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Medical Diagnosis Using Distance-Based Similarity Measures of Single Valued Neutrosophic Multisets

Abstract.

This presentation proposes a generalized distance measure and its similarity measures between single valued neutrosophic multisets (SVNMs).

Then, the similarity measures are applied to a medical diagnosis problem with incomplete, indeterminate and inconsistent information.

This diagnosis method can deal with the diagnosis problem with indeterminate and inconsistent information which cannot be handled by the diagnosis method based on intuitionistic fuzzy multisets (IFMs).

1. Introduction.

The vagueness or uncertainty representation of imperfect knowledge becomes a crucial issue in the areas of computer science and artificial intelligence.

To deal with the uncertainty, the fuzzy set proposed by Zadeh [1] allows the uncertainty of a set with a membership degree between 0 and 1.

[1] L. A. Zadeh. Fuzzy Sets. Information and Control, 8 (1965), 338-353.

Then, Atanassov [2] introduced an intuitionistic Fuzzy set (IFS) as a generalization of the Fuzzy set.

[2] K. Atanassov. Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20 (1986), 87-96.

1. Introduction – cont.

The IFS represents the uncertainty with respect to both membership and non-membership.

However, it can only handle incomplete information but not the indeterminate and inconsistent information which exists commonly in real situations.

Therefore, Smarandache [3] proposed a neutrosophic set.

It can independently express truth-membership degree, indeterminacy-membership degree, and false-membership degree and deal with incomplete, indeterminate, and inconsistent information.

[3] F. Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

1. Introduction – cont.

After that, Wang et al [4] introduced a single valued neutrosophic set (SVNS), which is a subclass of the neutrosophic set. S

VNS is a generalization of the concepts of the classic set, fuzzy set, and IFS.

The SVNS should be used for better representation as it is a more natural and justified estimation [4].

[4] H. Wang, F. Y. Smarandache, Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4 (2010), 410-413.

1. Introduction – cont.

All the factors described by the SVNS are very suitable for human thinking due to the imperfection of knowledge that human receives or observes from the external world.

For example, for a given proposition “Movie X would be hit”, in this situation human brain certainly cannot generate precise answers in terms of yes or no, as indeterminacy is the sector of unawareness of a proposition’s value between truth and falsehood.

Obviously, the neutrosophic components are best fit in the representation of indeterminacy and inconsistent information.

1. Introduction – cont.

Recently, Ye [5-7] proposed some similarity measures of SVNSs and applied them to decision making and clustering analysis.

[5] J. Ye and Q. S. Zhang. Single valued neutrosophic similarity measures for multiple attribute decision making. Neutrosophic Sets and Systems, 2 (2014), 48-54.

[6] J. Ye. Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment. Journal of Intelligent and Fuzzy Systems, (2014).

[7] J. Ye. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. Journal of Intelligent Systems, (2014).

Based on multiset theory, Yager [8] introduced a fuzzy multiset concept, which allows the repeated occurrences of any element.

[8] R. R. Yager. On the theory of bags (Multi sets). International Journal of General System, 13 (1986), 23-37.

1. Introduction – cont.

Thus, the fuzzy multiset can occur more than once with the possibility of the same or different membership values.

Then, Shinoj and Sunil [9] extended the fuzzy multiset to the intuitionistic fuzzy multiset (IFM) and presented some basic operations and a distance measure for IFMs, and then applied the distance measure to medical diagnosis.

[9] T. K. Shinoj and J. J. Sunil. Intuitionistic fuzzy multi sets and its application in medical diagnosis. World Academy of Science, Engineering and Technology, 6(1) (2012), 1418-1421.

Rajarajeswari and Uma [10] put forward the Hamming distance-based similarity measure for IFMs and its application in medical diagnosis.

[10] P. Rajarajeswari and N. Uma. Normalized Hamming Similarity Measure for Intuitionistic Fuzzy Multi Sets and Its Application in Medical diagnosis. Intl. Journal of Mathematics Trends and Technology, 5(3) (2014), 219-225.

1. Introduction – cont.

Recently, Ye et al. [11] presented a single valued neutrosophic multiset (SVNM) as a generalization of IFM and the Dice similarity measure between SVNMs, and then applied it to medical diagnosis.

[11] S. Ye and J. Ye. Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. Neutrosophic Sets and Systems, 2014.

