Simplified neutrosophic exponential similarity measures for the initial evaluation/diagnosis of benign prostatic hyperplasia symptom

Jing Fu[a], Jun Ye[b*]
[a]Shaoxing Second Hospital, 123 Yanan Road, Shaoxing, Zhejiang 312000, P.R. China, E-mail: jingsaox@163.com
[b]Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing, Zhejiang 312000, P.R. China

Abstract

When physician carries out the clinical survey of a patient with benign prostatic hyperplasia (BPH) symptom to reach the initial evaluation/diagnosis of BPH symptom, the existing initial evaluation method of BPH based on the international prostate symptom score (I-PSS) usually use the objective evaluation/diagnosis method with crisp values without considering fuzzy information. However, this normal evaluation/diagnosis method may lose a lot of incomplete, uncertainty, and inconsistent information in the clinical survey and initial evaluation process of the BPH symptom for a patient and result in the unreasonable evaluation/diagnosis of the BPH symptom. To overcome this drawback, this paper aims to propose new exponential similarity measures between SNSs, including single valued neutrosophic exponential similarity measures and interval neutrosophic exponential similarity measures, and their initial evaluation/diagnosis method of the BPH symptom with simplified neutrosophic information. Finally, two evaluation/diagnosis examples on the BPH symptom are provided to demonstrate the effectiveness and rationality of the proposed method.

Keywords: Simplified neutrosophic set; Single valued neutrosophic set; Interval neutrosophic set; Exponential similarity measure; Benign prostatic hyperplasia; Medical diagnosis

1. Introduction

Benign prostatic hyperplasia (BPH) is a common medical problem encountered in our aging men, who lead to obstructive and irritative voiding symptom. Then, the American Urological Association (AUA) uses seven questions as the AUA symptom indices [1, 2] for BPH scored on a scale from 0 to 5 points. The international prostate symptom score (I-PSS) [1, 2] offers an objective documentation of symptoms: totally scoring 0-7 is mildly symptomatic, 8-19 moderately symptomatic, and 20-35 severely symptomatic. However, the objective evaluation is a non-fuzzy evaluation method (a normal evaluation method) in I-PSS.

The initial evaluation of the BPH symptom is obtained by means of clinical survey for a patient to select further examinations (e.g., creatinine, intravenous urograpgy, urethrogram, urodynamics, urethrocystoscopy, etc.) and suitable treatment alternatives (e.g., watchful waiting, medical, surgical,

*Corresponding author: Jun Ye
Tel.: +86 575 88327323; E-mail: yehjun@aliyun.com

or minimally invasive surgical treatments, etc.). Then the choice of treatment is reached in a shared decision-making process between the physician and the patient. When physician carries out the clinical survey of a patient to reach the initial evaluation of the BPH symptom, the patient gives the responses of the seven questions which usually contain a “grey zone” of the uncertainty for the patient about the BPH symptom. Thus the clinical data of the BPH symptom obtained by physician are incomplete, uncertainty or contradictory. In this case, fuzzy expression is a suitable tool. Zadeh [3] firstly introduced the degree of membership/truth in 1965 and defined the fuzzy set. Based on a generalization of the fuzzy set, Atanassov [4] introduced the degree of nonmembership/falsity in 1986 and defined the intuitionistic fuzzy set. Further, Atanassov and Gargov [5] introduced an interval-valued intuitionistic fuzzy set. Smarandache [6] introduced the degree of indeterminacy/neutrality as independent component in 1995 and defined the neutrosophic set as a generalization of the intuitionistic fuzzy set and the interval-valued intuitionistic fuzzy set. He has coined the words “neutrosophy” and “neutrosophic”. The neutrosophic set introduced from philosophical point of view can represent uncertainty, imprecise, incomplete and inconsistent information. From science and engineering point of view, the neutrosophic set will be difficult to be applied in real science and engineering fields [7, 8]. Therefore, Wang et al. [7, 8] proposed the concepts of a single valued neutrosophic set (SVNS) and an interval neutrosophic set (INS) as the subclasses of the neutrosophic set. Further, Ye [9] introduced the concept of a simplified neutrosophic set (SNS), including the concepts of SVNS and INS, which is a subclass of the neutrosophic set. A SNS is very suitable for handling medical diagnosis problems since a symptom usually implies a lot of incomplete, uncertainty, and inconsistent information for a disease, which characterizes a relation between symptoms and a disease. Recently, SNSs have been applied to medical diagnosis problems. Ye [10] presented the improved cosine similarity measures between SNSs for medical diagnoses. As a generalization of SVNS, Ye et al. [11, 12] introduced a single valued neutrosophic multiset and the Dice similarity measure and distance-based similarity measures of single valued neutrosophic multisets, and then applied them to medical diagnoses. Ye and Fu [13] put forward a single valued neutrosophic similarity measure based on tangent function and the tangent similarity measure-based multi-period medical diagnosis method (a dynamic medical diagnosis method).

