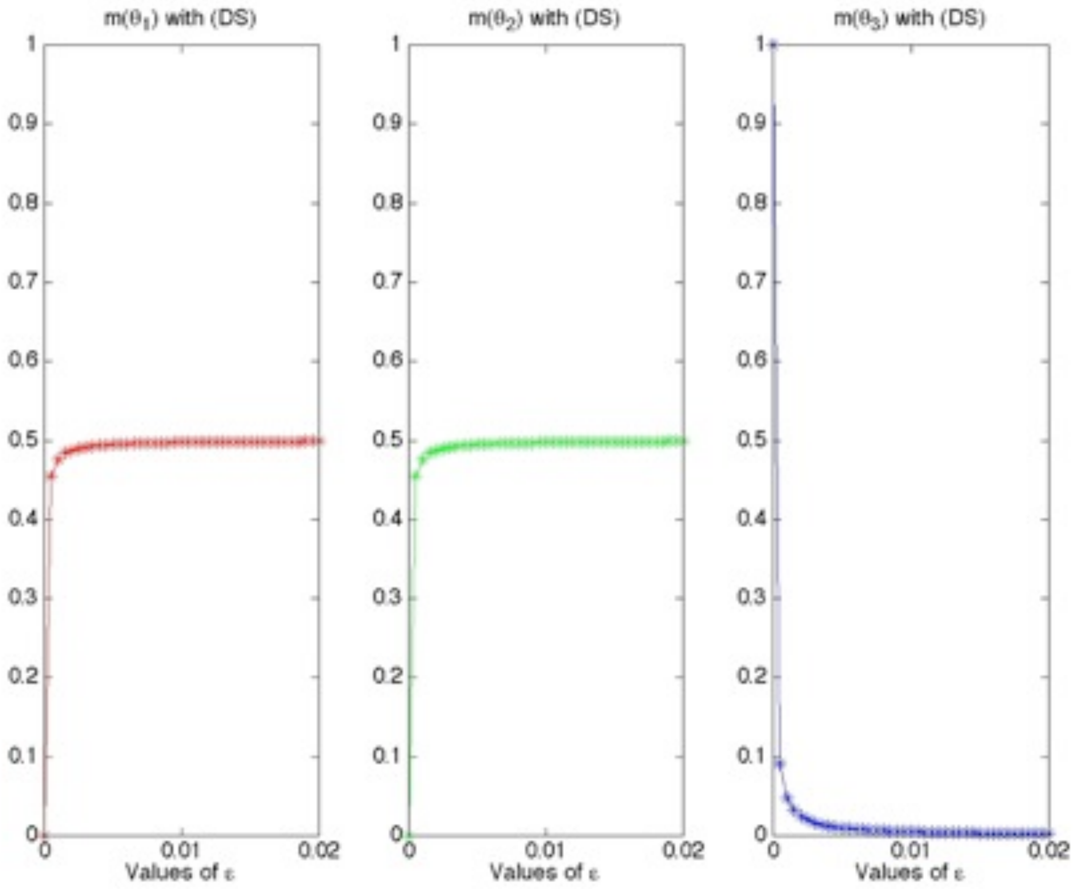
Zadeh’s example — Numerical robustness analysis of DS rule

	θ_1	θ_2	θ_3
Source 1	$m_1(\theta_1) = 0.99 - \epsilon$	$m_1(\theta_2) = \epsilon$	$m_1(\theta_3) = 0.01$
Source 2	$m_2(\theta_1) = \epsilon$	$m_2(\theta_2) = 0.99 - \epsilon$	$m_2(\theta_3) = 0.01$

DS result

$$\text{For } \epsilon = 0, m(\theta_3) = 1$$

$$\text{For } \epsilon = 0.0005, \begin{cases} m(\theta_1) = 0.45410 \\ m(\theta_2) = 0.45410 \\ m(\theta_3) = 0.0918 \end{cases}$$



DS rule is not robust to slight input changes

Belief functions and DST

Dezert-Tchamova example (2011) Low conflict case $\Theta = \{\theta_1, \theta_2, \theta_3\}$

Non-Bayesian BBAs

Focal elem. \ bba's	$m_1(.) \neq m_V(.)$	$m_2(.) \neq m_V(.)$
A	a	0
$A \cup B$	$1 - a$	b_1
C	0	$1 - b_1 - b_2$
$A \cup B \cup C$	0	b_2

Conjunctive fusion

$$m_{12}(A) = m_1(A)m_2(A \cup B) + m_1(A)m_2(A \cup B \cup C) = a(b_1 + b_2)$$

$$m_{12}(A \cup B) = m_1(A \cup B)m_2(A \cup B) + m_1(A \cup B)m_2(A \cup B \cup C) = (1 - a)(b_1 + b_2)$$

Conflicting mass

$$\begin{aligned} K_{12} = m_{12}(\emptyset) &= m_1(A)m_2(C) + m_1(A \cup B)m_2(C) \\ &= a(1 - b_1 - b_2) + (1 - a)(1 - b_1 - b_2) \\ &= 1 - b_1 - b_2 \end{aligned}$$

The conflict can be chosen as low as we want.

Belief functions and DST

Dezert-Tchamova example (cont'd)

$$m_{12}(A) = m_1(A)m_2(A \cup B) + m_1(A)m_2(A \cup B \cup C) = a(b_1 + b_2)$$

$$m_{12}(A \cup B) = m_1(A \cup B)m_2(A \cup B) + m_1(A \cup B)m_2(A \cup B \cup C) = (1 - a)(b_1 + b_2)$$

After normalization by $1 - K_{12} = b_1 + b_2$ one gets, with DS rule

$$m_{DS}(A) = \frac{m_{12}(A)}{1 - K_{12}} = \frac{a(b_1 + b_2)}{b_1 + b_2} = a = m_1(A)$$

$$m_{DS}(A \cup B) = \frac{m_{12}(A \cup B)}{1 - K_{12}} = \frac{(1 - a)(b_1 + b_2)}{b_1 + b_2} = 1 - a = m_1(A \cup B)$$

- $m_{DS}(.) = [m_1 \oplus m_2](.) = m_1(.)$ even if $m_2(.) \neq m_v(.)$
- The informative source $m_2(.)$ doesn't count $\Rightarrow$ Dictatorial power of DS rule
- The level of conflict K_{12} doesn't matter in the result.

Such fusion result is very counter intuitive

Belief functions and DST

Example where DS rule is incompatible with Bayes rule

Bayesian BBA

$$\begin{cases} m_1(x_1) = P(X = x_1|Z_1) = 0.2 \\ m_1(x_2) = P(X = x_2|Z_1) = 0.3 \\ m_1(x_3) = P(X = x_3|Z_1) = 0.5 \end{cases} \quad \text{and} \quad \begin{cases} m_2(x_1) = P(X = x_1|Z_2) = 0.5 \\ m_2(x_2) = P(X = x_2|Z_2) = 0.1 \\ m_2(x_3) = P(X = x_3|Z_2) = 0.4 \end{cases}$$

with **informative prior** bba/pmf:

$$\begin{cases} m_0(x_1) = P(X = x_1) = 0.6 \\ m_0(x_2) = P(X = x_2) = 0.3 \\ m_0(x_3) = P(X = x_3) = 0.1 \end{cases}$$

Bayes rule

$$\begin{cases} P(x_1|Z_1 \cap Z_2) & = \frac{0.2 \cdot 0.5/0.6}{2.2667} = \frac{0.1667}{2.2667} \approx 0.0735 \\ P(x_2|Z_1 \cap Z_2) & = \frac{0.3 \cdot 0.1/0.3}{2.2667} = \frac{0.1000}{2.2667} \approx 0.0441 \\ P(x_3|Z_1 \cap Z_2) & = \frac{0.5 \cdot 0.4/0.1}{2.2667} = \frac{2.0000}{2.2667} \approx 0.8824 \end{cases}$$

DS rule

$$\begin{cases} m_{DS}(x_1) & = \frac{1}{1-0.9110} \cdot 0.2 \cdot 0.5 \cdot 0.6 = \frac{0.060}{0.089} \approx 0.6742 \\ m_{DS}(x_2) & = \frac{1}{1-0.9110} \cdot 0.3 \cdot 0.1 \cdot 0.3 = \frac{0.009}{0.089} \approx 0.1011 \\ m_{DS}(x_3) & = \frac{1}{1-0.9110} \cdot 0.5 \cdot 0.4 \cdot 0.1 = \frac{0.020}{0.089} \approx 0.2247 \end{cases}$$

DS rule is incompatible with Bayes rule in general. [Dezert/Tchamova/Han/Tacnet 2013]

DS rule is compatible with Bayes rule **only** if the prior is uniform or vacuous.

Belief functions and DST

Major innovations of DST

- Important paradigm shift for modeling uncertainty
- New appealing mathematical formalism of (quantitative) belief functions
- New combination rule for belief functions (DS rule)

... but BF and DST have never been fully accepted by a part of scientific community mainly because

- Independence between sources of evidence has never been well defined

- Doubts on the validity of DS rule

- Good experimental protocol to validate DST and DS rule is lacking

See Zadeh 1979,Yager 1983, Lemmer 1985, Dubois 1986, Pearl 1988,Voorbraak 1991, Wang 1994, Walley 1996, Fixsen et al. 1997, Gelman 2006, Dezert & al. 2012, etc

Belief functions and DST

What we have shown

- the dictatorial power of DS rule to fuse equi-reliable sources of evidence.
- the conflict (high or low) can be totally ignored through DS rule.
- the problem of validity of DST is not due to conflict level, but the absolute truth interpretation of proposition by Shafer for each source.
- In [Dezert-Tchamova 2014], we have proved a logical contradiction in the foundations of DST.

Recommendation

BF are mathematically appealing and well defined, but don't use DS rule to combine them, even in low conflicting situations.

Belief functions and DST

Some tricks to reduce troubles with DS rule

1) Apply ad-hoc thresholdings on the conflict to accept (or reject) DS result.

2) Modify input BBAs, or apply discounting techniques on sources.
- How to be sure that no problem will occur with DS rule after discounting ?
- How to discount sources when no statistical data is available ?

3) Mix the two previous strategies.

How to better prevent troubles in fusion of sources of evidence ?

Switch to new better (more efficient) techniques to fusion vague, uncertain, imprecise, conflicting quantitative and qualitative information fusion for static or dynamic problematics.

This is what DSmT proposes.

Part 2

Introduction to DSmT (Dezert-Smarandache Theory)

Introduction to DSmT

DSmT versus DST in short

Shafer's interpretation: A source can provide absolute truth from partial knowledge, observation, experience, ...

... but such interpretation yields a logical contradiction in DST foundations and counter-intuitive/disputable results in applications.

Our interpretation: A source can provide only a relative truth from partial knowledge, observation, experience, ...

This new interpretation makes differences in the way to process belief functions.

Introduction to DSmT

Main references

F. Smarandache, J. Dezert (Eds), Advances and applications of DSmT for information fusion, Vols. 1-4, 2004, 2006, 2009 & 2015.

Free e-books

http://www.onera.fr/fr/staff/jean-dezert
http://www.smarandache.com/DSmT.htm

Free toolboxes

http://bfasp.iutlan.univ-rennes1.fr/wiki/index.php/Toolboxes
http://martin.iutlan.univ-rennes1.fr/Doc/GeneralBeliefFunctionsFramework.tar

Introduction to DSmT

What is DSmT

It is a natural extension of the belief function framework to work with

- different models for the frame (not only Shafer's model)
- with (possibly imprecise) quantitative belief functions
- with qualitative belief functions (expressed as labels)
- new PCR rules of combination, and conditioning
- new probabilistic transformation for decision-making support

Why to use DSmT

- provides better results in fusion applications than DST
- deals with static and dynamic frames in a same general framework
- can cover broader fields of applications (because of more flexibility)

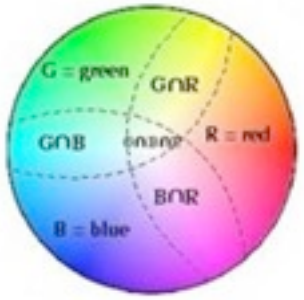
Drawback of DSmT

- its higher complexity (from theoretical and implementation standpoints)

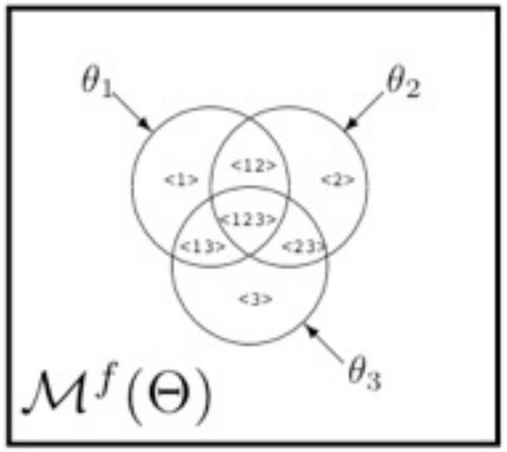
Introduction to DSmT

Free DSm model

No constraint on elements of the frame

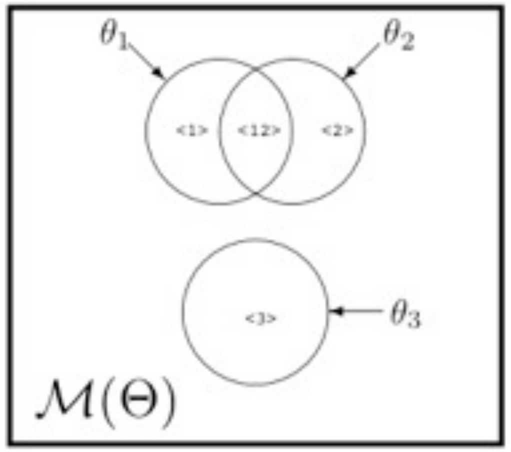


Parts can have vague boundaries



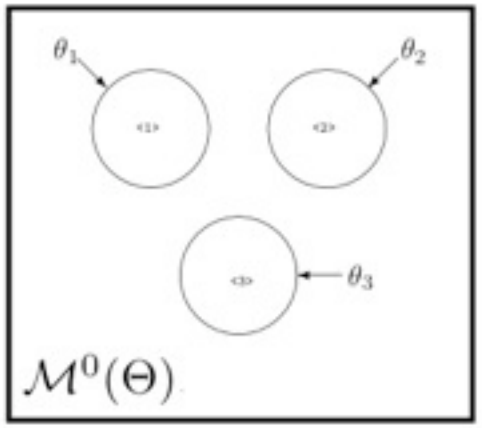
Hybrid DSm model

We introduce integrity constraints into the free DSm model.



