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Soft Neutrosophic Loops and Their Generalization

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Abstract. Soft set theory is a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. In this paper we introduced soft neutrosophic loop,soft neutosophic biloop, soft neutrosophic N -loop with the discuission of some of their characteristics. We also introduced a new type of soft neutrophic loop, the so called soft strong neutrosophic loop which is of pure neutrosophic character. This notion also found in all the other corresponding notions of soft neutrosophic thoery. We also given some of their properties of this newly born soft structure related to the strong part of neutrosophic theory.

Keywords: Neutrosophic loop, neutrosophic biloop, neutrosophic N-loop, soft set, soft neutrosophic loop,soft neutrosophic biloop, soft neutrosophic N-loop.

1 Introduction

Florentin Smarandache for the first time intorduced the concept of neutrosophy in 1995, which is basically a new branch of philosophy which actually studies the origion, nature, and scope of neutralities. The neutrosophic logic came into being by neutrosophy. In neutrosophic logic each proposition is approximated to have the percentage of truth in a subset T , the percentage of indeterminacy in a subset I , and the percentage of falsity in a subset F . Neutrosophic logic is an extension of fuzzy logic. In fact the neutrosophic set is the generalization of classical set, fuzzy conventional set, intuitionistic fuzzy set, and interal valued fuzzy set. Neutrosophic logic is used to overcome the problems of imperciseness, indeterminate, and inconsistentness of date etc. The theoy of neutrosophy is so applicable to every field of agebra. W.B Vasantha Kandasamy and Florentin Smarandache introduced neutrosophic fields, neutrosophic rings, neutrosophic vectorspaces, neutrosophic groups, neutrosophic bigroups and neutrosophic N -groups, neutrosophic semigroups, neutrosophic bisemigroups, and neutrsosophic N -semigroups, neutrosophic loops, nuetrosophic biloops, and neutrosophic N -loops, and so on. Mumtaz ali et.al. introduced nuetosophic LA -semigoups.

Molodtsov intorduced the theory of soft set. This mathematical tool is free from parameterization inadequacy, syndrome of fuzzy set theory, rough set theory, probability theory and so on. This theory has been applied successfully in many fields such as smoothness of functions, game theory, operation reaserch, Riemann integration, Perron integration, and probability. Recently soft set theory attained much attention of the researchers since its appearance and the work based on several operations of soft set introduced in $\left[2,9,10\right]$. Some properties and algebra may be found in $\left[1\right]$. Feng et.al. introduced soft semirings in $\left[5\right]$. By means of level soft sets an adjustable approach to fuzzy soft set can be seen in $\left[6\right]$. Some other concepts together with fuzzy set and rough set were shown in $\left[7,8\right]$.

This paper is about to introduced soft nuetrosophic loop, soft neutrosphic biloop, and soft neutrosophic N -loop and the related strong or pure part of neutrosophy with the notions of soft set theory. In the proceeding section, we define soft neutrosophic loop, soft neutrosophic strong loop, and some of their properties are discuissed. In the next section, soft neutrosophic biloop are presented with their strong neutrosophic part. Also in this section some of their characterization have been made. In the last section soft neutrosophic N -loop and their coresponding strong theory have been constructed with some of their properties.

2 Fundamental Concepts

Neutrosophic Loop

Definition 1. A neutrosophic loop is generated by a loop L and I denoted by $\langle L \cup I \rangle$. A neutrosophic loop in general need not be a loop for $I^2 = I$ and I may not have an inverse but every element in a loop has an inverse.

Definition 2. Let $\langle L \cup I \rangle$ be a neutrosophic loop. A proper subset $\langle P \cup I \rangle$ of $\langle L \cup I \rangle$ is called the neutrosophic subloop, if $\langle P \cup I \rangle$ is itself a neutrosophic loop under the operations of $\langle L \cup I \rangle$.

Definition 3. Let $(\langle L \cup I \rangle, \circ)$ be a neutrosophic loop of finite order. A proper subset P of $\langle L \cup I \rangle$ is said to be Lagrange neutrosophic subloop, if P is a neutrosophic subloop under the operation $\circ$ and $o(P) / o\langle L \cup I \rangle$.

Definition 4. If every neutrosophic subloop of $\langle L \cup I \rangle$ is Lagrange then we call $\langle L \cup I \rangle$ to be a Lagrange neutrosophic loop.

Definition 5. If $\langle L \cup I \rangle$ has no Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ to be a Lagrange free neutrosophic loop.

Definition 6. If $\langle L \cup I \rangle$ has atleast one Lagrange neutrosophic subloop then we call $\langle L \cup I \rangle$ to be a weakly Lagrange neutrosophic loop.

Neutrosophic Biloops

Definition 6. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a non-empty neutrosophic set with two binary operations $*_1, *_2$, $\langle B \cup I \rangle$ is a neutrosophic biloop if the following conditions are satisfied.

1. $\langle B \cup I \rangle = P_1 \cup P_2$ where P_1 and P_2 are proper subsets of $\langle B \cup I \rangle$.
2. $(P_1, *_1)$ is a neutrosophic loop.
3. $(P_2, *_2)$ is a group or a loop.

Definition 7. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop. A proper subset P of $\langle B \cup I \rangle$ is said to be a neutrosophic subbiloop of $\langle B \cup I \rangle$ if $(P_1, *_1, *_2)$ is itself a neutrosophic biloop under the operations of $\langle B \cup I \rangle$.

Definition 8. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a finite neutrosophic biloop. Let $P = (P_1 \cup P_2, *_1, *_2)$ be a neutrosophic biloop. If $o(P) / o(B)$ then we call P, a Lagrange neutrosophic subbiloop of B.

Definition 9. If every neutrosophic subbiloop of B is Lagrange then we call B to be a Lagrange neutrosophic biloop.