However, because the existing initial evaluation method of BPH based on I-PSS [1, 2] usually use the objective evaluation/diagnosis method with crisp values without considering fuzzy information, this normal evaluation/diagnosis method may lose a lot of incomplete, uncertainty, and inconsistent information in the clinical survey and initial evaluation process of the BPH symptom for a patient and result in the unreasonable evaluation/diagnosis of the BPH symptom. To overcome this drawback, this paper aims to propose new exponential similarity measures between SNSs, including single valued neutrosophic exponential similarity measures and interval neutrosophic exponential similarity measures, and their initial evaluation/diagnosis method of the BPH symptom with simplified neutrosophic information.

The rest of the article is structured as follows. In Section 2, we briefly introduce some basic concepts of SNSs. Section 3 proposes exponential similarity measures between SNSs based on exponential function, including single valued neutrosophic exponential similarity measures and interval neutrosophic exponential similarity measures, and investigates their properties. In Section 4, the initial evaluation/diagnosis methods of the BPH symptom are presented based on the exponential similarity measures under a simplified neutrosophic environment, and then two evaluation examples on the BPH symptom are given to show the effectiveness and rationality of the proposed evaluation method. Conclusions and further research are given in Section 5.

2. **Basic concepts of SNSs**

The SNS introduced by Ye [9] is a generalization of the intuitionistic fuzzy set and the interval-valued intuitionistic fuzzy set and gives us an additional possibility to represent incomplete, uncertainty and inconsistent information, which exists in real world. Therefore, it is more suitable for applications in an indeterminate and inconsistent environment. The definition of SNS is introduced as follows.

Definition 1 [9]. Let X be a space of points (objects) with generic elements in X denoted by x. A SNS N in X is characterized by a truth-membership function $T_N(x)$, an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$. Then, a SNS N can be expressed as $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$, where the sum of $T_N(x)$, $I_N(x)$, $F_N(x) \subseteq [0, 1]$ satisfies the condition $0 \le \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \le 3$ for each point x in X.

Then, SNS is a subclass of the neutrosophic set and includes the concepts of SVNS and INS.

Assume that $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle \mid x \in X\}$ and $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$ are two SNSs, where $T_M(x)$, $I_M(x)$, $F_M(x) \in [0, 1]$, $0 \le T_M(x) + I_M(x) + F_M(x) \le 3$, $T_N(x)$, $I_N(x)$, $F_N(x) \in [0, 1]$, and $0 \le T_N(x) + I_N(x) + F_N(x) \le 3$ for each point x in X, i.e., M and N are two SVNSs. Then, the inclusion, equation, complement for SNSs M and N are defined, respectively, as follows [9]:

(1) $N \subseteq M$ if and only if $T_N(x) \le T_M(x)$, $I_N(x) \ge I_M(x)$, $F_N(x) \ge F_M(x)$ for any x in X,

(2) $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$,

(3) $M^c = \{\langle x, F_M(x), 1 - I_M(x), T_M(x)\rangle \mid x \in X\}$ and $N^c = \{\langle x, F_N(x), 1 - I_N(x), T_N(x)\rangle \mid x \in X\}$.

Assume that $M = \{\langle x, T_M(x), I_M(x), F_M(x)\rangle \mid x \in X\}$ and $N = \{\langle x, T_N(x), I_N(x), F_N(x)\rangle \mid x \in X\}$ are two SNSs, where $T_M(x)$, $I_M(x)$, $F_M(x) \subseteq [0, 1]$, $0 \le \sup T_M(x) + \sup I_M(x) + \sup F_M(x) \le 3$, $T_N(x)$, $I_N(x)$, $F_N(x) \subseteq [0, 1]$, and $0 \le \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \le 3$ for each point x in X, i.e., M and N are two INSs. Then, the inclusion, equation, complement for SNSs N and M are defined, respectively, as follows [9]:

(1) $N \subseteq M$ if and only if $\inf T_N(x) \le \inf T_M(x)$, $\inf I_N(x) \ge \inf I_M(x)$, $\inf F_N(x) \ge \inf F_M(x)$, $\sup T_N(x) \le \sup T_M(x)$, $\sup I_N(x) \ge \sup I_M(x)$, $\sup F_N(x) \ge \sup F_M(x)$ for any x in X,

(2) $N = M$ if and only if $N \subseteq M$ and $M \subseteq N$,

(3) $M^c = \{\langle x, [\inf F_M(x), \sup F_M(x)], [1 - \sup I_M(x), 1 - \inf I_M(x)], [\inf T_M(x), \sup T_M(x)]\rangle \mid x \in X\}$

and $N^c = \{\langle x, [\inf F_N(x), \sup F_N(x)], [1 - \sup I_N(x), 1 - \inf I_N(x)], [\inf T_N(x), \sup T_N(x)]\rangle \mid x \in X\}$.

