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A NEW APPROACH IN FAILURE MODES AND EFFECTS ANALYSIS BASED ON COMPROMISE SOLUTION BY CONSIDERING OBJECTIVE AND SUBJECTIVE WEIGHTS WITH INTERVAL-VALUED INTUITIONISTIC FUZZY SETS

Z. HAJIGHASEMI AND S. M. MOUSAVI

Abstract. Failure modes and effects analysis (FMEA) is a well-known risk analysis approach that has been conducted to distinguish, analyze and mitigate serious failure modes. It demonstrates the effectiveness and the ability of understanding and documenting in a clear manner; however, the FMEA has weak points and it has been criticized by some authors. For example, it does not consider relative importance among three risk factors (i.e., O, S and D). Different sequences of O, S and D may result in exactly the same value of risk priority number (RPN), but their semantic risk implications may be totally different and these three risk factors are difficult to be precisely expressed. This study introduces a new interval-valued intuitionistic fuzzy (IVIF)-decision approach based on compromise solution concept that defeats the above weak points and improves the traditional FMEA's results. This study firstly employs both subjective and objective weights in the decision process simultaneously. Secondly, there are two kinds of subjective weights performed in the study: aggregated weights obtained by experts' assessments as well as entropy measure. Thirdly, this approach is defined under an IVIF-environment to ensure that the evaluation information would be preserved, and the uncertainties could be handled during the computations. Hence, it considers uncertainty in experts' judgments as well as reduces the probability of obtaining two ranking orders with the same value. Finally, the alternatives are ranked with a new collective index according to the compromise solution concept. To show the effectiveness of the proposed approach, two practical examples are solved from the recent literature in engineering applications. The proposed decision approach has an acceptable performance. Also, its advantages have been mentioned in comparison with other decision approaches.

1. **Introduction**

Failure modes and effects analysis (FMEA) is an analytical approach to define, identify and eliminate the known and potential failures, errors, and problems from system, design, process or service before reaching to customers. When FMEA is applied to criticality analysis, it is assigned as failure modes, effects and criticality analysis (FMECA). Firstly, in the 1960, the aerospace industry had used the tool as

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an official method to evaluate the effects of system failures on mission accomplishments, staff and equipment safety. It is a bottom-up approach and help designers and analysts to identify and correct failure modes and improve the system in design and manufacturing levels [8, 27, 30, 32, 35]. Representing its capability, it has been applied to other industries, such as car manufacturing, electronics and medical industries. Feili et al. [14] have employed FMEA in renewable energy industry and have used it as a powerful tool for identifying and eliminating potential failures of geothermal power plants (GPPs). Zeng et al. [43] have employed FMEA as a risk assessment tool for achieving continuous improvement in implementing an integrated management system (IMS). Healthcare is another field that received researchers' attention [34]. Chanamool and Naenna [6] have studied decision-making processes in health organizations and improved it by applying FMEA for the risk assessment of work processes. Kumru and Kumru [17] have also employed FMEA for continuous improvement of purchase process in hospitals. Increasing reliability and safety of medical products through managing the risks during medical product development process [16], and analyzing and mitigating the risks of computer-controlled systems [3] are some of new applications of FMEA in different industries in the recent years.

Because of diversity of failures in system and resource restriction in organizations to perform corrective actions, it is important to prioritize the failure modes and devote resources to the high risk failure modes [9, 19]. To rank failure modes, risk assessment in FMEA is performed by developing a risk priority number (RPN) to determine the most serious risks for corrective actions. RPN is aggregation index which is obtained by multiplying three risk factors, denoted by O, S and D.

$$RPN = S^* D^* O \tag{1}$$

where O is probability of occurrence, S is related to severity of the failure mode and D is probability of not detecting the failure. These three factors are expressed in 1-10 rating scale and are used to calculate the RPN. The more the RPN, the more the risk of each failure mode and the corrective actions should be done. After performing the corrective actions, the RPN numbers should be recalculated to check the efficiency of the performed actions and to know whether the risks are removed.

Although the FMEA is regarded as structured, capable and well-documented method, it has been criticized for several weak points: (1) This method employs 3 risk factors without considering the difference in factors' relative importance. It is proved by several researchers and practitioners that the importance and effect of severity is much more than loss of detection; however, both of these factors have the same weight and effect in final result. This leads to the other drawback of FMEA. (2) Different sequences of O, S and D may result in exactly the same value of RPN, but their semantic risk implications may be totally different. The failure with safety rate 9 and O and D rates of 1 has the same priority as failure mode with S and O rates 9 and D rate of 9; however, the first one is much more critical. (3) Despite the general idea about FMEA, its scores are not distributed uniformly between 0 and 1000. There are many holes and only 120 different combinations of risk factors are possible. (4) Due to ordinal scale for factors' evaluations, the final score is not capable of comparing and measuring in terms of corrective actions. In fact, after prioritizing the failure modes and performing corrective actions, RPN does not have the capability of measuring the actions' effectiveness, and the reduction in RPN after performing corrective action does not show amount of reduced risk in the system under consideration. (5) RPN is sensitive to variations in risk factors' weights and this makes some doubts about the RPN method robustness. (6) Finally, these three risk factors are difficult to be precisely expressed.

