Research Article

A Hybrid BSC-DEA Model with Indeterminate Information

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Strategy is the main source of long-term growth for organizations, and if it is not successfully implemented, even if appropriate ones are adopted, the process is futile. The balanced scorecard which focuses on four aspects such as growth and learning, internal processes, customer, and financial is considered as a comprehensive framework for assessing performance and the progress of the strategy. Moreover, the data envelopment analysis is one of the best mathematical methods to compute the efficiency of organizations. The combination of these two techniques is a significant quantitative measurement with respect to the organization's performance. However, in the real world, determinate and indeterminate information exists. Henceforth, the indeterminate issues are inescapable and must be considered in the performance evaluation. Neutrosophic number is a helpful tool for dealing with information that is indeterminate and incomplete. In this paper, we propose a new model of data envelopment analysis in the neutrosophic number environment. Furthermore, we attempt to combine the new model with the balanced scorecard to rank different decision-making units. Finally, the proposed method is illustrated by an empirical study involving 20 banking branches. The results show the effectiveness of the proposed method and indicate that the model has practical outcomes for decision-makers.

1. Introduction

All organizations whether governmental or private require an effective performance assessment for development, growth, and sustainability in the competitive world of today. In other words, senior executive managers have always been seeking a solution to ensure that their strategies are executed and, hence, have selected performance assessment methods as tools to implement their strategies.

The balanced scorecard (BSC) has been introduced as a comprehensive framework for performance assessment and advancement of strategy, which balances the short- and long-term goals, financial and nonfinancial measures, internal and external performance, internal and external stakeholders, and the occurring progressive and nonprogressive performance indexes. BSC is a proven framework that describes and operates the organization's strategy [1].

Data envelopment analysis (DEA) is a mathematical programming for measuring the relative efficiencies of homogeneous decision-making units (DMUs) without knowing production functions, just by utilizing input and output information [2, 3]. The first models in DEA are the CCR and BCC models in which the efficiency of each DMU obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to that of the similar ratio for all DMUs is less than or equal to one [2, 3]. DEA technique has just been effectively connected in various cases such as broadcasting companies [4], banking institutions [5–8], R&D organizations [9, 10], health care services [11], manufacturing [12, 13], telecommunication [14], and supply chain management [15].

One of the disadvantages of the BSC is the lack of a quantitative measurement of the organization's performance using the mathematical method. Therefore, the integrated BSC-DEA approach can be used to provide a mathematical model of performance measurement for macrogoals, which is complete than the separate models [16]. In the hybrid BSC-DEA technique, BSC is utilized as a tool for designing the assessment indexes for performance, whereas the DEA is used as a tool for performance evaluation. This approach has

drawn the attention of several researchers within a very short period of time.

Eilat et al. [17–19] used the BSC-DEA method for the first time in R&D projects. Min et al. [20] applied the BSC model and the DEA technique for the efficiency of Korean hotels. Chen et al. [21, 22], based on the four perspectives of the BSC, with the help of the quantitative DEA tools, carried out the efficiency evaluation of semiconductor industries and cooperative credit banks in Taiwan. Macedo et al. [23] applied a hybrid BSC-DEA model to performance measurement of the bank branches in Brazil with 6 indexes. García-Valderrama et al. [24] proposed a framework for the analysis of the relationships between the four perspectives of the BSC, and utilizing DEA developed several different models of efficiency. Chiang et al. [25] attempted to develop an integrated framework to encompass the BSC and DEA for measuring management performance and selected auto and commercial bank industries as the targets for empirical investigation. Amado et al. [26] presented the development of a conceptual framework which aims to assess DMUs from multiple perspectives. The proposed conceptual framework combines the BSC method with DEA by using various interconnected models which try to encapsulate four perspectives of BSC. Wu and Liao [27] proposed an integrated DEA-BSC model to evaluate the operational efficiency of airlines. To adapt this model, 38 major airlines in the world were selected to assess their relative performance. In [28], the information technology (IT) project has been evaluated by using a hybrid DEA-BSC model. This approach uses BSC as a comprehensive framework for defining IT project evaluation criteria and uses DEA as a nonparametric technique for ranking IT projects. For illustrations of the other researches which have been executed in relative to assessing the efficiency of organizations by utilizing the DEA and the BSC, refer to [29–32].

However, data in real world are imprecise and vague, and one of the main tools for description of this kind of data is fuzzy number. Since Zadeh [33] presented fuzzy sets (FS), fuzzy theory has been applied effectively in an extensive variety of subject fields [34–37]. Some researchers also considered the BSC-DEA models under fuzzy environment [38–42]. Since the fuzzy set considers only the degree of membership and has not the degree of nonmembership, Atanassov [43] made an enhancement to overcome this weakness and presented the intuitionist fuzzy set (IFS) consisting of the degree of membership and the degree of nonmembership. There are various models of DEA with IFSs (see [44–46]). However, the IFS did not consider the degree of indeterminacy. We know that the incomplete, indeterminate, and inconsistent information in real life often exists. Because of an absence of data, estimation mistakes, or the limited attention and knowledge of decision-makers, in numerous circumstances, the obtained information might be partial determinacy and/or partial indeterminacy. Fuzzy and intuitionist fuzzy sets cannot therefore represent data with both determined and indefinite data.

To express this kind of information, Smarandache [47–49] originally established the neutrosophic logic, which generalizes the concept of the classic set, fuzzy set, interval-valued fuzzy set, and intuitionistic fuzzy set. This logic divided into two categories of the neutrosophic sets (NSs) and the neutrosophic numbers (NNs).

