A Critical Path Problem Using Triangular Neutrosophic Number

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Abstract— **The Critical Path Method (CPM) is one of several related techniques for planning and managing of complicated projects in real world applications. In many situations, the data obtained for decision makers are only approximate, which gives rise of neutrosophic critical path problem. In this paper, the proposed method has been made to find the critical path in network diagram, whose activity time uncertain. The vague parameters in the network are represented by triangular neutrosophic numbers, instead of crisp numbers. At the end of paper, two illustrative examples are provided to validate the proposed approach.**

Keywords— **Neutrosophic Sets, Project Management, CPM, Score and Accuracy Functions**.

I.   INTRODUCTION

Project management is concerned with selecting, planning, execution and control of projects in order to meet or exceed stakeholders' need or expectation from project. Two techniques of project management, namely Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) where developed in 1950s. [1] The successful implementation of CPM requires clear determined time duration for each activity.

Steps involved in CPM include [2]:

1.   Develop Work Breakdown Structure of a project, estimate the resources needed and establish precedence relationship among activities.
2.   Translate the activities into network.
3.   Carry out network computation and prepare schedule of the activities.

In CPM the main problem is wrongly calculated activity durations, of large projects that have many activities. The planned value of activity duration time may change under certain circumstances and may not be presented in a precise manner due to the error of the measuring technique or instruments etc. It has been obvious that neutrosophic set theory is more appropriate to model uncertainty that is associated with parameters such as activity duration time and resource availability in CPM.

This paper is organized as follows:

In section 2, the basic concepts neutrosophic sets are briefly reviewed. In section 3, the mathematical model of neutrosophic CPM and the proposed algorithm is presented. In section 4, two numerical examples are illustrated. Finally section 5 concludes the paper with future work.

II.   PRELIMINARIES

In this section, the basic definitions involving neutrosophic set, single valued neutrosophic sets, triangular neutrosophic numbers and operations on triangular neutrosophic numbers are outlined.

Definition 1. [3,5-7] Let X be a space of points (objects) and $x \in X$. A neutrosophic set A in X is defined by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or real nonstandard subsets of $]^-0, 1^+[$. That is $T_A(x):X \rightarrow]^-0, 1^+[$, $I_A(x):X \rightarrow]^-0, 1^+[$ and $F_A(x):X \rightarrow]^-0, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$.

Definition 2. [3,7] Let X be a universe of discourse. A single valued neutrosophic set A over X is an object having the form $A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X\}$, where $T_A(x):X \rightarrow [0,1]$, $I_A(x):X \rightarrow [0,1]$ and $F_A(x):X \rightarrow [0,1]$ with $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ for all $x \in X$. The intervals $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively. For convenience, a SVN number is denoted by $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a+b+c \leq 3$.

Definition 3. [4,5] Let $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0,1]$ and $a_1, a_2, a_3 \in R$ such that $a_1 \leq a_2 \leq a_3$. Then a single valued triangular neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a special neutrosophic set on the real line set R, whose truth-membership,

indeterminacy-membership, and falsity-membership functions are given as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\dfrac{x - a_1}{a_2 - a_1}\right) & if\ a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}} & if\ x = a_2 \\ \alpha_{\tilde{a}}\left(\dfrac{a_3 - x}{a_3 - a_2}\right) & if\ a_2 < x \leq a_3 \\ 0 & otherwise \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \theta_{\tilde{a}}(x - a_1))}{(a_2 - a_1)} & if\ a_1 \leq x \leq a_2 \\ \theta_{\tilde{a}} & if\ x = a_2 \\ \dfrac{(x - a_2 + \theta_{\tilde{a}}(a_3 - x))}{(a_3 - a_2)} & if\ a_2 < x \leq a_3 \\ 1 & otherwise \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \beta_{\tilde{a}}(x - a_1))}{(a_2 - a_1)} & if\ a_1 \leq x \leq a_2 \\ \beta_{\tilde{a}} & if\ x = a_2 \\ \dfrac{(x - a_2 + \beta_{\tilde{a}}(a_3 - x))}{(a_3 - a_2)} & if\ a_2 < x \leq a_3 \\ 1 & otherwise \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}$, $\theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$ denote the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively. A single valued triangular neutrosophic number $\tilde{a} = \langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ may express an ill-defined quantity about a, which is approximately equal to a.