To overcome mentioned weak points, new methods and techniques are proposed to improve the FMEA results. The technique for order of preference by similarity to ideal solution (TOPSIS) is a extensively used method for conducting multiple criteria decision-making (MCDM) problems [11]. Recently, Vahdani et al. [32] and Kuo et al. [18] performed FMEA analysis using TOPSIS approach to rank the alternatives. Also, analytical hierarchy process (AHP) by Braglia [5], decision making trial and evaluation laboratory technique (DEMATEL) by Seyed-Hosseini et al. [30] are proposed and applied to perform FMEA to overcome RPN's weak points. Moreover, some researchers focused on how to determine risk factors' weights. Generally, subjective weights based on experts' judgments are used to determine relative weights among risk factors. Also, Ye [39] introduced entropy operator to determine subjective weights based on performance matrix. Liu et al. [27] argued that considering both subjective and objective weights makes the FMEA more practical and sensible. They used intuitionistic fuzzy hybrid weighted Euclidean distance (IFHWED) operator to simultaneously consider both objective and subjective weights to obtain results. Recently, Liu et al. [22] have also employed both the subjective and objective weights in the process of risk and failure analysis.

On the other hand, it is not easy to judge the alternatives with respect to criteria in a precise manner. Surveys in different industries and applications show that in practice the data available for failures analysis are incomplete due to data loss, unreliability, data complexity or imprecision [3, 10, 16, 41]. Therefore, different uncertainty approaches are used to enable the FMEA to analyze the uncertain data. On this basis, several approaches are suggested for considering uncertainty in the FMEA. Fuzzy sets theory, evidence theory and grey theory are well-known theories applied to cover this deficiency. Du et al. [13] have used evidence theory to express the experts' imprecise opinions. Then, they have transformed them to crisp ones by employing weighted averaging. The crisp data are analyzed and aggregated using TOPSIS method. Grey theory has also been employed to aggregate and make a final conclusion from uncertain and imprecise experts' opinion [7, 20]. Zhou and Thai [45] have employed fuzzy sets theory to calculate the failures' risks expressed by linguistic terms. Then, they have prioritized the failures based on grey relational coefficient and found that the result of two methods are quite similar.

Meanwhile, fuzzy sets theory is the most famous theory for handling the information under uncertain environments and has been employed by several researchers. The triangular fuzzy numbers and trapezoidal fuzzy numbers were employed, respectively, by Kuo et al. [18] to consider uncertainty in FMEA. Kirkire et al. [16] have used triangular fuzzy numbers to manage the risks of failures during each phase of medical product development process and the risks have been mapped back to development phases. Yeh and Chen [41] have mentioned the traditional

FMEA deficiency in precisely expressing the experts' knowledge and have overcome by using linguistic fuzzy variables. Baek et al. [3] have applied fuzzy FMEA in offshore project. They have used linguistic fuzzy variables to confront the missing or unreliable data of SCADA and imprecise experts' opinions. Rachieru et al. [29] have also shown that the fuzzy FMEA can help experts prioritize the failures more precisely. Recently, 2-dimension uncertain linguistic generalized average operators are developed to consider 2 dimensions of human judgments, and hence, to improve decision-making process under uncertain environments [21, 23, 42].

After introducing intuitionistic fuzzy set (IFS) notions by Atanassov [2], it has been extensively applied to consider uncertainty in different situations [7, 8, 22]. This is because of IFS's feature in assigning membership and non-membership values in order to represent vagueness hidden in terms in more detail compared with previous fuzzy types. Chang and Cheng [8] proposed a risk assessment methodology using the IFSs in FMEA. They have applied DEMATEL method to calculate the final rank of failures. In addition, Chang et al. [8] has analyzed the system reliability and evaluation of redundancy place in system using linguistic intuitionistic fuzzy variables. Liu et al. [27] have proposed a method based on IFSs theory to improve scores' aggregation in FMEA. They have applied intuitionistic fuzzy weighted averaging (IFWA) operator to combine the experts' judgments and then using an intuitionistic fuzzy hybrid weighted Euclidean distance (IFHWED) as an improved aggregation operator, prioritized the failures' risks. Recently, Liu et al. [22] have also proposed an intuitionistic fuzzy hybrid TOPSIS approach to prioritize the failures risks and have highlighted the most serious failures.

