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ROUGH STANDARD NEUTROSOPHIC SETS:

An aplication on standard neutrosophic information systems

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I.Introduction

- Fuzzy set (L. Zadeh, 1965): a useful method study in the problems of imprecision and uncertainty.

We denote $(A, \mu_A(x))$ is a fuzzy set A on universe set U.

$$\mu: U \to [0,1]$$
$$u \mapsto \mu_A(u)$$

- Intuitionistic set (Atanassov, 1986): $(A, \mu_A(u), \gamma_A(u))$ where μ_A is a membership, γ_A is a non-membership of A and

$$\mu_A(u) + \gamma_A(u) \leq 1, \forall u \in U$$

I. Introduction

- Neutrosophic set (NS) (F. Smarandache, 1999): $(A, \mu_A(u), \eta_A(u), \gamma_A(u))$ where $\mu_A(u)$ is a degree of truth (T), $\eta_A(u)$ is a degree of indeterminacy (I) and $\gamma_A(u)$ is a degree of falsity (F) statisfy $0 \leq \mu_A(u), \eta_A(u), \gamma_A(u) \leq 1$.
- A tandard neutrosophic set (SNS) (Picture fuzzy set) (B.C. Cuong, 2013): $(A, \mu_A(u), \eta_A(u), \gamma_A(u))$ in which $0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1$.

The family of all standard neutrosophic set in U is denoted by $\mathrm{PFS(U)}$

- Neutrosophic set and standard neutrosophic set have many application, see [7],[8],...

I. Introduction

- An information system (IS) is any organized system for the collection, organization, storage and communication of information.
- Rough set (Z. Pawlak, 1980s): a usefully mathematical tool for data mining, especially for information systems.
- A standard neutrosophic information system (SNIS) is an information system in which have using standard neutrosophic values, such as voting information systems,...
- Rough standard neutrosophic set (RSNS) is a usefully mathematical tool for SNIS,...

II. Basic notions of standard neutrosophic and rough set

Definition 2. (Lattice $(\mathrm{D}^*, \leq_{\mathrm{D}^*})$). Let $\mathrm{D}^* = \{(\mathrm{x}_1, \mathrm{x}_2, x_3) \in [0,1]^3 : \mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3 \leq 1\}$.

We define a relation $\leq_{\mathrm{D}^*}$ on D^* as follows: $\forall (\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3), (\mathrm{x}_1', \mathrm{x}_2', \mathrm{x}_3') \in \mathrm{D}^*$ then

$(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) \leq_{\mathrm{D}^*} (\mathrm{x}_1', \mathrm{x}_2', \mathrm{x}_3')$iff (or $(\mathrm{x}_1 < \mathrm{x}_1', \mathrm{x}_3 \geq \mathrm{x}_3')$ or $(\mathrm{x}_1 = \mathrm{x}_1', \mathrm{x}_3 > x_3')$ or$(\mathrm{x}_1 = \mathrm{x}_1', \mathrm{x}_3 = x_3', \mathrm{x}_2 \leq \mathrm{x}_2'))$ and $(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3) =_D^* (\mathrm{x}_1', \mathrm{x}_2', \mathrm{x}_3') \Leftrightarrow (\mathrm{x}_1 = \mathrm{x}_1', \mathrm{x}_2 = \mathrm{x}_2', \mathrm{x}_3 = \mathrm{x}_3')$.