Especially when the upper and lower ends of the interval numbers $T_M(x)$, $I_M(x)$, $F_M(x)$ in M and $T_N(x)$, $I_N(x)$, $F_N(x)$ in N are equal, the INSs M and N degrade to the SVNSs M and N. Therefore, the SVNSs are the special cases of the INSs, and also both are the special cases of the SNSs.

3. Exponential similarity measures of SNSs

Based on exponential function, this section proposes exponential similarity measures between

SNSs, including single valued neutrosophic exponential similarity measures and interval neutrosophic exponential similarity measures, and investigates their properties.

Definition 2. Let $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle | x_j \in X\}$ and $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle | x_j \in X\}$ be any two SVNSs in $X = \{x_1, x_2, \ldots, x_n\}$. Thus, we can define an exponential similarity measure between N and M as follows:

$$E_1(M,N) = \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right) - \exp(-1)}{1 - \exp(-1)}. \quad (1)$$

Obviously, the exponential similarity measure has the following proposition.

Proposition 1. For two SVNSs M and N in $X = \{x_1, x_2, \ldots, x_n\}$, the exponential similarity measure $E_1(M, N)$ should satisfy the following properties (1)-(4):

(1) $0 \le E_1(M, N) \le 1$;

(2) $E_1(M, N) = 1$ if and only if $M = N$;

(3) $E_1(M, N) = E_1(N, M)$;

(4) If P is a SVNS in X and $M \subseteq N \subseteq P$, then $E_1(M, P) \le E_1(M, N)$ and $E_1(M, P) \le E_1(N, P)$.

Proof:

(1) Since there are $T_M(x_j)$ $I_M(x_j)$ $F_M(x_j) \in [0, 1]$ and $T_N(x_j)$ $I_N(x_j)$ $F_N(x_j) \in [0, 1]$ in two SVNSs M and N, the distance $\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)/3$ lies between 0 and 1. By applying Eq. (1), the exponential similarity measure also lies between 0 and 1. Hence, there is $0 \le E_1(M, N) \le 1$.

(2) For the two SVNSs M and N, if $M = N$, this implies $T_M(x_j) = T_N(x_j)$, $I_M(x_j) = I_N(x_j)$, $F_M(x_j) = F_N(x_j)$ for $x_j \in X$ and $j = 1, 2, \ldots, n$. Hence $\left|T_M(x_j) - T_N(x_j)\right| = 0$, $\left|I_M(x_j) - I_N(x_j)\right| = 0$ and $\left|F_M(x_j) - F_N(x_j)\right| = 0$. Thus there is $E_1(M,N) = (1 - \exp(-1))/(1 - \exp(-1)) = 1$.

If $E_1(M, N) = 1$, this implies $E_1(M,N) = (1 - \exp(-1))/(1 - \exp(-1)) = 1$, and then there are $\left|T_M(x_j) - T_N(x_j)\right| = 0$, $\left|I_M(x_j) - I_N(x_j)\right| = 0$ and $\left|F_M(x_j) - F_N(x_j)\right| = 0$. Thus, these equalities indicate that $T_M(x_j) = T_N(x_j)$, $I_M(x_j) = I_N(x_j)$, $F_M(x_j) = F_N(x_j)$ for $x_j \in X$ and $j = 1, 2, \ldots, n$. Hence $M = N$.

(3) Proof is straightforward.

(4) If $M \subseteq N \subseteq P$, then this implies $T_M(x_j) \le T_N(x_j) \le T_P(x_j)$, $I_M(x_j) \ge I_N(x_j) \ge I_P(x_j)$, $F_M(x_j) \ge F_N(x_j) \ge F_P(x_j)$ for $j = 1, 2, \ldots, n$ and $x_j \in X$. Then, we have

$$\left|T_M(x_j) - T_N(x_j)\right| \le \left|T_M(x_j) - T_P(x_j)\right|,\ \left|T_N(x_j) - T_P(x_j)\right| \le \left|T_M(x_j) - T_P(x_j)\right|,$$

$$\left|I_M(x_j) - I_N(x_j)\right| \le \left|I_M(x_j) - I_P(x_j)\right|,\ \left|I_N(x_j) - I_P(x_j)\right| \le \left|I_M(x_j) - I_P(x_j)\right|,$$

$$\left|F_M(x_j) - F_N(x_j)\right| \le \left|F_M(x_j) - F_P(x_j)\right|,\ \left|F_N(x_j) - F_P(x_j)\right| \le \left|F_M(x_j) - F_P(x_j)\right|.$$

Hence, $E_1(M, P) \le E_1(M, N)$ and $E_1(M, P) \le E_1(N, P)$ since the exponential function $\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right)$ is a decreasing function.