Neutrosophic set (NS) is another uncertainty concept that is a generalization of IFSs and introduced by Smarandache [31]. This concept enables decision makers to use truth membership, indeterminacy-membership, and falsity-membership to describe their opinions. Consequently, Liu and Wang [26] have proposed a single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM) operator on the basis of Bonferroni mean, the weighted Bonferroni mean (WBM), and the normalized WBM to aggregate the single-valued neutrosophic numbers.

In addition, the concept of generalized interval-valued trapezoidal fuzzy numbers is another extension of fuzzy sets theory which is based on the concepts of generalized trapezoidal fuzzy numbers and interval-valued fuzzy numbers. Liu and Jin [28] have introduced some operations and rules of generalized interval-valued trapezoidal fuzzy numbers (GITFNs) and have proposed three generalized interval-valued trapezoidal fuzzy weighted aggregation operators that can handle the information hidden in uncertain data during aggregation phase of decision-making. Liu et al. [28] have also proposed a method based on ordered weighted harmonic averaging operators to improve decision-making process under generalized interval-valued trapezoidal fuzzy environment. A power generalized average (PGA) operator and an intuitionistic trapezoidal fuzzy power generalized weighted average (ITFPGWA) operator have been proposed by Liu and Liu [25], which extend the aggregation operators used in decision-making problems.

The concept of interval-valued intuitionistic fuzzy sets (IVIFSs) is also generalized form of IFSs theory that can have a meaningful role in improving decision modeling. It considers membership and non-membership functions as interval values; and hence, has more potential to handle and trail vague situations rather than IFSs [33, 36, 44]. As IVIFSs can properly handle the vagueness and ambiguity of data, it can be a powerful way to deal with real-life problems. Recently, Hashemi et al. [15] proposed an extended compromise ratio model under an interval-valued intuitionistic fuzzy (IVIF)-environment.

Based on the above remarks, this paper presents a new IVIF-decision approach based on compromise solution concept under an IVIF-environment. The main drawbacks of FMEA that this paper aims to solve them are as follows: (1) Difficulty in precise assessment of experts' judgments. Because the FMEA result is directly dependent on input data, and the data is obtained from an uncertain and vague source of experts' judgments, considering the uncertainty of judgments as complete as possible, is a very important issue. (2) Relative importance weights of three risk factors are assumed to be the same; Moreover, the traditional PRN formula is very sensitive to variations of risk factors. Considering proper risk factors weighting method, besides of taking into account uncertainty hidden in relative factors' weights simultaneously, can insure overcoming these weak points. (3) The way in which the RPN is obtained, is questionable. Since the risk factors are ordered numbers, they cannot be multiplied. This issue makes the RPN prioritization results debatable. Moreover, if a new risk factor with different scales is added to risk factors, the traditional RPN formula cannot be used anymore.

Based on mentioned traditional FMEA's weak points, the main advantages of proposed IVIF-decision approach can be described as below:

(1) The analysis is performed under an IVIF-environment to handle uncertainties: this study uses IVIFSs that can possess more information about human judgments by contemplating membership and non-membership functions as interval values. Thus, it is less likely to lose information duration calculations and analysis. Moreover, by considering the compromise solution concept, these characteristics help the proposed approach to have a clear computational image, easy to understand steps and the ability to determine the best option quickly.

(2) Objective weights are considered as well as subjective weights concurrently: many authors have regarded only one of subjective or objective weights in conducting FMEA (e.g., [7, 8, 9]); however, it is noticeable that none of them without another is complete. Therefore, in this study, both subjective and objective weights are concurrently employed.

(3) There are two kinds of subjective weights employed in this paper: this study considers direct assessment as well as entropy measure to ensure that the performance matrix is conforming to direct weights evaluation, and therefore, results are more trustworthy.

(4) A new collective index is presented to rank failure modes in the final decision process. This index ranks alternatives on the basis of their weighted Euclidean distances to positive and negative ideal points. In addition, this new collective index provides capability of considering new risk factors in

prioritizing of failure modes. It will be shown that this new index has an acceptance performance and the ability to help decision makers in evaluating failure modes and choosing the most important ones.

The rest of this paper is arranged as follows: Section 2 introduces IVIFS and relevant preliminaries, and section 3 presents the proposed IVIF-decision approach. Section 4 illustrates two practical examples to show the performance of new approach, and finally the discussion and conclusions are expressed in sections 5 and 6.