Shafer's model = specific hybrid model

All exhaustive elements of the frame are known to be truly exclusive (i.e. a «refinement» is implicitly done)



Parts have precise boundaries

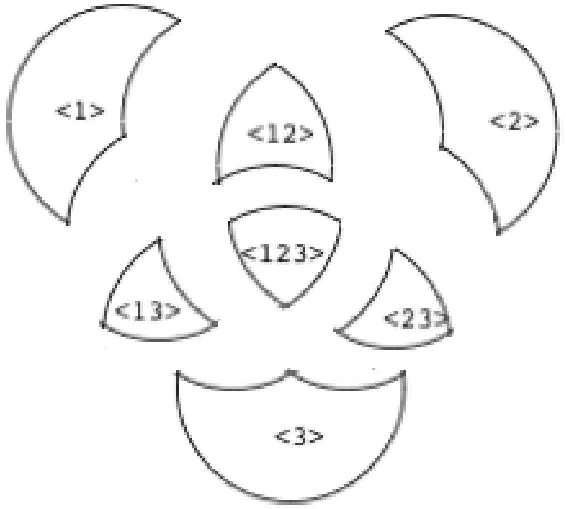
Introduction to DSmT

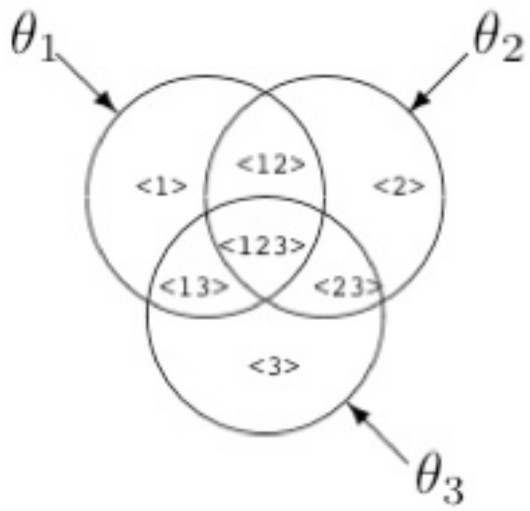
FoD $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$. Finite set of exhaustive elements (discrete/continuous/fuzzy/relative concepts)

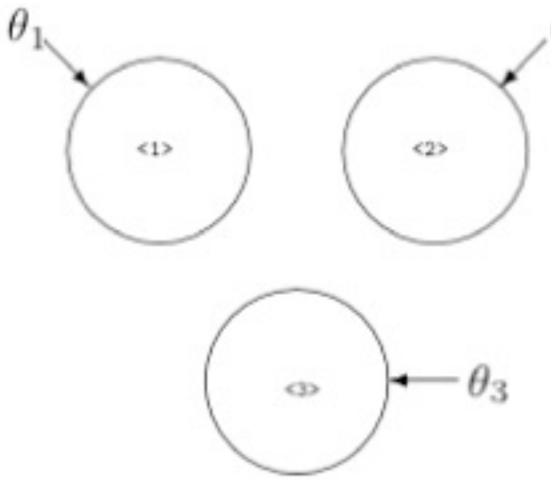
Fusion spaces

Power sets, Hyper-power set (Dedekind's lattice) and Super-power sets

$$|2^{\Theta_{ref}} = S^{\Theta} \triangleq (\Theta, \cup, \cap, c(.))| > |D^{\Theta} = (\Theta, \cup, \cap)| > |2^{\Theta} = (\Theta, \cup)|$$







Super-power set = power set of the refined frame

Introduction to DSmT

Belief functions in DSmT

$$m(.) : G^\Theta \rightarrow [0, 1] \qquad m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in G^\Theta} m(A) = 1$$

$$\text{Bel}(A) = \sum_{\substack{B \subseteq A \\ B \in G^\Theta}} m(B) \qquad \text{and} \qquad \text{Pl}(A) = \sum_{\substack{B \cap A \neq \emptyset \\ B \in G^\Theta}} m(B)$$

where G^Θ is the fusion space (i.e. 2^Θ, D^Θ, or $S^\Theta = 2^{\Theta_{refined}}$)

One can also define **qualitative** BBA's (using labels), and **imprecise** admissible (quantitative or qualitative) BBA's - see [DSmTBooks]

Introduction to DSmT

Fact: Decision-makers/humans don't like to take decision under uncertainty. Uncertainty reduction is sought thank to an efficient fusion process.

Why using new fusion rules in DSmT

To circumvent problems of DS rule

To **not** increase the uncertainty in the fusion of BBAs more than justified

Proportional Conflict Redistribution (PRC) rules of DSmT

Exploit separately information entailed in all partial conflicts (and not use directly the whole conflicting mass).

PCR5/6 transfers the partial conflicting masses to the elements involved in the partial conflict proportionally to masses $m_1(.)$ and $m_2(.)$ of elements involved in the partial conflict ONLY.

Introduction to DSmT

Principle of PCR rules of combination

1 - Apply the conjunctive rule

2 - Calculate the total or partial conflicting masses

3 - Redistribute the (total or partial) conflicting mass proportionally on non-empty sets according to the integrity constraints one has for the FoD

The proportional transfer of conflicting mass can be done in many ways.

- PCR rule #5 (PCR5) proposed by Smarandache & Dezert [DSmTBook3]
- PCR rule #6 (PCR6) proposed by Martin & Osswald [DSmTBook3]

PCR5 = PCR6 for combining 2 sources

PCR5 ≠ PCR6 for combining *s>2* sources

Which one is better? Why?

Introduction to DSmT

Combining two BBAs with PCR5/6 rules

See [DSmTBooks] for general formulas

$$m_{PCR5/6}(X) = \sum_{\substack{X_1,X_2\in 2^\Theta \\ X_1\cap X_2 = X}} m_1(X_1)m_2(X_2) + \sum_{\substack{Y\in 2^\Theta\setminus\{X\} \\ X\cap Y=\emptyset}} [\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)}]$$

Example $\Theta = \{A, B\}$

	A	B	$A\cup B$
$m_1(.)$	0.6	0.3	0.1
$m_2(.)$	0.2	0.3	0.5
$m_{12}(.)$	0.44	0.27	0.05

$$m_{12}(A\cap B = \emptyset) = m_1(A)m_2(B) + m_1(B)m_2(A)$$
$$= 0.18 + 0.06 = 0.24$$

$$x_1/0.6 = y_1/0.3 = (x_1+y_1)/(0.6+0.3) = 0.18/0.9 = 0.2 \longrightarrow \begin{cases} x_1 = 0.6\cdot 0.2 = 0.12 \\ y_1 = 0.3\cdot 0.2 = 0.06 \end{cases}$$

$$x_2/0.2 = y_2/0.3 = (x_2+y_2)/(0.2+0.3) = 0.06/0.5 = 0.12 \longrightarrow \begin{cases} x_2 = 0.2\cdot 0.12 = 0.024 \\ y_2 = 0.3\cdot 0.12 = 0.036 \end{cases}$$

$$m_{PCR5/6}(A) = 0.44 + \boxed{0.12} + \boxed{0.024} = 0.584$$
$$m_{PCR5/6}(B) = 0.27 + \boxed{0.06} + \boxed{0.036} = 0.366$$
$$m_{PCR5/6}(A\cup B) = 0.05 + 0 = 0.05$$

With Dempster's rule
$m_{DS}(A) \approx 0.579$
$m_{DS}(B) \approx 0.355$
$m_{DS}(A\cup B) \approx 0.066$

The mass put on ignorance with PCR5/6 is lower than with DST

Introduction to DSmT

Example of difference between PCR5 and PCR6 rules

$\Theta = \{A, B\}$

Shafer's model

$$m_1(A) = 0.6 \quad m_1(B) = 0.3 \quad m_1(A \cup B) = 0.1$$

$$m_2(A) = 0.2 \quad m_2(B) = 0.3 \quad m_2(A \cup B) = 0.5$$

$$m_3(A) = 0.7 \quad m_3(B) = 0.1 \quad m_3(A \cup B) = 0.2$$

Let's consider the partial conflicting mass.

$$m_1(A)m_2(B)m_3(B) = 0.6 \cdot 0.3 \cdot 0.1 = 0.018$$

With PCR5, one takes

$$\frac{x_A^{PCR5}}{m_1(A)} = \frac{x_B^{PCR5}}{m_2(B)m_3(B)} = \frac{m_1(A)m_2(B)m_3(B)}{m_1(A) + m_2(B)m_3(B)}$$

$$\frac{x_A^{PCR5}}{0.6} = \frac{x_B^{PCR5}}{0.03} = \frac{0.018}{0.6 + 0.03} \approx 0.02857 \longrightarrow \begin{cases} x_A^{PCR5} = 0.60 \cdot 0.02857 \approx 0.01714 \\ x_B^{PCR5} = 0.03 \cdot 0.02857 \approx 0.00086 \end{cases}$$

With PCR6, one takes

$$\frac{x_A^{PCR6}}{m_1(A)} = \frac{x_B^{PCR6}}{m_2(B) + m_3(B)} = \frac{m_1(A)m_2(B)m_3(B)}{m_1(A) + (m_2(B) + m_3(B))}$$

$$\frac{x_A^{PCR6}}{0.6} = \frac{x_B^{PCR6}}{0.3 + 0.1} = \frac{0.018}{0.6 + (0.3 + 0.1)} = 0.018 \longrightarrow \begin{cases} x_A^{PCR6} = 0.6 \cdot 0.018 = 0.0108 \\ x_B^{PCR6} = (0.3 + 0.1) \cdot 0.018 = 0.0072 \end{cases}$$

Martin & Osswald have shown in their application that PCR6 result is more stable than PCR5 result for decision making.

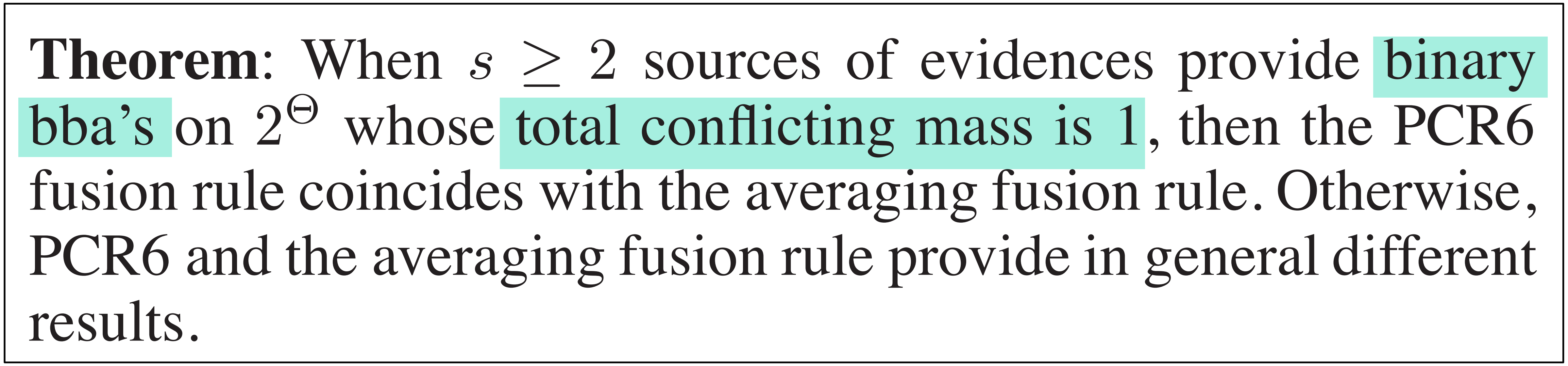
Introduction to DSmT

Advantages PCR5/6 rules work with any conflict, and outperfom DS rule.

Drawbacks Complexity, non-associativity

Why PCR6 is better than PCR5 and DS rule

PCR6 can be used to estimate correctly frequentist probas in random binary experiment. DS and PCR5 do not work.



Theorem: When $s \geq 2$ sources of evidences provide binary bba's on 2^{Θ} whose total conflicting mass is 1, then the PCR6 fusion rule coincides with the averaging fusion rule. Otherwise, PCR6 and the averaging fusion rule provide in general different results.

This theorem does not hold for PCR5 (but in s=2 case), nor for DS rule.

