Neutrosophic Masses & Indeterminate Models. Applications to Information Fusion

Florentin Smarandache
Mathematics Department
The University of New Mexico
705 Gurley Ave., Gallup, NM 87301, USA
E-mail: smarand@unm.edu
http://fs.gallup.unm.edu/neutrosophy.htm

The 2012 International Conference on
Advanced Mechatronic System (ICAMechS2012)

Tokyo, Japan, 18-22 September 2012

Contents

Introduction

- In this paper we introduce for the first time the notions of indeterminate mass (bba), indeterminate element, indeterminate intersection, and so on. We give an example of neutrosophic dynamic fusion using two classical masses, defined on a determinate frame of discernment, but having indeterminate intersections in the super-power set *S* (the fusion space). We also adjust several classical fusion rules (*PCR5* and *DSmH*) to work for indeterminate intersections instead of empty intersections.

Neutrosophic Logic

- Neutrosophic Logic (NL) [1] started in 1995 as a generalization of the fuzzy logic, especially of the intuitionistic fuzzy logic. A logical proposition *P* is characterized by three neutrosophic components:
- $$NL(P) = (T, I, F) \qquad (3)$$
- where *T* is the degree of truth, *F* the degree of falsehood, and *I* the degree of indeterminacy (or <u>neutral</u>, where the name “neutro-sophic” comes from, i.e. neither truth nor falsehood but in between – or included-middle principle), and with:
- $$T, I, F\ ]^{-}0,1^{+}[\qquad (4)$$
- where $]^{-}0,1^{+}[$ is a non-standard interval.
- In this paper, for technical proposal, we can reduce this interval to the standard interval $[0, 1]$.
- The main distinction between neutrosophic logic and intuitionistic fuzzy logic (IFL) is that in NL the sum $T+I+F$ of the components, when *T, I*, and *F* are crisp numbers, does not need to necessarily be *1* as in IFL, but it can also be less than *1* (for incomplete/missing information), equal to *1* (for complete information), or greater than *1* (for paraconsistent/contradictory information).
- The combination of neutrosophic propositions is done using the neutrosophic operators (especially \/, /\ ).

Classical Mass (bba)

- Let Θ be a frame of discernment, defined as:

$\Theta = \{\phi_1, \phi_2, \ldots, \phi_n\}$, $n \geq 2$,

and its Super-Power Set (or fusion space):

$S^\Theta = (\Theta, \cup, \cap, C)$

which means the set Θ closed under union, intersection, and respectively complement.

We recall that a classical mass *m(.)* is defined as:

$m : S^\Theta \rightarrow [0,1]$

such that

$$\sum_{X \in S^\Theta} m(X) = 1.$$

Neutrosophic Mass (nbba)

- We extend this classical basic belief assignment (mass) *m(.)* to a neutrosophic basic belief assignment (nbba) (or neutrosophic mass) $m_n(\,.)$ in the following way.

$$m_n : S^{\Theta} \rightarrow [0,1]^3$$

with

$$m_n(A) = (T(A), I(A), F(A))$$

where *T(A)* means the (local) chance that hypothesis *A* occurs, *F(A)* means the (local) chance that hypothesis *A* does not occur (nonchance), while *I(A)* means the (local) indeterminate chance of *A* (i.e. knowing neither if *A* occurs nor if *A* doesn't occur),

such that:

$$\sum_{X \in S^{\Theta}} [T(X) + I(X) + F(X)] = 1 .$$

Neutrosophic Mass (nbba) - 2

- In a more general way, the above summation can be less than 1 (for incomplete neutrosophic information), equal to 1 (for complete neutrosophic information), or greater than 1 (for paraconsistent/conflicting neutrosophic information). But in this paper we only present the case when the above summation is equal to 1.
- Of course,

$$0 \leq T(A), I(A), F(A) \leq 1.$$

Indeterminate Mass (ibba)

- A *basic belief assignment* (or *mass*) is considered *indeterminate* if there exist at least an element such that $I(A) > 0$, i.e. there exists some indeterminacy in the chance of at least an element A for occurring or for not occurring. Therefore, a neutrosophic mass which has at least one element A with $I(A) > 0$ is an *indeterminate mass*.
- A classical mass $m(.)$ can be extended under the form of a neutrosophic mass $m_n'(.)$ in the following way:

$$m_n': S^{\Theta} \to [0,1]^3$$

with

$$m_n'(A) = (m(A), 0, 0)$$

but reciprocally it does not work since $I(A)$ has no correspondence in the definition of the classical mass.

Indeterminate Mass (ibba) - 2

- We just have $T(A) = m(A)$ and $F(A) = m(C(A))$, where $C(A)$ is the complement of A. The non-null $I(A)$ can, for example, be roughly approximated by the total ignorance mass m(Θ), or better by the partial ignorance mass $m(\Theta_I)$ where Θ_I is the union of all singletons that have some non-zero indeterminacy, but these mean less accuracy and less refinement in the fusion.