We put $\mathrm{x}_4 = 1 - (\mathrm{x}_1 + \mathrm{x}_2 + \mathrm{x}_3)$

II. Basic notions of standard neutrosophic and rough set

Previous surveys of voters in the US presidential election of 2017. Many people believe that Mrs Clinton will win. But, when the election results were announced, Mr Trumpt win. Those who carried out the survey has no statistical omission to those who have not been surveyed or comments about the survey. These people in the elections could actually participate very strong decision to actually vote results. It is $x_4 = 1 - (x_1 + x_2 + x_3)$, where $(\mathrm{x}_1, \mathrm{x}_2, x_3) \in D^*$

II. Basic notions of standard neutrosophic and rough set

- Standard neutrosophic set (SNS) $(A, \mu_A(u), \eta_A(u), \gamma_A(u))$ in which

$$0 \leq \mu_A(u) + \eta_A(u) + \gamma_A(u) \leq 1.$$

- Level set of SNS: (α, β, θ) −level of a SNS A defined by

$$A_{\beta}^{\alpha,\theta} = \{x \in U | (\mu_A(x), \eta_A(x), \gamma_A(x)) \geq (\alpha, \beta, \theta)\}$$

II. Basic notions of standard neutrosophic and rough set

- Let U be a nonempty universe of discourse. A subset $\mathrm{R} \in \mathrm{P}(\mathrm{U} \times \mathrm{U})$ is referred to as a (crisp) binary relation on U.
- Denote $\mathrm{R_s}(\mathrm{x}) = \{\mathrm{y} \in \mathrm{U} | (\mathrm{x}, \mathrm{y}) \in \mathrm{R}\}, \mathrm{x} \in \mathrm{U}$.
- **Rough set**: Let (U, R) be a crisp approximation space. For each crisp set $\mathrm{A} \subseteq \mathrm{U}$, we define the upper and lower approximations of A (w.r.t) (U, R) denoted by $\overline{\mathrm{R}}(\mathrm{A})$ and $\underline{\mathrm{R}}(\mathrm{A})$, respectively, are defined as follows

$$\overline{\mathrm{R}}(\mathrm{A}) = \{\mathrm{x} \in \mathrm{U}: \mathrm{R_s}(\mathrm{x}) \cap \mathrm{A} \neq \emptyset \}$$

$$\underline{\mathrm{R}}(\mathrm{A}) = \{\mathrm{x} \in \mathrm{U}: \mathrm{R_s}(\mathrm{x}) \subseteq \mathrm{A}\}$$

III. Rough standard neutrosophic set

- **RSNS**: Let (U,R) be a crisp approximation space. For $A \in PFS(U)$, the upper and lower approximations of A (w.r.t) (U,R) denoted by $\overline{RP}(A)$ and $\underline{RP}(A)$, respectively, are defined as follows:

$$\overline{RP}(A) = \{(x, \mu_{\overline{RP}(A)}(x), \eta_{\overline{RP}(A)}(x), \gamma_{\overline{RP}(A)}(x)) | x \in U\}$$

$$\underline{RP}(A) = \{(x, \mu_{\underline{RP}(A)}(x), \eta_{\underline{RP}(A)}(x), \gamma_{\underline{RP}(A)}(x)) | x \in U\}$$

Where

$$\mu_{\overline{RP}(A)}(x) = \vee_{y \in R_s(x)} \mu_A(y), \ \gamma_{\overline{RP}(A)}(x) = \wedge_{y \in R_s(x)} \gamma_A(y),$$

$$\eta_{\overline{RP}(A)}(x) = \wedge_{y \in R_s(x)} \eta_A(y),$$

and

$$\mu_{\underline{RP}(A)}(x) = \wedge_{y \in R_s(x)} \mu_A(y), \ \gamma_{\underline{RP}(A)}(x) = \vee_{y \in R_s(x)} \gamma_A(y),$$

$$\eta_{\underline{RP}(A)}(x) = \wedge_{y \in R_s(x)} \eta_A(y)$$

- Some properties of RSNS are studied in full paper.

IV. The standard neutrosophic information systems (SNIS)

- Information systems (IS)

Let (U, A, F, D, G) be a information system. Here U is the (nonempty) set of objects, i.e., $U = \{u_1, u_2, \dots, u_n\}$, $A = \{a_1, a_2, \dots, a_m\}$ is the conditional attribute set, and F is the relation set of U and A, i.e., $F = \{f_j : U \rightarrow V_j, j = 1,2, \dots, m\}$ where V_j is the domain of the attribute a_j $(j = 1,2, \dots, m)$; $D = \{d_1, d_2, \dots, d_p\}$ is the decision attribute set; G is the relation set of U and D.