Therefore, the proofs of these properties are completed. □

Generally, one takes the weight of each element x_j for $x_j \in X$ into account and assumes that the weight of an element x_j is w_j (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Hence, we can introduce the following weighted exponential similarity measure between M and N:

$$W_1(M,N) = \sum_{j=1}^{n} w_j \frac{\exp\left(-\frac{1}{3}\left(\left|T_M(x_j) - T_N(x_j)\right| + \left|I_M(x_j) - I_N(x_j)\right| + \left|F_M(x_j) - F_N(x_j)\right|\right)\right) - \exp(-1)}{1 - \exp(-1)}. \quad (2)$$

Clearly, the exponential similarity measure $W_1(M, N)$ should satisfy the properties (1)-(4) in Proposition 1. Especially when $w_j = 1/n$ for j = 1, 2, …, n, Eq. (2) reduces to Eq. (1).

Similarly, we can extend the exponential similarity measures of SVNSs to propose exponential similarity measures between INSs.

Let $M = \{\langle x_j, T_M(x_j), I_M(x_j), F_M(x_j)\rangle | x_j \in X\}$ and $N = \{\langle x_j, T_N(x_j), I_N(x_j), F_N(x_j)\rangle | x_j \in X\}$ be any two INSs in $X = \{x_1, x_2, \ldots, x_n\}$, where $T_M(x_j) = [\inf T_M(x_j), \sup T_M(x_j)]$, $I_M(x_j) = [\inf I_M(x_j), \sup I_M(x_j)]$, $F_M(x_j) = [\inf F_M(x_j), \sup F_M(x_j)] \subseteq [0, 1]$ in M for any $x_j \in X$ are denoted by $T_M(x_i) = [T_M^L(x_i), T_M^U(x_i)]$, $I_M(x_i) = [I_M^L(x_i), I_M^U(x_i)]$ and $F_M(x_i) = [F_M^L(x_i), F_M^U(x_i)]$, respectively, and $T_N(x_j) = [\inf T_N(x_j), \sup T_N(x_j)]$, $I_N(x_j) = [\inf I_N(x_j), \sup I_N(x_j)]$ and $F_N(x_j) = [\inf F_N(x_j), \sup F_N(x_j)] \subseteq [0, 1]$ in N for any $x_j \in X$ are denoted by $T_N(x_i) = [T_N^L(x_i), T_N^U(x_i)]$, $I_N(x_i) = [I_N^L(x_i), I_N^U(x_i)]$ and $F_N(x_i) = [F_N^L(x_i), F_N^U(x_i)]$, respectively, for convenience. Then, based on the extension of the above similarity measure equations (1) and (2), we can introduce the following two exponential similarity measures between M and N:

$$E_2(M,N) = \frac{1}{n}\sum_{j=1}^{n} \frac{\exp\left(-\frac{1}{6}\left(\begin{array}{l}\left|T_M^L(x_j) - T_N^L(x_j)\right| + \left|I_M^L(x_j) - I_N^L(x_j)\right| + \left|F_M^L(x_j) - F_N^L(x_j)\right| \\ + \left|T_M^U(x_j) - T_N^U(x_j)\right| + \left|I_M^U(x_j) - I_N^U(x_j)\right| + \left|F_M^U(x_j) - F_N^U(x_j)\right|\end{array}\right)\right) - \exp(-1)}{1 - \exp(-1)}, \quad (3)$$

$$W_2(M,N) = \sum_{j=1}^{n} w_j \frac{\exp\left(-\frac{1}{6}\left(\begin{array}{l}\left|T_M^L(x_j) - T_N^L(x_j)\right| + \left|I_M^L(x_j) - I_N^L(x_j)\right| + \left|F_M^L(x_j) - F_N^L(x_j)\right| \\ + \left|T_M^U(x_j) - T_N^U(x_j)\right| + \left|I_M^U(x_j) - I_N^U(x_j)\right| + \left|F_M^U(x_j) - F_N^U(x_j)\right|\end{array}\right)\right) - \exp(-1)}{1 - \exp(-1)}, \quad (4)$$

where w_j is the weight of an element x_j (j = 1, 2, …, n) with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Obviously, Eqs. (1) and (2) are the special cases of Eqs. (3) and (4) when the upper and lower ends of the interval numbers $T_M(x_j)$, $I_M(x_j)$, $F_M(x_j)$ in M and $T_N(x_j)$, $I_N(x_j)$, $F_N(x_j)$ in N are equal. Therefore, the above exponential similarity measures of INSs also satisfy the properties (1)-(4) in Proposition 1. The proof is similar to that of Proposition 1, and then it is not repeated here.