Definition 4. [4] Let $\tilde{a} = \langle(a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ and $\tilde{b} = \langle(b_1, b_2, b_3); \alpha_{\tilde{b}}, \theta_{\tilde{b}}, \beta_{\tilde{b}}\rangle$ be two single valued triangular neutrosophic numbers and $\gamma \neq 0$ be any real number. Then,

$$\tilde{a} + \tilde{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

$$\tilde{a} - \tilde{b} = \langle(a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle$$

$$\tilde{a}\tilde{b} = \begin{cases} \langle(a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle \\ \qquad if\ (a_3 > 0, b_3 > 0) \\ \langle(a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle \\ \qquad if\ (a_3 < 0, b_3 > 0) \\ \langle(a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle \\ \qquad if\ (a_3 < 0, b_3 < 0) \end{cases}$$

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle\left(\dfrac{a_1}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & if(a_3 > 0, b_3 > 0) \\ \langle\left(\dfrac{a_3}{b_3}, \dfrac{a_2}{b_2}, \dfrac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & if(a_3 < 0, b_3 > 0) \\ \langle\left(\dfrac{a_3}{b_1}, \dfrac{a_2}{b_2}, \dfrac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}\rangle & if(a_3 < 0, b_3 < 0) \end{cases}$$

$$\gamma\tilde{a} = \begin{cases} \langle(\gamma a_1, \gamma a_2, \gamma a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle\ if\ (\gamma > 0) \\ \langle(\gamma a_3, \gamma a_2, \gamma a_1); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle\ if\ (\gamma < 0) \end{cases}$$

$$\tilde{a}^{-1} = \langle\left(\dfrac{1}{a_3}, \dfrac{1}{a_2}, \dfrac{1}{a_1}\right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle, \quad Where\ (\tilde{a} \neq 0)$$

III. CRITICAL PATH METHOD IN NEUTROSOPHIC ENVIRONMENT AND THE PROPOSED ALGORITHM

Project network is a set of activities that must be performed according to precedence constraints determining which activities must start after the completion of specified other activities. Let us define some terms used in drawing network diagram of CPM:

1. Activity: It is any portion of a project that has a definite beginning and ending and may use some resources such as time, labor, material, equipment, etc.
2. Event or Node: Beginning and ending points of activities denoted by circles are called nodes or events.
3. Critical Path: Is the longest path in the network.

The problems of determining critical activities, events and paths are easy ones in a network with deterministic (crisp) duration of activities and for this reason; in this section we convert the neutrosophic CPM to its equivalent crisp model.

The CPM in neutrosophic environment takes the following form:

A network N= $\langle E, A, \tilde{T}\rangle$, being a project model, is given. E is asset of events (nodes) and A $\subset E \times E$ is a set of activities. $\tilde{T}$ is a triangular neutrosophic number and stand for activity duration.

To obtain crisp model of neutrosophic CPM we should use the following equations:

We defined a method to compare any two single valued triangular neutrosophic numbers which is based on the score function and the accuracy function. Let $\tilde{a} = \langle(a_1, b_1, c_1), \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}}\rangle$ be a single valued triangular neutrosophic number, then

$$S(\tilde{a}) = \frac{1}{16}[a_1 + b_1 + c_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} - \beta_{\tilde{a}}) \quad (4)$$

And

$$A(\tilde{a}) = \frac{1}{16}[a_1 + b_1 + c_1] \times (2 + \alpha_{\tilde{a}} - \theta_{\tilde{a}} + \beta_{\tilde{a}}) \quad (5)$$

Is called the score and accuracy degrees of $\tilde{a}$, respectively. The neutrosophic CPM model can be represented by a crisp model using truth membership, indeterminacy membership, and falsity membership functions and the score and accuracy degrees of $\tilde{a}$, using equations (1), (2), (3) and (4), (5) respectively.

Then the CPM with crisp activity times becomes:
A network N= $\langle E, A, T\rangle$, being a project model, is given. E is asset of events (nodes) and A $\subset E \times E$ is a set of activities. The set E= $\{1, 2, \ldots, n\}$ is labeled in such a way that the following

condition holds: $(i, j) \in A$ and $i < j$. The activity times in the network are determined by T_{ij}.