2. Preliminaries

2.1. Basic Concepts and Operations of Interval-valued Intuitionistic Fuzzy Sets.

In this section, the key notions for IVIFSs theory that will be useful throughout this study are presented. Atanassov and Gargov [1] introduced the notion of IVIFSs, which is defined as [15]:

Definition 2.1. Let X be an ordinary finite, nonempty set. An IVIFS in X is defined as

$$\tilde{A} = \{\langle x, \bar{\mu}_{\tilde{A}}(x), \bar{v}_{\tilde{A}}(x)\rangle\} \tag{2}$$

where $\bar{\mu}_{\tilde{A}}(x) \subset [0,1], \bar{v}_{\tilde{A}}(x) \subset [0,1]$, and $\sup \bar{\mu}_{\tilde{A}}(x) + \sup \bar{v}_{\tilde{A}}(x) \leq 1$, $\forall x \in X$.

In this study, an IVIF $\tilde{A}$ is denoted by $\langle [a_{\tilde{A}}, b_{\tilde{A}}], [c_{\tilde{A}}, d_{\tilde{A}}]\rangle$, where $[a_{\tilde{A}}, b_{\tilde{A}}] \subset [0,1], [c_{\tilde{A}}, d_{\tilde{A}}] \subset [0,1]$. For each element x, $\bar{\pi} = [1 - b_{\tilde{A}}(x) - d_{\tilde{A}}(x), 1 - a_{\tilde{A}}(x) - c_{\tilde{A}}(x)]$, is called hesitancy degree of an IVIF of $x \in X$ in $\tilde{A}$. It can be seen that $\bar{\pi} \subset [0,1]$.

Atanassov [2] and Xu [37] defined four operational laws of IVIFNs, each operator is introduced as follows:

$$\tilde{A} \oplus \tilde{B} = \langle [a_{\tilde{A}} + a_{\tilde{B}} - a_{\tilde{A}}.a_{\tilde{B}}, b_{\tilde{A}} + b_{\tilde{B}} - b_{\tilde{A}}.b_{\tilde{B}}], [c_{\tilde{A}}.c_B, d_{\tilde{A}}.d_{\tilde{B}}]\rangle \tag{3}$$

$$\tilde{A} \otimes \tilde{B} = \langle [a_{\tilde{A}}.a_{\tilde{B}}, b_{\tilde{A}}.b_{\tilde{B}}], [c_{\tilde{A}} + c_{\tilde{B}} - c_{\tilde{A}}.c_{\tilde{B}}, d_{\tilde{A}} + d_{\tilde{B}} - d_{\tilde{A}}.d_{\tilde{B}}]\rangle \tag{4}$$

$$\lambda\tilde{A} = \langle [1 - (1 - a_{\tilde{A}})^{\lambda}, 1 - (1 - b_{\tilde{A}})^{\lambda}], [(c_{\tilde{A}})^{\lambda}, (d_{\tilde{A}})^{\lambda}]\rangle, \quad \lambda > 0 \tag{5}$$

$$\tilde{A}^{\lambda} = \langle [(a_{\tilde{A}})^{\lambda}, (b_{\tilde{A}})^{\lambda}], [1 - (1 - c_{\tilde{A}})^{\lambda}, 1 - (1 - d_{\tilde{A}})^{\lambda}]\rangle, \quad \lambda > 0 \tag{6}$$

These operations can guarantee that the operational results are IVIFNs. Also, Yu et al. [42] and Ye [40] introduced the score and accuracy functions to measure and evaluate an IVIF as follows.

Definition 2.2. Let $\tilde{A}_1$ and $\tilde{A}_2$ be two IVIFNs, then:

$$S(\tilde{A}) = \frac{1}{4}[2 + a_{\tilde{A}} - c_{\tilde{A}} + b_{\tilde{A}} - d_{\tilde{A}}] \tag{7}$$

$$H(\tilde{A}) = a_{\tilde{A}} + b_{\tilde{A}} - 1 + \frac{c_{\tilde{A}} + d_{\tilde{A}}}{2} \tag{8}$$

where $S(\tilde{A}) \in [0,1]$ and $H(\tilde{A}) \in [-1,1]$. The larger value of $S(\tilde{A})$ the higher the IVIF is. On this basis, the largest and the smallest IVIFN are $\langle [1,1],[0,0]\rangle$ and $\langle [0,0],[1,1]\rangle$, respectively. If $S(\tilde{A}) < S(\tilde{B})$, then $\tilde{A} < \tilde{B}$

If $S(\tilde{A}) = S(\tilde{B})$, then:
If $H(\tilde{A}) = H(\tilde{B})$, then $\tilde{A} = \tilde{B}$
If $H(\tilde{A}) < H(\tilde{B})$, then $\tilde{A} < \tilde{B}$