Introduction to DSmT

Compatibility of PCR6 with frequentist probabilities

n(A) = number of successes of event A

n>0 is the number of random experiments

$$P(A) = \lim_{n\to\infty} \frac{n(A)}{n} \qquad \hat{P}(A|n(A), n) = \frac{n(A)}{n}$$

The coin random flip experiment $\Theta = \{H = \text{Head}, T = \text{Tail}\}$

We observe $\{o_1 = H, o_2 = H, o_3 = T, o_4 = H, o_5 = T, o_6 = H, o_7 = H, o_8 = T\} \rightarrow \begin{cases} n(H) = 5 \\ n(T) = 3 \end{cases}$

Shafer's model

bba's \ Focal elem.	H	T
$m_1(.)$	1	0
$m_2(.)$	1	0
$m_3(.)$	0	1
$m_4(.)$	1	0
$m_5(.)$	0	1
$m_6(.)$	1	0
$m_7(.)$	1	0
$m_8(.)$	0	1

$$\hat{P}(H|\{o_1, o_2, \ldots, o_8\}) = \frac{n(H)}{n} = \frac{5}{8} = \frac{1}{8}(1+1+0+1+0+1+1+0) = m^{Average}_{1,2,\ldots,8}(H)$$

$$\hat{P}(T|\{o_1, o_2, \ldots, o_8\}) = \frac{n(T)}{n} = \frac{3}{8} = \frac{1}{8}(0+0+1+0+1+0+0+1) = m^{Average}_{1,2,\ldots,8}(T)$$

Theorem 1 applies →

$$\begin{cases} m^{PCR6}_{1,2,\ldots,8}(H) = m^{Average}_{1,2,\ldots,8}(H) = \hat{P}(H|\{o_1, o_2, \ldots, o_8\}) = 5/8 \\ m^{PCR6}_{1,2,\ldots,8}(T) = m^{Average}_{1,2,\ldots,8}(T) = \hat{P}(T|\{o_1, o_2, \ldots, o_8\}) = 3/8 \end{cases}$$

DS rule doesn't apply because the conflict is total, and PCR5 gives

$$\frac{x_H}{1\cdot 1\cdot 1\cdot 1\cdot 1} = \frac{y_T}{1\cdot 1\cdot 1} = \frac{m_{1,2,\ldots,8}(\emptyset)}{(1\cdot 1\cdot 1\cdot 1\cdot 1) + (1\cdot 1\cdot 1)} = \frac{1}{1+1} = 0.5$$

$$\begin{cases} m^{PCR5}_{1,2,\ldots,8}(H) = x_H = 0.5 \neq (m^{PCR6}_{1,2,\ldots,8}(H) = 5/8) \\ m^{PCR5}_{1,2,\ldots,8}(T) = y_T = 0.5 \neq (m^{PCR6}_{1,2,\ldots,8}(T) = 3/8) \end{cases}$$

Introduction to DSmT

Complexity of BF

$$(|2^\Theta| = 2^n) < (|D^\Theta| = d(n)) < (|2^{\Theta_{ref}}| = 2^{2^n-1})$$

$\|\Theta\| = n$	$\|2^\Theta\| = 2^n$	$\|D^\Theta\| = d(n)$	$\|2^{\Theta_{ref}}\| = 2^{2^n-1}$
2	4	5	$2^3 = 8$
3	8	19	$2^7 = 128$
4	16	167	$2^{15} = 32768$
5	32	7580	$2^{31} = 2147483648$

How to reduce complexity for combining BF

Approximate BBA by simpler ones

Implement fusion rules with sampling techniques [DSmT Book 3,Chap6]

Use simpler fusion rules

Introduction to DSmT

Approximate a BBA by a simpler one (probabilistic transforms)

Simplest method keeps only singletons as focal elements and normalize, but we loose information on partial ignorances

Pignistic transform redistributes mass of partial ignorances equally to singletons included in them [Smets 1990]

$$P\{A\} = \sum_{X\in 2^\Theta} \frac{|X\cap A|}{|X|} m(X)$$

DSmP transform redistributes mass of partial ignorances proportionally to masses of singletons included in them [Dezert-Smarandache 2008]

$$\forall X \in G^\Theta \setminus \{\emptyset\}$$

$$\boxed{DSmP_\epsilon(X) = \sum_{Y\in G^\Theta} \frac{\sum_{\substack{Z\subseteq X\cap Y\\ \mathcal{C}(Z)=1}} m(Z) + \epsilon\cdot\mathcal{C}(X\cap Y)}{\sum_{\substack{Z\subseteq Y\\ \mathcal{C}(Z)=1}} m(Z) + \epsilon\cdot\mathcal{C}(Y)} m(Y)}$$

$\epsilon \geq 0$ is a tuning parameter

Qualitative BetP and DSmP are possible. Other transforms exist.

Introduction to DSmT

Example BetP versus DSmP

	θ_1	θ_2	$\theta_1 \cup \theta_2$
$m(.)$	0.3	0.1	0.6

Shafer's model

Shannon's entropy $H(P) \triangleq -\sum_{i=1}^{n} P\{\theta_i\} \log_2(P\{\theta_i\})$

With BetP

$$\begin{cases} BetP(\emptyset) = 0 \\ BetP(\theta_1 \cap \theta_2) = 0 \\ BetP(\theta_1) = m(\theta_1) + \frac{1}{2} m(\theta_1 \cup \theta_2) = 0.6 \\ BetP(\theta_2) = m(\theta_2) + \frac{1}{2} m(\theta_1 \cup \theta_2) = 0.4 \\ BetP(\theta_1 \cup \theta_2) = m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1 \end{cases}$$

H(BetP)=0.9710 bits **Bigger entropy**

With DSmP

$$\begin{cases} DSmP_{\epsilon=0.001}(\emptyset) = 0 \\ DSmP_{\epsilon=0.001}(\theta_1 \cap \theta_2) = 0 \\ DSmP_{\epsilon=0.001}(\theta_1) = m(\theta_1) + \frac{m(\theta_1)+\epsilon}{m(\theta_1)+m(\theta_2)+2\epsilon} \cdot m(\theta_1 \cup \theta_2) = 0.7492 \\ DSmP_{\epsilon=0.001}(\theta_2) = m(\theta_2) + \frac{m(\theta_2)+\epsilon}{m(\theta_1)+m(\theta_2)+2\epsilon} \cdot m(\theta_1 \cup \theta_2) = 0.2508 \\ DSmP_{\epsilon=0.001}(\theta_1 \cup \theta_2) = m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1 \end{cases}$$

H(DSmP)=0.8125 bits **Lower entropy**

Introduction to DSmT

Approximate BBA using distances

1993 - Tessem's distance - Not a strict metric [Han et al. 2012]

$$d_T(m_1, m_2) = \max_{A \subseteq \Theta} \{|\mathrm{BetP}_1(A) - \mathrm{BetP}_2(A)|\}$$

2001 - Jousselme's distance - A strict metric proved in [Bouchard et al. in 2013]

$$d_J(m_1, m_2) = \sqrt{\frac{1}{2} \cdot (m_1 - m_2)^T \mathbf{Jac}\ (m_1 - m_2)} \qquad \mathbf{Jac}(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

2011 - Dissimilarity based on Fuzzy-Membership Function (FMF)

$$d_F(m_1, m_2) = 1 - \frac{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \wedge \mu^{(2)}(\theta_i))}{\sum_{i=1}^{n} (\mu^{(1)}(\theta_i) \vee \mu^{(2)}(\theta_i))}$$

$$\mu^{(i)} = [\,\mu^{(i)}(\theta_1), \mu^{(i)}(\theta_2), \cdots \mu^{(i)}(\theta_n)\,] = [\,Pl^{(i)}(\theta_1), Pl^{(i)}(\theta_2), \cdots, Pl^{(i)}(\theta_n)\,]$$

Introduction to DSmT

2014 - Euclidean belief interval based distance [Han-Dezert-Yang 2014]

$$d^E_{BI}(m_1,m_2) = \sqrt{N_c \cdot \sum_{i=1}^{2^n-1} [d^I(BI_1(A_i),BI_2(A_i))]^2} \qquad N_c = 1/2^{n-1}$$

2014 - Chebyshev belief interval based distance

$$d^C_{BI}(m_1,m_2) = \max_{A_i \subseteq \Theta, i=1,\dots,2^n-1} \left\{ d^I(BI_1(A_i),BI_2(A_i)) \right\}$$

using **Wasserstein's distance of interval numbers**

$$d^I([a_1,b_1],[a_2,b_2]) = \sqrt{\left[\frac{a_1+b_1}{2} - \frac{a_2+b_2}{2}\right]^2 + \frac{1}{3}\left[\frac{b_1-a_1}{2} - \frac{b_2-a_2}{2}\right]^2}$$

because belief intervals BI=[Bel(.),Pl(.]=[a,b] are just interval numbers.

Introduction to DSmT

Example

$$m_1(\{\theta_1\}) = m_1(\{\theta_2\}) = m_1(\{\theta_3\}) = 1/3;$$
$$m_2(\{\theta_1\}) = m_2(\{\theta_2\}) = m_2(\{\theta_3\}) = 0.1, m_2(\Theta) = 0.7;$$
$$m_3(\{\theta_1\}) = m_3(\{\theta_2\}) = 0.1, m_2(\theta_3) = 0.8.$$

Distance types	d_J	d_T	d_F	d_C	d_{BI}^E	d_{BI}^C
$d(m_1, m_2)$	0.4041	0	0.5833	0.2000	0.2858	0.2333
$d(m_1, m_3)$	0.4041	0.4667	0.6364	0.6667	0.4041	0.4667

Jousselme distance

seems not very reasonable (m2 makes no preference for choice, whereas m3 prefers the 3rd element)

Tessem's (BetP) distance

not intuitively acceptable because m1 different of m2 but dT(m1,m2)=0.

New belief interval distances

result makes more sense because $d(m_1, m_2) < d(m_1, m_3)$

Introduction to DSmT

Decision-making using belief functions

Pessimistic attitude: Max of Bel(.) **Optimistic attitude:** Max of Pl(.)

Common attitude: Use a probabilistic transformation to estimate a subjective proba measure P(.) in [Bel(.),Pl(.)]. Typically max of BetP, or max of DSmP.

General decision-making problem

States of the nature

Alternatives

$$\begin{array}{cc} & \begin{array}{ccccc} S_1 & \cdots & S_j & \cdots & S_n \end{array} \\ \begin{array}{c} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_q \end{array} & \begin{pmatrix} C_{11} & \cdots & C_{11} & \cdots & C_{1n} \\ & & \vdots & & \\ C_{i1} & \cdots & C_{ij} & \cdots & C_{in} \\ & & \vdots & & \\ C_{q1} & \cdots & C_{qj} & \cdots & C_{qn} \end{pmatrix} = C \end{array}$$

← benefit/payoff matrix

How to select the best alternative A^* given C matrix and the knowledge one has on the states of the nature?

Introduction to DSmT

Decision under certainty

If we know the true state of nature is S_j take

$$A^* = A_{i^*} \quad \text{with} \quad i^* \triangleq \arg\max_i \{C_{ij}\}$$

Decision under risk

If we know all probas $p_j=P(S_j)$, then compute expected benefits $E[C_i] = \sum_j p_j \cdot C_{ij}$ and take

$$A^* = A_{i^*} \quad \text{with} \quad i^* \triangleq \arg\max_i \{E[C_i]\}$$

Decision under ignorance

If we don't know probabilities $p_j=P(S_j)$, use Yager's OWA (Ordered Weighted Averaging) approach (1988).

Decision under uncertainty

If we have only a BBA defined on the power-set 2^S, where $S=\{S_1, S_2,..., S_n\}$, Yager proposed extended OWA.

Introduction to DSmT

Yager's OWA for decision under ignorance

$p_j = P(S_j)$ are unknown

Step 1 (Decisional attitude) Choose a normalized set of weights $w_{i1}, \ldots, w_{in}$ with $w_{i1} + \ldots + w_{in} = 1$

Step 2 (Evaluation) Compute the weighted average of ordered benefits for each row (alternative) i=1,2, ...q

$$V_i \triangleq \text{OWA}(C_{i1}, C_{i2}, \ldots, C_{in}) = \sum_j w_{ij} \cdot b_{ij}$$

b_{ij} is the jth largest element in the collection of benefit $\{C_{i1}, \ldots C_{in}\}$

Step 3 (Decision) Select $A^* = A_{i^*} \quad \text{with} \quad i^* \triangleq \arg\max_i \{V_i\}$

Introduction to DSmT

Example of OWA for decision under ignorance

$p_j = P(S_j)$ are unknown

$$C = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \\ \end{array}$$

$$C = \begin{array}{ccccc} & S_1 & S_2 & S_3 & S_4 \\ A_1 & 10 & 0 & 20 & 30 \\ A_2 & 1 & 10 & 20 & 30 \\ A_3 & 30 & 10 & 2 & 5 \end{array}$$

① **Pessimistic choice** (we take the min value per row)

$$W = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$V_1 = \text{OWA}(10, 0, 20, 30) = 0$$
$$V_2 = \text{OWA}(1, 10, 20, 30) = 1$$
$$V_3 = \text{OWA}(30, 10, 2, 5) = 2$$

Best choice = A_3

② **Optimitic choice**

$$W = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_1 = \text{OWA}(10, 0, 20, 30) = 30$$
$$V_2 = \text{OWA}(1, 10, 20, 30) = 30$$
$$V_3 = \text{OWA}(30, 10, 2, 5) = 30$$

All alternatives have same score

③ **Hurwicz choice** $\alpha = 0.3$ (balance between min and max values per row)

$$W = \begin{bmatrix} \alpha \\ 0 \\ 0 \\ 1-\alpha \end{bmatrix} = \begin{bmatrix} 0.3 \\ 0 \\ 0 \\ 0.7 \end{bmatrix}$$

$$V_1 = \text{OWA}(10, 0, 20, 30) = 9$$
$$V_2 = \text{OWA}(1, 10, 20, 30) = 9.7$$
$$V_3 = \text{OWA}(30, 10, 2, 5) = 10.4$$

Best choice = A_3

④ **Normative choice**

$$W = \begin{bmatrix} 1/4 \\ 1/4 \\ 1/4 \\ 1/4 \end{bmatrix}$$

$$V_1 = \text{OWA}(10, 0, 20, 30) = 60/4 = 15$$
$$V_2 = \text{OWA}(1, 10, 20, 30) = 61/4$$
$$V_3 = \text{OWA}(30, 10, 2, 5) = 47/4$$

Best choice = A_2

Introduction to DSmT

OWA for decision under uncertainty

$p_j=P(S_j)$ are unknown, but we have a BBA m(.) defined on the powerset 2^S, of states $S=\{S_1, S_2,..., S_n\}$

Step 1 (Decisional attitude) Choose a normalized set of weights $w_{i1}, ..., w_{in}$ with $w_{i1} +...+ w_{in}=1$

Step 2 (Evaluation) For each benefit subrow M_{ik} associated to a focal element X_k of BBA m(.) compute the benefit of V_{ik} of A_i by

$$V_{ik} = \text{OWA}(M_{ik}) \quad \text{and} \quad M_{ik} = \{C_{ij}|S_j \in X_k\}$$

Compute generalized expected benefits $$E[C_i] = \sum_{k=1}^{r} m(X_k)V_{ik}$$

Step 3 (Decision) Select $$A^* = A_{i^*} \quad \text{with} \quad i^* = \arg\max_i E[C_i]$$

Introduction to DSmT

Example of OWA for decision under uncertainty

States of the world

$S = \{S_1, S_2, S_3, S_4, S_5\}$

Alternatives

$A = \{A_1, A_2, A_3, A_4\}$

$$\underbrace{m(S_1 \cup S_3 \cup S_4)}_{X_1} = 0.6 \qquad \underbrace{m(S_2 \cup S_5)}_{X_2} = 0.3 \qquad \underbrace{m(S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5)}_{X_3} = 0.1$$