- The (U, A, F) is called a classical information system.
- Relation $R_B = IND(B)$(where $B \subset A$), as follows, $\forall x, y \in U$:

$$x\ IND(B)\ y \iff f_j(x) = f_j(y) \text{ for all } j \in \{j : a_j \in B\}.$$

IV. The standard neutrosophic information systems (SNIS)

- **SNIS**: Let (U, A, F, D, G) be the information system. If $D = \{D_k | k = 1,2, \dots, q\}$ where D_k is a standard neutrosophic subset of U and G is the relation set of U and D, then (U, A, F, D, G) is called a standard neutrosophic information system.
- **Example**: A SNIS (see Tabble 1) $U = \{u_1, u_2, \dots, u_{10}\}$, condition attribute set is $A = \{a_1, a_2, a_3\}$ and the decision attribute set is $D = \{D_1, D_2, D_3\}$, where $D_k (k = 1,2,3)$ is a standarf neutrosophic subset of U.

IV. The standard neutrosophic information systems (SNIS)

Table 1: *A standard neutrosophic information system*

U	a_1	a_2	a_3	D_1	D_2	D_3
u_1	3	2	1	(0.2,0.5,0.3)	(0.15,0.2,0.6)	(0.4,0.5,0.05)
u_2	1	3	2	(0.3,0.5,0.1)	(0.3,0.3,0.3)	(0.35,0.4,0.1)
u_3	3	2	1	(0.6,0.4,0)	(0.3,0.6,0.05)	(0.1,0.4,0.45)
u_4	3	3	1	(0.15,0.7,0.1)	(0.1,0.8,0.05)	(0.2,0.3,0.4)
u_5	2	2	4	(0.05,0.7,0.2)	(0.2,0.3,0.4)	(0.05,0.5,0.4)
u_6	2	3	4	(0.1,0.5,0.3)	(0.2,0.4,0.3)	(1,0,0)
u_7	1	3	2	(0.25,0.4,0.3)	(1,0,0)	(0.3,0.4,0.3)
u_8	2	2	4	(0.1,0.2,0.6)	(0.25,0.4,0.3)	(0.4,0.6,0)
u_9	3	2	1	(0.45,0.45,0.1)	(0.25,0.3,0.4)	(0.2,0.3,0.5)
u_{10}	1	3	2	(0.05,0.9,0.05)	(0.4,0.3,0.2)	(0.05,0.2,0.7)

V. The knowledge discovery in SNIS

- Let (U, A, F, D, G) be the NSIS and $B \subseteq A$, we denote $\underline{RP_B}(D_j)$ is the lower rough SN approximation of $D_j \in PFS(U)$ on approximation space (U, R_B).
- **Theorem 5**: Let (U, A, F, D, G) be the SNIS and $B \subseteq A$. If for any $x \in U$: $\left(\mu_{D_i}(x), \eta_{D_i}(x), \gamma_{D_i}(x)\right) \geq \left(\alpha(x), \theta(x), \beta(x)\right) =$ $\underline{RP_B}(D_i)(x) > \underline{RP_B}(D_j)(x)(i \neq j)$,

then $[x]_B \cap (\sim D_j)_{\alpha(x)}^{\beta(x),0} \neq \emptyset$ and

$[x]_B \subseteq (D_i)_{\beta(x)}^{\alpha(x),\theta(x)}$ where $\left(\alpha(x), \theta(x), \beta(x)\right) \in D^*$.