Definition 2.3. Let $\tilde{a}_1$ and $\tilde{a}_2$ be two IVIFNs, then the Euclidean distance operator can be defined as [44]:

$$d(\tilde{a}_1, \tilde{a}_2) = \sqrt{\frac{1}{4}((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2)} \tag{9}$$

Definition 2.4. Let $\tilde{x}_j = \langle [a_j, b_j], [c_j, d_j] \rangle (j \in N)$ be a collection of IVIFNs, and $\lambda_j = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ be the weight vector of $\tilde{x}_j (j \in N)$, where λ_j indicates the importance degree of $\tilde{x}_j$, satisfying $\lambda_j \geq 0 (j \in N)$ and $\sum_(j = 1)^n \lambda_j = 1$, and let interval-valued intuitionistic fuzzy weighted averaging (IVIFWA): $\Phi^n \to \Phi$ if [44]:

$$\begin{aligned} \tilde{x}_j &= IVIFWA(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) = \sum_{j=1}^{n} \lambda_j . \tilde{x}_j \\ &= \langle \Big[1 - \Pi_{j=1}^{n} (1 - a_j)^{\lambda_j}, 1 - \Pi_{j=1}^{n} (1 - b_j)^{\lambda_j}], [\Pi_{j=1}^{n} (c_j)^{\lambda_j}, \Pi_{j=1}^{n} (d_j)^{\lambda_j} \Big] \rangle \end{aligned} \tag{10}$$

Definition 2.5. Let $\tilde{x}_j (j = 1, 2, \ldots, n)$ be a collection of IVIFNs, the interval-valued intuitionistic fuzzy weighted geometric aggregation operator of the IVIF-numbers is obtained by [15]:

$$\begin{aligned} \tilde{x}_{ij} &= IVIFWGA(\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n) = \Pi_{j=1}^{n} \tilde{x}_j^{\omega_j} \\ &= \langle \Big[\Pi_{k=1}^{l} (a_j^k)^{\omega_k}, \Pi_{k=1}^{l} (b_j^k)^{\omega_k} \Big] \Big[1 - \Pi_{k=1}^{l} (1 - c_{ij}^k)^{\omega_k}, 1 - \Pi_{k=1}^{l} (1 - d_{ij}^k)^{\omega_k} \Big] \rangle \end{aligned} \tag{11}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of $\tilde{x}_j (j = 1, 2, \ldots, n), \omega \in [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$.

2.2. **The OWA operator.** The OWA operator is a parameterized aggregation operator and first suggested by Yager [38]. This operator characterized by its input that is rearranged in the decrease order. The OWA's weights are the weights of ordered positions of input instead of each of them and can be defined as follows [36].

Definition 2.6. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting vector $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)^T$, with $\omega_j \subset [0, 1]$ and $\sum_{j=1}^{n} \omega_j$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} \omega_j b_j \tag{12}$$

where b_j is the jth largest of a_i.

For determining the OWA weights, much work has been done. To eliminate the effect of unfair judgments on the decision results, Xu [36] suggested a normal distribution-based method to generate the weights of the OWA operator. In this manner, the associated weighting vector is calculated by:

$$\omega_i = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-[(i-\mu_n)^2 / 2\sigma_n^2]} = \frac{e^{-[(i-\mu_n)^2 / 2\sigma_n^2]}}{\sum_{i=1}^{n} e^{-[(i-\mu_n)^2 / 2\sigma_n^2]}} \quad i = 1, 2, \ldots, n. \tag{13}$$

where $\omega_j = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of the OWA operator, μ_n is the mean, and σ_n is the standard deviation of the collection of $1, 2, \ldots, n$ and calculated by:

$$\mu_n = \frac{1}{n} \frac{n(1+n)}{2} = \frac{1+n}{2} \tag{14}$$

$$\sigma_n = \left(\frac{1}{n} \sum_{i=1}^{n} (i - \mu_n)^2 \right)^{\frac{1}{2}} \tag{15}$$

2.3. **Entropy weights.** After introducing the IVIF notions by Atanassov and Gargov [1], much work has been done on operators over IVIFSs. Recently, Ye [40] proposed a MCDM method using the entropy weights-based correlation coefficient of IVIFSs. This study has used entropy measures to determine the factors' weights when they are completely unknown. In fact, this operator determines the weights based on decision matrix provided by experts.

In this study, factors' weights are determined by experts and hence it is likely that their assessments in comparison with their ratings' evaluations result in biased judgment. Therefore, entropy method is used to obtain risk factors' weights as well as direct experts' assessments.