Based on SVNMs, this paper further develops a generalized distance measure and the distance-based similarity measures between SVNMs, and then applies the similarity measures to medical diagnosis.

2. Preliminaries.

2.1 Some concepts of SVNSs

Smarandache [3] originally presented the concept of a neutrosophic set.

[3] F. Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

A neutrosophic set A in a universal set X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$.

The functions $T_A(x)$, $I_A(x)$, $F_A(x)$ in X are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$, and $F_A(x): X \to]^-0, 1^+[$.

Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ is no restriction, i.e. $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2. Preliminaries – cont.

2.1 Some concepts of SVNSs

However, Smarandache [3] introduced the neutrosophic set from philosophical point of view.

[3] F. Smarandache. A unifying field in logics. neutrosophy: Neutrosophic probability, set and logic. Rehoboth: American Research Press, 1999.

Therefore, it is difficult to apply the neutrosophic set to practical problems.

To easily apply in science and engineering areas, Wang et al. [4] introduced the concept of SVNs, which is a subclass of the neutrosophic set and gave the following definition.

[4] H. Wang, F. Y. Smarandache, Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4 (2010), 410-413.

2. Preliminaries – cont.

2.1 Some concepts of SVNSs

Definition 1 [4]. Let X be a universal set. A SVNs A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by the following form:

$$\Big\langle \quad {}_{A} \quad {}_{A} \quad {}_{A} \quad \Big\rangle \qquad ,$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For two SVNs $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, T_B(x), I_B(x), F_B(x)\rangle \mid x \in X\}$, there are the following relations [4]:

2. Preliminaries – cont.

2.1 Some concepts of SVNSs

(1) Complement:

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle \mid x \in X\};$$

(2) Inclusion:

$A \subseteq B$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X;

(3) Equality:

$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(4) Union:

$$A \cup B = \{\langle x, T_A(x) \vee T_B(x), I_A(x) \wedge I_B(x), F_A(x) \wedge F_B(x)\rangle \mid x \in X\};$$

2. Preliminaries – cont.

2.1 Some concepts of SVNSs

(5) Intersection:

$$A \cap B = \left\{ \left\langle x, T_A(x) \wedge T_B(x), I_A(x) \vee I_B(x), F_A(x) \vee F_B(x) \right\rangle \mid x \in X \right\};$$

(6) Addition:

$$A + B = \left\{ \left\langle \begin{matrix} x, T_A(x) + T_B(x) - T_A(x)T_B(x), \\ I_A(x)I_B(x), F_A(x)F_B(x) \end{matrix} \right\rangle \mid x \in X \right\};$$

(7) Multiplication:

$$A \times B = \left\{ \left\langle \begin{matrix} x, T_A(x)T_B(x), I_A(x) + I_B(x) - I_A(x)I_B(x), \\ F_A(x) + F_B(x) - F_A(x)F_B(x) \end{matrix} \right\rangle \mid x \in X \right\}.$$

2. Preliminaries.

2.2 Some concepts of SVNMs

As a generalization of the concept of IFM, a concept of SVNM and some basic operational relations for SVNMs [11] are introduced below.

[11] S. Ye and J. Ye. Dice similarity measure between single valued neutrosophic multisets and its application in medical diagnosis. Neutrosophic Sets and Systems, 2014.

Definition 2 [11]. Let X be a nonempty set with generic elements in X denoted by x. A single valued neutrosophic multiset (SVNM) A drawn from X is characterized by three functions: count truth-membership of CT_A, count indeterminacy-membership of CI_A, and count falsity-membership of CF_A such that $CT_A(x): X \to Q$, $CI_A(x): X \to Q$, $CF_A(x): X \to Q$ for $x \in X$, where Q is the set of all real number multisets in the real unit interval [0, 1]. Then, a SVNM A is denoted by

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

$$A = \left\{ \left\langle \begin{matrix} x, (T_A^1(x), T_A^2(x), \ldots, T_A^q(x)), \\ (I_A^1(x), I_A^2(x), \ldots, I_A^q(x)), \\ (F_A^1(x), F_A^2(x), F_A^q(x)) \end{matrix} \right\rangle \mid x \in X \right\},$$

where the truth-membership sequence $(T_A^1(x), T_A^2(x), \ldots, T_A^q(x))$, the indeterminacy-membership sequence $(I_A^1(x), I_A^2(x), \ldots, I_A^q(x))$, and the falsity-membership sequence $(F_A^1(x), F_A^2(x), \ldots, F_A^q(x))$ may be in decreasing or increasing order, and the sum of $T_A^i(x)$, $I_A^i(x)$, $F_A^i(x)$ $\in [0, 1]$ satisfies the condition $0 \le T_A^i(x) + I_A^i(x) + F_A^i(x) \le 3$ for $x \in X$ and $i = 1, 2, \ldots, q$.