Definition 10. If B has atleast one Lagrange neutrosophic subbiloop then we call B to be a weakly Lagrange neutrosophic biloop.

Definition 11. If B has no Lagrange neutrosophic subbiloops then we call B to be a Lagrange free neutrosophic biloop.

Neutrosophic N-loop

Definition 12. Let $S(B) = \{S(B_1) \cup S(B_2) \cup ... \cup S(B_n), *_1, *_2, ..., *_N\}$ be a non-empty neutrosophic set with N-binary operations. $S(B)$ is a neutrosophic N-loop if $S(B) = S(B_1) \cup S(B_2) \cup ... \cup S(B_n)$, $S(B_i)$ are proper subsets of $S(B)$ for $1 \le i \le N$ and some of $S(B_i)$ are neutrosophic loops and some of the $S(B_i)$ are groups.

Definition 13. Let $S(B) = \{S(B_1) \cup S(B_2) \cup ... \cup S(B_n), *_1, *_2, ..., *_N\}$ be a neutrosophic N-loop. A proper subset $(P, *_1, *_2, ..., *_N)$ of $S(B)$ is said to be a neutrosophic sub N-loop of $S(B)$ if P itself is a neutrosophic N-loop under the operations of $S(B)$.

Definition 14. Let $(L = L_1 \cup L_2 \cup ... \cup L_N, *_1, *_2, ..., *_N)$ be a neutrosophic N-loop of finite order. Suppose P is a proper subset of L, which is a neutrosophic sub N-loop. If $o(P) / o(L)$ then we call P a Lagrange neutrosophic sub N-loop.

Definition 15. If every neutrosophic sub N-loop is Lagrange then we call L to be a Lagrange neutrosophic N-loop.

Definition 16. If L has atleast one Lagrange neutrosophic sub N -loop then we call L to be a weakly Lagrange neutrosophic N -loop.

Definition 17. If L has no Lagrange neutrosophic sub N -loop then we call L to be a Lagrange free neutrosophic N -loop.

Soft Sets

Throughout this subsection U refers to an initial universe, E is a set of parameters, $P(U)$ is the power set of U, and $A, B \subset E$. Molodtsov defined the soft set in the following manner:

Definition 7. A pair (F, A) is called a soft set over U where F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $a \in A$, $F(\mathrm{a})$ may be considered as the set of a -elements of the soft set (F, A), or as the set of a -approximate elements of the soft set.

Example 1. Suppose that U is the set of shops. E is the set of parameters and each parameter is a word or sentence. Let

$$E = \begin{Bmatrix} \text{high rent,normal rent,} \\ \text{in good condition,in bad condition} \end{Bmatrix}.$$

Let us consider a soft set (F, A) which describes the attractiveness of shops that Mr. Z is taking on rent. Suppose that there are five houses in the universe $U = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration, and that $A = \{a_1, a_2, a_3\}$ be the set of parameters where

a_1 stands for the parameter 'high rent,

a_2 stands for the parameter 'normal rent,

a_3 stands for the parameter 'in good condition.

Suppose that

$$F(a_1) = \{s_1, s_4\},$$
$$F(a_2) = \{s_2, s_5\},$$
$$F(a_3) = \{s_3\}.$$

The soft set (F, A) is an approximated family $\{F(a_i), i = 1, 2, 3\}$ of subsets of the set U which gives us a collection of approximate description of an object. Then (F, A) is a soft set as a collection of approximations over U, where

$$F(a_1) = high\ rent = \{s_1, s_2\},$$
$$F(a_2) = normal\ rent = \{s_2, s_5\},$$
$$F(a_3) = in\ good\ condition = \{s_3\}.$$

Definition 8. For two soft sets (F, A) and (H, C) over U, (F, A) is called a soft subset of (H, C) if

1. $A \subseteq C$ and
2. $F(a) \subseteq H(a)$, for all $x \in A$.

This relationship is denoted by $(F, A) \subset (H, \mathrm{C})$. Similarly (F, A) is called a soft superset of (H, C) if (H, C) is a soft subset of (F, A) which is denoted by $(F, A) \supset (H, \mathrm{C})$.

Definition 9. Two soft sets (F, A) and (H, C) over U are called soft equal if (F, A) is a soft subset of (H, C) and (H, C) is a soft subset of (F, A).

Definition 10. Let (F, A) and (K, C) be two soft sets over a common universe U such that $A \cap C \neq \phi$. Then their restricted intersection is denoted by $(F, A) \cap_R (K, \mathrm{C}) = (H, \mathrm{D})$ where (H, D) is defined as $H(c) = F(c) \cap \mathrm{K}(c)$ for all $c \in \mathrm{D} = A \cap C$.

Definition 11. The extended intersection of two soft sets (F, A) and (K, C) over a common universe U is the soft set (H, D), where $D = A \cup C$, and for all $c \in C$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - C, \\ \mathrm{K}(c) & \text{if } c \in C - A, \\ F(c) \cap \mathrm{K}(c) & \text{if } c \in A \cap C. \end{cases}$$

We write $(F, A) \cap_\varepsilon (K, C) = (H, \mathrm{D})$.

Definition 12. The restricted union of two soft sets (F, A) and (K, C) over a common universe U is the soft set (H, D), where $D = A \cup C$, and for all $c \in D$, $H(c)$ is defined as $H(c) = F(c) \cup \mathrm{K}(c)$ for all $c \in D$. We write it as

$(F,A) \cup_R (K,C) = (H,D)$.

Definition 13. The extended union of two soft sets (F,A) and (K,C) over a common universe U is the soft set (H,D), where $D = A \cup C$, and for all $c \in D$, $H(c)$ is defined as

$$H(c) = \begin{cases} F(c) & \text{if } c \in A - C, \\ K(c) & \text{if } c \in C - A, \\ F(c) \cup K(c) & \text{if } c \in A \cap C. \end{cases}$$

We write $(F,A) \cup_\varepsilon (K,C) = (H,D)$.