The neutrosophic sets (NSs) are represented by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree and have some subclasses such as single valued neutrosophic sets [50–60], interval neutrosophic sets [61–65], and simplified neutrosophic sets [64, 66–68]. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or nonstandard subsets of $]0^-, 1^+[$; that is, $T_A(x): X \longrightarrow]0^-, 1^+[$, $I_A(x): X \longrightarrow]0^-, 1^+[$, and $F_A(x): X \longrightarrow]0^-, 1^+[$. There is no restriction on the sum of $T_A(x)$, $I_A(x)$, and $F_A(x)$, so

$$0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+. \qquad (1)$$

The neutrosophic number (NN) introduces a concept of indeterminacy, denoted by $A = m + nI (m, n \in R)$, and consists of its determinate part m and its indeterminate part nI. In the worst scenario, A can be unknown, i.e., $A = nI$. However, when there is no indeterminacy related to A, in the best scenario, there is only its determinate part, i.e., $A = m$. Smarandache also refined the NNs by decomposition of the indeterminacy I into different types of indeterminacies such as $I_1, I_2, \ldots, I_n$, and extended the neutrosophic number to the refined neutrosophic number [69, 70].

It is worth mentioning that the neutrosophic sets (NSs) cannot deal with decision-making problems with neutrosophic numbers, as NSs and NNs are two different branches in neutrosophic theory and indicate different forms and concepts of information.

It is clear that the NNs are very practical for conveying information about indeterminate evaluations in complex decision-making problems. For example, Ye [71] provided a neutrosophic number tool for a multiple attribute group decision-making (MAGDM) problem with NNs. He presented a de-neutrosophication method and a possibility degree ranking method for NNs as a methodological support for group decision-making problems. Additionally, Ye [72] developed a bidirectional projection measure of NNs for MAGDM problems. Under a NN environment, Chen and Ye [73] presented a projection model of NNs and its decision-making method for the selecting problems of clay-bricks. Kong et al. [74] presented a distance measure and cosine similarity measure between NNs and applied it to the misfire fault diagnosis of gasoline engines. Furthermore, Smarandache [75] introduced the concept of a neutrosophic linguistic number (NLN) in symbolic neutrosophic theory. Based on this concept, Ye [76] proposed basic operational laws of NLNs. Zhang et al. [77] proposed an extend TODIM method to handle multiple attribute group decision-making problems in which the evaluation information is expressed by NNs. Zheng et al. [78] presented some aggregation operators based on NNs, which are used to handle MAGDM problems. Furthermore, under this environment, Liu and Liu [79] proposed some generalized weighted power

aggregation operators that are used to deal with MAGDM problems more effectively. Jiang and Ye [80] defined a new concept of neutrosophic number functions for the objective functions and constraints in engineering optimization design problems with determinate and indeterminate information and obtained a general NN optimization model of truss structure design. To overcome the complex calculation and difficult solution problems in methods of [80], Ye [81] proposed an improved NN optimization method and applied it to a three-bar planar truss structural design with indeterminate information. Furthermore, Ye et al. [82] using the neutrosophic number functions investigated the anisotropy and scale effect of indeterminate joint roughness coefficient (JRC), which is a quite crucial parameter for determining the shear strength in rock mechanics. Recently, some scholars also under NN environment proposed some models for various optimization problems such as linear programming [83], multiobjective programming [84], nonlinear programming [85], and bilevel linear programming problem [86].

The first model of DEA with NS was established by Edalatpanah [87], and additional investigations have been accessible in [88–95]. However, these models are formulated solely for NSs. In real-life situations, some inputs/outputs in DEA may also be indeterminate and inconsistent, and considering neutrosophic number for each input/output of DMUs helps decision-makers to obtain a better interpretation of information. In addition, by using the NNs in DEA, analysts can obtain a better representation of reality through considering all aspects of the decision-making process. Unfortunately, in the current literature, there is no study of data envelopment analysis (DEA) models and also BSC-DEA methodology in the NN environment. It is clear that the conventional fuzzy sets cannot express neutrosophic DEA with both determinate and indeterminate information. Therefore, it is necessary to propose a new method based on the neutrosophic numbers to BSC-DEA methodology. The main purposes of this paper are as follows: (1) to develop a new model for DEA within the NN environment and (2) to combine the new model with BSC to rank different decision-making units. There are usually many qualitative ways to evaluate an organization's performance. One of these tools is the balanced scorecard approach that separates the organization from the operational point of view. By examining the organization using this segmentation, one can usually gain an understanding of organizational performance, but in quantitative terms, approaches are always needed to evaluate activities separately and to provide accurate benchmarks for different decisions. Data envelopment analysis approach is one of the tools that can be helpful and provides a little understanding of the various points of the scorecard. Understanding the need and the precise relationship between these two concepts illustrates the importance of the subject and led us to present this hybrid model for ranking the decision-making units in the organization.

This hybrid model is presented in an innovative way and demonstrates the significant relationship between the qualitative concepts in the BSC and the quantitative concepts in data envelopment analysis for the purpose of decision-making strategy and ultimately enhancing organizational performance.

The rest of the paper is organized as follows: Section 2 presents some essential concepts regarding neutrosophic numbers and BSC and DEA models. Section 3 proposes a new model of DEA in neutrosophic number environment. Section 4 explains a hybrid BSC-DEA model with NNs. An empirical study involving 20 banking branches and conclusions are given in Sections 5 and 6, respectively.

2. Preliminary Concepts

In this section, we present several basic discussion concerning neutrosophic numbers, balance scorecard, and data envelopment analysis.