For convenience, a SVNM A can be denoted by the following simplified form:

$$A = \left\{ \left\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \right\rangle \mid x \in X, i = 1,2,\ldots,q \right\}.$$

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

Definition 3 [11]. The length of an element x in a SVNM is defined as the cardinality of $CT_A(x)$ or $CI_A(x)$, or $CF_A(x)$ and is denoted by $L(x\colon A)$. Then $L(x\colon A) = |CT_A(x)| = |CI_A(x)| = |CF_A(x)|$.

Definition 4 [11]. Let A and B be two SVNMs in X, then the length of an element x in A and B is denoted by $l_x = L(x\colon A, B) = \max\{L(x\colon A), L(x\colon B)\}$.

Example 1. Consider two SVNMs in the set $X = \{x, y, z\}$:

$A = \{<x, (0.3, 0.2), (0.4, 0.3), (0.6, 0.8)>, <y, (0.5, 0.4, 0.3), (0.1, 0.2, 0.3), (0.3, 0.4, 0.5)>\}$,

$B = \{<x, (0.3), (0.4), (0.6)>, <z, (0.5, 0.4, 0.3, 0.2), (0.0, 0.1, 0.2, 0.3), (0.2, 0.3, 0.4, 0.5)>\}$.

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

Thus, there are $L(x: A) = 2$, $L(y: A) = 3$, $L(z: A) = 0$; $L(x: B) = 1$, $L(y: B) = 0$, $L(z: B) = 4$, $l_x = L(x: A, B) = 2$, $l_y = L(y: A, B) = 3$, and $l_z = L(z: A, B) = 4$.

For convenient operation between SVNMs A and B in X, one can make $L(x: A) = L(x: B)$ by appending sufficient minimal number for the truth-membership value and sufficient maximum number for the indeterminacy-membership and falsity-membership values.

Definition 5 [11]. Let $A = \{\langle x, T_A^i(x), I_A^i(x), F_A^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x, T_B^i(x), I_B^i(x), F_B^i(x) \mid x \in X, i = 1, 2, \ldots, q\}$ be any two SVNMs in X. Then, there are the following relations:

(1) Inclusion: $A \subseteq B$ if and only if $T_A^i(x) \le T_B^i(x)$, $I_A^i(x) \ge I_B^i(x)$, $F_A^i(x) \ge F_B^i(x)$ for $i = 1, 2, \ldots, q$ and $x \in X$;

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

(2) Equality: $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;

(3) Complement:

$$A^c = \left\{ \left\langle x, F_A^i(x), \left(1 - I_A(x)\right), T_A^i(x) \right\rangle \mid x \in X, i = 1,2,\ldots,q \right\};$$

(4) Union:

$$A \cup B = \left\{ \left\langle \begin{matrix} x, T_A^i(x) \vee T_B^i(x), \\ I_A^i(x) \wedge I_B^i(x), \\ F_A^i(x) \wedge F_B^i(x) \end{matrix} \right\rangle \mid x \in X, i = 1,2,\ldots,q \right\};$$

(5) Intersection:

$$A \cap B = \left\{ \left\langle \begin{matrix} x, T_A^i(x) \wedge T_B^i(x), \\ I_A^i(x) \vee I_B^i(x), \\ F_A^i(x) \vee F_B^i(x) \end{matrix} \right\rangle \mid x \in X, i = 1,2,\ldots,q \right\}.$$

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

For convenience, we can use $a = \langle (T^1, T^2, \ldots, T^q), (I^1, I^2, \ldots, I^q), (F^1, F^2, \ldots, F^q) \rangle$ to represent an element in a SVNM A and call it a single valued neutrosophic multiset value (SVNMV).