Definition 2.7. For any $A \in IVIF(X)$ the entropy measure is proposed as follows [40]:

$$\begin{aligned} I(A) = \Bigg\{ & \sin \frac{\pi \times [1 + a_{\tilde{A}}(x) + pW_{\mu}A(x) - c_{\tilde{A}}(x) - qW_{v}A(x)]}{4} \\ & + \sin \frac{\pi \times [1 - a_{\tilde{A}}(x) - pW_{\mu}A(x) + c_{\tilde{A}}(x) + qW_{v}A(x)]}{4} - 1 \Bigg\} \\ & \times \frac{1}{\sqrt{2} - 1} \end{aligned} \tag{16}$$

where $W_{\mu A}(x) = b_{\tilde{A}}(x) - a_{\tilde{A}}(x)$ and $W_v A(x) = d_{\tilde{A}}(x) - c_{\tilde{A}}(x)$.

2.4. **The IVIFHWED Operator.** In order to use several weights in determining the distance, Liu et al. [21] have employed IFHWED operator to aggregate both the objective and subjective results. The IVIFHWED operator can be defined as follows.

Definition 2.8. An interval-valued intuitionistic fuzzy hybrid weighted Euclidean distance (IVIFHWED) operator of dimension n is a mapping IVIFHWED: $R^n \times R^n \to R$ and can be calculated as follows [21]:

$$\tilde{D}_{ij}^{\varepsilon} = IVIFHWED(\tilde{A}, \tilde{B}) = \sum_{r=1}^{R} \left[\varphi_r (\sum_{j=1}^{l} d(\tilde{A}, \tilde{B}).w_j^r) \right] \tag{17}$$

where r is the number of different relative weights devoted to each criterion, RF_j.

3. Proposed IVIF-decision Approach for the FMEA

Conventional FMEA approach is criticized for considerable weak points. In the vast majority of cases, data is imprecise and involved with vagueness and it is rather unlikely that one can evaluate risk factors with precise numbers. By employing the IVIFSs developed by Atanassov and Gargov [1], not only the membership and non-membership degrees of opinions are available, but also the decision makers are enable to judge these functions in the imprecise manner and estimate them instead of expressing precisely. In practice, IVIFNs have been broadly applied in real-life decision making problems, and studies of application of IVIFs in decision-making processes as received extensive attention [44]. Therefore, the risk parameters are described as linguistic IVIF variables, and then they are converted to IVIFNs. Chen [10] extended the linguistic transformation standards developed by Boran et al. [4] to the IVIF-environment. This study provides a nine-point rating scale and modifies the linguistic descriptions of each term as provided in Table 1, [10].

Linguistic terms	Interval-valued intuitionistic fuzzy numbers
Extremely Good/High (EL)	$\langle[0.02, 0.05], [0.90, 0.95]\rangle$
Very Good/High (VL)	$\langle[0.10, 0.15], [0.70, 0.75]\rangle$
Good/High (L)	$\langle[0.25, 0.30], [0.55, 0.60]\rangle$
Medium Good/high (ML)	$\langle[0.40, 0.45], [0.45, 0.50]\rangle$
Fair/Medium (M)	$\langle[0.50, 0.55], [0.35, 0.40]\rangle$
Medium Bad/low (MH)	$\langle[0.60, 0.65], [0.25, 0.30]\rangle$
Bad/low (H)	$\langle[0.70, 0.75], [0.15, 0.20]\rangle$
Very Bad/low (VH)	$\langle[0.80, 0.85], [0.05, 0.10]\rangle$
Extremely Bad/Low (EH)	$\langle[0.90, 0.95], [0.02, 0.05]\rangle$

Table 1. Linguistic Terms and Their Corresponding IVIF-values

Traditional RPN method does not consider the relative importance between risk factors. In many cases, only subjective or objective weights are mentioned [30, 32]. As Chin et al. [12] have discussed, due to multi-disciplinary nature of FMEA team, determining the risk factors subjectively may be not easy, and different experts may have different opinions about factors' importance. On the other hand, by employing only objective weights without considering subjective weights, the results may be questionable [21]. Hence, this study considers both subjective and objective weights for the computations. Subjective weights are obtained by experts' assessments as well as entropy weights calculated by equation (14), and objective weights are determined by ordered weights according to equation (13).

Table 2 shows linguistic variables used for rating relative importance weights and their relevant IVIF-numbers [15].