3 Soft Neutrosophic Loop

Definition 14. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L \cup I \rangle$. Then (F,A) is called soft neutrosophic loop if and only if $F(a)$ is neutrosophic subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Example 2. Let $\langle L \cup I \rangle = \langle L_7(4) \cup I \rangle$ be a neutrosophic loop where $L_7(4)$ is a loop. Then (F,A) is a soft neutrosophic loop over $\langle L \cup I \rangle$, where

$$F(a_1) = \{\langle e, eI, 2, 2I \rangle\}, F(a_2) = \{\langle e, 3 \rangle\},$$
$$F(a_3) = \{\langle e, eI \rangle\}.$$

Theorem 1. Every soft neutrosophic loop over $\langle L \cup I \rangle$ contains a soft loop over L.

Proof. The proof is straightforward.

Theorem 2. Let (F,A) and (H,A) be two soft neutrosophic loops over $\langle L \cup I \rangle$. Then their intersection $(F,A) \cap (H,A)$ is again soft neutrosophic loop over $\langle L \cup I \rangle$.

Proof. The proof is staightforward.

Theorem 3. Let (F,A) and (H,C) be two soft neutrosophic loops over $\langle L \cup I \rangle$. If $A \cap C = \phi$, then $(F,A) \cup (H,C)$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 1. The extended union of two soft neutrosophic loops (F,A) and (K,C) over $\langle L \cup I \rangle$ is not a soft neutrosophic loop over $\langle L \cup I \rangle$.

With the help of example we can easily check the above remark.

Proposition 1. The extended intersection of two soft neutrosophic loopps over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 2. The restricted union of two soft neutrosophic loops (F,A) and (K,C) over $\langle L \cup I \rangle$ is not a soft neutrosophic loop over $\langle L \cup I \rangle$.

One can easily check it by the help of example.

Proposition 2. The restricted intersection of two soft neutrosophic loops over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Proposition 3. The AND operation of two soft neutrosophic loops over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$.

Remark 3. The OR operation of two soft neutosophic loops over $\langle L \cup I \rangle$ may not be a soft nuetrosophic loop over $\langle L \cup I \rangle$.

Definition 15. Let $\langle L_n(m) \cup I \rangle = \{e, 1, 2, ..., n, eI, 1I, 2I, ..., nI\}$ be a new class of neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L_n(m) \cup I \rangle$. Then (F,A) is called soft new class neutrosophic loop if $F(a)$ is a neutrosophic subloop of $\langle L_n(m) \cup I \rangle$ for all $a \in A$.

Example 3. Let

$\langle L_5(3)\cup I\rangle = \{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a new class of neutrosophic loop. Let $A=\{a_1,a_2,a_3,a_4,a_5\}$ be a set of parameters. Then (F,A) is soft new class neutrosophic loop over $\langle L_5(3)\cup I\rangle$, where

$$F(a_1)=\{e,eI,1,1I\}, F(a_2)=\{e,eI,2,2I\},$$
$$F(a_3)=\{e,eI,3,3I\}, F(a_3)=\{e,eI,4,4I\},$$
$$F(a_5)=\{e,eI,5,5I\}.$$

Theorem 4. Every soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$ is a soft neutrosophic loop over $\langle L_n(m)\cup I\rangle$ but the converse is not true.

Proposition 4. Let (F,A) and (K,C) be two soft new class neutrosophic loops over $\langle L_n(m)\cup I\rangle$. Then

1) Their extended intersection $(F,A)\cap_E(K,C)$ is a soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$.
2) Their restricted intersection $(F,A)\cap_R(K,C)$ is a soft new classes neutrosophic loop over $\langle L_n(m)\cup I\rangle$.
3) Their AND operation $(F,A)\wedge(K,C)$ is a soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$.

Remark 4. Let (F,A) and (K,C) be two soft new class neutrosophic loops over $\langle L_n(m)\cup I\rangle$. Then

1) Their extended union $(F,A)\cup_E(K,C)$ is not a soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$.
2) Their restricted union $(F,A)\cup_R(K,C)$ is not a soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$.
3) Their OR operation $(F,A)\vee(K,C)$ is not a soft new class neutrosophic loop over $\langle L_n(m)\cup I\rangle$.

One can easily verify $(1),(2)$, and (3) by the help of examples.

Definition 16. Let (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called the identity soft neutrosophic loop over $\langle L\cup I\rangle$ if $F(a)=\{e\}$ for all $a\in A$, where e is the identity element of $\langle L\cup I\rangle$.

Definition 17. Let (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called an absolute soft neutrosophic loop over $\langle L\cup I\rangle$ if $F(a)=\langle L\cup I\rangle$ for all $a\in A$.

Definition 18. Let (F,A) and (H,C) be two soft neutrosophic loops over $\langle L\cup I\rangle$. Then (H,C) is callsed soft neutrosophic subloop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic subloop of $F(a)$ for all $a\in A$.

Example 4. Consider the neutrosophic loop

$$\langle L_{15}(2)\cup I\rangle=\{e,1,2,3,4,...,15,eI,1I,2I,...,14I,15I\}$$

of order 32. Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then (F,A) is a soft neutrosophic loop over $\langle L_{15}(2)\cup I\rangle$, where

$$F(a_1)=\{e,2,5,8,11,14,eI,2I,5I,8I,11I,14I\},$$
$$F(a_2)=\{e,2,5,8,11,14\},$$
$$F(a_3)=\{e,3,eI,3I\}.$$

Thus (H,C) is a soft neutrosophic subloop of (F,A) over $\langle L_{15}(2)\cup I\rangle$, where

$$H(a_1)=\{e,eI,2I,5I,8I,11I,14I\},$$
$$H(a_2)=\{e,3\}.$$

Theorem 5. Every soft loop over L is a soft neutrosophic subloop over $\langle L\cup I\rangle$.