Definition 6. Let $a_1 = \langle (T_1^1, T_1^2, \ldots, T_1^q), (I_1^1, I_1^2, \ldots, I_1^q), (F_1^1, F_1^2, \ldots, F_1^q) \rangle$ and $a_2 = \langle (T_2^1, T_2^2, \ldots, T_2^q), (I_2^1, I_2^2, \ldots, I_2^q), (F_2^1, F_2^2, \ldots, F_2^q) \rangle$ be two SVNMVs and $\lambda \geq 0$, then the operational rules of SVNMVs are defined as follows:

$$(1)\ a_1 \oplus a_2 = \left\langle \begin{array}{l} (T_1^1 + T_2^1 - T_1^1 T_2^1, \\ T_1^2 + T_2^2 - T_1^2 T_2^2, \\ \ldots, T_1^q + T_2^q - T_1^q T_2^q), \\ (I_1^1 I_2^1, I_1^2 I_2^2, \ldots, I_1^q I_2^q), \\ (F_1^1 F_2^1, F_1^2 F_2^2, \ldots, F_1^q F_2^q) \end{array} \right\rangle;$$

2. Preliminaries – cont.

2.2 Some concepts of SVNMs

$$(2)\ a_1 \otimes a_2 = \left\langle \begin{matrix} (T_1^1 T_2^1, T_1^2 T_2^2, \ldots, T_1^q T_2^q), \\ (I_1^1 + I_2^1 - I_1^1 I_2^1, I_1^2 + I_2^2 - I_1^2 I_2^2, \\ \ldots, I_1^q + I_2^q - I_1^q I_2^q), \\ (F_1^1 + F_2^1 - F_1^1 F_2^1, F_1^2 + F_2^2 - F_1^2 F_2^2, \\ \ldots, F_1^q + F_2^q - F_1^q F_2^q) \end{matrix} \right\rangle;$$

$$(3)\ \lambda a_1 = \left\langle \begin{matrix} \left(1-(1-T_1^1)^\lambda, 1-(1-T_1^2)^\lambda, \ldots, 1-(1-T_1^q)^\lambda\right), \\ \left((I_i^1)^\lambda, (I_i^2)^\lambda, \ldots, (I_i^q)^\lambda\right), \left((F_i^1)^\lambda, (F_i^2)^\lambda, \ldots, (F_i^q)^\lambda\right) \end{matrix} \right\rangle;$$

$$(4)\ a_1^\lambda = \left\langle \begin{matrix} \left((T_i^1)^\lambda, (T_i^2)^\lambda, \ldots, (T_i^q)^\lambda\right), \\ \left(1-(1-I_1^1)^\lambda, 1-(1-I_1^2)^\lambda, \ldots, 1-(1-I_1^q)^\lambda\right), \\ \left(1-(1-F_1^1)^\lambda, 1-(1-F_1^2)^\lambda, \ldots, 1-(1-F_1^q)^\lambda\right) \end{matrix} \right\rangle.$$

3. Distance and similarity measures of SVNMs.

The distance measure and similarity measure are usually used in real science and engineering applications.

Therefore, the section proposes a generalized distance measure between SVNMs and the distance-based similarity measures between SVNMs.

However, the distance and similarity measures in SVNSs are considered for truth-membership, indeterminacy-membership, and falsity-membership functions only once, while the distance and similarity measures in SVNMs should be considered more than once because their functions are multi-values.

3. Distance and similarity measures of SVNMs – cont.

Definition 7. Let $A = \{\langle x_j, T_A^i(x_j), I_A^i(x_j), F_A^i(x_j)\rangle \mid x_j \in X, i = 1, 2, \ldots, q\}$ and $B = \{\langle x_j, T_B^i(x_j), I_B^i(x_j), F_B^i(x_j)\rangle \mid x_j \in X, i = 1, 2, \ldots, q\}$ be any two SVNMs in $X = \{x_1, x_2, \ldots, x_n\}$. Then, we define the following generalized distance measure between A and B:

$$D_p(A,B) = \left[\frac{1}{n} \sum_{j=1}^{n} \frac{1}{3l_j} \sum_{i=1}^{l_j} \begin{pmatrix} \left|T_A^i(x_j) - T_B^i(x_j)\right|^p + \\ \left|I_A^i(x_j) - I_B^i(x_j)\right|^p + \\ \left|F_A^i(x_j) - F_A^i(x_j)\right|^p \end{pmatrix} \right]^{1/p}, \quad (1)$$

where $l_j = L(x_j: A, B) = \max\{L(x_j: A), L(x_j: B)\}$ for $j = 1, 2, \ldots, n$. If $p = 1, 2$, Eq. (1) reduces to the Hamming distance and the Euclidean distance, which are usually applied to real science and engineering areas.