Notations of CPM solution:

T_i^e =Earliest occurrence time of predecessor event i,

T_i^l = Latest occurrence time of predecessor event i,

T_j^e =Earliest occurrence time of successor event j,

T_j^l = Latest occurrence time of successor event j,

T_{ij}^e/Start= Earliest start time of an activity ij,

T_{ij}^e/Finisht=Earliest finish time of an activity ij,

T_{ij}^l/Start=Latest start time of an T_i^l activity ij,

T_{ij}^l/Finisht= Latest finish time of an activity ij,

T_{ij} = Duration time of activity ij,

Earliest and Latest occurrence time of an event:

T_j^e =maximum $(T_j^e + T_{ij})$, calculate all T_j^e for jth event, select maximum value.

T_i^l =minimum $(T_j^l - T_{ij})$, calculate all T_i^l for ith event, select minimum value.

T_{ij}^e/Start=T_i^e,

T_{ij}^e/Finisht=$T_i^e + T_{ij}$,

T_{ij}^l/Finisht=T_j^l,

T_{ij}^l/Start=$T_j^l - T_{ij}$,

Critical path is the longest path in the network. At critical path, T_i^e=T_i^l, for all i.

Slack or Float is cushion available on event/ activity by which it can be delayed without affecting the project completion time. Slack for ith event = $T_i^l - T_i^e$, for events on critical path, slack is zero.

From the previous steps we can conclude the proposed algorithm as follows:

1. To deal with uncertain, inconsistent and incomplete information about activity time, we considered activity time of CPM technique as triangular neutrosophic number.
2. Calculate membership functions of each triangular neutrosophic number, using equation 1, 2 and 3.
3. Obtain crisp model of neutrosophic CPM using equation (4) and (5) as we illustrated previously.
4. Draw CPM network diagram.
5. Determine floats and critical path, which is the longest path in network.
6. Determine expected project completion time.

IV.   ILLUSTRATIVE EXAMPLES

To explain the proposed approach in a better way, we solved two numerical examples and steps of solution are determined clearly.

A.   NUMERICAL EXAMPLE 1

An application deals with the realization of a road connection between two famous cities in Egypt namely Cairo and Zagazig. Linguistics terms such as "approximately between" and "around" can be properly represented by approximate reasoning of neutrosophic set theory. Here triangular neutrosophic

numbers are used to describe the duration of each task of project. As a real time application of this model, the following example is considered. The project manager wishes to construct a possible route from Cairo (*s*) to Zagazig (*d*). Given a road map of Egypt on which the times taken between each pair of successive intersection are marked, to determine the critical path from source vertex (s) to the destination vertex (*d*). Activities and their neutrosophic durations are presented in table 1.

TABLE 1. INPUT DATA FOR NEUTROSOPHIC CPM.

Activity	Neutrosophic Activity Time(days)	Immediate predecessors
A	About 2 days (1,2,3;0.8,0.5,0.3)	-
B	About 3 days (2,3,8;0.6,0.3,0.5)	-
C	About 3 days (1,3,10;0.9,0.7,0.6)	A
D	About 2 days (1,2,6;0.5,0.6,0.4)	B
E	About 5 days (2,5,11;0.8,0.6,0.7)	B
F	About 4 days (1,4,8;0.4,0.6,0.8)	C
G	About 5 days (3,5,20;0.8,0.3,0.2)	C
H	About 6 days (4,6,10;0.8,0.5,0.3)	D
I	About 7 days (5,7,15;0.3,0.5,0.4)	F,E
J	About 5 days (3,5,7;0.8,0.5,0.7)	H,G

Step 1: Neutrosophic model of project take the following form: N= $\langle E, A, \tilde{T} \rangle$, where E is asset of events (nodes) and A$\subset E \times E$ is a set of activities. $\tilde{T}$ is a triangular neutrosophic number and stand for activity time.