V. The knowledge discovery in SNIS

- Let (U, A, F, D, G) be a SNIS, $R_A = IND(A)$. The universe is divided by R_A as following: $U/R_A = \{X_1, X_2 \dots, X_k\}$. Then the approximation of the SN decision denoted as, for all $i = 1,2, \dots, k$,
$$\underline{RP_A}\left(D(X_i)\right) = \left(\underline{RP_A}(D_1(X_i)), \underline{RP_A}(D_2(X_i)), \dots, \underline{RP_A}\left(D_q(X_i)\right)\right)$$
- **Example 3**. The SNIS in Table 1. The equivalent classes
$$U/R_A = \{X_1 = \{u_1, u_3, u_9\}, X_2 = \{u_2, u_7, u_{10}\},$$
$$X_3 = \{u_4\}, X_4 = \{u_5, u_8\}, X_5 = \{u_6\}\}$$

The approximation of the standard neutrosophic decision is in Table 2.

V. The knowledge discovery in SNIS

U/R_A	$\underline{RP}_A(D_1(X_i))$	$\underline{RP}_A(D_2(X_i))$	$\underline{RP}_A(D_3(X_i))$
X_1	(0.2,0.5,0)	(0.15,0.6,0,05)	(0.1,0.5,0.05)
X_2	(0.05,0.9,0.05)	(0.3,0.3,0.1)	(0.05,0.4,0.1)
X_3	(0.15, 0.7, 0.1)	(0.1,0.8,0.05)	(0.2,0.3,0.4)
X_4	(0.05,0.7,0.2)	(0.2,0.4,0.3)	(0.05,0.6,0)
X_5	(0.1,0.5,0.3)	(0.2,0.4,0.3)	(1,0,0)

Table 2: *The approximation of the Standard neutrosophic decision*

VI. The knowledge reduction and extension of SNIS

Definition 7. Let (U, A, F) be the classical IS and $B \subseteq A$.

(i) B is called the SN reduction of (U, A, F), if B is the minimum set which satisfies the following relations: $\forall X \in PFS(U), x \in U$,

$$\underline{RP_A}(X) = \underline{RP_B}(X), \qquad \overline{RP}_A(X) = \overline{RP}_B(X)$$

(ii) B is called the SN lower approximation reduction of (U, A, F), if B is the minimum set which satisfies the following relations: $\forall X \in PFS(U), x \in U:$

$$\underline{RP_A}(X) = \underline{RP_B}(X)$$

(iii) B is called the SN upper approximation reduction of (U, A, F), if B is the minimum set which satisfies the following relations: $\forall X \in PFS(U), x \in U\ \overline{RP}_A(X) = \overline{RP}_B(X)$

Where $\underline{RP_A}(X), \underline{RP_B}(X)$, $\overline{RP}_A(X), \overline{RP}_B(X)$ are SN lower and SN upper approximation sets of SN set $X \in PFS(U)$ based on R_A, R_B, respectively

VI. The knowledge reduction and extension of SNIS

- **Definition 8**. Let (U, A, F, D, G) be the SNIS

$$D_{ij} = \begin{cases} \{a_l \in A\colon\ f_l(X_i) \neq f_l(X_j)\}; & g_{X_i}(D_k) \neq g_{X_j}(D_k) \\ A & ;\ g_{X_i}(D_k) = g_{X_j}(D_k) \end{cases}$$

is called the discernibility matrix of (U, A, F, D, G) (where $g_{X_i}(D_k)$ is the maximum of $\underline{RP_A}(D(X_i))$ obtained at D_k, i.e., $g_{X_i}(D_k) = \underline{RP_A}(D_k(X_i)) = \max\{\underline{RP_A}(D_t(X_i)), t = 1,2, \ldots, q\}$.

VI. The knowledge reduction and extension of SNIS

Definition 9. Let (U, A, F, D, G) be the standard neutrosophic information system, for any $B \subseteq A$, if the following relations holds, for any $x \in U$:

$$\underline{RP_B}(D_i)(x) > \underline{RP_B}(D_j)(x) \Leftrightarrow \underline{RP_A}(D_i)(x) > \underline{RP_A}(D_j)(x)(i \neq j)$$

then B is called the consistent set of A.