$X_1 \leftarrow$ partial ignorances $\rightarrow X_2$

full ignorance $\rightarrow$ X_3

$$C = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{pmatrix} 7 & 5 & 12 & 13 & 6 \\ 12 & 10 & 5 & 11 & 2 \\ 9 & 13 & 3 & 10 & 9 \\ 6 & 9 & 11 & 15 & 4 \end{pmatrix} \end{array}$$

$$M(X_1) = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \\ M_{41} \end{bmatrix} = \begin{array}{c} \begin{array}{ccc} S_1 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 7 & 12 & 13 \\ 12 & 5 & 11 \\ 9 & 3 & 10 \\ 6 & 11 & 15 \end{bmatrix} \end{array}$$

$$M(X_2) = \begin{bmatrix} M_{12} \\ M_{22} \\ M_{32} \\ M_{42} \end{bmatrix} = \begin{array}{c} \begin{array}{cc} S_1 & S_5 \end{array} \\ \begin{bmatrix} 5 & 6 \\ 10 & 2 \\ 13 & 9 \\ 9 & 4 \end{bmatrix} \end{array}$$

$$M(X_3) = \begin{bmatrix} M_{13} \\ M_{23} \\ M_{33} \\ M_{43} \end{bmatrix} = \begin{array}{c} \begin{array}{ccccc} S_1 & S_2 & S_3 & S_4 & S_5 \end{array} \\ \begin{bmatrix} 7 & 5 & 12 & 13 & 6 \\ 12 & 10 & 5 & 11 & 2 \\ 9 & 13 & 3 & 10 & 9 \\ 6 & 9 & 11 & 15 & 4 \end{bmatrix} \end{array} = C$$

sub-payoff matrices

Introduction to DSmT

OWA Example (cont'd)

Pessimistic attitude One takes the min value by row

$$M(X_1) = \begin{bmatrix} M_{11} \\ M_{21} \\ M_{31} \\ M_{41} \end{bmatrix} = \begin{array}{c} \begin{matrix} S_1 & S_3 & S_4 \end{matrix} \\ \begin{bmatrix} 7 & 12 & 13 \\ 12 & 5 & 11 \\ 9 & 3 & 10 \\ 6 & 11 & 15 \end{bmatrix} \end{array} \quad W = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad V(X_1) = \begin{bmatrix} 7 \\ 5 \\ 3 \\ 6 \end{bmatrix}$$

$X_1 = S_1 \cup S_3 \cup S_4$

$m(X_1) = 0.6$

$$M(X_2) = \begin{bmatrix} M_{12} \\ M_{22} \\ M_{32} \\ M_{42} \end{bmatrix} = \begin{array}{c} \begin{matrix} S_1 & S_5 \end{matrix} \\ \begin{bmatrix} 5 & 6 \\ 10 & 2 \\ 13 & 9 \\ 9 & 4 \end{bmatrix} \end{array} \quad W = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad V(X_2) = \begin{bmatrix} 5 \\ 2 \\ 9 \\ 4 \end{bmatrix}$$

$X_2 = S_2 \cup S_5$

$m(X_2) = 0.3$

$$M(X_3) = \begin{bmatrix} M_{13} \\ M_{23} \\ M_{33} \\ M_{43} \end{bmatrix} = \begin{array}{c} \begin{matrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{matrix} \\ \begin{bmatrix} 7 & 5 & 12 & 13 & 6 \\ 12 & 10 & 5 & 11 & 2 \\ 9 & 13 & 3 & 10 & 9 \\ 6 & 9 & 11 & 15 & 4 \end{bmatrix} \end{array} = C \quad W = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad V(X_3) = \begin{bmatrix} 5 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$X_3 = S_1 \cup S_2 \cup S_3 \cup S_4 \cup S_5$

$m(X_3) = 0.1$

$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{bmatrix} E[C_1] \\ E[C_2] \\ E[C_3] \\ E[C_4] \end{bmatrix} = \sum_{k=1}^{r=3} m(X_k) \cdot V(X_k) = \begin{bmatrix} 6.2 \\ 3.8 \\ 4.8 \\ 5.2 \end{bmatrix}$$

$7 \cdot 0.6 + 5 \cdot 0.3 + 5 \cdot 0.1 = 6.2 \Leftarrow \boxed{A_1}$ Best choice = A₁

Introduction to DSmT

OWA Example (cont'd)

Optimistic attitude

One takes the max value by row

$$M(X_1)=\begin{bmatrix}M_{11}\\M_{21}\\M_{31}\\M_{41}\end{bmatrix}=\begin{array}{c}\begin{array}{ccc}S_1&S_3&S_4\end{array}\\\begin{bmatrix}7&12&13\\12&5&11\\9&3&10\\6&11&15\end{bmatrix}\end{array}\quad W=\begin{bmatrix}1\\0\\0\end{bmatrix}\quad V(X_1)=\begin{bmatrix}13\\12\\10\\15\end{bmatrix}$$

$$X_1=S_1\cup S_3\cup S_4$$

$$m(X_1)=0.6$$

$$M(X_2)=\begin{bmatrix}M_{12}\\M_{22}\\M_{32}\\M_{42}\end{bmatrix}=\begin{array}{c}\begin{array}{cc}S_1&S_5\end{array}\\\begin{bmatrix}5&6\\10&2\\13&9\\9&4\end{bmatrix}\end{array}\quad W=\begin{bmatrix}1\\0\end{bmatrix}\quad V(X_2)=\begin{bmatrix}6\\10\\13\\9\end{bmatrix}$$

$$X_2=S_2\cup S_5$$

$$m(X_2)=0.3$$

$$M(X_3)=\begin{bmatrix}M_{13}\\M_{23}\\M_{33}\\M_{43}\end{bmatrix}=\begin{array}{c}\begin{array}{ccccc}S_1&S_2&S_3&S_4&S_5\end{array}\\\begin{bmatrix}7&5&12&13&6\\12&10&5&11&2\\9&13&3&10&9\\6&9&11&15&4\end{bmatrix}\end{array}=C\quad W=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}\quad V(X_3)=\begin{bmatrix}13\\12\\13\\15\end{bmatrix}$$

$$X_3=S_1\cup S_2\cup S_3\cup S_4\cup S_5$$

$$m(X_3)=0.1$$

$$\begin{array}{c}A_1\\A_2\\A_3\\A_4\end{array}\begin{bmatrix}E[C_1]\\E[C_2]\\E[C_3]\\E[C_4]\end{bmatrix}=\sum_{k=1}^{r=3}m(X_k)\cdot V(X_k)=\begin{bmatrix}10.9\\11.4\\11.2\\13.2\end{bmatrix}\Leftarrow\boxed{A_4}$$

$$13\cdot 0.6+6\cdot 0.3+13\cdot 0.1=10.9$$

Best choice = A_4

(A_4 is also chosen with normative attitude)

Introduction to DSmT

Problem with Yager's OWA approach

The final result strongly depends on the decisional attitude.
How to choose it among the infinite number of possible attitudes?

Yager defined an index of optimism and proposed to compute w_i from it using max-entropy principle.

How to deal with decisional attitude choice ?

Use jointly the two most extreme attitudes (pessimistic and optimistic) to be more cautious.

Cautious OWA (COWA) method [Tacnet-Dezert 2011]

Pessimistic and optimistic generalized expected benefits allow to build belief intervals, and to get BBAs that are combined with PCR6 to get combined BBA to take final decision..

Improvement of COWA (having lower complexity) [Han-Dezert-Tacnet 2012]

Introduction to DSmT

DSmT for Multi-criteria decision-making

DSm-AHP method

Extension of Saaty's Analytic Hierarchy Process (AHP) with BF, PCR rules and importance discounting technique.

Dezert J., Tacnet J.-M., Evidential Reasoning for Multi-Criteria Analysis based on DSmT-AHP, ISAHP 2011, Italy, June 2011.

Dezert J, Tacnet J.-M., Batton-Hubert M., Smarandache F., Multi-criteria decision making based on DSmT/AHP, Proc. of International Workshop on Belief Functions, Brest, France, April 2-4, 2010.

Soft-ELECTRE method

Improvement of Roy's ELECTRE method to assign alternatives into a set of predetermined categories based on BF and PCR rules.

Dezert J., Tacnet J.-M., Sigmoidal Model for Belief Function-based Electre Tri method, Belief 2012, Compiègne, May 2012.

Dezert J., Tacnet J.-M., Soft ELECTRE TRI outranking method based on belief functions, Proc. Of Fusion 2012, Singapore, July 2012.

Introduction to DSmT

DSmT for quality assessment of optimal data association

Basic idea: Find the 1st and 2nd best optimal optimal assigments. Detect the instability of solutions and use them to estimate the quality of 1st best optimal assignment thanks to BF and PCR6 rule of combination.

Dezert J., Benameur K., On the quality of optimal assignment for data association, Proc. of Belief 2014 Conf. Oxford, UK, Sept. 26-29, 2014.

J. Dezert, K. Benameur, L. Ratton, J.-F. Grandin, On the Quality Estimation of Optimal Multiple Criteria Data Association Solutions, in Proc. of Fusion 2015, Washington D.C, USA, July 6-9, 2015.

Dezert J., Tchamova A., Konstantinova P., The Impact of the Quality Estimation of Optimal Assignment for Data Association in a Multitarget Tracking Context, in preparation for CYBERNETICS AND INFORMATION TECHNOLOGIES Journal.

Part 3

Some applications of DSmT

Applications of DSmT

see http://www.onera.fr/staff/jean-dezert?page=3

~ 25 Ph.D Thesis, and 220 papers by colleagues during 2004--2014

Target tracking and recognition

Satellite imaging (classification and change detection)

Medical imaging (classification and diagnosis)

Biometrics (fingerprint and face recognition)

Robotics (SLAM)

OCR (Signature verification)

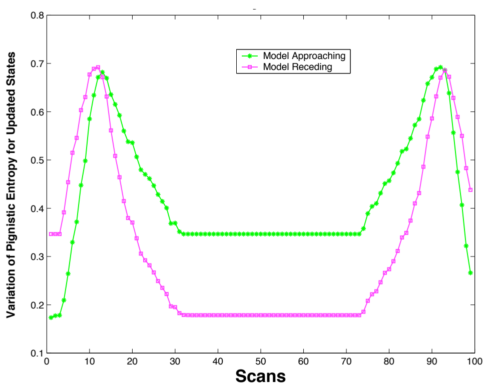
MCDM and risk management

Image fusion

Failure detection

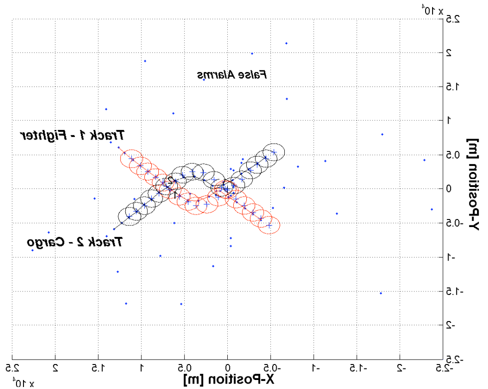
Applications of DSmT

DSmT for Target tracking and recognition



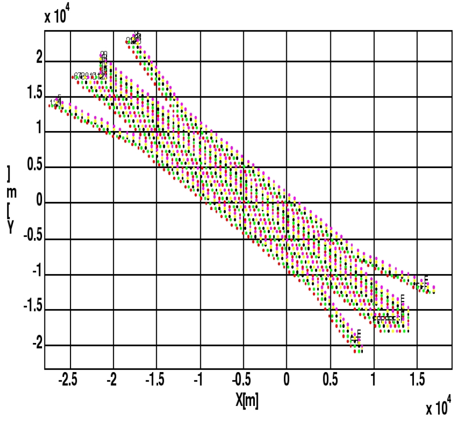
Estimation of target behavior tendencies
[Tchamova et. al 2003]

Sonar amplitude meas+ + fuzzy rules + DSmT for updating

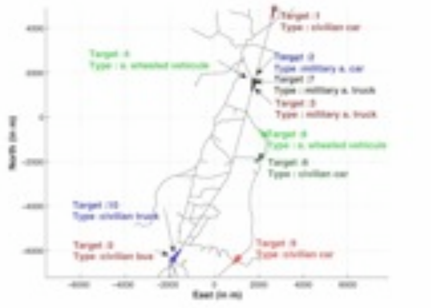


Generalized data association for MTT in clutter
[Tchamova et. al 2004-2006]

MTT with kinematics and attribute measurements



Performance improvment of Multitarget Tracking using DSmT [Tchamova et al. 2005-2006]



Improvement of Multiple Ground Targets Tracking with GMTI Sensor and Fusion of Identification Attributes [B. Pannetier et al. 2008]

Applications of DSmT

DSmT for Target tracking and recognition

Multiple Ground Target Tracking and Classification with DSmT
[B. Pannetier et al. 2010]

A PCR-BIMM filter For Maneuvering Target Tracking
[Dezert-Pannetier 2010]

Tracking Applications with Fuzzy-Based Fusion Rules
[Tchamova-Dezert 2013]

