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A Multi-attribute VIKOR Decision-making Method Based on Hesitantneutrosophic Sets

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Abstract
By combining neutrosophic sets and hesitant fuzzy sets, the definition of hesitant neutrosophic sets is presented. Based on the Euclidean distance and the cosine similarity formula, the similarity formula of neutrosophic sets is constructed. Then its similarity formula is also provided. According to the similarity formula of hesitantneutrosophic sets, a new multi-attribute VIKOR decision-making method for hesitant neutrosophic information is develop. At last an illustrative example is given to verify its practicality and effectiveness.
Key words: Hesitant Neutrosophic Set, VIKOR Method, Similarity Measure.

1. INTRODUCTION

In the real world, the information on attributes of the object is often imprecise or uncertain. Due to the shortage of human understanding of uncertainty problems, the attribute values in the decision-making problems are usually incomplete. In order to solve this problem, in 1965, Zadeh(1969), put forward the concept of fuzzy set it to a certain extent, make up the shortcoming of the classical set theory. However, because of the complexity of the human mind and personal qualities, decision makers have different opinions for the same problems, and are often difficult to reach an agreement. Fuzzy sets are unable to describe this phenomenon. In order to deal with the complicated situation, Torra(2010) proposed hesitant fuzzy sets which were the expansion form of fuzzy sets. The hesitant fuzzy set allowed one element belonging to a collection of membership could be several possible values, which could reflect the different preference. The hesitant fuzzy set are applied into multi-attribute decision-making (MADM). Xu and Xia(2011) gave the concepts of distance, correlation and similarity between the two hesitation fuzzy sets, and then discussed the corresponding relationship between them. Yager RR (1986) defined the correlation coefficient between the two hesitation fuzzy sets, which could be used to measure the relationship between them. With the complex evolution of processing problems, there are many forms of hesitant fuzzy sets and other mathematical processing tools, which make the description of real-world objects more powerful. Based on the sets, Atanassov (1989) proposed the intuitionistic fuzzy sets(IFS) which considered a membership function $T_A(x)$ and a non-membership function $F_A(x)$. However, IFSs could only deal with incomplete information, but could not do the indeterminate information and inconsistent information.

In 1999, Smarandache developed the neutrosophic set(NS) from a philosophical perspective which composed of three parts $T_A(x), I_A(x), F_A(x)$. In neutrosophic sets, the uncertainty can be quantitative and clear, where uncertainty degree and non-truth degree are independent of each other. And this assumption is very important in information fusion. Broumi and Smarandache put forward a new method calculating the distance between neutrosophic set based on Hausdorff distance, at the same time constructed similarity measurement method by the distance of induction. Under the environment of neutrosophic set, Wang (2010) put forward a single-valued neutrosophic set which the three function satisfied $T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$; Ye gave that the cosine similarity degree is a special case of coefficient in a single-valued neutrosophic set. Based on those, Wang proposed the concept of the interval neutrosophic sets. Ye defined the measures between them and constructed a new method of MADM.

Inspired by articles, YANG yong-wei combined the two concepts and proposed the definition of hesitantneutrosophic sets which make them more capable of dealing with uncertain information. At the same time, the similarity function of neutrosophic sets is given by the Euclidean distance and the cosine similarity formula.

In real decision-making environment, sometime MADM has some problems: First, the existing decision-making methods tend to focus on the absolute value of attributes, ignoring the psychological effectofthe relative value of attribute on decision maker; Second, in the existing decision-making methods, the attribute index values are often regarded as a set of isolated vector values, ignoring the synergistic effect of the index values on the overall scheme. Therefore, in the real decision-making, the attribute value should be evaluated according to the reference value, and the scheme is modified simultaneously, in order to prevent the negative effects of individual indicators of poor performance from being neutralized or overshadowed by other indicators and not being objectively reflected. Some schemes with extremely unbalanced performance may be selected as excellent schemes without fully reflecting the synergy between the attributes, resulting in evaluation or decision-making errors.

In order to make up for the shortcomings of the above multi-attribute group decision-making methods and improve the scientific and reliability of the decision-making results, this paper introduces the collaborative thinking into the multi-attribute group decision-making model and proposes a new multi-attribute VIKOR decision-making method based on hesitantneutrosophic sets.

2. THE DEFINITIONS AND THE MULTI-ATTRIBUTE VIKOR METHOD

2.1 The definitions of hesitantneutrosophic sets

The first we give the definitions about the hesitantneutrosophic sets. From philosophical point of view, Smarandache originally presented the concept of a neutrosophic set A in a universal set X.