2.1. Neutrosophic Number Concept. A neutrosophic number (NN) is represented by $A = m + nI$ $(m, n \in \mathbb{R})$, where m and nI are determinate and indeterminate parts, respectively; for example, consider a NN as $A = 3 + 4I$. Then, it indicates that its determinate value is 3, and its indeterminate value is $4I$. Assume that the indeterminacy I is considered as such a possible interval $[0, 2]$, and then, it is equivalent to $A = [3, 11]$, where A is within the interval $[3, 11]$. For the best case, we have $nI = 0$ and A can be expressed as the determinate part $A = m$, whereas in the worst case $m = 0$ and A expressed as the indeterminate part, $A = nI$. For convenience, let $\aleph$ be the set of all NNs, and then, a NN is denoted by $A = m + nI = [m + n(\inf(I)), m + n(\sup(I))]$ for $I \subseteq [\inf(I), \sup(I)]$ and $A \in \aleph$.

Definition 1 (see [80, 83]).
Let $A_1 = m_1 + n_1 I$ and $A_2 = m_2 + n_2 I$ for $m_i, n_i \in R$, $A_i \in \aleph$, and $I \in [I^L, I^U]$ be two NNs, then they contain the following arithmetic laws:

(i) $A_1 + A_2 = (m_1 + m_2) + (n_1 + n_2)I$,

(ii) $A_1 - A_2 = (m_1 - m_2) + (n_1 - n_2)I$, $\qquad(2)$

(iii) $A_1 \times A_2 = m_1 m_2 + (m_1 n_2 + m_2 n_1)I + n_1 n_2 I^2$.

Definition 2 (see [81]). A NN function with n variables and $\aleph$ domain is defined as $f(x, I): \aleph^n \longrightarrow \aleph$, where, $x = [x_1, \ldots, x_n]^T \in \aleph^n$ and $I \in [I^L, I^U]$. Moreover, I is indeterminacy and $f(x, I)$ can be an NN linear/ nonlinear function; for example, $f(x, I): (5 + 4I)x_1 + (1 + 2I)x_2 + 3I$ for $x = [x_1, x_2]^T \in \aleph^2$ is an NN linear function.

2.2. Balance Scorecard (BSC). Kaplan and Norton proposed the BSC model as a method to evaluate the performance of an organization. The traditional performance assessment systems are more prominently based on financial indexes, whereas successful companies rely not only on financial indicators to evaluate their performance but they also considered their performance from three other BSC perspectives; i.e., customer, internal processes, learning, and growth [96, 97]. The BSC method is a performance measurement framework that provides a complete overview of

an organization's performance with a set of financial and nonfinancial scales. The BSC model has been utilized effectively in manufacturing, service, nonprofitable, and government organizations. Many applications for a balanced scorecard have defined from a business perspective [98, 99]. In Figure 1, the four aspects of the balanced scorecard have been depicted.

According to Figure 2, we must create value for our customers (the customer perspective) in order to achieve financial (the financial perspective). This would not be feasible unless we excel in our operational processes and adapt them to the needs of our customers (the internal processes perspective). It is not possible to gain excellence in operation and process of value unless we create the right work environment for employees and strengthen the innovation and creativity in learning and growth (the learning and growth perspective).

2.3. Data Envelopment Analysis (DEA).

Data envelopment analysis (DEA) is a linear programming method for assessing the efficiency and productivity of decision-making units (DMUs). In the traditional DEA literature, various well-known DEA approaches can be found such as CCR and BCC models [2, 3]. The efficiency of a DMU is established as the ratio of sum weighted output to sum weighted input, subjected to happen between one and zero. The outputs and inputs are known, and the weighted value of the inputs and outputs is selected in such a manner that the efficiency of that DMU is maximized.

Let us assume that n DMU's are present as {DMUj: $j = 1$, ..., n}, which utilize m inputs x_{ij} $(i = 1, 2, \ldots, m)$ to produce s outputs x_{ij} $(i = 1, 2, \ldots, m)$. Here, u_r $(r = 1, 2, \ldots, s)$ and v_i $(i = 1, 2, \ldots, m)$ are the weights of the ith input and rth output. Then, the CCR model is as follows:

$$\theta_p^* = \max \sum_{r=1}^{s} u_r y_{rp},$$

$$\text{s.t}:$$

$$\sum_{i=1}^{m} v_i x_{ip} = 1, \tag{3}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad \forall j,$$

$$u_r, v_i \ge 0, \quad \forall r, i.$$

We solve model (3) n-times to work out the efficiency of n DMUs. If $\theta_p^* = 1$, we say that the DMUp is efficient; otherwise, it is inefficient.

3. New Model of DEA in NN Environment

In this section, we propose a new model of DEA in the neutrosophic number environment. Let us consider the CCR model (3) under the environment of the neutrosophic number. Then, we have

$$\theta_p^* = \max \sum_{r=1}^{s} u_r \widetilde{y}_{rp},$$

$$\text{s.t}:$$

$$\sum_{i=1}^{m} v_i \widetilde{x}_{ip} = 1, \tag{4}$$

$$\sum_{r=1}^{s} u_r \widetilde{y}_{rj} - \sum_{i=1}^{m} v_i \widetilde{x}_{ij} \le 0, \quad \forall j,$$

$$u_r, v_i \ge 0, \quad \forall r, i,$$

where $\widetilde{x}_{ij} = x_{ij} + \eta_{ij} I (i = 1, 2, \ldots, m)$ and $\widetilde{y}_{rj} = y_{rj} + \gamma_{rj} I$ $(r = 1, 2, \ldots, s)$ are neutrosophic numbers of the input and output for the jth DMU and also $I \subseteq [\inf(I), \sup(I)]$. We propose a new model to solve (4).

Theorem 1 shows the feasibility and boundedness of model (4).