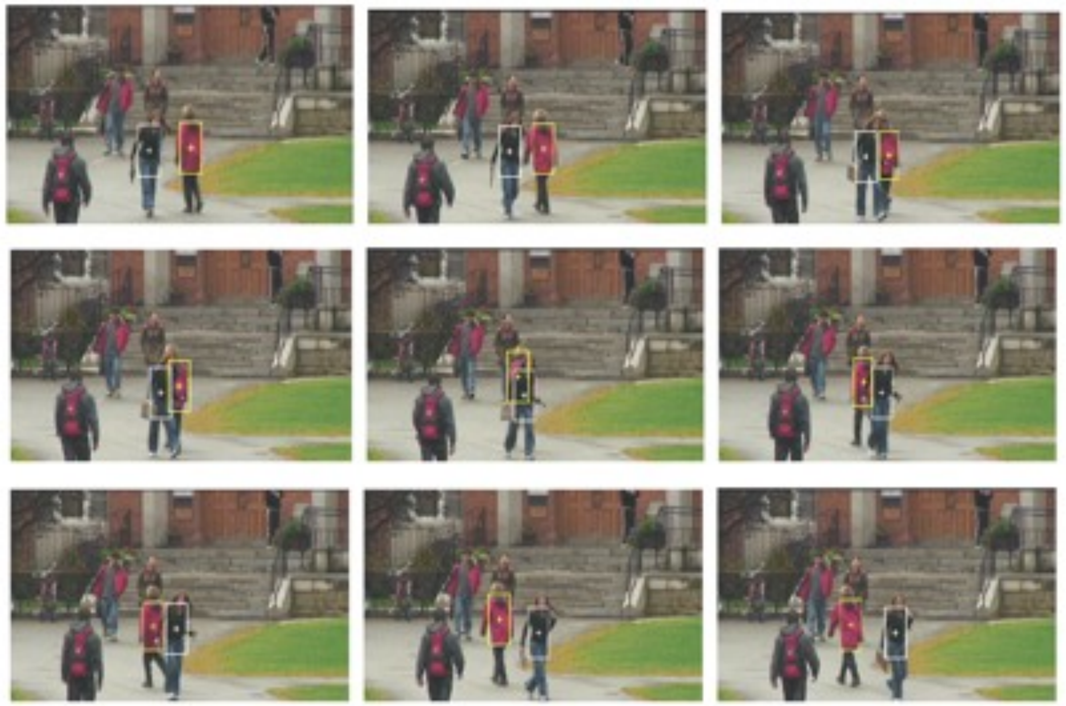
On the Quality of Optimal Assignment for data association
[J. Dezert,et al. Belief 2014]

On the Quality Estimation of Optimal Multiple Criteria Data Association Solutions
[J. Dezert,et al. Fusion2015]

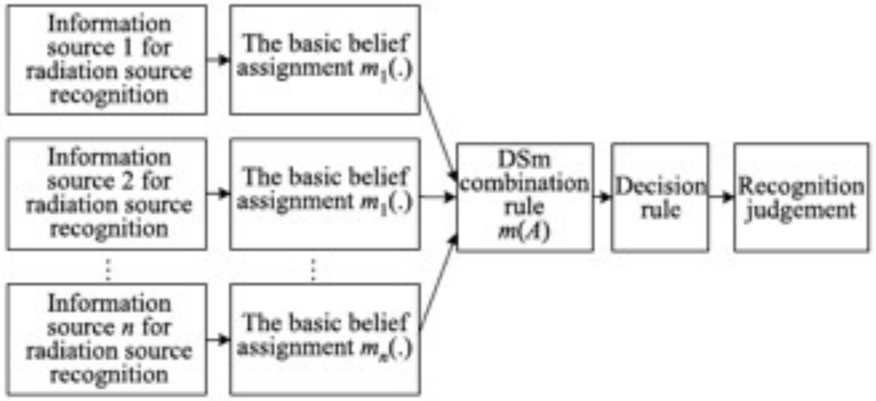
Applications of DSmT

DSmT for Target tracking and recognition

A Sequential Monte-Carlo and DSmT Based Approach for Conflict Handling in case of Multiple target Tracking [Sun,Bentabet 2008]



An Improved Radar Emitter Recognition Method Based on Dezert-Smarandache Theory [Chen et al. 2015]

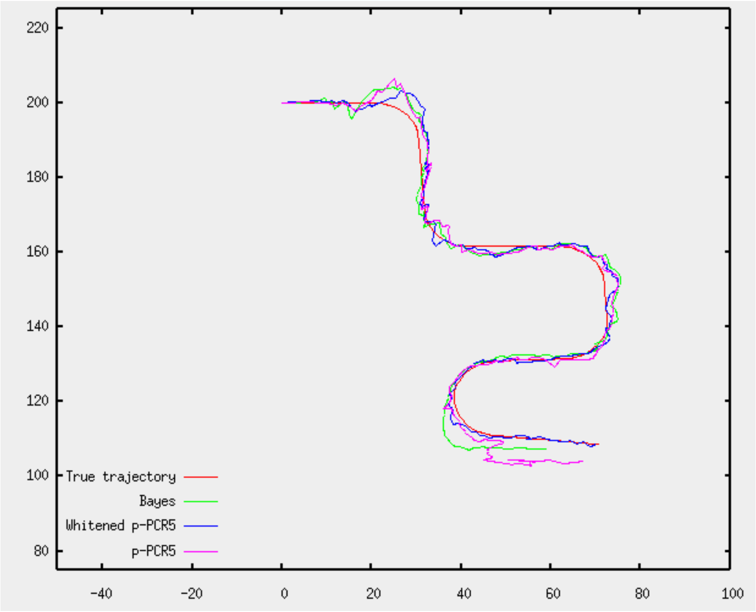


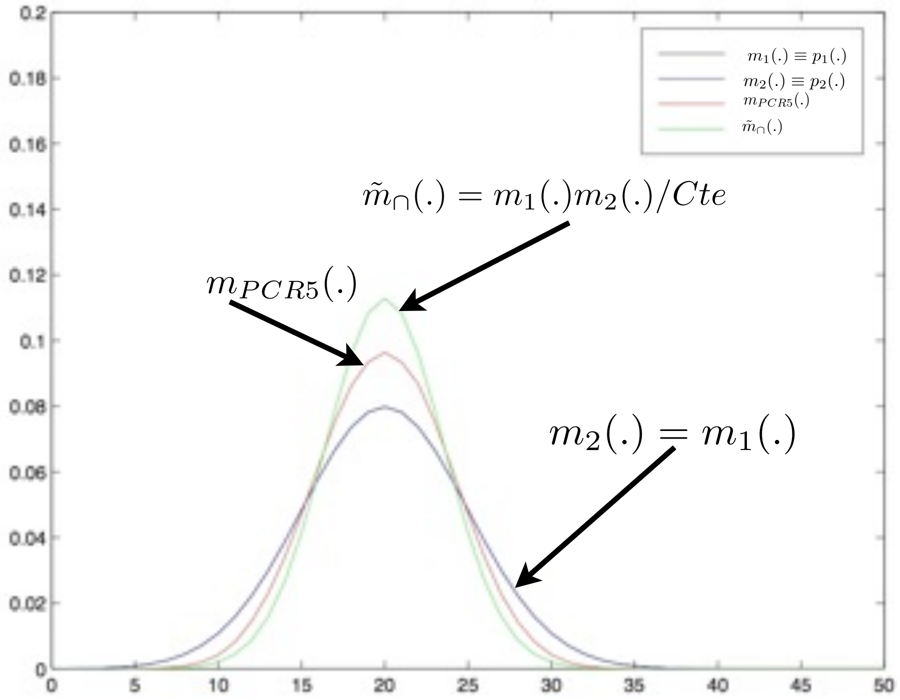
Applications of DSmT

DSmT for Target tracking and recognition

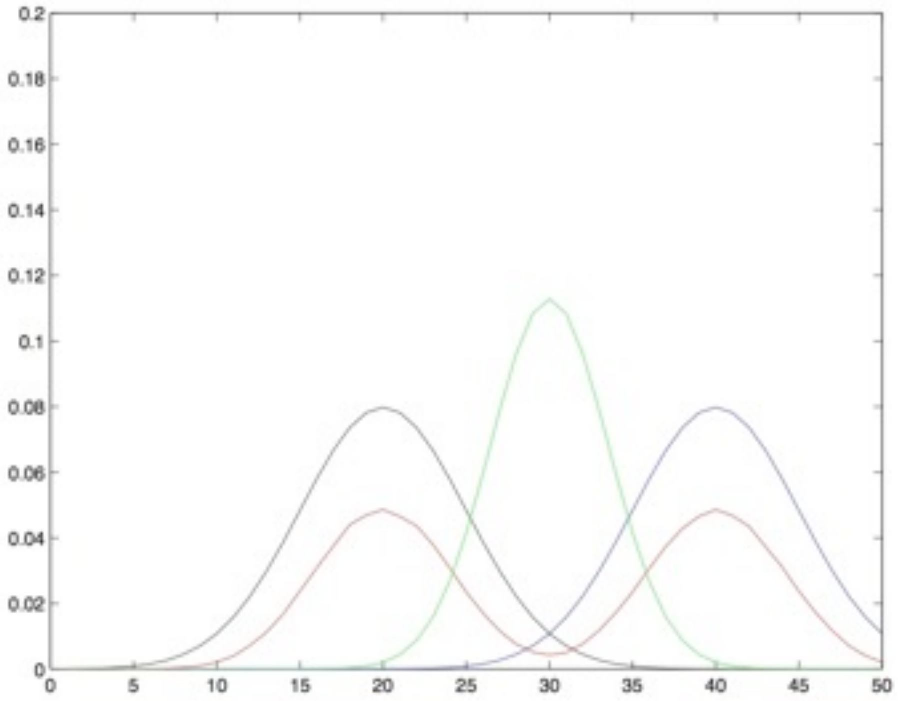
MS Particle filtering with PCR5 for target tracking
[Kirchner & al. 2007]

Distributed passive sensor tracking context.
Robustness to bad initialization





$$\text{Case } 1 : \; m_2(.) = m_1(.)$$



$$\text{Case } 2 : \; m_2(.) \neq m_1(.)$$

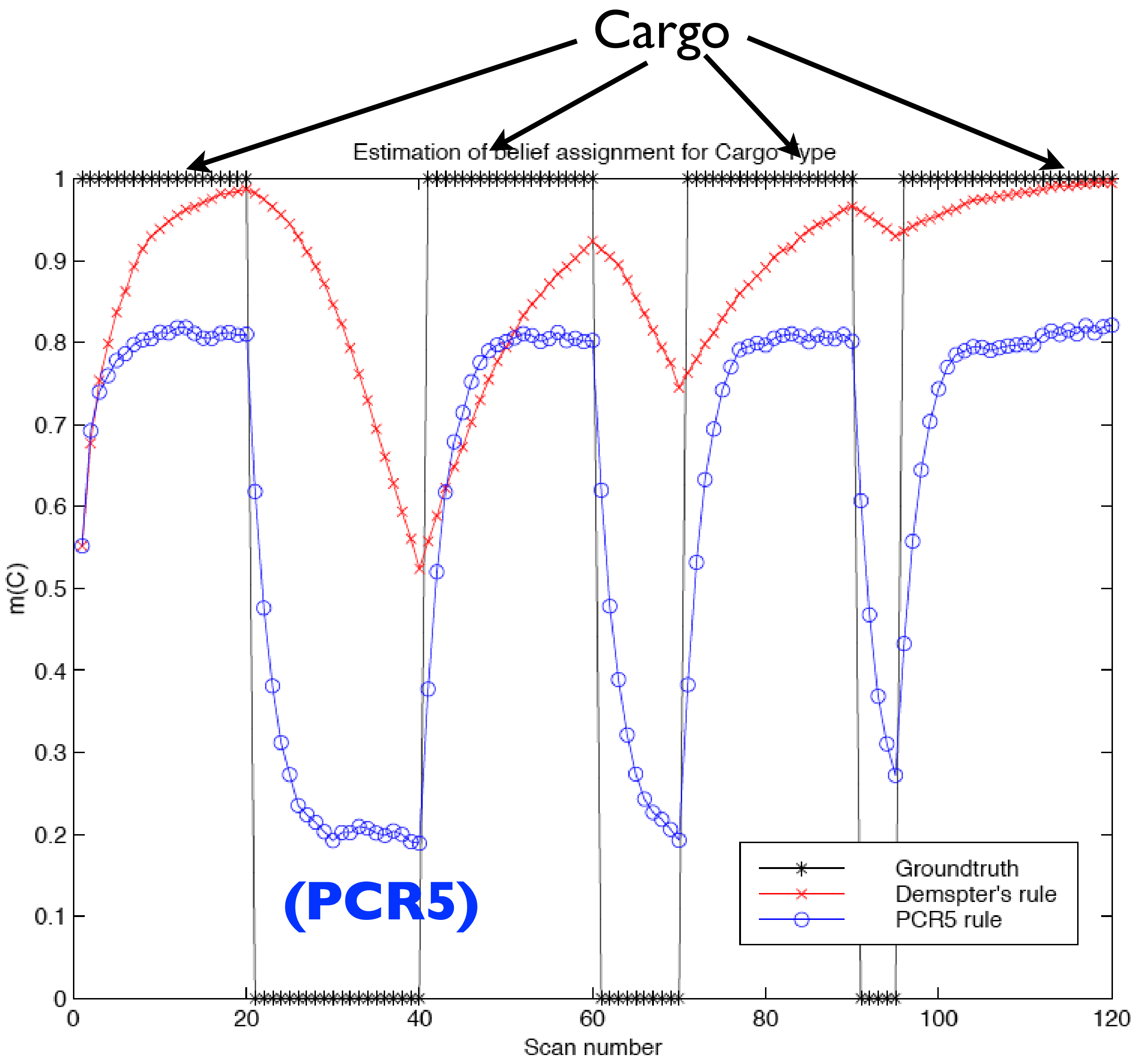
We restrict bba to be Bayesian and we extend PCR5 to work on a continuous frame