Definition 1 Let X be a universe of discourse, for any $x \in X$, the set $A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\}$ is called a neutrosophic set, which is characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ in X are real standard or nonstandard subsets $]0^0, 1^+[$, such that $T_A(x): X \to [0,1]$, $I_A(x): X \to [0,1]$ and $F_A(x): X \to [0,1]$. Then, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$.

But it is very difficult to apply the neutrosophic set to science and engineering fields. Wang et al. introduced the concept of a single-valued neutrosophic set as a subclass of the neutrosophic set and gave the following definition.

Definition 2 Let X be a universe of discourse, then a single-valued neutrosophic set is defined as $A = \{x(T_A(x), I_A(x), F_A(x)) \mid x \in X\}$, where $T_A(x) \to [0,1]$, $I_A(x) \to [0,1]$, $F_A(x) \to [0,1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for any $x \in X$. The value denote the truth characterized independently by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$.

Let A and B be two neutrosophic sets on a universe of discourse $X = \{x_1, x_2, \cdots x_n\}$, the Euclidean distance

$$d(A,B) = \left\{ \frac{1}{3n} \sum_{i=1}^{n} [(T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2]^{1/2} \right\} \tag{1}$$

However, sometimes it is very difficult to give the membership of an element, not because we have errors, or because we have some values for the probability distribution, but because we have many possible values. For this situation, Torra introduced a set of hesitation set.

Definition 3 Let X be a universe of discourse, then a hesitation fuzzy set (HFS) is defined as $A = \{(x, h_A(x)) \mid x \in X\}$, where $h_A(x) \to [0,1]$. For confidence, Xu and Xia call $h_A(x)$ be hesitation fuzzy element (HFE).

In practical decision-making problems, due to the insufficience of the effective information, it is difficult for decision makers to express their views with exact values, so YANG Yong-wei introduced the definition of hesitation hesitantneutrosophic set.

Definition 4 Let X be a universe of discourse, a hesitantneutrosophic set(HNS) is defined as $A = \{(x, h_A(x)) \mid x \in X\}$, where $h_A(x) = \{(h_A^T(x), h_A^I(x), h_A^F(x)) \mid x \in X\}$, and $h_A^T(x): X \to [0,1]$, $h_A^I(x): X \to [0,1], h_A^F(x): X \to [0,1]$ are the truth-membership function, indeterminacy-membership function and falsity-membership function respectively satisfying $0 \le h_A^T(x) + h_A^I(x) + h_A^F(x) \le 3$.

Definition 5 Let X be a universe of discourse, $A = \{(x_i, h_A(x_i)) \mid x_i \in X\}$ be a hesitantneutrosophic set, where hesitantneutrosophic element $h_A(x_i) = \{h_{AK} = (h_{Ak}^T(x), h_{Ak}^I(x), h_{Ak}^F(x)) \mid k = 1, 2, \cdots, l_i\}$. We define the average value $\bar{h}_A(x_i)$ as the following:

$$\bar{h}_A(x_i) = (\bar{h}_A^T(x_i), \bar{h}_A^I(x_i), \bar{h}_A^F(x_i)) \tag{2}$$

where $\bar{h}_A^T(x_i) = \frac{1}{l_i} \sum_{i=1}^{l_i} h_{Ak}^T(x_i)$, $\bar{h}_A^I(x_i) = \frac{1}{l_i} \sum_{i=1}^{l_i} h_{Ak}^I(x_i)$, $\bar{h}_A^F(x_i) = \frac{1}{l_i} \sum_{i=1}^{l_i} h_{Ak}^F(x_i)$.

And the variance $\delta_{hA}(x_i)$ of hesitantneutrosophic value $h_A(x_i)$ is

$$\delta_{hA}(x_i) = (\delta_{hA}^T(x_i), \delta_{hA}^I(x_i), \delta_{hA}^F(x_i)) \tag{3}$$

where $\delta_{hA}^T(x_i) = \sqrt{\frac{1}{l_i}\sum_{i=1}^{l_i}(h_{Ak}^T(x_i) - \bar{h}_A^T(x_i))^2}$, $\delta_{hA}^I(x_i) = \sqrt{\frac{1}{l_i}\sum_{i=1}^{l_i}(h_{Ak}^I(x_i) - \bar{h}_A^I(x_i))^2}$,

$\delta_{hA}^F(x_i) = \sqrt{\frac{1}{l_i}\sum_{i=1}^{l_i}(h_{Ak}^F(x_i) - \bar{h}_A^F(x_i))^2}$.