Definition 19. Let $\langle L\cup I\rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft normal neutrosophic loop if and only if $F(a)$ is normal neutrosophic subloop of $\langle L\cup I\rangle$ for all

$a \in A$.

Example 5. Let $\langle L_5(3) \cup I \rangle = \{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\}$ be a neutrosophic loop. Let $A = \{a_1, a_2, a_3\}$ be a set of parameters. Then clearly (F, A) is soft normal neutrosophic loop over $\langle L_5(3) \cup I \rangle$, where

$$F(a_1) = \{e, eI, 1, 1I\}, F(a_2) = \{\mathrm{e}, \mathrm{eI}, 2, 2\mathrm{I}\},$$
$$F(a_3) = \{\mathrm{e}, \mathrm{eI}, 3, 3\mathrm{I}\}.$$

Theorem 6. Every soft normal neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Proposition 5. Let (F, A) and (K, C) be two soft normal neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ is a soft normal neutrosophic loop over $\langle L \cup I \rangle$.

Remark 5. Let (F, A) and (K, C) be two soft normal neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted union $(F, A) \cup_R (K, \mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.
3. Their OR operation $(F, A) \vee (K, \mathrm{C})$ is not a soft normal neutrosophic loop over $\langle L \cup I \rangle$.

One can easily verify $(1), (2)$, and (3) by the help of examples.

Definition 20. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F, A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F, A) is called soft Lagrange neutrosophic loop if $F(a)$ is a Lagrange neutrosophic subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Example 6. In Example (1), (F, A) is a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.

Theorem 7. Every soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 8. If $\langle L \cup I \rangle$ is a Lagrange neutrosophic loop, then (F, A) over $\langle L \cup I \rangle$ is a soft Lagrange neutrosophic loop but the converse is not true.

Remark 6. Let (F, A) and (K, C) be two soft Lagrange neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F, A) \cap_E (K, \mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F, A) \cap_R (K, \mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F, A) \wedge (K, \mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
4. Their extended union $(F, A) \cup_E (K, \mathrm{C})$ is not a soft Lagrnage neutrosophic loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F, A) \cup_R (K, \mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F, A) \vee (K, \mathrm{C})$ is not a soft Lagrange neutrosophic loop over $\langle L \cup I \rangle$.

One can easily verify $(1), (2), (3), (4), (5)$ and (6) by the help of examples.

Definition 21. Let $\langle L \cup I \rangle$ be a neutrosophic loop and

(F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft weak Lagrange neutrosophic loop if atleast one $F(a)$ is not a Lagrange neutrosophic subloop of $\langle L \cup I \rangle$ for some $a \in A$.

Example 7. Consider the neutrosophic loop

$$\langle L_{15}(2) \cup I \rangle = \{e,1,2,3,4,...,15,eI,1I,2I,...,14I,15I\}$$

of order 32. Let $A=\{a_1,a_2,a_3\}$ be a set of parameters. Then (F,A) is a soft weakly Lagrange neutrosophic loop over $\langle L_{15}(2) \cup I \rangle$, where

$$F(a_1)=\{e,2,5,8,11,14,eI,2I,5I,8I,11I,14I\},$$
$$F(a_2)=\{e,2,5,8,11,14\},$$
$$F(a_3)=\{e,3,eI,3I\}.$$

Theorem 9. Every soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 10. If $\langle L \cup I \rangle$ is weak Lagrange neutrosophic loop, then (F,A) over $\langle L \cup I \rangle$ is also soft weak Lagrange neutrosophic loop but the converse is not true.

Remark 7. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft weak Lagrnage neutrosophic loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A) \vee (K,C)$ is not a soft weak Lagrange neutrosophic loop over $\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 22. Let $\langle L \cup I \rangle$ be a neutrosophic loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft Lagrange free neutrosophic loop if $F(a)$ is not a lagrange neutrosophic subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Theorem 11. Every soft Lagrange free neutrosophic loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 12. If $\langle L \cup I \rangle$ is a Lagrange free neutrosophic loop, then (F,A) over $\langle L \cup I \rangle$ is also a soft Lagrange free neutrosophic loop but the converse is not true.

Remark 8. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft Lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft Lagrange soft neutrosophic loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft Lagrnage soft neutrosophic loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L \cup I \rangle$.

6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

4 Soft Neutrosophic Strong Loop

Definition 23. Let $\langle L\cup I\rangle$ be a neutrosophic loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft neutrosophic strong loop if and only if $F(a)$ is a strong neutrosophic subloop of $\langle L\cup I\rangle$ for all $a\in A$.

Proposition 6. Let (F,A) and (K,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is a soft neutrosophic strong loop over $\langle L\cup I\rangle$.

Remark 9. Let (F,A) and (K,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended union $(F,A)\cup_E(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic strong loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 24. Let (F,A) and (H,C) be two soft neutrosophic strong loops over $\langle L\cup I\rangle$. Then (H,C) is called soft neutrosophic strong subloop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong subloop of $F(a)$ for all $a\in A$.

Definition 25. Let $\langle L\cup I\rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange neutrosophic strong loop if $F(a)$ is a Lagrange neutrosophic strong subloop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 13. Every soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$ is a soft neutrosophic loop over $\langle L\cup I\rangle$ but the converse is not true.

Theorem 14. If $\langle L\cup I\rangle$ is a Lagrange neutrosophic strong loop, then (F,A) over $\langle L\cup I\rangle$ is a soft Lagrange neutrosophic loop but the converse is not true.