4. Initial evaluation/diagnosis method of BPH using the exponential similarity measures

According to seven questions in the AUA symptom indexes [1, 2] for BPH, we can consider a set of the seven questions Q = {Q_1(Over the past month, how often have you had a sensation of not emptying your bladder completely after you finished urinating?), Q_2(Over the past month, how often have you had to urinating again less than two hours after you finished urinating?), Q_3(Over the past month, how often have you found you stopped and started again several times when you urinated?), Q_4(Over the past month, how often have you found it difficult to postpone urination?), Q_5(Over the past month, how often have you had a week urinary stream?), Q_6(Over the past month, how often have you had to push or strain to begin urination?), Q_7(Over the past month, how many times did you most typically get up to urinate from the time you went to bed at night until the time you got up in the morning?)} for physician to survey the BPH symptom of patients. The clinical convey of BPH symptom responses in 5 times for a patient P_k (k =1, 2, …, t) can be constructed by Table 1.

Table 1 BPH symptom responses in 5 times for a patient P_k

Question	Truth	Indeterminacy	Falsity
Q_1: Over the past month, how often have you had a sensation of not emptying your bladder completely after you finished urinating?			
Q_2: Over the past month, how often have you had to urinating again less than two hours after you finished urinating?			
Q_3: Over the past month, how often have you found you stopped and started again several times when you urinated?			
Q_4: Over the past month, how often have you found it difficult to postpone urination?			
Q_5: Over the past month, how often have you had a week urinary stream?			
Q_6: Over the past month, how often have you had to push or strain to begin urination?			
Q_7: Over the past month, how many times did you most typically get up to urinate from the time you went to bed at night until the time you got up in the morning?			

Based on I-PSS [1, 2], BPH can be divided into the four types of symptoms, which are represented by a set of the four types of symptoms S = {S_1(Normal symptom), S_2(Mild symptom), S_3(Moderate symptom), S_4(Severe symptom)} as the symptom knowledge for the initial evaluation of BPH patients, as shown in Table 2.

Table 2 Four types of the BPH symptom with simplified neutrosophic information

S_i (Symptom type)	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
S_1 (Normal symptom)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>
S_2 (Mild symptom)	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>

S_3 (Moderate symptom)	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>
S_4 (Severe symptom)	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>

From Table 2, the BPH symptom types of patients with respect to all the questions can be represented by the following SNS information:

$S_1 = \{\langle Q_1, 0, 0, 1\rangle, \langle Q_2, 0, 0, 1\rangle, \langle Q_3, 0, 0, 1\rangle, \langle Q_4, 0, 0, 1\rangle, \langle Q_5, 0, 0, 1\rangle, \langle Q_6, 0, 0, 1\rangle, \langle Q_7, 0, 0, 1\rangle\}$,

$S_2 = \{\langle Q_1, 0, 0.2, 0.8\rangle, \langle Q_2, 0, 0.2, 0.8\rangle, \langle Q_3, 0, 0.2, 0.8\rangle, \langle Q_4, 0, 0.2, 0.8\rangle, \langle Q_5, 0, 0.2, 0.8\rangle, \langle Q_6, 0, 0.2, 0.8\rangle, \langle Q_7, 0, 0.2, 0.8\rangle\}$,

$S_3 = \{\langle Q_1, 0.2, 0.4, 0.4\rangle, \langle Q_2, 0.2, 0.4, 0.4\rangle, \langle Q_3, 0.2, 0.4, 0.4\rangle, \langle Q_4, 0.2, 0.4, 0.4\rangle, \langle Q_5, 0.2, 0.4, 0.4\rangle, \langle Q_6, 0.2, 0.4, 0.4\rangle, \langle Q_7, 0.2, 0.4, 0.4\rangle\}$.

$S_4 = \{\langle Q_1, 0.6, 0.4, 0\rangle, \langle Q_2, 0.6, 0.4, 0\rangle, \langle Q_3, 0.6, 0.4, 0\rangle, \langle Q_4, 0.6, 0.4, 0\rangle, \langle Q_5, 0.6, 0.4, 0\rangle, \langle Q_6, 0.6, 0.4, 0\rangle, \langle Q_7, 0.6, 0.4, 0\rangle\}$.