Theorem 1. *Model (4) is always feasible and bounded. Furthermore, its optimal objective function is 1.*

Proof. With the solution $u^t = (0, \ldots, 0)$ and $v^t = (0, \ldots, (1/\widetilde{x}_{ip}), \ldots, 0)$, it is easy to see that model (11) is always feasible. Thus, regardless of the values of inputs and outputs, there is always at least one feasible solution for model (11). On the other hand, by this solution, we have

$$\sum_{i=1}^{m} v_i \widetilde{x}_{ip} = 1,$$

$$\sum_{r=1}^{s} u_r \widetilde{y}_{rj} \le 1. \tag{5}$$

Because the above solution is feasible along with the objective function of model (4) is maximization, the best value regarding the objective function is certainly equal to 1.   □

4. A Hybrid BSC-DEA Model with Neutrosophic Numbers

In this section, we attempt to combine the new neutrosophic DEA model proposed in Section 3 with the BSC to rank different DMUs. Since the BSC model evaluates the performance of an organization in the grounds of macrogoals and model of DEA with neutrosophic numbers is also a method to measure efficiency or performance with indeterminate information, therefore, by combining the two-abovementioned measuring methods, the performance is measured and aligned with strategic goals. In the hybrid of DEA and BSC models, the BSC is utilized as a tool for the assessment of performance indexes and the neutrosophic DEA model is used as a tool to evaluate the efficiency in this model. The entire structure of the hybrid DEA-BSC model is shown in Figure 3.

Figure 4 also denotes the four aspects of the performance of the BSC with specific organizational strategies, and in each of the domains, the relative indexes have been defined.

FIGURE 1: Transforming the vision and strategy into the four BSC aspects.

FIGURE 2: Causal correlation in BSC.

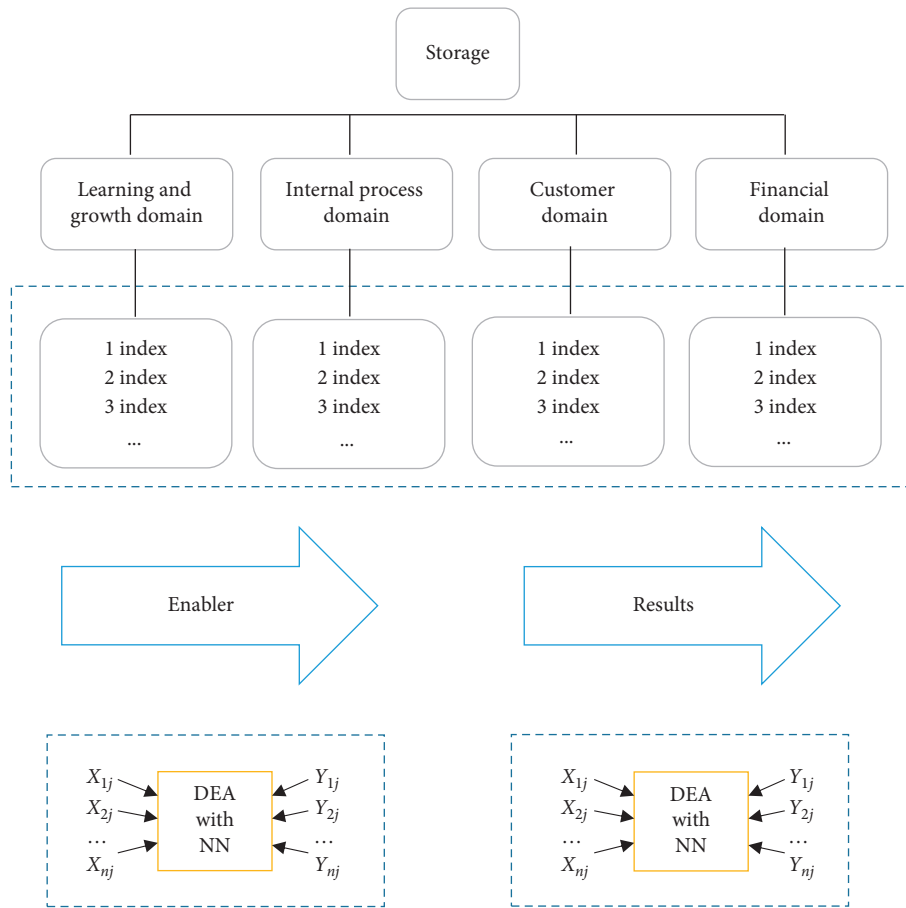FIGURE 3: Hybrid BSC-DEA model with neutrosophic numbers.

Figure 4: A hybrid BSC-DEA strategy with neutrosophic numbers.

A number of these indexes could be inputs and or outputs. Following Figure 4, we have defined two areas of "Enablers" and "Results."

The *enabler's field*, which consists of the two parts of "learning and growth" and "internal process," is an area in which every organization should invest in order to build strong and motivated personnel alongside a secure process. The *results' field* which includes "customer" and "financial" is the indicator that provides the benefit of customers to realize the financial goals. There is no doubt that the profit and loss of the organization are determined in the field of *results*, but creating a margin of profit is surely the yield of efforts and investment in the field of *enablers*.

The steps of indexing and performance evaluation using the two techniques BSC and DEA can be expressed as follows:

(i) Organization identification: at this stage, the goals and strategies of the organization are identified, and using the BSC technique, indicators are created in a balanced manner with different perspectives.

(ii) Performance evaluation: the indicators evaluated by the BSC are classified in two areas of enablers and results, and each area is classified into both input and output groups and used by the neutrosophic DEA in horizontal assessment (overtime periods) or vertical evaluation (in comparison with similar DMUs).

(iii) The correction and improvement of path design: by the neutrosophic DEA, the path to correction and improvement for each indicator is determined.

(iv) Determination of target goals for the next period.