Based on the Euclidean distance of neutrosophic set, we give the Euclidean distance of two hesitantneutrosophic elements:

$$d(h_1, h_2) = \sqrt{\frac{1}{3l}\sum_{k=1}^{l_i}\left\| h_{1(k)} - h_{2(k)} \right\|} \tag{4}$$

where $\left\| h_{1\tau(k)} - h_{2\tau(k)} \right\| = (h_{1\tau(k)}^T - h_{2\tau(k)}^T)^2 + (h_{1\tau(k)}^I - h_{2\tau(k)}^I)^2 + (h_{1\tau(k)}^F - h_{2\tau(k)}^F)^2, l = \max\{l_1, l_2\}$, $h_{i\tau(k)}$ is the kth element in h_i, and the elements are arranged in descending order, satisfying $(h_{i\tau(k)}^T - h_{i\tau(k)}^F)^2(h_{i\tau(k)}^T + h_{i\tau(k)}^F + h_{i\tau(k)}^I) \geq (h_{i\tau(k+1)}^F - h_{i\tau(k+1)}^F)(h_{i\tau(k+1)}^T + h_{i\tau(k+1)}^F + h_{i\tau(k+1)}^I)$. So $\tau : (1, 2, \cdots, l) \to (1, 2, \cdots, l)$ is regarded as permutation.

2.2 The similarity of hesitantneutrosophic sets

Wang proposed the similarity of hesitantneutrosophic sets based on the cosine similarity.

Definition 6 Let X be a universe of discourse, the functions $S : NS(X) \times NS(X) \to [0,1]$ is called the similarity function on $NS(X)$, if it satisfies: $\forall A, B, C \in NS(X)$,

(S1) $0 \leq S(A, B) \leq 1$;

(S2) $S(A, B) = 1$ if and only if $A = B$;

(S3) if $A \subseteq B \subseteq C$, then $S(A, C) \leq \min\{S(A, B), S(B, C)\}$

Theorem 1 Suppose A, B be two hesitantneutrosophic sets on a universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, then the Euclidean distance of them is

(1) cosine similarity:

$$\rho(A, B) \cong S_1(A, B) = \frac{1}{n}\sum_{i=1}^{n}\cos[\frac{\pi(|T_A(x_i) - T_A(x_i)| \vee |I_A(x_i) - I_A(x_i)| \vee |F_A(x_i) - F_A(x_i)|)}{2}],$$

(2) distance-induced similarity: $S_2(A, B) = 1 - d(A, B)$,

(3) Hausdorff similarity: $S_3(A, B) = \frac{1}{1 + d(A, B)}$.

For hesitantneutrosophic information, cosine similarity measurement formula focuses on the component where the truth-membership function, indeterminacy-membership function and falsity-membership function play a major role, while ignoring the influence of attribute information as a whole in decision - making. So we give distance - induced similarity using Hausdorff similarity.

Theorem 2 Suppose A, B be two hesitantneutrosophic sets on a universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, $d(A, B)$ and $\rho(A, B)$ are the Euclidean distance and cosine similarity, so the similarity formula of A, B is

$$S_4(A, B) = \frac{\rho(A, B)}{\rho(A, B) + d(A, B)}. \tag{5}$$

Definition 7 Suppose $h_1 = \{h_{1K} = (h_{1k}^T(x), h_{1k}^I(x), h_{1k}^F(x))\}$ and $h_2 = \{h_{2K} = (h_{2k}^T(x), h_{2k}^I(x), h_{2k}^F(x))\}$ are two hesitantneutrosophic numbers on a universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$, then the similarity of h_1 and h_2 is

$$S(h_1, h_2) \cong S_i(h_{1\tau}, h_{2\tau}), (i = 1, 2, 3, 4) \tag{6}$$

Where $h_{1\tau} = \{h_{1\tau(k)} = (h^T{}_{1\tau(k)}, h^I{}_{1\tau(k)}, h^F{}_{1\tau(k)})\}$ and $h_{2\tau} = \{h_{2\tau(k)} = (h^T{}_{2\tau(k)}, h^I{}_{2\tau(k)}, h^F{}_{2\tau(k)})\}$, $l = \max\{l_1, l_2\}$, $\tau : (1, 2, \cdots, l) \to (1, 2, \cdots, l)$ is a permutation.