Applications of DSmT

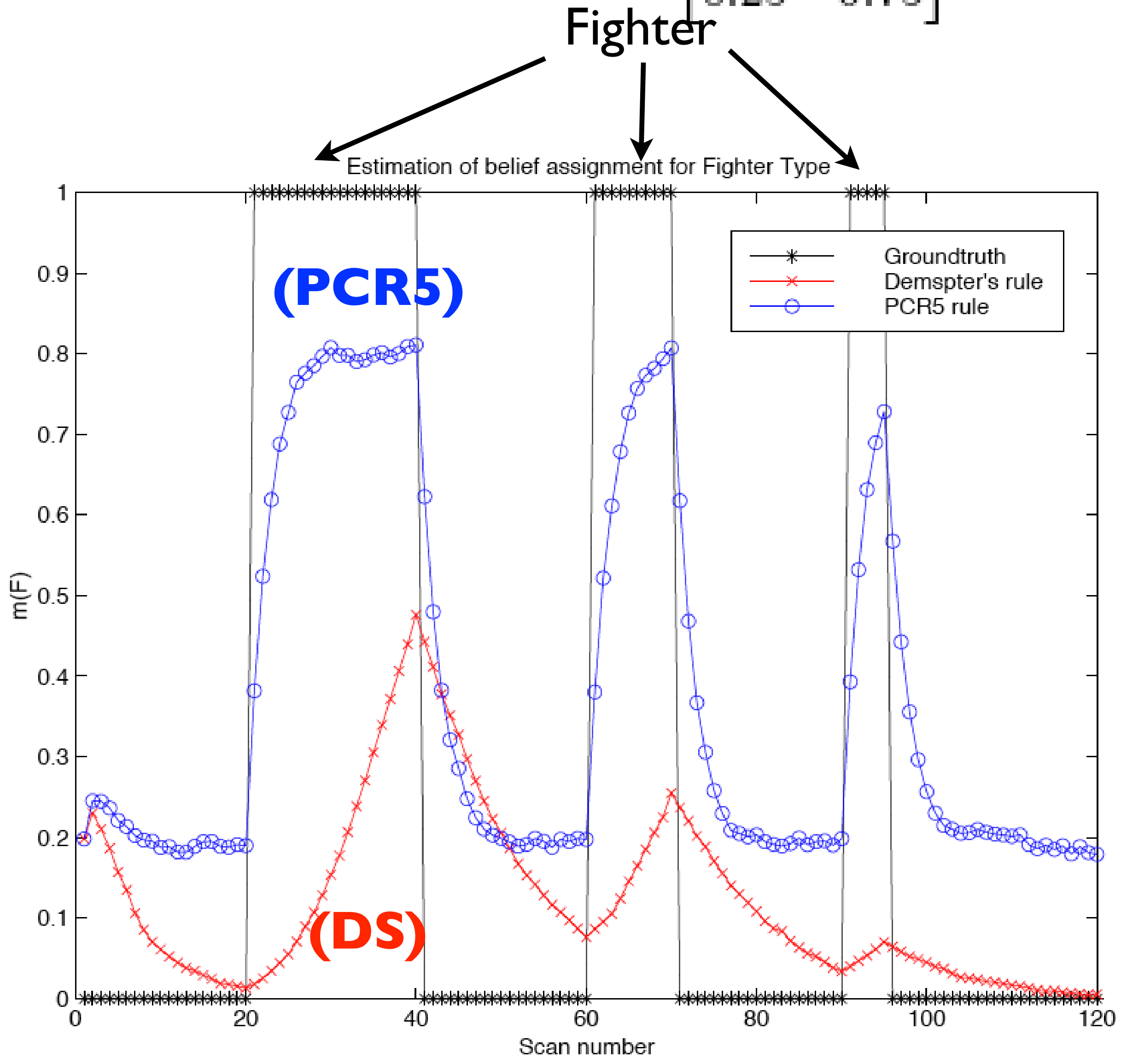
DSmT for Target tracking and recognition

Target Type Tracking [Dezert, Tchamova et al. 2006]

2 targets sequentially observed and classified with $\mathbf{C_2} = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$



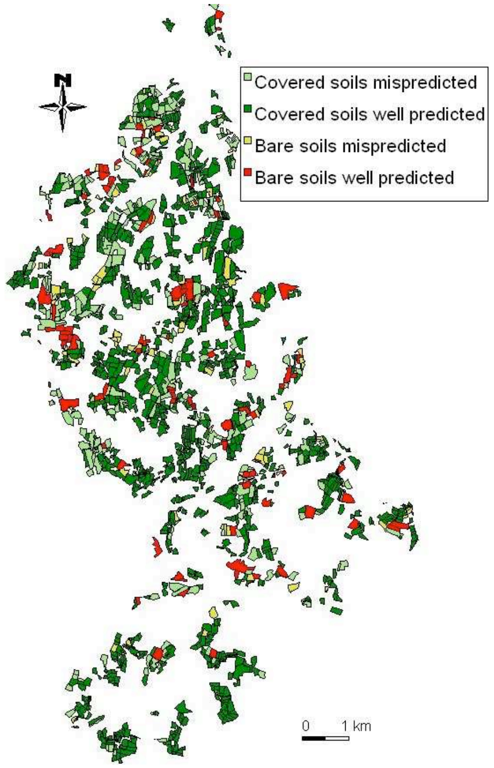
Cargo Type Tracking



Fighter Type Tracking

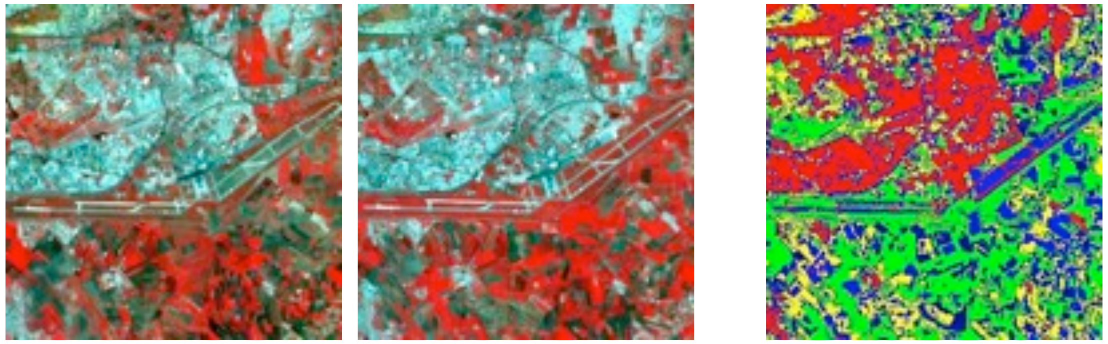
Applications of DSmT

DSmT for Satellite imaging (classification and change detection)



Land cover change prediction for pollution prevention
[Corgne et al. 2004 + Ph D Thesis]

Application of DSm Theory for SAR image change detection
[Hachicha et al. 2009]



Satellite image fusion using DSmT
[Bouakache,Belhadj-Aissa,Mercier 2009]

Applications of DSmT

DSmT for Satellite imaging (classification and change detection)

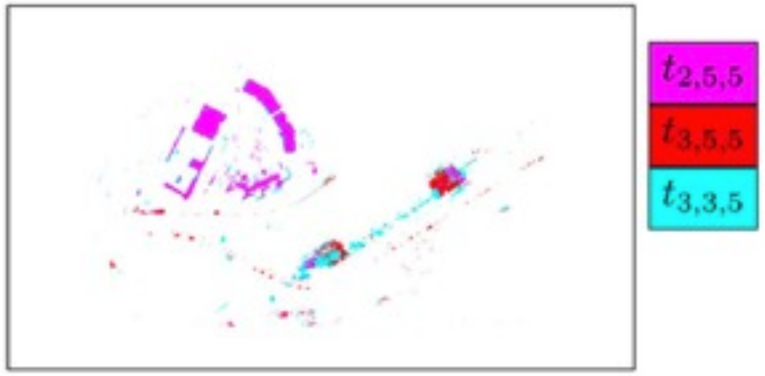
Dynamic Evidential Reasoning for Change Detection in Remote Sensing Images [Liu et al. 2011]

Transition in $T^{\Theta}_{n=2}$	interpretation
$t_{2,5}$	farmland → building
$t_{3,5}$	normal building → destroyed building
$t_{2,5,5}$	farmland → building → building
$t_{3,5,5}$	normal building → destroyed building → destroyed building
$t_{3,3,5}$	normal building → normal building → destroyed building


Before Earthquake


After Earthquake at t1

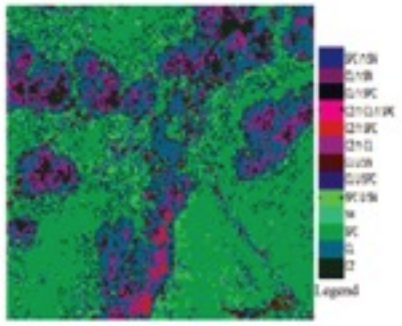

After Earthquake at t2



Multisource Fusion/Classification Using ICM* and DSmT with New Decision Rule [Elhassouny et al. 2012] * ICM = Iterated Conditional Mode

On the SAR change detection review and optimal decision [Hachicha et al. 2014]

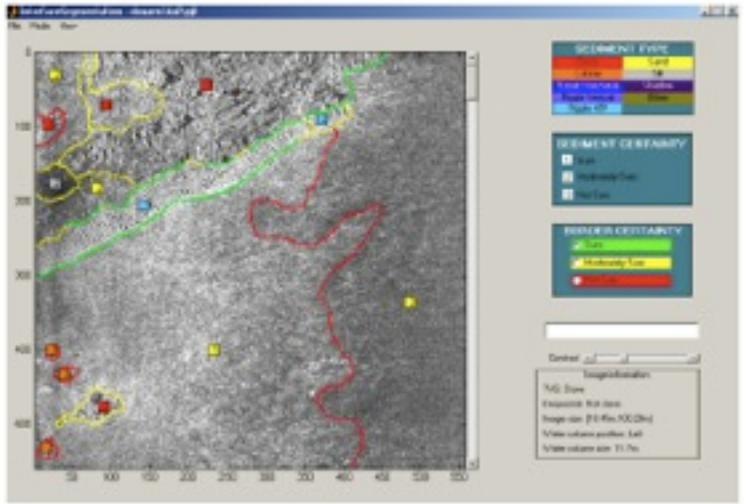
New contributions into the Dezert-Smarandache theory: Application to remote sensing image classification [Haouas et al. 2014]


forest classification

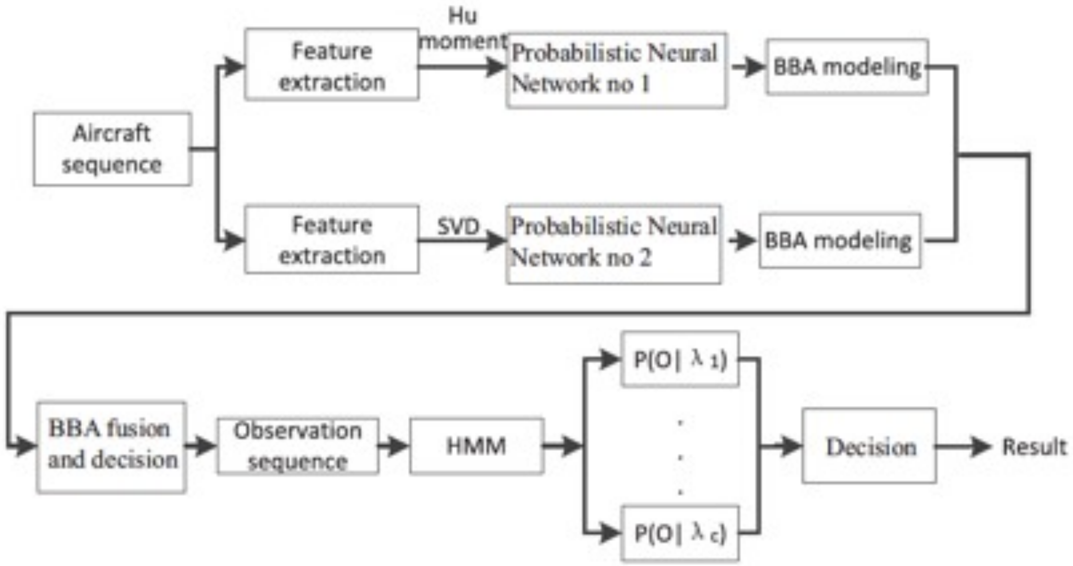
Applications of DSmT

DSmT for recognition and classification

Image segmentation and target classification based on real radar data and PCR rules [Martin, Osswald 2006]



Automatic Aircraft Recognition using DSmT and HMM [Li et al. 2014]



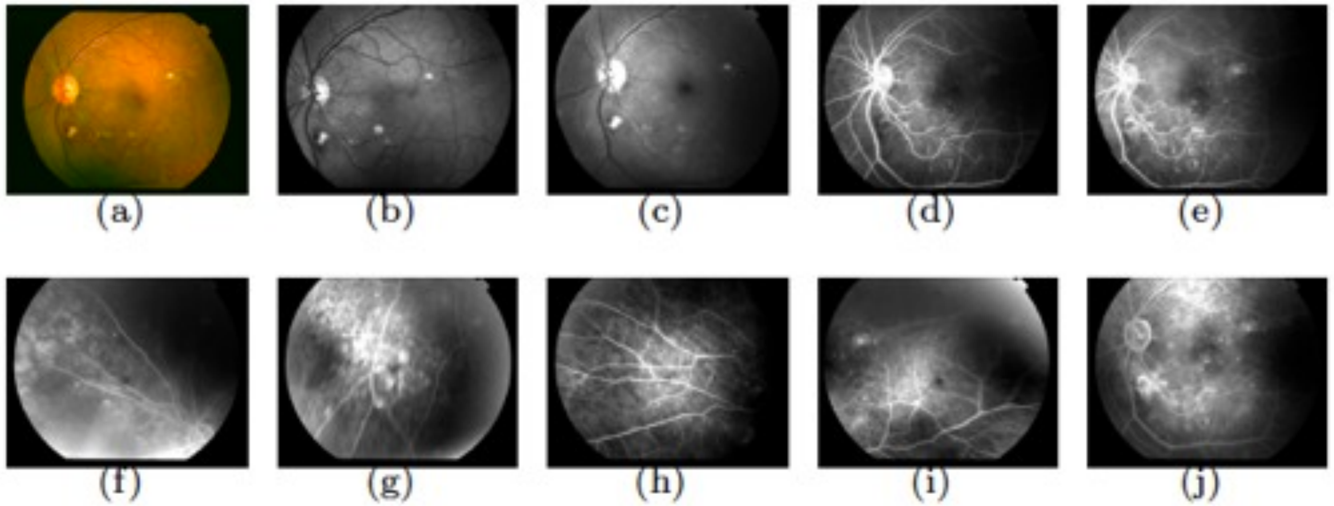
Applications of DSmT

DSmT for Medical imaging (classification and diagnosis)