Then, the defined distance measure has the following Proposition 1:

3. Distance and similarity measures of SVNMs – cont.

Proposition 1. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the generalized distance measure $D_p(A, B)$ should satisfy the following properties (D1-D4):

(D1) $0 \le D_p(A, B) \le 1$;

(D2) $D_p(A, B) = 0$ if and only if $A = B$;

(D3) $D_p(A, B) = D_p(B, A)$;

(D4) If C is a SVNM in X and $A \subseteq B \subseteq C$, then $D_p(A, C) \le D_p(A, B) + D_p(B, C)$ for $p > 0$.

Proofs:

(D1) Proof is straightforward.

3. Distance and similarity measures of SVNMs – cont.

(D2) If $A = B$, then there are $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for $i = 1, 2, \ldots, l_j$, $j = 1, 2, \ldots, n$, and $x_j \in X$. Hence $\left|T_A^i(x_j) - T_B^i(x_j)\right|^p = 0$, $\left|I_A^i(x_j) - I_B^i(x_j)\right|^p = 0$, and $\left|F_A^i(x_j) - F_B^i(x_j)\right|^p = 0$. Thus $D_p(A, B) = 0$. When $D_p(A, B) = 0$, there are $\left|T_A^i(x_j) - T_B^i(x_j)\right|^p = 0$, $\left|I_A^i(x_j) - I_B^i(x_j)\right|^p = 0$, and $\left|F_A^i(x_j) - F_B^i(x_j)\right|^p = 0$. Then, one can obtain $T_A^i(x_j) = T_B^i(x_j)$, $I_A^i(x_j) = I_B^i(x_j)$, $F_A^i(x_j) = F_B^i(x_j)$ for $i = 1, 2, \ldots, l_j$, $j = 1, 2, \ldots, n$, and $x_j \in X$. Hence $A = B$.

(D3) Proof is straightforward.

3. Distance and similarity measures of SVNMs – cont.

(D4) Since $T_A^i(x_j) - T_c^i(x_j) = T_A^i(x_j) - T_B^i(x_j) + T_B^i(x_j) - T_c^i(x_j)$, It is obvious that

$$\left|T_A^i(x_j) - T_c^i(x_j)\right| \le \left|T_A^i(x_j) - T_B^i(x_j)\right| + \left|T_B^i(x_j) - T_C^i(x_j)\right|,$$
$$\left|I_A^i(x_j) - I_c^i(x_j)\right| \le \left|I_A^i(x_j) - I_B^i(x_j)\right| + \left|I_B^i(x_j) - I_C^i(x_j)\right|,$$
$$\left|F_A^i(x_j) - F_c^i(x_j)\right| \le \left|F_A^i(x_j) - F_B^i(x_j)\right| + \left|F_B^i(x_j) - F_C^i(x_j)\right|.$$

For $p > 0$, we have

$$\left|T_A^i(x_j) - T_c^i(x_j)\right|^p \le \left|T_A^i(x_j) - T_B^i(x_j)\right|^p + \left|T_B^i(x_j) - T_C^i(x_j)\right|^p,$$
$$\left|I_A^i(x_j) - I_c^i(x_j)\right|^p \le \left|I_A^i(x_j) - I_B^i(x_j)\right|^p + \left|I_B^i(x_j) - I_C^i(x_j)\right|^p,$$
$$\left|F_A^i(x_j) - F_c^i(x_j)\right|^p \le \left|F_A^i(x_j) - F_B^i(x_j)\right|^p + \left|F_B^i(x_j) - F_C^i(x_j)\right|^p.$$

Considering the above inequalities and Eq. (1), one can obtain that $D_p(A, C) \le D_p(A, B) + D_p(B, C)$ for $p > 0$.

Therefore, the proofs of these properties are completed.