Assume that we give the clinical survey for t BPH patients by using Table 1 to obtain the t patients' responses of the BPH symptom which are represented by the form of truth, indeterminacy and falsity values. For a patient P_k (k = 1, 2, …, t) with SNS information, we can give the following evaluation/diagnosis method.

To give a proper evaluation/diagnosis for a patient P_k with the BPH symptom, we can calculate the similarity measure $W_q(P_k, S_i)$ for q = 1 or 2, i = 1, 2, 3, 4 and k = 1, 2, …, t. The proper BPH symptom evaluation S_{i^*} for the patient P_k is derived by $i^* = \arg\max_{1\le i\le 4}\{W_q(P_k, S_i)\}$.

To illustrate the evaluation/diagnosis process of the BPH symptom, we provide two evaluation/diagnosis examples on the BPH symptom to demonstrate the applications and effectiveness of the proposed evaluation/diagnosis method under simplified neutrosophic (single valued neutrosophic and interval neutrosophic) environments.

4.1 Initial evaluation of the BPH symptom under a single valued neutrosophic environment

In some case, we can obtain that data collected from the clinical convey of patients are single values rather than interval values. In this case, the exponential similarity measure of SVNSs is a better tool to give a proper initial evaluation of the BPH symptom for a patient.

Example 1. Assume that we give the clinical survey for three BPH patients by using Table 1, and then we can obtain the three patients' responses of the BPH symptom which are represented by the form of truth, indeterminacy and falsity values as shown in Table 3.

Table 3 BPH symptom responses (single values) in 5 times for three patients

Question	P_1			P_2			P_3		
	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity
Q_1	2/5	1/5	2/5	1/5	1/5	3/5	3/5	0/5	2/5
Q_2	2/5	2/5	1/5	2/5	1/5	2/5	3/5	1/5	1/5
Q_3	2/5	1/5	2/5	1/5	0/5	4/5	3/5	1/5	1/5

Q_4	2/5	1/5	2/5	2/5	1/5	2/5	4/5	1/5	0/5
Q_5	3/5	2/5	0/5	1/5	2/5	2/5	3/5	1/5	1/5
Q_6	2/5	0/5	3/5	2/5	0/5	3/5	4/5	1/5	0/5
Q_7	3/5	0/5	2/5	1/5	1/5	3/5	2/5	2/5	1/5

From Table 3, the BPH symptom responses of the patient P_k (k = 1, 2, 3) with respect to all the questions can be represented by the following SVNS information:

P_1 = {⟨Q_1, 0.4, 0.2, 0.4⟩, ⟨Q_2, 0.4, 0.4, 0.2⟩, ⟨Q_3, 0.4, 0.2, 0.4⟩, ⟨Q_4, 0.4, 0.2, 0.4⟩, ⟨Q_5, 0.6, 0.4, 0.0⟩, ⟨Q_6, 0.4, 0.0, 0.6⟩, ⟨Q_7, 0.6, 0.0, 0.4⟩},

P_2 = {⟨Q_1, 0.2, 0.2, 0.6⟩, ⟨Q_2, 0.4, 0.2, 0.4⟩, ⟨Q_3, 0.2, 0.0, 0.8⟩, ⟨Q_4, 0.4, 0.2, 0.4⟩, ⟨Q_5, 0.2, 0.4, 0.4⟩, ⟨Q_6, 0.4, 0.0, 0.6⟩, ⟨Q_7, 0.2, 0.2, 0.6⟩},

P_3 = {⟨Q_1, 0.6, 0.0, 0.4⟩, ⟨Q_2, 0.6, 0.2, 0.2⟩, ⟨Q_3, 0.6, 0.2, 0.2⟩, ⟨Q_4, 0.8, 0.2, 0.0⟩, ⟨Q_5, 0.6, 0.2, 0.2⟩, ⟨Q_6, 0.8, 0.2, 0.0⟩, ⟨Q_7, 0.4, 0.4, 0.2⟩}.

Assume that the weight of each element Q_j is w_j = 1/7 for j = 1, 2, …, 7. By applying Eq. (2), we can obtain the results of the similarity measure between the patient P_k (k = 1, 2, 3) and the considered symptom S_i (i = 1, 2, 3, 4), as shown in Table 4.

Table 4 Similarity measure values between P_k and S_i with SVNSs

	S_1	S_2	S_3	S_4
$W_1(P_1, S_i)$	0.4457	0.5460	**0.7285**	0.6857
$W_1(P_2, S_i)$	0.5896	0.7038	**0.7814**	0.5244
$W_1(P_3, S_i)$	0.3319	0.4406	0.6112	**0.7778**

In Table 4, the largest similarity measure indicates the proper evaluation/diagnosis. In clinical initial evaluations for the three patients, therefore, Patients P_1 and P_2 have moderate symptoms, and then Patient P_3 has severe symptom.