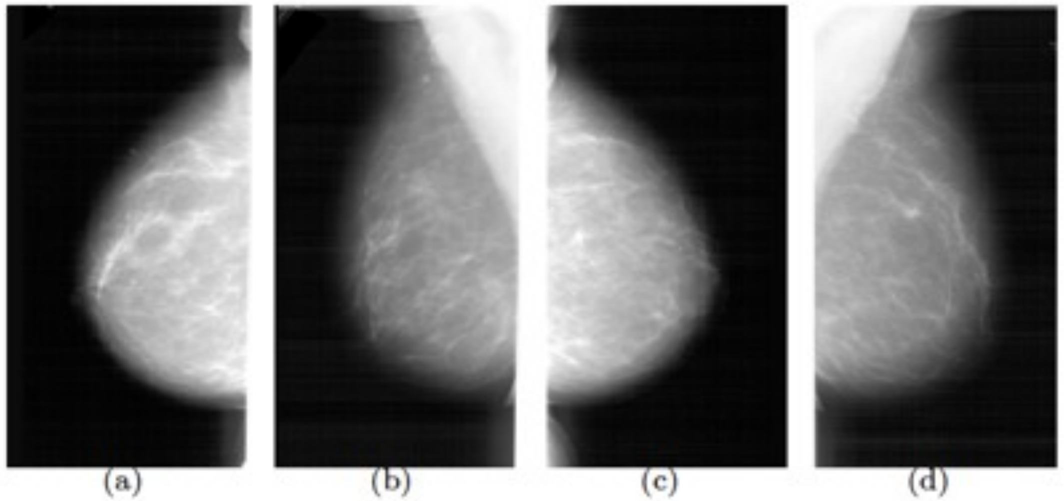
Applications: Retinopathy and breast cancer detection

Multimodal information retrieval based on DSmT. Application to computer-aided medical diagnosis [Quellec et al. 2008-2009]

Case retrieval in medical databases by fusing heterogeneous information [Quellec et al.2001]



Diabetic Retinopathy Database



Digital Database for Screening Mammography

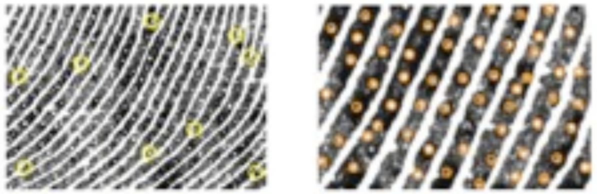
Applications of DSmT

DSmT for Biometrics (fingerprint and face recognition)

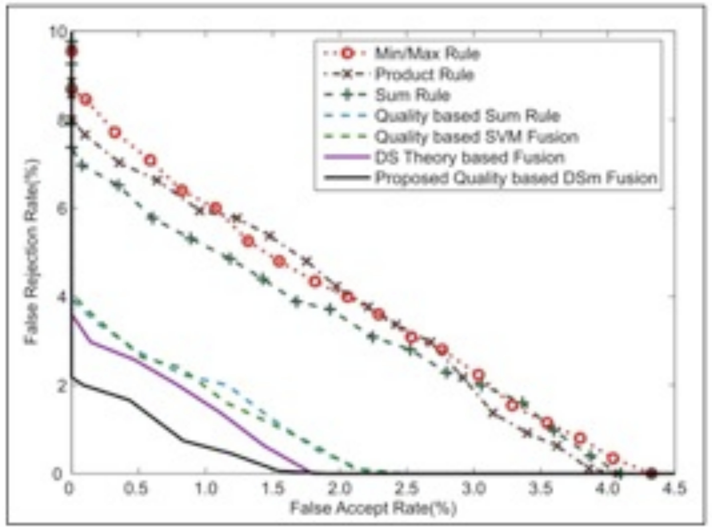
Unification of Evidence Theoretic Fusion Algorithms: A Case Study in Level-2 and Level-3 Fingerprint Feature [Vatsa et al. 2008]

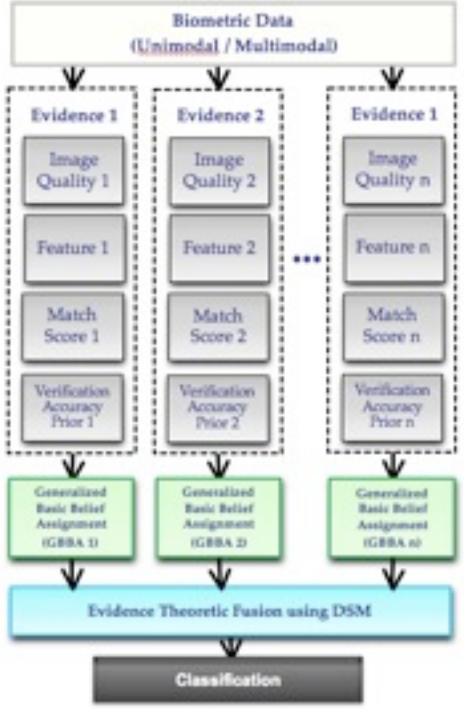
Example of conflicting data – Face recognition algorithm *accepts* and fingerprint recognition algorithm *rejects*

Biometric match score fusion based on DSmT [Vatsa 2008]



Integrated Multilevel Image Fusion and Match Score Fusion of Visible and Infrared Face Images for Robust Face Recognition [Singh et al. 2008]





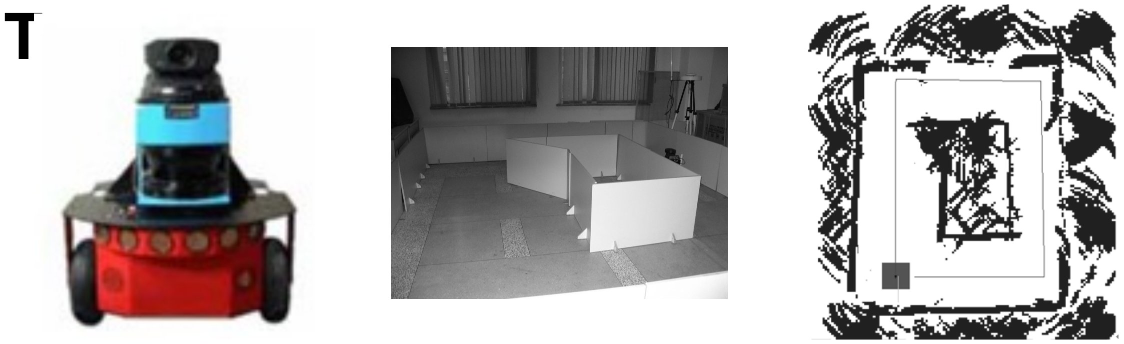
Quality-Augmented Fusion of Level-2 and Level-3 Fingerprint Information using DSm Theory [Vatsa et al. 2008]

Applications of DSmT

DSmT for Robotics (SLAM)

Robot Map building from Sonar Sensors and DSmT [Li, Dezert et al. 2006]

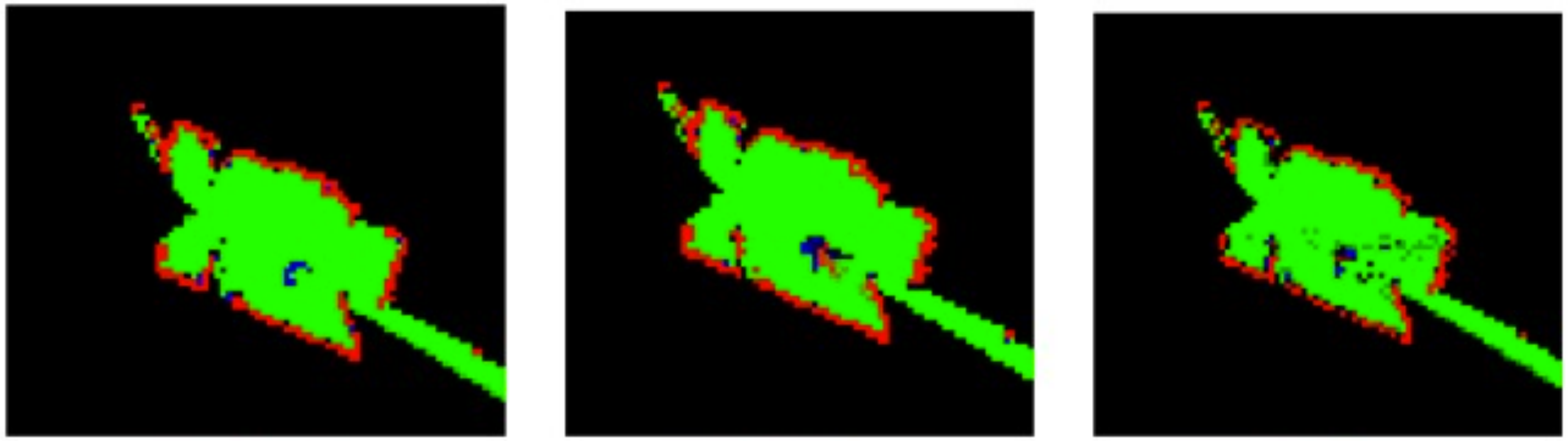
Robot Map building and self Localization on real sonar data based on PCR5 [Li & al. 2007]



Occupancy Grid Mapping Based on DSmT for Dynamic Environment Perception [Zhou et al. 2013, 2015]

Environment Perception Using Grid Occupancy Estimation with Belief Functions [Dezert,Moras Pannetier 2015]

We use (Z)PCR6 to update grid perception to make mapping for long-term navigation and detect mobile objects. Real experiment with LIDAR sensor.



With DS rule With PCR6 rule With ZPCR6 rule

- DS works well for the static part of environment, but not near the person.
- PCR6 works well for the static part and detects well the walking person.
- ZPCR6 $\sim$ PCR6 but $m_{ZPCR6}(\Omega) > m_{PCR6}(\Omega)$ for cells behind the person

Applications of DSmT

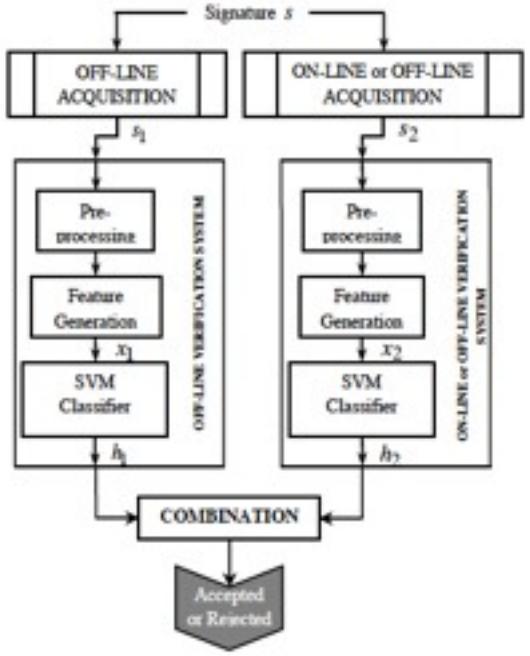
DSmT for OCR (signature & handwritten address verification)

Handwritten Digit Recognition Based On a DSmT-SVM Parallel Combination [Abbas et al. 2012]

A DSmT Based Combination Systems for Handwritten Signature Verification [Abbas et al. 2012]

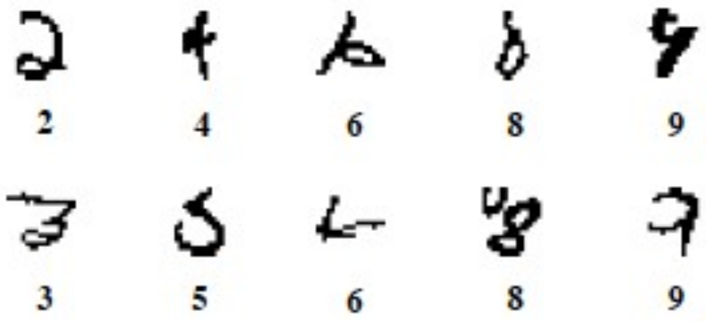
SVM-DSmT Combination for Off-Line Signature Verification [Abbas et al. 2012]

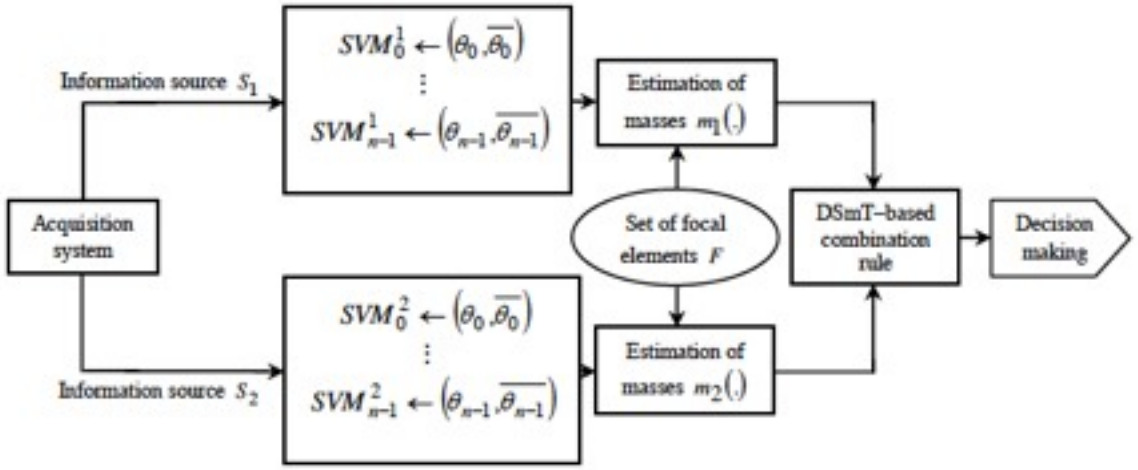
The Effective Use of the DSmT for Multi-Class Classification [Abbas et al. 2015]



(a) Genuine signatures.

(b) Forgery signatures.