The goals of the indicators that set by the neutrosophic DEA are set as the goals of the indicators for the next period of BSC implementation. In this method, each time the BSC is executed, *i.e.*, at each period when the organization's data are entered into the BSC system, and the results are presented, the neutrosophic DEA model assesses the organization, and the objectives of the indices are determined in the next period. If the goals are met, the organization will achieve the desired and expected efficiency. In the next period of performance evaluation, the organization's condition is compared with the expected conditions from the previous period and the new efficiency is determined. This method is executed periodically, and after each implementation, the manager is expected to lead the organization into the desired optimal efficiency.

5. Numerical Experiment

In Section 4, phases that must be considered in the designing of a hybrid BSC and DEA system were explained. In this section, a case study of this combined system, which has been executed on 20 branches of one of the Iranian banks,

will be described. It is worth emphasizing that, due to the privacy policies, the names of these branches are not shared. Furthermore, for each branch of the bank, we gather the related data from the records unit of the bank, the statistical center of Iran, the reliable library, online resources, and the judgments of some experts. After collecting data, we found that the information is sometimes inconsistent, indeterminate, and incomplete. The investigation revealed that several reforms of the mentioned bank and other issues have led to considerable uncertainty and indeterminacy about the data. As a result, we identified them as NNs. According to the presented model, information has been gathered in the two domains of the enablers and the results. Tables 1 and 2 show the indicators and their information in these two domains. As can be seen, the percentage of banking services and the growth rate of services have grown dramatically.

According to Algorithm 1, we can obtain the relative efficiency of DMUs. In this paper, we consider $\lambda = 0, 0.5, 1$ and $I = [0,1.2]$. For example, in the enablers' stage, the relative efficiency of DMU1 can be used as follows:
$$\theta_1^* = \max z_1 = (47 + 3I)u_1 + (3.5 + 2I)u_2 + (83.5 + I)u_3,$$

s.t :

$$(12 + 2I)v_1 + 12.2v_2 + (50 + I)v_3 = 1,$$
$$(47 + 3I)u_1 + (3.5 + 2I)u_2 + (83.5 + I)u_3-$$
$$(12 + 2I)v_1 - 12.2v_2 - (50 + I)v_3 \le 0,$$
$$(45 + I)u_1 + (3.4 + 4I)u_2 + (80.4 + 3I)u_3-$$
$$(63 + 3I)v_1 - (10.6 + I)v_2 - 16.2v_3 \le 0,$$
$$(43.8 + 3I)u_1 + 6.2u_2 + (80.7 + I)u_3-$$
$$(14 + I)v_1 - 10.7v_2 - (65.5 + 5I)v_3 \le 0,$$
$$(65 + 2I)u_1 + (8.5 + I)u_2 + 93u_3-$$
$$(15.3 + 6I)v_1 - (11.6 + I)v_2 - 32.5v_3 \le 0,$$
$$(40.4 + I)u_1 + (5.2 + 2I)u_2 + 84.6u_3-$$
$$(17.8 + I)v_1 - (11.3 + I)v_2 - (38 + 3I)v_3 \le 0,$$
$$(65.8 + I)u_1 + (3.7 + 2I)u_2 + (88.2 + I)u_3-$$
$$(14.8 + I)v_1 - 10.6v_2 - (38 + 3I)v_3 \le 0,$$
$$(47.3 + 3I)u_1 + (8.5 + 2I)u_2 + (91.1 + 4I)u_3-$$
$$18.4v_1 - (13.2 + I)v_2 - 67v_3 \le 0,$$
$$(55.4 + 4I)u_1 + (8.2 + 2I)u_2 + (83.6 + I)u_3-$$
$$16.9v_1 - 12.5v_2 - (65.8 + 2I)v_3 \le 0,$$
$$(58 + I)u_1 + 3.7u_2 + (76 + 4I)u_3-$$
$$(21.5 + I)v_1 - 11.9v_2 - 73.5v_3 \le 0,$$
$$(54.7 + I)u_1 + (8.4 + 4I)u_2 + (79.8 + 5I)u_3-$$
$$12.2v_1 - (10.5 + 2I)v_2 - (65 + I)v_3 \le 0,$$
$$(69.2 + 6I)u_1 + (4.6 + I)u_2 + (96.3 + 3I)u_3-$$
$$(19.7 + 4I)v_1 - (10.7 + 3I)v_2 - (60 + I)v_3 \le 0,$$
$$(64 + I)u_1 + (4.3 + I)u_2 + (94 + I)u_3-$$

$$(15.3 + 2I)v_1 - 12.2v_2 - 87v_3 \le 0,$$
$$(58.3 + 2I)u_1 + (5.9 + 3I)u_2 + (96.2 + 2I)u_3-$$
$$(18.7 + I)v_1 - 14v_2 - (71.2 + I)v_3 \le 0,$$
$$(55.7 + 2I)u_1 + (5.5 + 5I)u_2 + (81 + I)u_3-$$
$$(25 + 4I)v_1 - (14 + 3I)v_2 - (78.6 + 2I)v_3 \le 0,$$
$$47u_1 + (9.4 + 3I)u_2 + (84.1 + 5I)u_3-$$
$$(19.3 + 3I)v_1 - 10.8v_2 - (65.6 + 2I)v_3 \le 0,$$
$$(67.1 + I)u_1 + (5.3 + 4I)u_2 + (85.7 + I)u_3-$$
$$(18 + I)v_1 - (12 + 5I)v_2 - 72.8v_3 \le 0,$$
$$(59.5 + 2I)u_1 + (6.9 + 6I)u_2 + (90 + 4I)u_3- \qquad (6)$$
$$(17.8 + 4I)v_1 - (11 + 4I)v_2 - 64v_3 \le 0,$$
$$(65.3 + I)u_1 + (7 + I)u_2 + (86.4 + I)u_3-$$
$$(20 + 2I)v_1 - (11 + I)v_2 - (62 + I)v_3 \le 0,$$
$$(49 + I)u_1 + (4.1 + I)u_2 + (91.4 + 2I)u_3-$$
$$(22 + 2I)v_1 - (12.5 + 3I)v_2 - (74.5 + I)v_3 \le 0,$$
$$(65 + I)u_1 + (6.4 + 2I)u_2 + (95 + I)u_3-$$
$$(15.4 + I)v_1 - (18 + 4I)v_2 - (70 + I)v_3 \le 0,$$
$$I - 1.2\lambda = 0,$$
$$u_1, u_2, u_3, , v_1, v_2, v_3 \ge 0.$$