Remark 10. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong loops over $\langle L\cup I\rangle$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange strong neutrosophic loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage neutrosophic strong loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange neutrosophic strong loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic strong loop over

$\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 26. Let $\langle L \cup I \rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft weak Lagrange neutrosophic strong loop if atleast one $F(a)$ is not a Lagrange neutrosophic strong subloop of $\langle L \cup I \rangle$ for some $a \in A$.

Theorem 15. Every soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 16. If $\langle L \cup I \rangle$ is weak Lagrange neutrosophic strong loop, then (F,A) over $\langle L \cup I \rangle$ is also soft weak Lagrange neutrosophic strong loop but the converse is not true.

Remark 11. Let (F,A) and (K,C) be two soft weak Lagrange neutrosophic strong loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft weak Lagrnage neutrosophic strong loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A) \vee (K,C)$ is not a soft weak Lagrange neutrosophic strong loop over $\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 27. Let $\langle L \cup I \rangle$ be a neutrosophic strong loop and (F,A) be a soft neutrosophic loop over $\langle L \cup I \rangle$. Then (F,A) is called soft Lagrange free neutrosophic strong loop if $F(a)$ is not a lagrange neutrosophic strong subloop of $\langle L \cup I \rangle$ for all $a \in A$.

Theorem 17. Every soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$ is a soft neutrosophic loop over $\langle L \cup I \rangle$ but the converse is not true.

Theorem 18. If $\langle L \cup I \rangle$ is a Lagrange free neutrosophic strong loop, then (F,A) over $\langle L \cup I \rangle$ is also a soft Lagrange free neutrosophic strong loop but the converse is not true.

Remark 12. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong loops over $\langle L \cup I \rangle$. Then

1. Their extended intersection $(F,A) \cap_E (K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
2. Their restricted intersection $(F,A) \cap_R (K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
3. Their AND operation $(F,A) \wedge (K,C)$ is not a soft Lagrange free neutrosophic strong loop over $\langle L \cup I \rangle$.
4. Their extended union $(F,A) \cup_E (K,C)$ is not a soft Lagrnage free strong neutrosophic strong loop over $\langle L \cup I \rangle$.
5. Their restricted union $(F,A) \cup_R (K,C)$ is not a soft Lagrange free neutrosophic loop over $\langle L \cup I \rangle$.
6. Their OR operation $(F,A) \vee (K,C)$ is not a soft Lagrange free neutrosophic strong loop over

$\langle L \cup I \rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Biloop

Definition 27. Let $(\langle B \cup I \rangle, *_1, *_2)$ be a neutrosophic biloop and (F, A) be a soft set over $(\langle B \cup I \rangle, *_1, *_2)$. Then (F, A) is called soft neutrosophic biloop if and only if $F(a)$ is a neutrosophic subbiloop of $(\langle B \cup I \rangle, *_1, *_2)$ for all $a \in A$.

Example 8. Let $(\langle B \cup I \rangle, *_1, *_2) = (\{e,1,2,3,4,5,eI,1I,2I,3I,4I,5I\} \cup \{g : g^6 = e\}$ be a neutrosophic biloop. Let $A = \{a_1, a_2\}$ be a set of parameters. Then (F, A) is clearly soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$, where

$$F(a_1) = \{e, 2, eI, 2I\} \cup \{g^2, g^4, e\},$$
$$F(a_2) = \{e, 3, eI, 3I\} \cup \{g^3, e\}.$$

Theorem 19. Let (F, A) and (H, A) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then their intersection $(F, A) \cap (H, A)$ is again a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Proof. Straightforward.

Theorem 20. Let (F, A) and (H, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$ such that $A \cap C = \phi$. Then their union is soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Proof. Straightforward.

Proposition 7. Let (F, A) and (K, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
3. Their AND operation $(F, A) \wedge (K, C)$ is a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

Remark 13. Let (F, A) and (K, C) be two soft neutrosophic biloops over $(\langle B \cup I \rangle, *_1, *_2)$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.
3. Their OR operation $(F, A) \vee (K, C)$ is not a soft neutrosophic biloop over $(\langle B \cup I \rangle, *_1, *_2)$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 28. Let $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop and (F, A) be a soft set over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is called soft new class neutrosophic subbiloop if and only if $F(a)$ is a neutrosophic subbiloop of $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ for all $a \in A$.

Example 9. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a new class neutrosophic biloop, where $B_1 = \langle L_5(3) \cup I \rangle = \{e,1,2,3,4,5,eI,2I,3I,4I,5I\}$ be a new class of neutrosophic loop and $B_2 = \{g : g^{12} = e\}$ is a group.

$\{e, eI, 1, 1I\} \cup \{1, g^6\}$,
$\{e, eI, 2, 2I\} \cup \{1, g^2, g^4, g^6, g^8, g^{10}\}$,
$\{e, eI, 3, 3I\} \cup \{1, g^3, g^6, g^9\}$,

$\{e, eI, 4, 4I\} \cup \{1, g^4, g^8\}$ are neutrosophic subloops of B. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then (F, A) is soft new class neutrosophic biloop over B, where

$$F(a_1) = \{e, eI, 1, 1I\} \cup \{e, g^6\},$$
$$F(a_2) = \{e, eI, 2, 2I\} \cup \{e, g^2, g^4, g^6, g^8, g^{10}\},$$
$$F(a_3) = \{e, eI, 3, 3I\} \cup \{e, g^3, g^6, g^6\},$$
$$F(a_4) = \{e, eI, 4, 4I\} \cup \{e, g^4, g^8\}.$$

Theorem 21. Every soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$ is trivially a soft neutrosophic biloop over but the converse is not true.