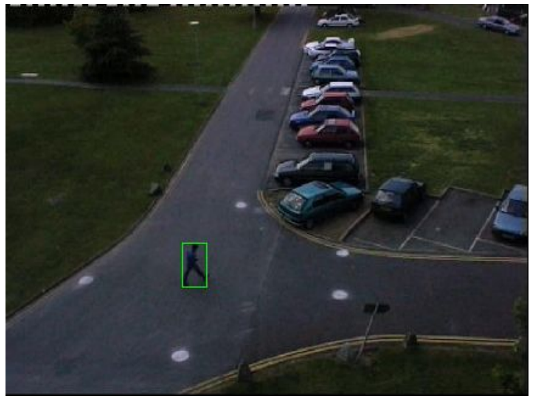


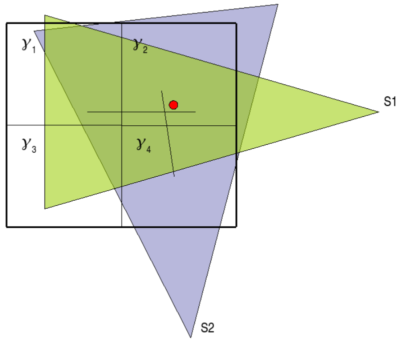


Applications of DSmT

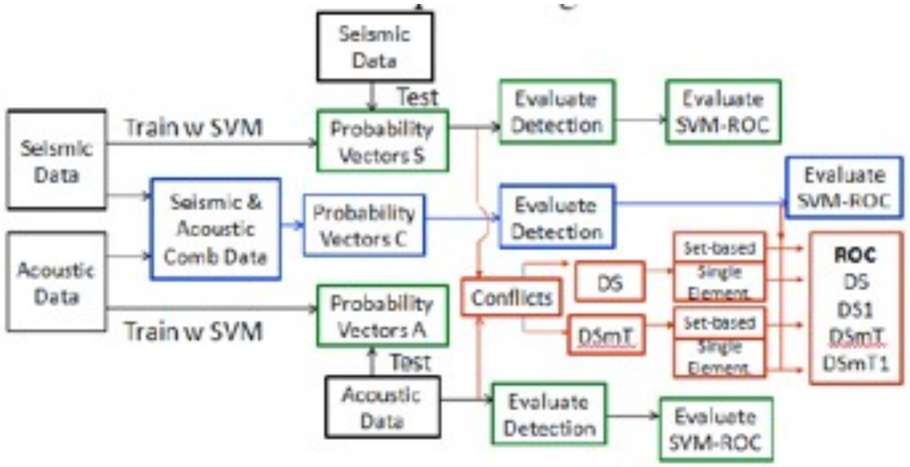
DSmT for sensor fusion

Decision Level Multiple Cameras Fusion Using Dezert-Smarandache Theory, [Garcia, Altamirano 2009]





DSmT Applied to Seismic and Acoustic Sensor Fusion [Blasch,Dezert,Valin 2011]



Seismic Data
Test
Evaluate Detection
Evaluate SVM-ROC
Seismic Data
Train w SVM
Probability Vectors S
Evaluate SVM-ROC
Seismic & Acoustic Comb Data
Probability Vectors C
Evaluate Detection
Acoustic Data
Set-based Single Element
ROC
DS
DS1
DSmT
DSmT1
Conflicts
DS
DSmT
Set-based Single Element
Probability Vectors A
Train w SVM
Test
Acoustic Data
Evaluate Detection
Evaluate SVM-ROC

Applications of DSmT

DSmT for image processing

Edge Detection in Color Images Based on DSmT [Dezert, Liu, Mercier 2011]

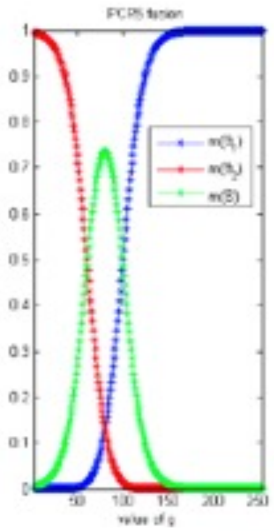
We use RGB channels of color image, and in each pixel of a layer we compute its BBA to belong (or not) to an edge thank to gradient values. $\Theta = \{\theta_1 \triangleq \text{Pixel} \in \text{Edge}, \theta_2 \triangleq \text{Pixel} \notin \text{Edge}\}$

We use sigmoidal modeling with chosen [te,tn] detection threshold uncertainty.

$$f_{\lambda,t}(g) \triangleq \frac{1}{1+e^{-\lambda(g-t)}}$$

focal element	$m_1(.)$	$m_2(.)$
θ_1	$f_{\lambda,t_e}(g)$	0
θ_2	0	$f_{-\lambda,t_n}(g)$
$\theta_1 \cup \theta_2$	$1 - f_{\lambda,t_e}(g)$	$1 - f_{-\lambda,t_n}(g)$

PCR5



We use PCR5 to combine the 3 BBA altogether.

We use max of DSmP to make final decision.


(a) Original Lena image


(b) Lena with noise

PCR5 Edge detector

Applications of DSmT

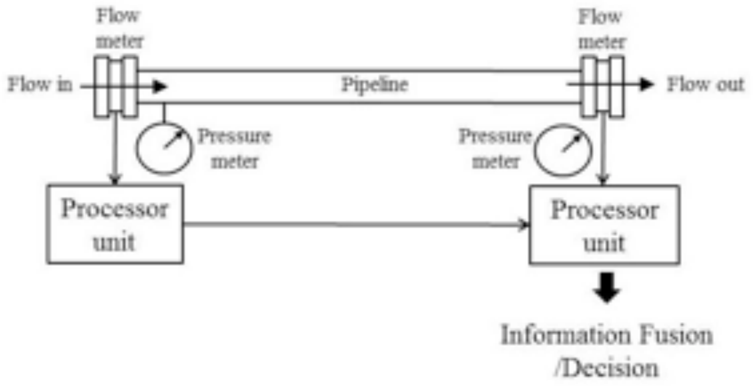
DSmT for failure detection

System and method for combining diagnostic evidences for turbine engine fault detection [US Patent 7337086, Honeywell Int. Inc., Feb, 2008]

One Fusion Approach of Fault Diagnosis Based on Rough Set Theory and Dezert-Smarandache Theory [Su et al. 2012]

Contextual reliability discounting in welding process diagnostic based on DSmT [Jamrozik 2014]

Developing a monitoring system for long-distance pipeline leakage incorporating fusion of conflicting evidences [Adair et al. 2015]

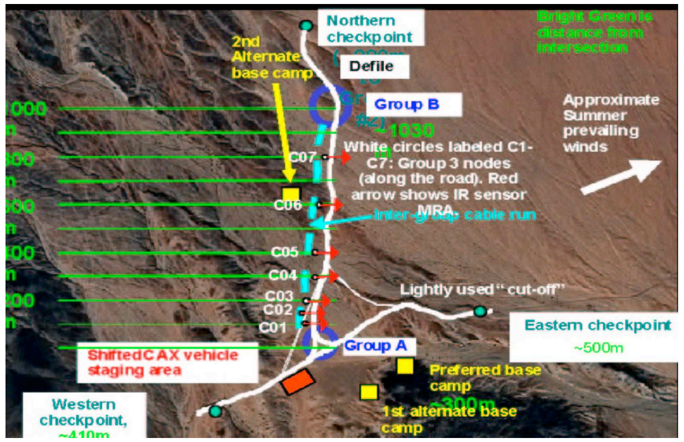


Applications of DSmT

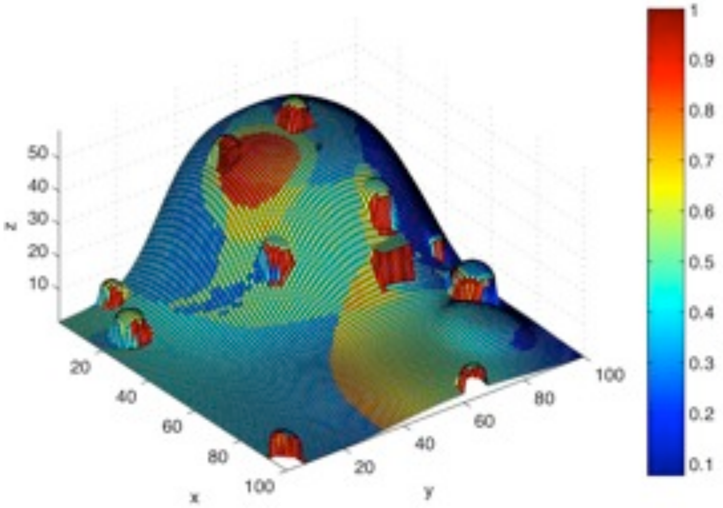
DSmT for resource/sensor management

Power and resource aware distributed smart fusion [Kadambe 2004]

Optimization of disparate DSN architecture to minimize power consumption and optimize target detection and classification.

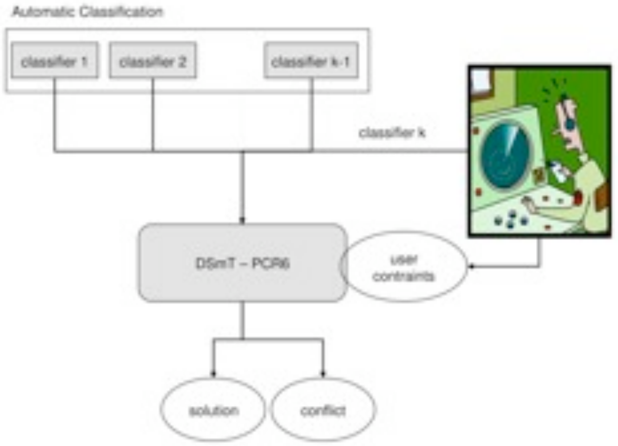


Map regenerating forest stands based on DST and DSmT combination rules [Mora,Fournier,Foucher 2009]



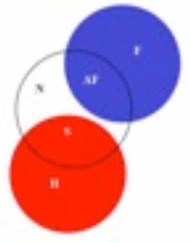
Automatic goal allocation for a planetary rover with DSmT [Vasile,Ceriotti 2009]

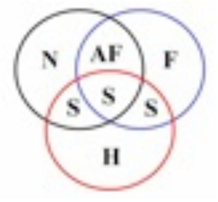
Utilizing classifier conflict for sensor management and user interaction [Van Norden, Jonker 2009]

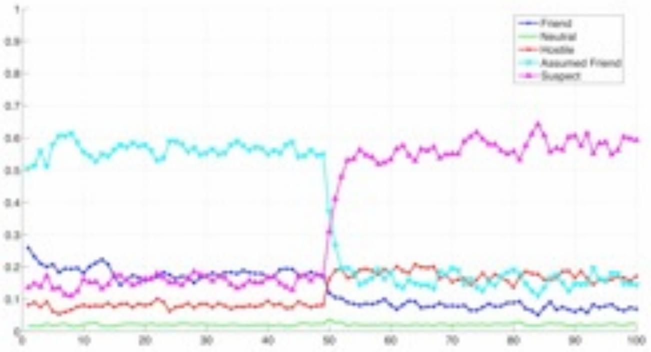


Applications of DSmT

Situation analysis and threat assessment

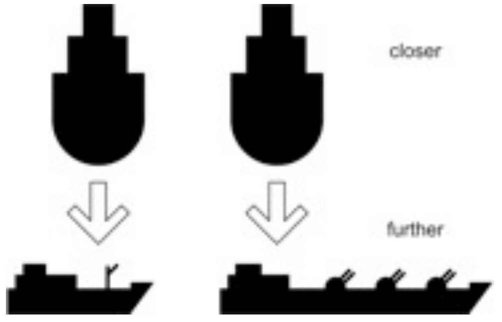


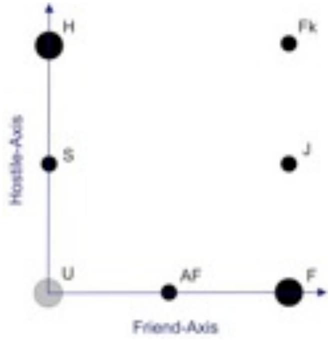




Fusion of ESM allegiance reports using DSmT [Djiknavorian,Valin, Grenier 2009]

Attribute information evaluation in C&C systems, [Krenc & Kawalec 2009]





Processing of information in C2 systems [Krenc 2010]

Maritime surveillance and threat assessment [Van Norden 2010]

Threat assessment of a possible Vehicle-Born Improvised Explosive Device using DSmT [Dezert,Smarandache 2010]

Intelligent Alarm Classification Based on DSmT [Tchamova,Dezert 2012]

Application of New Absolute and Relative Conditioning Rules in Threat Assessment [Krenc, Smarandache 2013]

Thank you for your attention.

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return on innovation

Some related references

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Dezert J., Tchamova A., Han D., Tacnet J.-M., Why Dempster's fusion rule is not a generalization of Bayes fusion rule, Proc. of Fusion 2013 Int. Conference on Information Fusion, Istanbul, Turkey, July 9-12, 2013.

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Much more at http://www.onera.fr/staff/jean-dezert?page=2

Short biography

Jean Dezert, The French Aerospace Lab (ONERA), Information Processing and Modeling Department
BP 80100, Chemin de la Hunière, FR-91123 Palaiseau Cedex, France.

Email: Jean.dezert@onera.fr

http://www.onera.fr/staff/jean-dezert

Jean Dezert was born in l'Hay les Roses, France, on August 25, 1962. He received the Electrical Engineering (EE) degree in 1985, and his Ph.D. from the University Paris XI, Orsay, in 1990 in Automatic Control and Signal Processing. During 1986-1990 he was with the Syst. Dept. at the French Aerospace Lab (ONERA) and did research in multi-sensor multi-target tracking (MS-MTT). During 1991-1992, he visited the Dept. of ESE, UConn, Storrs, USA as an ESA Postdoctoral Research Fellow under supervision of Prof. Bar-Shalom. During 1992-1993 he was teaching assistant in EE Dept, University of Orléans, France. Since 1993, he is Senior Research Scientist in the Information Processing and Modeling Department at the French Aerospace Lab. His current research interests include estimation theory, and information fusion (IF) and plausible reasoning and multi-criteria decision-making support with applications to MS-MTT, defense and security, robotics and risk assessment. Jean Dezert has been involved within International Society of Information Fusion (ISIF – www.isif.org) since its beginning and has been the Local Arrangements Co-Organizer of the first Fusion Conference in Europe in 2000. He is currently member of Executive board of ISIF (Vice-president 2004, President 2016). He has been involved in the Technical Program Committees of Fusion 2001-2015 Conf., and in several sessions and panel discussions on reasoning under uncertainty and data fusion. Jean Dezert is the co-founder with Prof. Smarandache of DSmT (Dezert-Smarandache Theory) of information fusion based on belief functions. Jean Dezert has published more than 150 papers in conferences and journals on tracking and information fusion and he has co-edited four books (collected works) in english (the first volume has been translated in chinese) devoted to DSmT. More than twenty theses related with DSmT and its applications have been defended so far in Europe, China, USA and Canada. Jean Dezert has given tutorials, seminars and workshops in the information fusion and target tracking fields in North America, Europe, Australia and China. Jean Dezert is reviewer for several international journals and Associate Editor of ISIF Journal of Advances in Information Fusion.