Theorem 6. Let (U, A, F, D, G) be the standard neutrosophic information system. If there exists a subset $B \subseteq A$ such that $B \cap D_{ij} \neq \emptyset$, then B is the consistent set of A.

VI. The knowledge reduction and extension of SNIS

Definition 11. Let (U, A, F) be the classical IS and $A \subseteq B$.

(i) B is called the SN extension of (U, A, F), if B satisfies the following relations: $\forall\ X \in PFS(U), x \in U$

$$\underline{RP}_A(X) = \underline{RP}_B(X), \qquad \overline{RP}_A(X) = \overline{RP}_B(X)$$

(ii) B is called the SN lower approximation extension of (U, A, F), if B satisfies the following relations: $\forall X \in PFS(U), x \in U$,

$$\underline{RP}_A(X) = \underline{RP}_B(X),$$

(iii) B is called the SN upper approximation extension of (U, A, F), if B satisfies the following relations: for any $X \in PFS(U), x \in U$

$$\overline{RP}_A(X) = \overline{RP}_B(X)$$

Theorem 8. Let (U, A, F) be the classical IS, for any hyper set B, such that $A \subseteq B$, if A is the SN reduction of the classical IS (U, B, F), then (U, B, F) is the SN extension of (U, A, F), but not conversely necessary.

VI. The knowledge reduction and extension of SNIS

Example 4. In the approximation of the SN decision in Table 1, Table 2. Let $B = \{a_1, a_2\}$, then we obtained the family of all equivalent classes of U based on the equivalent relation $R_B = IND(B)$ as follows

$$U/R_B = \begin{Bmatrix} X_1 = \{u_1, u_3, u_9\}, X_2 = \{u_2, u_7, u_{10}\}, \\ X_3 = \{u_4\}, X_4 = \{u_5, u_8\}, X_5 = \{u_6\} \end{Bmatrix}$$

We can get the approximation value given in Table 3. It is samed to the approximation value given in Table 2. It mean

$B = \{a_1, a_2\}$ is a reduction of (U, A, F)

The discernibility matrix of the standard neutrosophic information system (U, A, F, D, G) will be presented in Table 4.

VI. The knowledge reduction and extension of SNIS

U/R_B	$\underline{RP_A}(D_1(X_i))$	$\underline{RP_A}(D_2(X_i))$	$\underline{RP_A}(D_3(X_i))$
X_1	(0.2,0.5,0)	(0.15,0.6,0,05)	(0.1,0.5,0.05)
X_2	(0.05,0.9,0.05)	(0.3,0.3,0.1)	(0.05,0.4,0.1)
X_3	(0.15, 0.7, 0.1)	(0.1,0.8,0.05)	(0.2,0.3,0.4)
X_4	(0.05,0.7,0.2)	(0.2,0.4,0.3)	(0.05,0.6,0)
X_5	(0.1,0.5,0.3)	(0.2,0.4,0.3)	(1,0,0)

Table 3: *The approximation of the standard neutrosophic decision*

VI. The knowledge reduction and extension of SNIS

U/R_B	X_1	X_2	X_3	X_4	X_5
X_1	A				
X_2	A	A			
X_3	$\{a_2\}$	$\{a_1, a_3\}$	A		
X_4	$\{a_1, a_3\}$	A	A	A	
X_5	$\{a_1, a_3\}$	A	A	$\{a_2\}$	A

Table 4: *The discernibility matrix of the standard neutrosophic information system*

Conclusion

- We introduce the concept of standard neutrosophic information system
- We study the knowledge discovery of standard neutrosophic information system based on rough standard neutrosophic sets
- knowledge reduction and extension of the standard neutrosophic information system

THANK YOU FOR YOUR ATTENTION!

Question?

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