Now, by solving above problem, we can see that, for all values of λ, the relative efficiency of DMU1 is one. Furthermore, the relative efficiency of all DMUs for $\lambda = 0, 0.5, 1$ and $I = [0,1.2]$ was calculated, and the results are obtained in Tables 3 and 4 .

For better understanding, in Figure 5, we show the relative efficiency of DMUs for $I = [0,1.2]$ and different λ.

Form Tables 3–4 and Figure 5, it can be seen that obtaining the optimal results depends on the investment and effort in the enablers sector; that is, until the "learning and growth" and "internal processes" sections do not work well, gaining success is undoubtedly impossible. However, for the success of an organization, planning should be done in the two areas of enablers and results, but it can be clearly stated that the "results" sector requires appropriate measures in the field of enablers. In other words, efficiency in the field of results depends on the efficiency of enablers. Looking at Figure 5, we can infer the following:

(i) The DMUs 1,4,10, and 15, which were efficient in enablers sector, were also able to be efficient in the results section, using the capabilities they gained. It can be said that the efficiency condition in the field of results is efficiency in enablers sector.

(ii) Other DMUs that were not efficient in enablers sector could not be efficient in the results.

(iii) The DMUs 3, 5, 12, 16, 17, 18, and 20, despite the great efforts and obtaining privileges close to the efficient DMUs in the field of results, could not be efficient due to weaknesses in the enablers sector. It can be predicted that these DMUs will be efficient in the results sector if they are efficient in the field of

TABLE 1: Input and output of enablers.

| DMUs | Inputs | | | Outputs | | |
	Motivational costs (%)	Increasing expertise of employees (%)	Employee satisfaction (%)	Banking services (%)	Improvement of computer software (%)	Increasing speed of service (%)
1	$12 + 2I$	12.2	$50 + I$	$47 + 3I$	$3.5 + 2I$	$83.5 + I$
2	16.2	$10.6 + I$	$63 + 3I$	$45 + I$	$3.4 + 4I$	$80.4 + 3I$
3	$14 + I$	10.7	$65.5 + 5I$	$43.8 + 3I$	6.2	$80.7 + I$
4	$15.3 + 6I$	$11.6 + I$	32.5	$65 + 2I$	$8.5 + I$	93
5	$17.8 + I$	$11.3 + I$	$38 + 3I$	$40.4 + I$	$5.2 + 2I$	84.6
6	$14.8 + I$	10.6	$78 + I$	$65.8 + I$	$3.7 + 2I$	$88.2 + I$
7	18.4	$13.2 + I$	67	$47.3 + 3I$	$8.5 + 2I$	$91.1 + 4I$
8	16.9	12.5	$65.8 + 2I$	$55.4 + 4I$	$8.2 + 2I$	$83.6 + I$
9	$21.5 + I$	11.9	73.5	$58 + I$	3.7	$76 + 4I$
10	12.2	$10.5 + 2I$	$65 + I$	$54.7 + I$	$8.4 + 4I$	$79.8 + 5I$
11	$19.7 + 4I$	$10.7 + 3I$	$60 + I$	$69.2 + 6I$	$4.6 + I$	$96.3 + 3I$
12	$15.3 + 2I$	12.2	87	$64 + I$	$4.3 + I$	$94 + I$
13	$18.7 + I$	14	$71.2 + I$	$58.3 + 2I$	$5.9 + 3I$	$96.2 + 2I$
14	$25 + 4I$	$14 + 3I$	$78.6 + 2I$	$55.7 + 2I$	$5.5 + 5I$	$81 + I$
15	$19.3 + 3I$	10.8	$65.6 + 2I$	47	$9.4 + 3I$	$84.1 + 5I$
16	$18 + I$	$12 + 5I$	72.8	$67.1 + I$	$5.3 + 4I$	$85.7 + I$
17	$17.8 + 4I$	$11 + 4I$	64	$59.5 + 2I$	$6.9 + 6I$	$90 + 4I$
18	$20 + 2I$	$11 + I$	$62 + I$	$65.3 + I$	$7 + I$	$86.4 + I$
19	$22 + 2I$	$12.5 + 3I$	$74.5 + I$	$49 + I$	$4.1 + I$	$91.4 + 2I$
20	$15.4 + I$	$18 + 4I$	$70 + I$	$65 + I$	$6.4 + 2I$	$95 + I$

TABLE 2: Input and output of results part.