Proposition 8. Let (F, A) and (K, C) be two soft new class neutrosophic biloops over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then

1. Their extended intersection $(F, A) \cap_E (K, C)$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
2. Their restricted intersection $(F, A) \cap_R (K, C)$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
3. Their AND operation $(F, A) \wedge (K, C)$ is a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.

Remark 14. Let (F, A) and (K, C) be two soft new class neutrosophic biloops over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$. Then

1. Their extended union $(F, A) \cup_E (K, C)$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
2. Their restricted union $(F, A) \cup_R (K, C)$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.
3. Their OR operation $(F, A) \vee (K, C)$ is not a soft new class neutrosophic biloop over $B = (\langle L_n(m) \cup I \rangle \cup B_2, *_1, *_2)$.

One can easily verify $(1), (2)$, and (3) by the help of examples.

Definition 29. Let (F, A) be a soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (F, A) is called the identity soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ if $F(a) = \{e_1, e_2\}$ for all $a \in A$, where e_1, e_2 are the identities of $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ respectively.

Definition 30. Let (F, A) be a soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (F, A) is called an absolute-soft neutrosophic biloop over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ if $F(a) = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$ for all $a \in A$.

Definition 31. Let (F, A) and (H, C) be two soft neutrosophic biloops over $B = (\langle B_1 \cup I \rangle \cup B_2, *_1, *_2)$. Then (H, C) is called soft neutrosophic subbiloop of (F, A), if

1. $C \subseteq A$.
2. $H(a)$ is a neutrosophic subbiloop of $F(a)$ for all $a \in A$.

Example 10. Let $B = (B_1 \cup B_2, *_1, *_2)$ be a neutrosophic biloop, where $B_1 = \langle L_5(3) \cup I \rangle = \{e, 1, 2, 3, 4, 5, eI, 2I, 3I, 4I, 5I\}$ be a new class of neutrosophic loop and $B_2 = \{g : g^{12} = e\}$ is a group. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of parameters. Then (F, A) is soft neutrosophic biloop over B, where

$$F(a_1) = \{e, eI, 1, 1I\} \cup \{e, g^6\},$$
$$F(a_2) = \{e, eI, 2, 2I\} \cup \{e, g^2, g^4, g^6, g^8, g^{10}\},$$
$$F(a_3) = \{e, eI, 3, 3I\} \cup \{e, g^3, g^6, g^6\},$$
$$F(a_4) = \{e, eI, 4, 4I\} \cup \{e, g^4, g^8\}.$$

Then (H, C) is soft neutrosophic subbiloop of (F, A), where

$$H(a_1)=\{e,2)\cup\{e,g^2\},$$
$$H(a_2)=\{e,eI,3,3I\}\cup\{e,g^6\}.$$

Definition 32. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic biloop if and only if $F(a)$ is Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for all $a\in A$.

Example 11. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft Lagrange soft neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{e,eI,2,2I\}\cup\{e\},$$
$$F(a_2)=\{e,eI,3,3I\}\cup\{e\}.$$

Theorem 22. Every soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Remark 15. Let (F,A) and (K,C) be two soft Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 33. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not a Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for some $a\in A$.

Example 12. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft weakly Lagrange neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{e,eI,2,2I\}\cup\{e\},$$
$$F(a_2)=\{e,eI,3,3I\}\cup\{e,g^4\}.$$

Theorem 23. Every soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 24. If $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a weakly Lagrange neutrosophic biloop, then (F,A) over B is also soft weakly Lagrange neutrosophic biloop but the converse is not holds.

Remark 16. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$

is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 34. Let $(\langle B\cup I\rangle,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $(\langle B\cup I\rangle,*_1,*_2)$. Then (F,A) is called soft Lagrange free neutrosophic biloop if and only if $F(a)$ is not a Lagrange neutrosophic subbiloop of $(\langle B\cup I\rangle,*_1,*_2)$ for all $a\in A$.

Example 13. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop of order 20, where $B_1=\langle L_5(3)\cup I\rangle$ and $B_2=\{g:g^8=e\}$. Then clearly (F,A) is a soft Lagrange free neutrosophic biloop over $(\langle B\cup I\rangle,*_1,*_2)$, where

$$F(a_1)=\{e,eI,2,2I\}\cup\{e,g^2,g^4,g^6\},$$
$$F(a_2)=\{e,eI,3,3I\}\cup\{e,g^4\}.$$

Theorem 25. Every soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 26. If $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$ is a Lagrange free neutrosophic biloop, then (F,A) over B is also soft Lagrange free neutrosophic biloop but the converse is not holds.

Remark 17. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic biloops over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic biloop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Strong Biloop

Definition 35. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop where B_1 is a neutrosopphic biloop and B_2 is a neutrosophic group and (F,A) be soft set over B. Then (F,A) over B is called soft neutrosophic strong biloop if and only if $F(a)$ is a neutrosopchic strong subbiloop of B for all $a\in A$.

Example 14. Let $B=(B_1\cup B_2,*_1,*_2)$ where $B_1=\langle L_5(2)\cup I\rangle$ is a neutrosophic loop and $B_2=\{0,1,2,3,4,,1I,2I,3I,4I\}$ under multiplication modulo 5 is a neutrosophic group. Let $A=\{a_1,a_2\}$ be a set of parameters. Then (F,A) is soft neutrosophic strong biloop over B, where

$$F(a_1)=\{e,2,eI,2I\}\cup\{1,I,4I\},$$
$$F(a_2)=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{1,\mathrm{I},4\mathrm{I}\}.$$

Theorem 27. Every soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 28. If $B=(B_1\cup B_2,*_1,*_2)$ is a neutrosophic strong biloop, then (F,A) over B is also soft neutrosophic strong biloop but the converse is not holds.