If $I(X) = 0$ for all X, then the neutrosophic mass is simply reduced to a classical mass.

Indeterminate Element

- We have two types of elements in the fusion space S^{Θ}, *determinate elements* (which are well-defined), and *indeterminate elements* (which are not well-defined; for example: a geographical area whose frontiers are vague; or let's say in a murder case there are two suspects, *John* – who is known/determinate element – but he acted together with another man *X* (since the information source saw *John* together with an unknown/unidentified person) – therefore *X* is an indeterminate element).
- Herein we gave examples of singletons as indeterminate elements just in the frame of discernment , but indeterminate elements can also result from the combinations (unions, intersections, and/or complements) of determinate elements that form the super-power set S^{Θ}. For example, *A* and *B* can be determinate singletons (we call the elements in Θ as singletons), but their intersection $A \cap B$ can be an indeterminate (unknown) element, in the sense that we might not know if $A \cap B =$ *empty* or $A \cap B =$ *nonempty* .

Indeterminate Element - 2

- Or *A* can be a determinate element, but its complement *C(A)* can be indeterminate element (not well-known), and similarly for determinate elements *A* and *B*, but their union *A*∪*B* might be indeterminate.
- Indeterminate elements in S^{Θ} can, of course, result from the combination of indeterminate singletons too. All depends on the problem that is studied.
- A frame of discernment which has at least an indeterminate element is called *indeterminate frame of discernment*. Otherwise, it is called *determinate frame of discernment*. Similarly we call an *indeterminate fusion space* ( ) that fusion space which has at least one indeterminate element. Of course an indeterminate frame of discernment spans an indeterminate fusion space.
- An *indeterminate source of information* is a source which provides an indeterminate mass or an indeterminate fusion space. Otherwise it is called a *determinate source of information*.

Indeterminate Model

- An *indeterminate model* is a model whose fusion space is indeterminate, or a mass that characterizes the model is indeterminate.
- Such case has not been studied in the information fusion literature so far.

Classification of Models

- In the classical fusion theories all elements are considered determinate in the Closed World, except in Smets' Open World where there is some room (i.e. mass assigned to the empty set) for a possible unknown missing singleton in the frame of discernment. So, the Open World has a probable indeterminate element, and thus its frame of discernment is indeterminate. While the Closed World frame of discernment is determinate.
- In the Closed World in Dezert-Smarandache Theory there are three models classified upon the types of singleton intersections: Shafer's Model (where all intersections are empty), Hybrid Model (where some intersections are empty, while others are non-empty), and Free Model (where all intersections are non-empty).
- We now introduce a fourth category, called *Indeterminate Model* (where at least one intersection is indeterminate/unknown, and in general at least one element of the fusion space is indeterminate). We do this because in practical problems we don't always know if an intersection is empty or nonempty. As we still have to solve the problem in the real time, we have to work with what we have, i.e. with indeterminate models.

Classification of Models - 2

- The *indeterminate intersection* cannot be refined (because not knowing if *A B* is empty or nonempty, we'd get two different refinements: {*A*, *B*} when intersection is empty, and {*A**B*, *B**A*, *A B*} when intersection is nonempty).
- The *percentage of indeterminacy* of a model depends on the number of indeterminate elements and indeterminate masses.
- By default: the sources, the masses, the elements, the frames of discernment, the fusion spaces, and the models are supposed determinate.

Example of Information Fusion with an Indeterminate Model

- Suppose we have two sources, m1(.) and m2(.), such that:

	A	B	C	$A \cup B \cup C$	$A \cap B$ = Ind.	$A \cap C$ = ϕ	$B \cap C$ = Ind.
m_1	0.4	0.2	0.3	0.1			
m_2	0.1	0.3	0.2	0.4			
m_{12}	.21	.17	.20	.04	.14	.11	.13

Table 1

Example of Information Fusion with an Indeterminate Model - 2

	T(A)	T(B)	T(C)	$T(\Theta)$	I(A)	I(B)	I(C)
m_{12}	.21	.17	.20	.04			
addi- tions	.0075 .053 333		.022 5 .026 667		.068 572 .006 667	.051 428 .013 333 .02 .045	.04 .045
$m_{12PCR5I}$	.270 833	.17	.249 167	.04	.075 239	.129 761	.065

Table 2

BELIEF, DISBELIEF, AND UNCERTAINTY

In classical fusion theory there exist the following functions:

Belief in A with respect to the bba $m(.)$ is:

$$Bel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} m(X) \qquad (15)$$

Disbelief in A with respect to the bba $m(.)$ is:

$$Dis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} m(X) \qquad (16)$$

Uncertainty in A with respect to the bba $m(.)$ is:

$$U(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} m(X), \qquad (17)$$

where $C(A)$ is the complement of A with respect to the total ignorance Θ.