3. Distance and similarity measures of SVNMs – cont.

Based on the relationship between the distance measure and the similarity measure, we can introduce two distance-based similarity measures between *A* and *B*:

$$S_1(A,B) = 1 - D_p(A,B)$$

$$= 1 - \left[\frac{1}{n} \sum_{j=1}^{n} \frac{1}{3l_j} \sum_{i=1}^{l_j} \begin{pmatrix} \left| T_A^i(x_j) - T_B^i(x_j) \right|^p + \\ \left| I_A^i(x_j) - I_B^i(x_j) \right|^p + \\ \left| F_A^i(x_j) - F_A^i(x_j) \right|^p \end{pmatrix} \right]^{1/p}, \quad (2)$$

3. Distance and similarity measures of SVNMs – cont.

$$S_2(A,B) = \frac{1 - D_p(A,B)}{1 + D_p(A,B)}$$

$$= \frac{1 - \left[\frac{1}{n} \sum_{i=1}^{n} \frac{1}{3l_j} \sum_{j=1}^{l_j} \begin{pmatrix} \left| T_A^j(x_i) - T_B^j(x_i) \right|^p + \\ \left| I_A^j(x_i) - I_B^j(x_i) \right|^p + \\ \left| F_A^j(x_i) - F_A^j(x_i) \right|^p \end{pmatrix} \right]^{1/p}}{1 + \left[\frac{1}{n} \sum_{i=1}^{n} \frac{1}{3l_j} \sum_{j=1}^{l_j} \begin{pmatrix} \left| T_A^j(x_i) - T_B^j(x_i) \right|^p + \\ \left| I_A^j(x_i) - I_B^j(x_i) \right|^p + \\ \left| F_A^j(x_i) - F_A^j(x_i) \right|^p \end{pmatrix} \right]^{1/p}}. \quad (3)$$

Following *Proposition 1* for the defined distance measure and the relationship between the distance measure and the similarity measure, it is easy to obtain *Proposition 2* for the distance-based similarity measures.

3. Distance and similarity measures of SVNMs – cont.

Proposition 2. For two SVNMs A and B in $X = \{x_1, x_2, \ldots, x_n\}$, the distance-based similarity measure $S_k(A, B)$ (k =1, 2) should satisfy the following properties (S1-S4):

(S1) $0 \leq S_k(A, B) \leq 1$;

(S2) $S_k(A, B) = 1$ if and only if $A = B$;

(S3) $S_k(A, B) = S_k(B, A)$;

(S4) If C is a SVNM in X and $A \subseteq B \subseteq C$, , then $S_k(A, C) \leq S_k(A, B)$ and $S_k(A, C) \leq S_k(B, C)$.

By the similar proofs of *Proposition 1* and the relationship between the distance and the similarity measure, *Proofs* are straightforward.

3. Distance and similarity measures of SVNMs – cont.

Example 2: Let A and B be two SVNMs in $X = \{x_1, x_2\}$, which are given as follows:

$A = \{<x_1, (0.7, 0.8), (0.1, 0.2), (0.2, 0.3)>, <x_2, (0.5, 0.6), (0.2, 0.3), (0.4, 0.5)>\}$,

$B = \{<x_1, (0.5, 0.6), (0.1, 0.2), (0.4, 0.5)>, <x_2, (0.6, 0.7), (0.1, 0.2), (0.7, 0.8)>\}$.

The calculation process of the similarity measures between A and B is shown as follows:

(1) Using Hamming distance ($p = 1$):
By using Eq. (1) we obtain:

$D_1(A, B) = [(|0.7 - 0.5|+|0.1 - 0.1| + |0.2 - 0.4| + |0.8 - 0.6| + |0.2 - 0.2| + |0.3 - 0.5|)/6 + (|0.5 - 0.6| + |0.2 - 0.1| +|0.4 - 0.7| + |0.6 - 0.7| + |0.3 - 0.2| + |0.5 - 0.8|)/6]/2 = 0.15.$

3. Distance and similarity measures of SVNMs – cont.

Then, by applying *Eqs*. (2) and (3) we have the following result:

$S_1(A, B) = 1 - D_1(A, B) = 1 - 0.15 = 0.85$ and $S_2(A, B) = [1 - D_1(A, B)]/[1 + D_1(A, B)] = 0.7391$.