Proposition 9. Let (F,A) and (K,C) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

Remark 18. Let (F,A) and (K,B) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 36. Let (F,A) and (H,C) be two soft neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then (H,C) is called soft neutrosophic strong subbiloop of (F,A), if

3. $C\subseteq A$.
4. $H(a)$ is a neutrosophic strong subbiloop of $F(a)$ for all $a\in A$.

Definition 37. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1\cup B_2,*_1,*_2)$. Then (F,A) is called soft Lagrange neutrosophic strong biloop if and only if $F(a)$ is a Lagrange neutrosophic strong subbiloop of $B=(B_1\cup B_2,*_1,*_2)$ for all $a\in A$.

Theorem 29. Every soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Remark 19. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrnage neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 38. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1\cup B_2,*_1,*_2)$. Then (F,A) is called soft weakly Lagrange neutrosophic strong biloop if atleast one $F(a)$ is not a Lagrange neutrosophic strong subbiloop of $B=(B_1\cup B_2,*_1,*_2)$ for some $a\in A$.

Theorem 30. Every soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 31. If $B=(B_1\cup B_2,*_1,*_2)$ is a weakly Lagrange neutrosophic strong biloop, then (F,A) over B is also soft weakly Lagrange neutrosophic strong biloop but the converse does not holds.

Remark 20. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 39. Let $B=(B_1\cup B_2,*_1,*_2)$ be a neutrosophic biloop and (F,A) be a soft set over $B=(B_1\cup B_2,*_1,*_2)$. Then (F,A) is called soft Lagrange free neutrosophic strong biloop if and only if $F(a)$ is not a Lagrange neutrosophic subbiloop of $B=(B_1\cup B_2,*_1,*_2)$ for all $a\in A$.

Theorem 32. Every soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$ is a soft neutrosophic biloop but the converse is not true.

Theorem 33. If $B=(B_1\cup B_2,*_1,*_2)$ is a Lagrange free neutrosophic strong biloop, then (F,A) over B is also soft strong lagrange free neutrosophic strong biloop but the converse is not true.

Remark 21. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong biloops over $B=(B_1\cup B_2,*_1,*_2)$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic strong biloop over $B=(B_1\cup B_2,*_1,*_2)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic N-loop

Definition 40. Let $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft neutrosophic N -loop if and only if $F(a)$ is a neutrosopchic sub N -loop of $S(B)$ for all $a\in A$.

Example 15. Let $S(B)=\{S(B_1)\cup S(B_2)\cup S(B_3),*_1,*_2,*_3\}$ be a neutrosophic 3-loop, where $S(B_1)=\langle L_5(3)\cup I\rangle$, $S(B_2)=\{g:g^{12}=e\}$ and $S(B_3)=S_3$. Then (F,A) is sof neutrosophic N -loop over $S(B)$, where

$$F(a_1)=\{\mathrm{e},\mathrm{eI},2,2\mathrm{I}\}\cup\{\mathrm{e},\mathrm{g}^6\}\cup\{\mathrm{e},(12)\},$$
$$\mathrm{F}(a_2)=\{e,eI,3,3I\}\cup\{e,g^4,g^8\}\cup\{e,(13)\}.$$

Theorem 34. Let (F,A) and (H,A) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then their intersection $(F,A)\cap(H,A)$ is again a soft neutrosophic N -loop over $S(B)$.

Proof. Straightforward.

Theorem 35. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ such that $A\cap C=\phi$. Then their union is soft neutrosophic N -loop over $S(B)$.

Proof. Straightforward.

Proposition 10. Let (F,A) and (K,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic N -loop over $S(B)$.

Remark 22. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ Then

1. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.
2. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.
3. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft neutrosophic N -loop over $S(B)$.

One can easily verify (1), (2), and (3) by the help of examples.

Definition 41. Let (F,A) be a soft neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (F,A) is called the identity soft neutrosophic N -loop over $S(B)$ if $F(a)=\{e_1,e_2,...,e_N\}$ for all $a\in A$, where $e_1,e_2,...,e_N$ are the identities element of $S(B_1),S(B_2),...,S(B_N)$ respectively.

Definition 42. Let (F,A) be a soft neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (F,A) is called an absolute-soft neutrosophic N -loop over $S(B)$ if $F(a)=S(B)$ for all $a\in A$.

Definition 43. Let (F,A) and (H,C) be two soft neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then (H,C) is called soft neutrosophic sub N -loop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic sub N -loop of $F(a)$ for all $a\in A$.

Definition 45. Let $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft Lagrange neutrosophic N -loop if and only if $F(a)$ is Lagrange neutrosophic sub N -loop of $S(B)$ for all $a\in A$.

Theorem 36. Every soft Lagrange neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Remark 23. Let (F,A) and (K,C) be two soft Lagrange neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic N -loop over (B).
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage neutrosophic N -loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 46. Let $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $S(B)$. Then (F,A) is called soft weakly Lagrange neutrosophic biloop if atleast one $F(a)$ is not a Lagrange neutrosophic sub N -loop of $S(B)$ for some $a\in A$.

Theorem 37. Every soft weakly Lagrange neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Theorem 38. If $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ is a weakly Lagrange neutrosophic N -loop, then (F,A) over $S(B)$ is also soft weakly Lagrange neutrosophic N -loop but the converse is not holds.

Remark 24. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic N -loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 47. Let $S(B)=\{S(B_1)\cup S(B_2)\cup...\cup S(B_N),*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over

$S(B)$. Then (F,A) is called soft Lagrange free neutrosophic N -loop if and only if $F(a)$ is not a Lagrange neutrosophic sub N -loop of $S(B)$ for all $a \in A$.

Theorem 39. Every soft Lagrange free neutrosophic N -loop over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a soft neutrosophic biloop but the converse is not true.

Theorem 40. If $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$ is a Lagrange free neutrosophic N -loop, then (F,A) over $S(B)$ is also soft lagrange free neutrosophic N -loop but the converse is not hold.