Now the similarity of hesitantneutrosophic set is defined as:

Definition 8 Let $X = \{x_1, x_2, \cdots, x_n\}$ be a universe of discourse, and A, B be two hesitantneutrosophic sets, then $\bar{S}(A, B)$ is called as the similarity of A, B, if it satisfies:

(S1) $0 \leq \bar{S}(A, B) \leq 1$;

(S2) $\bar{S}(A,B)=1$ if and only if $A=B$;

(S3) $\bar{S}(A,B)=\bar{S}(B,A)$

In the process of multiple attribute decision making, the importance of attributes is often different, so we need to assign different weights. Therefore, the weights of elements should be considered, and we give under the weights of similarity of the hesitantneutrosophic set.

Theorem 3 Suppose X be a universe of discourse, $A=\{(x_i,h_A(x_i))\,|\,i=1,2,\cdots,n\}$ and $B=\{(x_i,h_B(x_i))\,|\,i=1,2,\cdots,n\}$ are the hesitantneutrosophic sets on X, $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ is a weight, where $\omega_i\in[0,1],\sum\omega_i=1$, then the formula $\bar{S}(A,B)=\sum\omega_i S(h_A(x_i),h_B(x_i))$ is the similarity measure satisfying the definition 8.

3. THE MULTI-ATTRIBUTE VIKOR METHOD BASED ON HESITANTNEUTROSOPHIC SETS

The VIKOR method is put forward by Opricovic in 1998, which is a kind of multiple attribute decision making method based on ideal point solution. And its basic idea is first to determine the ideal solution and negative ideal solution, then according to the value of each alternative, the schemes are preferred for the proximity to the ideal solution. Although the VIKOR method is similar to classic TOPSIS method, Opricovic by comparing the two methods, pointes out that the optimal solution by the TOPSIS is not necessarily the most close to the ideal point solution. The VIKOR method is a kind of compromise ranking method based on an ideal, which can achieve the optimal ranking of finite decision-making schemes by maximizing group utility and minimizing individual regret.

In this paper , we extend the traditional method to hesitantneutrosophic environment, and propose a new VIKOR method combing the grey correlation degree.

Let $Y=(Y_1,Y_2,\cdots,Y_m)$ be a discrete set of alternatives, $G=(G_1,G_2,\cdots,G_n)$ be a set of attributes, $\omega=(\omega_1,\omega_2,\cdots,\omega_n)$ be the weight of attributes, where $\omega_i\in[0,1],\sum\omega_i=1$. Then decision makers $D=(D_1,D_2,\cdots,D_l)$ provide his decision matrix $H^k=(h^k{}_{ij})_{m\times n}$, where h_{ij} for alternative Y_i under the attribute G_j. Suppose that the matrix $H^k=(h^k{}_{ij})_{m\times n}$ is hesitantneutrosophic decision matrix, where $h^k{}_{ij}$ is expressed by the hesitantneutrosophic fuzzy number.

In MADM process, we construct a new VIKOR method based on synergy degree.

Step1 Add average value $\bar{d}_{ij}$ in every hesitantneutrosophic fuzzy number, and the length is $l_{ij}=\max_j l_{ij}$.

Step 2 If the attribute types are the same type, the decision matrixes $H^k=(h^k{}_{ij})_{m\times n}$ do not need to be standardized; If the attribute types are different, they are normalized. The $H^k=(h^k{}_{ij})_{m\times n}$ is translated into $D^k=(d^k{}_{ij})_{m\times n}$,

$$d^k{}_{ij}=\begin{cases} h_{ij}, benefit \quad index \\ h^C{}_{ij}, \cos t \quad index \end{cases}. \tag{7}$$

Step 3 In the multi-attribute decision-making, ideal solutions are often used to determine the optimal solution decision set. Although ideal solutions may not exist in the real world, it provides a practical theoretical framework for the evaluation of alternatives. Determine the positive ideal solution and the negative ideal solution:

$Y^+=(d_1^+,d_2^+,\cdots,d_n^+)$, where $d_i^+=(\max_j\{d_{ij}^{\tau(i)}\},\min_j\{d_{ij}^{\tau(i)}\},\min_j\{d_{ij}^{\tau(i)}\})$, $Y^-=(d_1^-,d_2^-,\cdots,d_n^-)$, where $d_i^-=(\min_j\{d_{ij}^{\tau(i)}\},\max_j\{d_{ij}^{\tau(i)}\},\max_j\{d_{ij}^{\tau(i)}\})$.