| DMUs | Inputs | | Outputs | | |
	Improvement of operational processes (%)	Customer acquisition rate (%)	Customer satisfaction (%)	Profit margin (%)	Returns to investment (%)
1	$4.2 + I$	$20 + 2I$	$41 + I$	$6.5 + 6I$	$6.9 + 4I$
2	$5.5 + I$	$23.1 + 2I$	$31.4 + I$	$5.8 + 4I$	$5.6 + 3I$
3	$7.6 + 3I$	$21.4 + I$	47.1	$4.9 + 6I$	$7.5 + 3I$
4	3.3	$22.6 + 2I$	$46.4 + I$	$6.4 + 7I$	$7.9 + 3I$
5	$5.4 + 2I$	$17.9 + 3I$	$29 + I$	$4.8 + 4I$	$4 + 2I$
6	4.5	$28.1 + 2I$	43.2	$7.5 + 6I$	$8.9 + 2I$
7	7.2	20.9	$36.8 + I$	$4.4 + 3I$	$6.1 + 2I$
8	$5.4 + I$	$18.7 + 2I$	$39.4 + 2I$	$5.7 + 2I$	$3.8 + 3I$
9	$6.5 + I$	$28.4 + 2I$	$54.3 + I$	$5.4 + 3I$	$5.7 + 2I$
10	$5.8 + I$	$17.6 + 2I$	$42 + 2I$	$6.1 + 8I$	$7.5 + 3I$
11	$6.2 + I$	$23 + 4I$	$36.6 + 6I$	$5.9 + 3I$	$8.1 + 2I$
12	4.6	$19.4 + I$	$51.3 + 2I$	$5.8 + 3I$	$6.2 + I$
13	$5.9 + I$	$24 + 2I$	49.2	$6.2 + 3I$	$5.1 + 4I$
14	7.1	$46.2 + 2I$	57.5	$4.5 + I$	$3.5 + I$
15	6	$27.2 + I$	22.6	$8 + I$	$6.8 + 3I$
16	$3.5 + 3I$	$22.4 + 2I$	$47.5 + I$	$5.8 + 2I$	$7.4 + 3I$
17	$5.1 + I$	$17.3 + I$	$45.3 + I$	$5.6 + I$	$4.6 + 2I$
18	$4.4 + 3I$	$21.7 + I$	$34.6 + 2I$	$6.6 + I$	$7.1 + 2I$
19	$46 + I$	$28.4 + I$	$48.9 + 4I$	$6.9 + 4I$	$7 + I$
20	$4 + 2I$	$18 + 2I$	$52.2 + 2I$	$5.5 + I$	$6.3 + 3I$

enablers. Meanwhile, the behaviour of the DMU18 is interesting.

(iv) Before calculations, it was anticipated that inefficiencies in enablers sector would have the most ineffective outcomes in the results sector. These contents are seen in DMUs 8, 9, 13, 14, 17, and 18 with the efficiency level in the field of results, lower than the efficiency in the enablers sector.

To validate the proposed efficiencies, the proposed efficiencies are compared with the efficiencies of crisp CCR (model (3), or in our model when $I(\lambda) = 0$) that are given in Figure 5. In this figure, the efficiencies of DMUs are found to be smaller by our model compared to crisp CCR. It is interesting that DMU11 and DMU18 are efficient in crisp DEA, but they are inefficient using a new model. Therefore, the new neutrosophic DEA is more realistic rather than crisp CCR. Also, crisp CCR and the new neutrosophic DEA may

Step 1. Consider the DEA model that the inputs and outputs of each DMU are neutrosophic numbers.

Step 2. Using NN function, transform model (2) into the following model:

$\theta_p^* = \max f_p(u, I),$

s.t :

$g_p(v, I) = 1,$

$f_j(u, I) - g_j(v, I) \leq 0, \quad j = 1, \ldots, n,$

$u = [u_1, \ldots, u_s]^T,$

$v = [v_1, \ldots, u_m]^T \geq 0,$

where $f_j(u, I) = \sum_{r=1}^s u_r(y_{rj} + \gamma_{rj}I)$ and $g_j(v, I) = \sum_{i=1}^m v_i(x_{ij} + \eta_{ij}I)$.

Step 3. Consider $I \subseteq [\inf(I), \sup(I)]$ and using de-neutrosophication model of [77]; for $\lambda \in [0, 1]$, set $I(\lambda) = (1 - \lambda)\inf(I) + \lambda\sup(I)$; then, for $I(\lambda) \in [0, 1]$, transform model (3) into the following model:

$\theta_p^* = \max f_p(u, I(\lambda))$

s.t :

$g_p(v, I(\lambda)) = 1,$

$f_j(u, I(\lambda)) - g_j(v, I(\lambda)) \leq 0, \quad j = 1, \ldots, n,$

$u = [u_1, \ldots, u_s]^T,$

$v = [v_1, \ldots, u_m]^T \geq 0.$

Step 4. Obtain the corresponding optimal solutions of u and v for $I(\lambda) = 0, 0.5, 1$ that are considered as the minimum, the moderate, and the maximum indeterminacy, respectively, in the DEA problem (3).

ALGORITHM 1

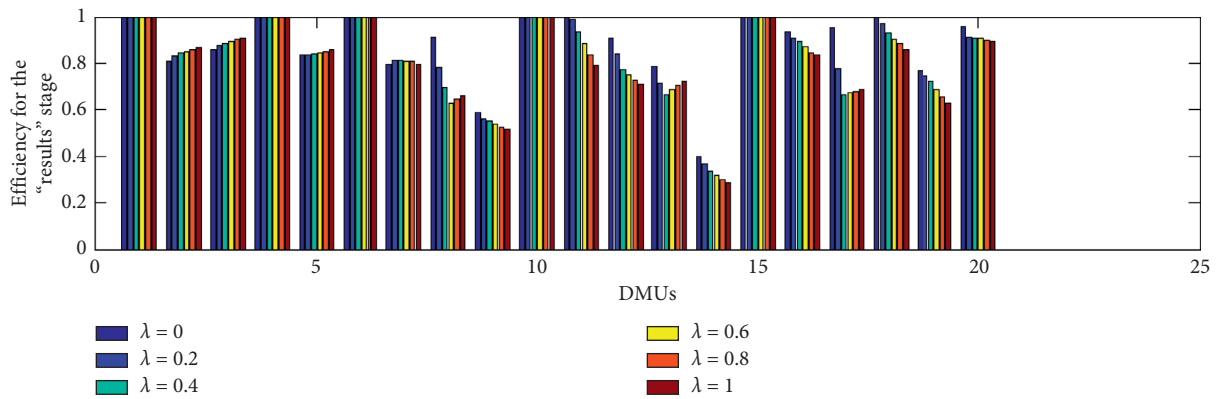TABLE 3: The relative efficiency of DMUs for enablers' stage.