Linguistic terms	Interval-valued intuitionistic fuzzy numbers
Very Good/High (VH)	$\langle[0.80, 0.90], [0.05, 0.10]\rangle$
Good/High (H)	$\langle[0.55, 0.70], [0.10, 0.20]\rangle$
Medium Good/high (MH)	$\langle[0.45, 0.60], [0.15, 0.30]\rangle$
Fair/Medium (M)	$\langle[0.30, 0.50], [0.20, 0.40]\rangle$
Medium Bad/low (ML)	$\langle[0.25, 0.40], [0.35, 0.50]\rangle$
Bad/low (L)	$\langle[0.10, 0.30], [0.45, 0.60]\rangle$
Very Bad/low (VL)	$\langle[0.00, 0.10], [0.70, 0.90]\rangle$

Table 2. Linguistic Terms for the Relative Importance of Criteria

Figure 1 illustrates the IVIF-decision approach based on compromise solution concept. The proposed flowchart for the FMEA can be described in the following steps:

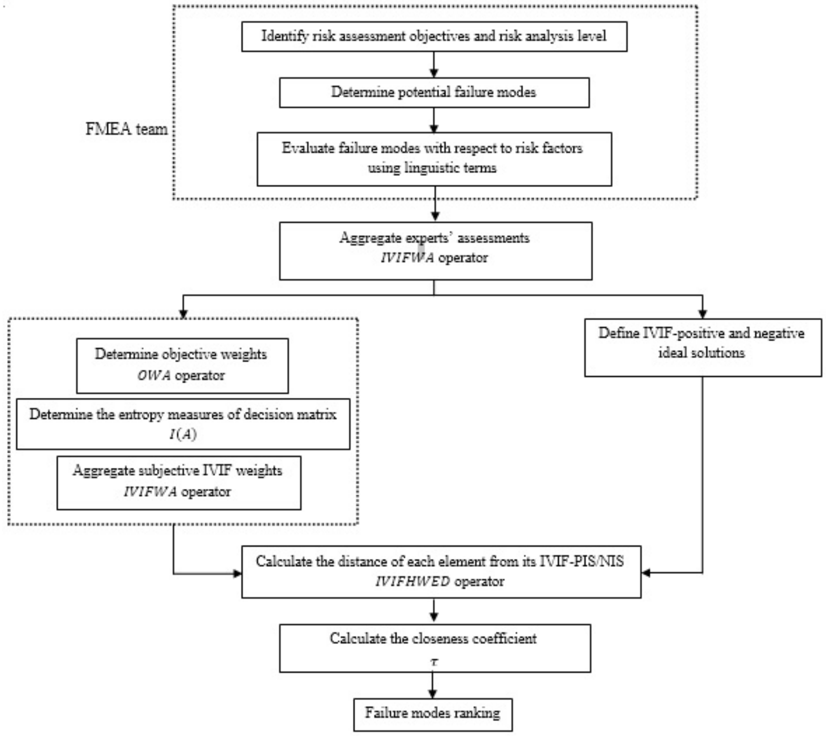

FIGURE 1. Steps of Proposed IVIF-FMEA

Step 1: Evaluate failure modes with respect to risk factors. Suppose that there is a cross-functional FMEA team including l members, $TM_k(k = 1, 2, \dots, l)$; and m potential failure modes, $FM_i(i = 1, 2, \dots, m)$, assessed by each of team members with respect to n risk factors, $RF_j(j = 1, 2, \dots, n)$. These assessments are represented by $\tilde{x}^k_{ij} = \langle [a^{(k)}_{ij}, b^{(k)}_{ij}], [c^{(k)}_{ij}, d^{(k)}_{ij}] \rangle$ as shown in the following. Moreover, there are relative weights for each risk factor,

$$\bar{w}^k_j = \langle [w_1 j^k, w_2 j^k], [w_3 j^k, w_4 j^k] \rangle$$

and relative importance of each member in FMEA team $\lambda\tilde{A}_k = \langle [\lambda^k_1, \lambda^k_2], [\lambda^k_3, \lambda^k_4] \rangle$. Each $\tilde{x}^k_{ij}$ and $\tilde{w}^k_j$ will be assessed by linguistic variables as mentioned in Tables 1 and 2.

$$X^{(k)} = (\tilde{x}^{(k)}_{ij})_{m\times n} = \begin{bmatrix} \langle \left[a^{(k)}_{11}, b^{(k)}_{11}\right], \left[c^{(k)}_{11}, d^{(k)}_{11}\right] \rangle & \cdots & \langle \left[a_1 m^{(k)}, b^{(k)}_{1m}\right], \left[c^{(k)}_{1m}, d^{(k)}_{1m}\right] \rangle \\ \vdots & \ddots & \vdots \\ \langle \left[a^{(k)}_{n1}, b^{(k)}_{n1}\right], \left[c^{(k)}_{n1}, d^{(k)}_{n1}\right] \rangle & \cdots & \langle \left[a^{(k)}_{mn}, b^{(k)}_{mn}\right], \left[c^{(k)}_{mn}, d^{(k)}_{mn}\right] \rangle \end{bmatrix}$$

Step 2: Aggregate experts' assessments using IVIFWA operator.