4.2 Initial evaluation of the BPH symptom under an interval neutrosophic environment

In some case, we can obtain that data collected from the clinical convey of patients are interval values rather than single values since patients easily express real situations by using the interval values. In this case, the exponential similarity measure of INSs is a better tool to give a proper initial evaluation of the BPH symptom.

Example 2. Assume that we give the clinical survey for three BPH patients by using Table 1, and then we can obtain the three patients' responses of the BPH symptom which are represented by the interval values of truth, indeterminacy and falsity, as shown in Table 5.

Table 5 BPH symptom responses (interval values) in 5 times for three patients

Question	P_1			P_2			P_3		
	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity
Q_1	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[0/5, 0/5]	[1/5, 2/5]
Q_2	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 2/5]	[0/5, 1/5]	[1/5, 2/5]	[3/5, 4/5]	[1/5, 1/5]	[0/5, 1/5]

Q_3	[2/5, 3/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[3/5, 4/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_4	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[2/5, 3/5]	[1/5, 2/5]	[0/5, 2/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_5	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]	[1/5, 2/5]	[2/5, 3/5]	[0/5, 1/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 1/5]
Q_6	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_7	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]

From Table 5, the BPH symptom responses of the patient P_k (k = 1, 2, 3) with respect to all the questions can be represented by the following INS information:

P_1 = {⟨Q_1, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_2, [0.2, 0.4], [0.4, 0.6], [0, 0.2]⟩, ⟨Q_3, [0.4, 0.6], [0.2, 0.4], [0.2, 0.4]⟩, ⟨Q_4, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_5, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_6, [0.4, 0.6], [0, 0.2], [0.4, 0.6]⟩, ⟨Q_7, [0.4, 0.6], [0, 0.2], [0.2, 0.4]⟩},

P_2 = {⟨Q_1, [0.2, 0.4], [0, 0.2],[0.4, 0.6]⟩, ⟨Q_2, [0.4, 0.4], [0, 0.2], [0.2, 0.4]⟩, ⟨Q_3, [0.2, 0.4], [0, 0.2], [0.6, 0.8]⟩, ⟨Q_4, [0.4, 0.6], [0.2, 0.4], [0, 0.4]⟩, ⟨Q_5, [0.2, 0.4], [0.4, 0.6], [0, 0.2]⟩, ⟨Q_6, [0.4, 0.6], [0, 0.2], [0.4,0.6]⟩, ⟨Q_7, [0.2, 0.4], [0.2, 0.4], [0.2, 0.4]⟩},

P_3 = {⟨Q_1, [0.6, 0.8], [0, 0], [0.2, 0.4]⟩, ⟨Q_2, [0.6, 0.8], [0.2, 0.2], [0, 0.2]⟩, ⟨Q_3, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_4, [0.6, 0.8], [0.2, 0.4], [0, 0]⟩, ⟨Q_5, [0.6, 0.8], [0.2, 0.4], [0, 0.2]⟩, ⟨Q_6, [0.6, 0.8,] [0.2, 0.4], [0, 0]⟩, ⟨Q_7, [0.4, 0.6], [0.4, 0.6], [0, 0.2]⟩}.

Assume that the weight of each element Q_j is w_j = 1/7 for j = 1, 2, …, 7. By applying Eq. (4), we can obtain the results of the similarity measure between the patient P_k (k = 1, 2, 3) and the considered symptom S_i (i = 1, 2, 3, 4), as shown in Table 6.

Table 6 Similarity measure values of between P_k and S_i with INSs

	S_1	S_2	S_3	S_4
$W_2(P_1, S_i)$	0.3956	0.4910	0.6784	**0.7164**
$W_2(P_2, S_i)$	0.4799	0.5831	**0.7331**	0.6297
$W_2(P_3, S_i)$	0.2718	0.3734	0.5741	**0.8322**

In Table 6, the largest similarity measure indicates the proper evaluation/diagnosis. In clinical initial evaluations for the three patients, therefore, Patients P_1 and P_3 have severe symptoms, and then Patient P_2 has moderate symptom.

Compared with the existing initial evaluation method based on I-PSS [1, 2], the proposed evaluation method demonstrates their effectiveness and rationality because the developed initial evaluation method with simplified neutrosophic information contain more evaluation information (truth, indeterminacy and falsity) than the existing initial evaluation method based on I-PSS (crisp values) [1, 2]. Therefore, the developed method is more suitable and more practical in the initial evaluation of BPH symptom and superior to the existing initial evaluation method [1, 2].