(2) Using the Euclidean distance ($p = 2$):

By using Eq. (1) we can obtain the following result:

$D_2(A, B) = \{[(|0.7 - 0.5|^2 + |0.1 - 0.1|^2 + |0.2 - 0.4|^2 + |0.8 - 0.6|^2 + |0.2 - 0.2|^2 + |0.3 - 0.5|^2)/6 + (|0.5 - 0.6|^2 + |0.2 - 0.1|^2 + |0.4 - 0.7|^2 + |0.6 - 0.7|^2 + |0.3 - 0.2|^2 + |0.5 - 0.8|^2)/6]/2\}^{1/2} = 0.178$.

Then, by applying *Eqs*. (2) and (3) we have the following result:

$S_1(A, B) = 1 - D_2(A, B) = 1 - 0.178 = 0.822$ and $S_2(A, B) = [1 - D_2(A, B)]/[1 + D_2(A, B)] = 0.6979$.

4. Medical diagnosis using the similarity measure.

Due to more and more complexity of real medical diagnosis, a lot of information available to physicians from modern medical technologies is often incomplete, indeterminate and inconsistent information.

Then, the SVNS proposed by Wang et al. [4] can be better to express this kind of information, but fuzzy sets and intuitionistic fuzzy sets cannot handle indeterminate and inconsistent information.

[4] H. Wang, F. Y. Smarandache, Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4 (2010), 410-413.

However, by only taking one time inspection, we wonder whether we can obtain a conclusion from a particular person with a particular decease or not.

4. Medical diagnosis using the similarity measure – cont.

Sometimes he/she may also show the symptoms of different diseases.

Then, how can we give a proper conclusion?

One solution is to examine the patient at different time intervals (e.g. two or three times a day).

Thus, authors present SVNMs as a better tool for reasoning such a situation.

The details of a typical example (adapted from [9]) are given below.

[9] T. K. Shinoj and J. J. Sunil. Intuitionistic fuzzy multi sets and its application in medical diagnosis. World Academy of Science, Engineering and Technology, 6(1) (2012), 1418-1421.

4. Medical diagnosis using the similarity measure – cont.

Let $P = \{P_1, P_2, P_3, P_4\}$ be a set of four patients, $D = \{D_1, D_2, D_3, D_4\}$ = {Viral fever, Tuberculosis, Typhoid, Throat disease} be a set of diseases, and $S = \{S_1, S_2, S_3, S_4, S_5\}$ = {Temperature, Cough, Throat pain, Headache, Body pain} be a set of symptoms.

Table 1 shows the characteristics between symptoms and the considered diseases represented by the form of single valued neutrosophic values (SVNVs).

In the medical diagnosis, if we have to take three different samples in three different times in a day (e.g., morning, noon and night), we can construct *Table 2*, in which the characteristics between patients and the indicated symptoms are represented by SVNMVs.

4. Medical diagnosis using the similarity measure – cont.

Table 1 Characteristics between symptoms and the considered diseases represented by SVNVs

	Temperature (S_1)	Cough (S_2)	Throat pain (S_3)	Headache (S_4)	Body pain (S_5)
Viral fever (D_1)	<0.8, 0.1, 0.1>	<0.2, 0.7, 0.1>	<0.3, 0.5, 0.2>	(0.5, 0.3, 0.2)	<0.5, 0.4, 0.1>
Tuberculosis (D_2)	<0.2, 0.7, 0.1>	<0.9, 0.0, 0.1>	<0.7, 0.2, 0.1>	(0.6, 0.3, 0.1)	<0.7, 0.2, 0.1>
Typhoid (D_3)	<0.5, 0.3, 0.2>	<0.3, 0.5, 0.2>	<0.2, 0.7, 0.1>	(0.2, 0.6, 0.2)	<0.4, 0.4, 0.2>
Throat disease(D_4)	<0.1, 0.7, 0.2>	<0.3, 0.6, 0.1>	<0.8, 0.1, 0.1>	(0.1, 0.8, 0.1)	<0.1, 0.8, 0.1>

Table 2 Characteristics between patients and the indicated symptoms represented by SVNMVs