Plausability of A with respect to the bba $m(.)$ is:

$$Pl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} m(X) \qquad (18)$$

NEUTROSOPHIC BELIEF, NEUTROSOPHIC DISBELIEF, AND NEUTROSOPHIC UNDECIDABILITY

Neutrosophic Belief in A with respect to the nbba $m_n(.)$ is:

$$NeutBel(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} F(X) \qquad (19)$$

Neutrosophic Disbelief in A with respect to the nbba $m_n(.)$ is:

$$NeutDis(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A = \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} F(X) \qquad (20)$$

Neutrosophic Uncertainty in A with respect to the nbba $m_n(.)$ is

$$NeutU(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} T(X) + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} F(X)$$

$$= \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} [T(X) + F(X)] \qquad (21)$$

We now introduce the **Neutrosophic Global Indeterminacy in** A with respect to the nbba $m_n(.)$ as a sum of local indeterminacies of the elements included in A:

$$NeutGlobInd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \qquad (22)$$

And afterwards we define another function called **Neutrosophic Undecidability about** A with respect to the nbba $m_n(.)$:

$$NeutUnd(A) = NeutU(A) + NeutGlobInd(A) \qquad (23)$$

or

$$NeutUnd(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi \\ X \cap C(A) \neq \phi}} [T(X) + F(X)] + \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \subseteq A}} I(X) \qquad (24)$$

Neutrosophic Plausability of A with respect to the nbba $m_n(.)$ is:

$$NeutPl(A) = \sum_{\substack{X \in S^{\Theta} \setminus \{\phi\} \\ X \cap A \neq \phi}} T(X) + \sum_{\substack{Y \in S^{\Theta} \setminus \{\phi\} \\ C(Y) \cap A \neq \phi}} F(Y) \qquad (25)$$

NEUTROSOPHIC DYNAMIC FUSION

- A Neutrosophic Dynamic Fusion is a dynamic fusion where some indeterminacy occurs: with respect to the mass or with respect to some elements.
- The solution of the above indeterminate model which has missing information, using the neutrosophic set, is consistent in the classical dynamic fusion in the case we receive part (or total) of the missing information.
- In the above example, let's say we find out later in the fusion process that $A \cap B = empty$. That means that the mass of indeterminacy of A, $I(A)=0.075239$, is transferred to A, and the masses of indeterminacy of B (resulted from $A \cap B$ only) - i.e. *0.051428* and *0.13333* - are transferred to B.
- Then one gets:

NEUTROSOPHIC DYNAMIC FUSION -2

	A	B	C	Θ	I(A)	I(B)	I(C)	A?B	A?C
m	.270833	.17	.249167	.04	0	.065	.065	0	0
+	.075239	.051428 .013333							
m_N	.346072	.234761	.249167	.04	0	.065	.065	0	0

NEUTROSOPHIC DYNAMIC FUSION - 3

- Let's suppose once more, considering the neutrosophic dynamic fusion, that afterwards we find out that B/\C=*nonempty*. Then, from previous *Table* the masses of indeterminacies of *B*, *I(B)* (0.065 = 0.02 + 0.045, resulted from which was considered indeterminate at the beginning of the neutrosophic dynamic fusion), and that of *C*, *I(C)*=0.065, go now to B/\C. Thus, we get:

	A	B	C	Θ	I(A)	I(B)	I(C)	A∩B	A∩C	B∩C
m_N	.346072	.234761	.249167	.04	0	.065	.065	0	0	0
-/+						-.065	-.065			+.065 +.065
m_{NN}	.346072	.234761	.249167	.04	0	0	0	0	0	.13

Conclusion

- In this paper we introduced for the first time the notions of indeterminate mass (bba), indeterminate element, indeterminate intersection, and so on. We gave an example of neutrosophic dynamic fusion using two classical masses, defined on a determinate frame of discernment, but having indeterminate intersections in the super-power set S^{Θ} (the fusion space). We adjusted several classical fusion rules (*PCR5* and *DSmH*) to work for indeterminate intersections instead of empty intersections.
- Then we extended the classical *Bel(.)*, *Dis(.)* {also called *Dou(.)*, i.e Dough} and the uncertainty *U(.)* functions to their respectively neutrosophic correspondent functions that use the neutrosophic masses, i.e. to the *NeutBel(.)*, *NeutDis(.)*, *NeutU(.)* and to the undecidability function *NeutUnd(.)* . We have also introduced the Neutrosophic Global Indeterminacy function, *NeutGlobInd(.)*, which together with *NeutU(.)* form the Neut*Und(.)* function.