5. Conclusions

Based on exponential function, this paper proposed the exponential similarity measures of SNSs, including single valued neutrosophic exponential similarity measures and interval neutrosophic exponential similarity measures. Then, an initial evaluation/diagnosis method for BPH symptom was established based on the exponential similarity measures under a simplified neutrosophic environment. Finally, two illustrative examples on the initial evaluations of BPH symptom are provided to demonstrate the application and effectiveness of the proposed evaluation method. The advantage of the evaluation method developed in this paper is that it can deal with medical diagnosis problems with incomplete, uncertainty and inconsistent information, while the existing initial evaluation method [1, 2] cannot handle them.

In further work, it is necessary to apply the exponential similarity measures of SNSs to other medical problems such as medical decision making, medical image processing, and medical clustering analysis.

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Highlights

- We proposed exponential similarity measures of simplified neutrosophic sets.
- We established the similarity measures-based initial evaluation method for BPH.
- Two evaluation examples on the BPH symptom demonstrated the effectiveness.

Table 1 BPH symptom responses in 5 times for a patient P_k

Question	Truth	Indeterminacy	Falsity
Q_1: Over the past month, how often have you had a sensation of not emptying your bladder completely after you finished urinating?			
Q_2: Over the past month, how often have you had to urinating again less than two hours after you finished urinating?			
Q_3: Over the past month, how often have you found you stopped and started again several times when you urinated?			
Q_4: Over the past month, how often have you found it difficult to postpone urination?			
Q_5: Over the past month, how often have you had a week urinary stream?			
Q_6: Over the past month, how often have you had to push or strain to begin urination?			
Q_7: Over the past month, how many times did you most typically get up to urinate from the time you went to bed at night until the time you got up in the morning?			

Table 2 Four types of the BPH symptom with simplified neutrosophic information

S_i (Symptom type)	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7
S_1 (Normal symptom)	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>	<0, 0, 1>
S_2 (Mild symptom)	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>	<0, 0.2, 0.8>
S_3 (Moderate symptom)	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>	<0.2, 0.4, 0.4>
S_4 (Severe symptom)	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>	<0.6, 0.4, 0>

Table 3 BPH symptom responses (single values) in 5 times for three patients

Question	P_1			P_2			P_3		
	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity
Q_1	2/5	1/5	2/5	1/5	1/5	3/5	3/5	0/5	2/5
Q_2	2/5	2/5	1/5	2/5	1/5	2/5	3/5	1/5	1/5
Q_3	2/5	1/5	2/5	1/5	0/5	4/5	3/5	1/5	1/5
Q_4	2/5	1/5	2/5	2/5	1/5	2/5	4/5	1/5	0/5
Q_5	3/5	2/5	0/5	1/5	2/5	2/5	3/5	1/5	1/5
Q_6	2/5	0/5	3/5	2/5	0/5	3/5	4/5	1/5	0/5
Q_7	3/5	0/5	2/5	1/5	1/5	3/5	2/5	2/5	1/5

Table 4 Similarity measure values between P_k and S_i with SVNSs

	S_1	S_2	S_3	S_4
$W_1(P_1, S_i)$	0.4457	0.5460	**0.7285**	0.6857
$W_1(P_2, S_i)$	0.5896	0.7038	**0.7814**	0.5244
$W_1(P_3, S_i)$	0.3319	0.4406	0.6112	**0.7778**

Table 5 BPH symptom responses (interval values) in 5 times for three patients

Question	P_1			P_2			P_3		
	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity	Truth	Indeterminacy	Falsity
Q_1	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[0/5, 0/5]	[1/5, 2/5]
Q_2	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 2/5]	[0/5, 1/5]	[1/5, 2/5]	[3/5, 4/5]	[1/5, 1/5]	[0/5, 1/5]
Q_3	[2/5, 3/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[0/5, 1/5]	[3/5, 4/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_4	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[2/5, 3/5]	[1/5, 2/5]	[0/5, 2/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_5	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]	[1/5, 2/5]	[2/5, 3/5]	[0/5, 1/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 1/5]
Q_6	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]	[2/5, 3/5]	[3/5, 4/5]	[1/5, 2/5]	[0/5, 0/5]
Q_7	[2/5, 3/5]	[0/5, 1/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[1/5, 2/5]	[2/5, 3/5]	[2/5, 3/5]	[0/5, 1/5]

Table 6 Similarity measure values of between P_k and S_i with INSs

	S_1	S_2	S_3	S_4
$W_2(P_1, S_i)$	0.3956	0.4910	0.6784	**0.7164**
$W_2(P_2, S_i)$	0.4799	0.5831	**0.7331**	0.6297
$W_2(P_3, S_i)$	0.2718	0.3734	0.5741	**0.8322**