Step 2: Obtaining crisp model of problem by using equations (4) and (5). Activities and their crisp durations are presented in table 2.
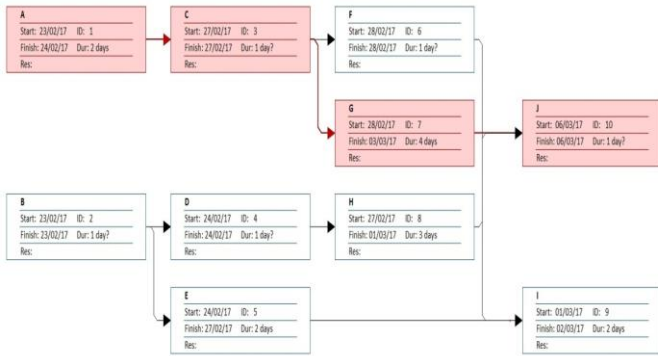
TABLE 2. INPUT DATA FOR CRISP CPM.

Activity	Activity Time(days)	Immediate predecessors
A	2	-
B	1	-
C	1	A
D	1	B
E	2	B
F	1	C
G	4	C
H	3	D
I	2	F,E
J	1	H,G

Step 3: Draw network diagram of CPM.
Network diagram of CPM using Microsoft Project 2010 presented in Fig.1.

Fig. 1. Network of activities with critical path

Step 4: Determine critical path, which is the longest path in the network.
From Fig.1, we find that the critical path is A-C-G-J and is denoted by red line.
Step 5: Calculate project completion time.
The expected project completion time $= t_A + t_C + t_G + t_J = 8$ days.

B. NUMERICAL EXAMPLE 2

Let us consider neutrosophic CPM and try to obtain crisp model from it. Since you are given the following data for a project.

TABLE 3. INPUT DATA FOR NEUTROSOPHIC CPM.

Activity	Neutrosophic Activity Time(days)	Immediate predecessors
A	$\tilde{2}$	-
B	$\tilde{4}$	A
C	$\tilde{5}$	A
D	$\tilde{8}$	B
E	$\tilde{6}$	C
F	$\widetilde{10}$	D,E

Time in the previous table considered as a triangular neutrosophic numbers.
Let,
$\tilde{2} = \langle(0,2,4); 0.8,0.6,0.4\rangle, \tilde{8} = \langle(4,8,15); 0.2,0.3,0.5\rangle,$
$\tilde{4} = \langle(1,4,12); 0.2,0.5,0.6\rangle, \tilde{6} = \langle(2,6,18); 0.5,0.4,0.9\rangle,$
$\tilde{5} = \langle(1,5,10); 0.8,0.2,0.4\rangle, \widetilde{10} = \langle(2,10,22); 0.7,0.2,0.5\rangle.$

To obtain crisp values of each triangular neutrosophic number, we should calculate score function of each neutrosophic number using equation (4).
The expected time of each activity are presented in table 4.

TABLE4. INPUT DATA FOR CRISP CPM.

Activity	Immediate Predecessors	Activity Time(days)
A	-	1
B	A	1
C	A	2
D	B	2
E	C	2
F	D,E	4

After obtaining crisp values of activity time we can solve the critical path method easily, and determine critical path efficiently.
To draw network of activities with critical path we used Microsoft project program.

Fig. 2. Network of activities with critical path
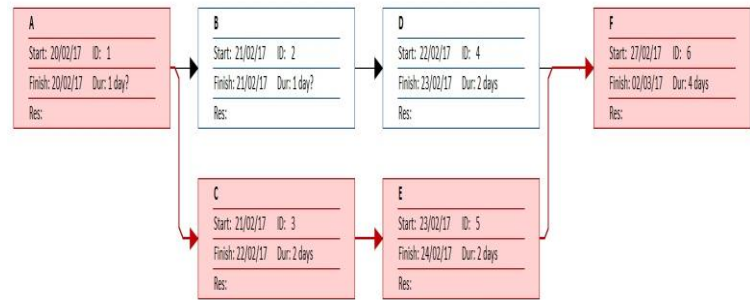
From Fig.2, we find that the critical path is A-C-E-F and is denoted by red line.
The expected project completion time $= t_A + t_C + t_E + t_F = 9$ days.

V. CONCLUSION

Neutrosophic set is a generalization of classical set, fuzzy set and intuitionistic fuzzy set because it not only considers the truth-membership and falsity- membership but also an indeterminacy function which is very obvious in real life situations. In this paper, we have considered activity time of CPM as triangular neutrosophic numbers and we used score function to obtain crisp values of activity time. In future, the research will be extended to deal with different project management techniques.

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