DMUs	λ		
	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1$
DMU1	1.0000	1.0000	1.0000
DMU2	0.8936	0.9140	0.9370
DMU3	0.9400	0.9534	0.9522
DMU4	1.0000	1.0000	1.0000
DMU5	0.9115	0.9196	0.9445
DMU6	1.0000	1.0000	1.0000
DMU7	0.8624	0.8987	0.9356
DMU8	0.8496	0.9072	0.9614
DMU9	0.7560	0.8137	0.8345
DMU10	1.0000	1.0000	1.0000
DMU11	1.0000	1.0000	0.9476
DMU12	0.9773	0.9622	0.9487
DMU13	0.8480	0.8984	0.9344
DMU14	0.6766	0.6831	0.6852
DMU15	1.0000	1.0000	1.0000
DMU16	0.9156	0.8867	0.9050
DMU17	0.9788	0.9090	0.9716
DMU18	1.0000	0.9983	0.9556
DMU19	0.8238	0.8060	0.7681
DMU20	0.9569	0.9622	0.9659

TABLE 4: The relative efficiency of DMUs for results' stage.

DMUs	λ		
	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1$
DMU1	1.0000	1.0000	1.0000
DMU2	0.8103	0.8477	0.8657
DMU3	0.8571	0.8898	0.9065
DMU4	1.0000	1.0000	1.0000
DMU5	0.8361	0.8403	0.8569
DMU6	1.0000	1.0000	1.0000
DMU7	0.7932	0.8100	0.7930
DMU8	0.9102	0.6600	0.6595
DMU9	0.5880	0.5434	0.5151
DMU10	1.0000	1.0000	1.0000
DMU11	1.0000	0.9078	0.7900
DMU12	0.9062	0.7606	0.7104
DMU13	0.7856	0.6722	0.7220
DMU14	0.3990	0.3264	0.2853
DMU15	1.0000	1.0000	1.0000
DMU16	0.9337	0.8832	0.8368
DMU17	0.9512	0.6678	0.6853
DMU18	1.0000	0.9133	0.8574
DMU19	0.7699	0.7066	0.6278
DMU20	0.9569	0.9080	0.8919
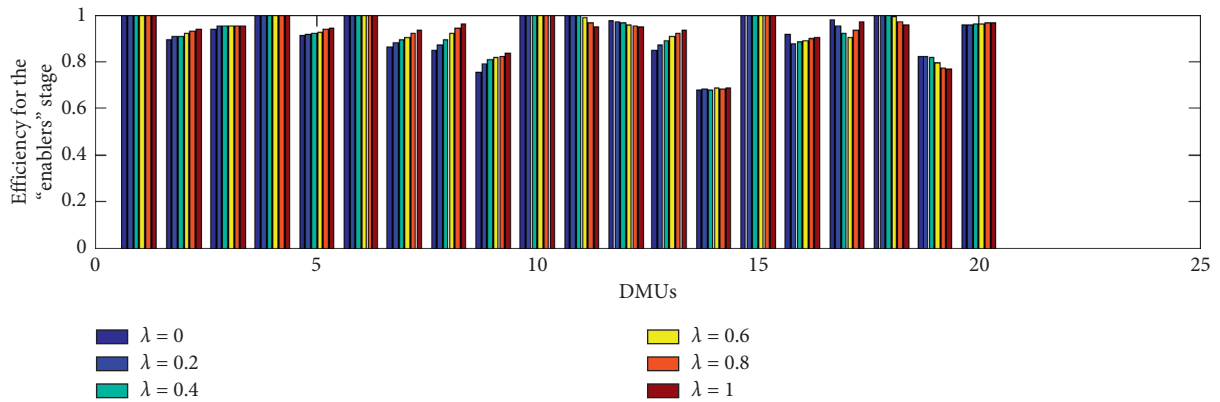
(a)

(b)

FIGURE 5: The relative efficiency of DMUs for enablers and results stages.

give the same efficiencies for certain data. However, crisp CCR model does not deal with the uncertain, indeterminate, and incongruous information. Therefore, the new model is more efficient rather than crisp CCR.

6. Conclusions

Specifying the various performance evaluation models and the appropriate utilization of these models in organizations is a crucial issue. In this paper, we proposed a new model of DEA in neutrosophic number environment and combined this model with BSC to rank different decision-making units. Finally, the proposed method is illustrated by an empirical study involving 20 banking branches. The results provide a more realistic framework and consider various aspects of indeterminate information. Moreover, although the new model and results presented here demonstrate the effectiveness of our approach, it could also be considered in other types of DEA models such as network DEA and their applications to banks, supplier selection, police stations, hospitals, tax offices, prisons, schools, and universities. However, developing data envelopment analysis models based on the plithogenic set, which is a generalization of neutrosophic set, and other perspectives of neutrosophic set is another area for further studies. As future researches, we intend to study these problems.

Abbreviations

BSC: Balanced scorecard
DEA: Data envelopment analysis
DMU: Decision-making units
CCR model: Charnes, Cooper, and Rhodes model
BCC model: Banker, Charnes, and Cooper model
MAGDM: Multiple attribute group decision-making
JRC: Joint roughness coefficient
FS: Fuzzy set
IFS: Intuitionistic fuzzy set
NS: Neutrosophic set
NN: Neutrosophic number
NLN: Neutrosophic linguistic number.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

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