Step 4 Caculate the group utility value V_i and individual regret value R_i based on similarity measure:

$$V_i=\sum_j\omega_i\left(\frac{d_j^+-d_{ij}}{d_j^+-d_j^-}\right)=\sum_j\omega_j\left(\frac{\bar{S}(d_j^+-d_{ij})}{\bar{S}(d_j^+-d_j^-)}\right)$$

$$R_i=\max_j\omega_i\left(\frac{d_j^+-d_{ij}}{d_j^+-d_j^-}\right)=\max_j\omega_j\left(\frac{\bar{S}(d_j^+-d_{ij})}{\bar{S}(d_j^+-d_j^-)}\right)$$

Step 5 Calculate the compromise evaluation value using the group utility value V_i and individual regret value:

$$Q_i=\lambda\left(\frac{V_i-V^+}{V^--V^+}\right)+(1-\lambda)\left(\frac{R_i-R^+}{R^--R^+}\right) \tag{8}$$

where $V^+ = \min V_i, V^- = \max V_i$, $R^+ = \min R_i, R^- = \max R_i$. $\lambda \in [0,1]$ is the maximum group utility weight, if $\lambda = 0.5$, then it indicates that decisions are made in a balanced compromise. In VIKOR, we get $\lambda = 0.5$.

Step 6 Rank the order of alternatives according to the Q_i, V_i, R_i, and get three orders Q_o, V_σ, R_σ.

Step 7 Determine the compromise solution.

4. NUMERICAL EXAMPLE

Suppose a software company wants to recruit a system administrator. After preliminary screening, three candidates Y_1, Y_2, Y_3 need to be final interviewed. Two experts give their decision-making matrix according to G_1 emotional stability, G_2 verbal communication skills, G_3 working experience and G_4 confidence, where the values of the evaluation are hesitantneutrosophic fuzzy numbers. Suppose the weight is $\omega = (0.30, 0.2, 0.15, 0.35)$.

Table 1. Decision making matrix of the first expert

	G1	G2	G3	G4
Y1	{(0.5,0.3,0.7),(0.9,0.6,0.4)}	{(0.8,0.3,0.5) }	{(0.4,0.8,0.5),(0.2,0.6,0.5)}	{(0.7,0.2,0.2) }
Y2	{(0.5,0.7,0.6),(0.4,0.6,0.9)}	{(0.6,0.7,0.3) }	{(0.8,0.4,0.7),(0.9,0.3,0.4),(0.7,0.5,0.6)}	{(0.9,0.3,0.5) }
Y3	{(0.7,0.2,0.8) }	{0.7,0.5,0.4)) }	{0.7,0.5,0.4) }	{0.8,0.5,0.3)) }

Table 2. Decision making matrix of the second expert

	G1	G2	G3	G4
Y1	{(0.5,0.2,0.6),(0.9,0.5,0.4)}	{(0.7,0.2,0.5) }	{(0.4,0.8,0.5),(0.2,0.6,0.5)}	{(0.8,0.1,0.2) }
Y2	{(0.4,0.7,0.6),(0.4,0.6,0.8)}	{(0.7,0.6,0.3) }	{(0.7,0.3,0.6),(0.8,0.3,0.4)}	{(0.7,0.3,0.5) }
Y3	{(0.6,0.2,0.7) }	{0.6,0.5,0.4)) }	{(0.8,0.5,0.4) }	{0.7,0.4,0.3)) }

Add the average value $\bar{d}_{ij}^k$ in the hesitantneutrosophic fuzzy numbers d_{ij}^k.Since all attributes are benefit indicators, it is not necessary to standardize the decision matrix. Then according to the steps, we calculate Group utility value, individual regret value and compromise evaluation value(seeing as table3).

Table 3. Group utility value, individual regret value and compromise evaluation value

	Qi	Vi	Ri
Y1	0.402	0.117	0.099
Y2	0.409	0.113	0.500
Y3	0.406	0.134	0.767

So we can obtain the rank order $Y_1 \succ Y_2 \succ Y_3$ according to the magnitude of the compromise evaluation value. The the company will choose the candidate Y1.The result satisfies the acceptable stability criteria and acceptable dominance criteria.

5. CONCLUSIONS

The multi-attribute VIKOR decision-making method of hesitantneutrosophic sets is proposed in this paper. By combining neutrosophic sets and hesitant fuzzy sets, the definition of hesitant neutrosophic sets is presented. Based on the Euclidean distance and the cosine similarity formula, the similarity formula of neutrosophic sets is constructed. Then its similarity formula is also provided. According to the similarity formula of hesitantneutrosophic sets, a new multi-attribute VIKOR decision-making method for hesitant neutrosophic information is develop. At last an illustrative example is given to verify its practicality and effectiveness.

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