	Temperature (S_1)	Cough (S_2)	Throat pain (S_3)	Headache (S_4)	Body pain (S_5)
P_1	<(0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1)>	<(0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.4)>	<(0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7)>	<(0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2)>	<(0.4, 0.3, 0.2), (0.6, 0.5, 0.5), (0.6, 0.4, 0.4)>
P_2	<(0.5, 0.4, 0.3), (0.3, 0.3, 0.2), (0.5, 0.4, 0.4)>	<(0.9, 0.8, 0.7), (0.2, 0.1, 0.1), (0.2, 0.2, 0.1)>	<(0.6, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.3, 0.3)>	<(0.6, 0.4, 0.3), (0.3, 0.1, 0.1), (0.7, 0.7, 0.3)>	<(0.8, 0.7, 0.5), (0.4, 0.3, 0.1), (0.3, 0.2, 0.1)>
P_3	<(0.2, 0.1, 0.1), (0.3, 0.2, 0.2), (0.8, 0.7, 0.6)>	<(0.3, 0.2, 0.2), (0.4, 0.2, 0.2), (0.7, 0.6, 0.5)>	<(0.8, 0.8, 0.7), (0.2, 0.2, 0.2), (0.1, 0.1, 0.0)>	<(0.3, 0.2, 0.2), (0.3, 0.3, 0.3), (0.7, 0.6, 0.6)	<(0.4, 0.4, 0.3), (0.4, 0.3, 0.2), (0.7, 0.7, 0.5)>
P_4	<(0.5, 0.5, 0.4), (0.3, 0.2, 0.2), (0.4, 0.4, 0.3)>	<(0.4, 0.3, 0.1), (0.4, 0.3, 0.2), (0.7, 0.5, 0.3)>	<(0.2, 0.1, 0.0), (0.4, 0.3, 0.3), (0.7, 0.7, 0.6)>	<(0.6, 0.5, 0.3), (0.2, 0.2, 0.1), (0.6, 0.4, 0.3)>	<(0.5, 0.4, 0.4), (0.3, 0.3, 0.2), (0.6, 0.5, 0.4)>

4. Medical diagnosis using the similarity measure – cont.

Then, by using *Eqs*. (1) and (2) and taking $p = 2$, we can obtain the similarity measure between each patient P_i (I = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in *Table 3*.

Similarly, by using *Eqs*. (1) and (3) and taking $p = 2$, we can obtain the similarity measure between each patient P_i (i = 1, 2, 3, 4) and the considered disease D_j (j = 1, 2, 3, 4), which are shown in *Table 4*.

In *Tables 3* and *4*, the largest similarity measure indicates the proper diagnosis.

Patient P_1 suffers from viral fever, *Patient* P_2 suffers from tuberculosis, *Patient* P_3 suffers from typhoid, and *Patient* P_4 also suffers from typhoid.

4. Medical diagnosis using the similarity measure – cont.

Table 3 Similarity measure values of $S_1(P_i, D_j)$

	Viral fever (D_1)	Tuberculosis (D_2)	Typhoid (D_3)	Throat disease(D_4)
P_1	**0.7358**	0.6101	0.7079	0.5815
P_2	0.6884	**0.7582**	0.6934	0.5964
P_3	0.6159	0.6141	**0.6620**	0.6294
P_4	0.7199	0.6167	**0.7215**	0.5672

Table 4 Similarity measure values of $S_2(P_i, D_j)$

	Viral fever (D_1)	Tuberculosis (D_2)	Typhoid (D_3)	Throat disease(D_4)
P_1	**0.5821**	0.4390	0.5478	0.4100
P_2	0.5248	**0.6106**	0.5307	0.4249
P_3	0.4450	0.4431	**0.4948**	0.4592
P_4	0.5624	0.4459	**0.5643**	0.3958

Conclusion.

This paper proposed the generalized distance and its two similarity measures.

Then, the two similarity measures of SVNMs were applied to medical diagnosis to demonstrate the effectiveness of the developed measure methods.

The medical diagnosis shows that the new measures perform well in the case of truth-membership, indeterminacy-membership, and falsity-membership functions and the example depicts that the proposed measure is effective with the three representatives of SVNMV – truth-membership, indeterminacy-membership and falsity-membership values.

Therefore, the measures of SVNMs make them possible to handle the diagnosis problems with indeterminate and inconsistent information, which cannot be handled by the measures of IFMs because IFMs cannot express and deal with the indeterminate and inconsistent information.

Conclusion – cont.

In further work, it is necessary and meaningful to extend SVNMs to propose interval neutrosophic multisets and their operations and measures and to investigate their applications such as decision making, pattern recognition, and medical diagnosis.

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