Remark 25. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic N -loops over $S(B)=\{S(B_1)\cup S(B_2)\cup ...\cup S(B_N),*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N -loop over $B=(\langle B_1\cup I\rangle\cup B_2,*_1,*_2)$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N -loop over $S(B)$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N -loop over $S(B)$.
4. Their extended union $(F,A)\cup_E(K,\mathrm{C})$ is not a soft Lagrnage free neutrosophic N -loop over $S(B)$.
5. Their restricted union $(F,A)\cup_R(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N -loop over $S(B)$.
6. Their OR operation $(F,A)\vee(K,\mathrm{C})$ is not a soft Lagrange free neutrosophic N -loop over $S(B)$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Soft Neutrosophic Strong N-loop

Definition 48. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then (F,A) is called soft neutrosophic strong N -loop if and only if $F(a)$ is a neutrosopchic strong sub N -loop of $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ for all $a\in A$.

Example 16. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup L_3,*_1,*_2,*_3\}$ where $L_1=\langle L_5(3)\cup I\rangle, L_2=\langle L_7(3)\cup I\rangle$ and $L_3=\{1,2,1I,2I\}$. Then (F,A) is a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$, where

$$F(a_1)=\{\mathrm{e},2,\mathrm{eI},2\mathrm{I}\}\cup\{\mathrm{e},2,\mathrm{eI},2\mathrm{I}\}\cup\{1,\mathrm{I}\},$$
$$\mathrm{F}(a_2)=\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{\mathrm{e},3,\mathrm{eI},3\mathrm{I}\}\cup\{1,2,2\mathrm{I}\}.$$

Theorem 41. All soft neutrosophic strong N -loops are soft neutrosophic N -loops but the converse is not true.

One can easily see the converse with the help of example.

Proposition 11. Let (F,A) and (K,C) be two soft neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,\mathrm{C})$ is a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,\mathrm{C})$ is a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,\mathrm{C})$ is a soft neutrosophic strong N -loop over $\langle L\cup I\rangle$.

Remark 26. Let (F,A) and (K,C) be two soft neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended union $(F,A)\cup_E (K,C)$ is not a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.
2. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.
3. Their OR operation $(F,A)\vee(K,C)$ is not a soft neutrosophic strong N-loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),$ and (3) by the help of examples.

Definition 49. Let (F,A) and (H,C) be two soft neutrosophic strong N-loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then (H,C) is called soft neutrosophic strong sub N-loop of (F,A), if

1. $C\subseteq A$.
2. $H(a)$ is a neutrosophic strong sub N-loop of $F(a)$ for all $a\in A$.

Definition 50. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N-loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange neutrosophic strong N-loop if and only if $F(a)$ is a Lagrange neutrosophic strong sub N-loop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 42. Every soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ is a soft neutrosophic N-loop but the converse is not true.

Remark 27. Let (F,A) and (K,C) be two soft Lagrange neutrosophic strong N-loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E (K,C)$ is not a soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R (K,C)$ is not a soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E (K,C)$ is not a soft Lagrnage neutrosophic strong N-loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R (K,C)$ is not a soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 51. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic strong N-loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft weakly Lagrange neutrosophic strong N-loop if atleast one $F(a)$ is not a Lagrange neutrosophic strong sub N-loop of $\langle L\cup I\rangle$ for some $a\in A$.

Theorem 43. Every soft weakly Lagrange neutrosophic strong N-loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ is a soft neutrosophic N-loop but the converse is not true.

Theorem 44. If $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$ is a weakly Lagrange neutrosophic strong N-loop, then (F,A) over $\langle L\cup I\rangle$ is also a soft weakly Lagrange neutrosophic strong N-loop but the converse is not true.

Remark 28. Let (F,A) and (K,C) be two soft weakly Lagrange neutrosophic strong N-loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup ...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft weakly Lagrnage neutrosophic strong N -loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft weakly Lagrange neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Definition 52. Let $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ be a neutrosophic N -loop and (F,A) be a soft set over $\langle L\cup I\rangle$. Then (F,A) is called soft Lagrange free neutrosophic strong N -loop if and only if $F(a)$ is not a Lagrange neutrosophic strong sub N -loop of $\langle L\cup I\rangle$ for all $a\in A$.

Theorem 45. Every soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ is a soft neutrosophic N -loop but the converse is not true.

Theorem 45. If $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$ is a Lagrange free neutrosophic strong N -loop, then (F,A) over $\langle L\cup I\rangle$ is also a soft Lagrange free neutrosophic strong N -loop but the converse is not true.

Remark 29. Let (F,A) and (K,C) be two soft Lagrange free neutrosophic strong N -loops over $\langle L\cup I\rangle=\{L_1\cup L_2\cup...\cup L_N,*_1,*_2,...,*_N\}$. Then

1. Their extended intersection $(F,A)\cap_E(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
2. Their restricted intersection $(F,A)\cap_R(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
3. Their AND operation $(F,A)\wedge(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
4. Their extended union $(F,A)\cup_E(K,C)$ is not a soft Lagrnage free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
5. Their restricted union $(F,A)\cup_R(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.
6. Their OR operation $(F,A)\vee(K,C)$ is not a soft Lagrange free neutrosophic strong N -loop over $\langle L\cup I\rangle$.

One can easily verify $(1),(2),(3),(4),(5)$ and (6) by the help of examples.

Conclusion

This paper is an extension of neutrosphic loop to soft neutrosophic loop. We also extend neutrosophic biloop, neutrosophic N -loop to soft neutrosophic biloop, and soft neutrosophic N -loop. Their related properties and results are explained with many illustrative examples. The notions related with strong part of neutrosophy also established within soft neutrosophic loop.

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