$$\begin{aligned}\tilde{x}_{ij} =& IVIFWA(\tilde{x}_{ij}^1, \tilde{x}_{ij}^2, \ldots, \tilde{x}_{ij}^l)\\ =& \sum_{k=1}^{l} \lambda^k . x_{ij}^k\\ =& \langle \Big[1 - \Pi_{k=1}^l (1 - a_{ij}^k)^{\lambda_k}, 1 - \Pi_{k=1}^l (1 - b_{ij}^k)^{\lambda_k}\Big],\\ & \Big[\Pi_{k=1}^l (c_{ij}^k)^{\lambda_k}, \Pi_{k=1}^l (d_{ij}^k)^{\lambda_k}\Big]\rangle\end{aligned} \tag{18}$$

$$\begin{aligned}\tilde{w}_j =& IVIFWA(\tilde{w}_j^1, \tilde{w}_j^2, \ldots, \tilde{w}_j^l)\\ =& \sum_{k=1}^{l} \lambda^k . w_j^k\\ =& \langle \Big[1 - \Pi_{k=1}^l (1 - w_{1j}^k)^{\lambda_k}, 1 - \Pi_{k=1}^l (1 - w_{2j}^k)^{\lambda_k}\Big],\\ & \Big[\Pi_{k=1}^l (w_{3j}^k)^{\lambda_k}, \Pi_{k=1}^l (w_{4j}^k)^{\lambda_k}\Big]\rangle\end{aligned} \tag{19}$$

where $\tilde{x}_{ij}$ is aggregated IVIF assessment of ith failure modes with respect to risk factors, RF_j; and $\tilde{w}_j$ is the aggregated subjective weights of risk factors, RF_j. Thus, the aggregated matrix of team assessments will be illustrated as follows:

$$\begin{aligned}X =& (\tilde{x}_{ij})_{m\times n}\\ =& \begin{bmatrix} \langle [a_{11}, b_{11}], [c_{11}, d_{11}]\rangle & \cdots & \langle [a_{1n}, b_{1n}], [c_{1n}, d_{1n}]\rangle \\ \langle [a_{21}, b_{21}], [c_{21}, d_{21}]\rangle & \cdots & \langle [a_{21}, b_{21}], [c_{21}, d_{21}]\rangle \\ \vdots & \ddots & \vdots \\ \langle [a_{m1}, b_{m1}], [c_{m1}, d_{m1}]\rangle & \cdots & \langle [a_{mn}, b_{mn}], [c_{mn}, d_{mn}]\rangle \end{bmatrix}\end{aligned}$$

Moreover, it is possible to consider experts' weights as IVIF-numbers. In this manner, the aggregated $\tilde{x}_{ij}$ can be calculated by IVIFWGA operator:

$$\begin{aligned}\tilde{x}_{ij} =& IVIFWGA(\tilde{x}_{ij}^1, \tilde{x}_{ij}^2, \cdots, \tilde{x}_{ij}^l) = \sum_{k=1}^{l} (\lambda^k . x_{ij}^k)^{\omega_k}\\ =& \langle \Big[\Pi_{k=1}^l (\lambda_1^k . a_{ij}^k)^{\omega_k}, \Pi_{k=1}^l (\lambda_2^k . b_{ij}^k)^{\omega_k}\Big],\\ & [1 - \Pi_{k=1}^l (1 - \lambda_3^k - c_{ij}^k + \lambda_3^k . c_{ij}^k)^{\omega_k}, 1\\ & - \Pi_{k=1}^l (1 - d_{ij}^k - \lambda_4^k + \lambda_4^k . c_{ij}^k)^{\omega_k}]\rangle\end{aligned} \tag{20}$$

$$\begin{aligned}\tilde{w}_j =& IVIFWGA(\tilde{w}_j^1, \tilde{w}_j^2, \ldots, \tilde{w}_j^l) = \sum_{k=1}^{l} \lambda^k . w_j^k\\ =& \langle \Big[\Pi_{k=1}^l \lambda_1^k . w_1 j^k)^{\omega_k}, \Pi_{k=1}^l (\lambda_2^k . w_2 j^k)^{\omega_k}\Big],\\ & [1 - \Pi_{k=1}^l (1 - \lambda_3^k - w_3 j^k + \lambda_3^k . w_3 j^k)^{\omega_k}, 1\\ & - \Pi_{k=1}^l (1 - w_3 j^k - \lambda_4^k + \lambda_4^k . w_3 j^k)^{\omega_k}]\rangle\end{aligned} \tag{21}$$

where, $\omega = (\omega_1, \omega_2, \ldots, \omega_l)^T = (\frac{1}{l}, \frac{1}{l}, \ldots, \frac{1}{l})^T$ is the weight vector of $\tilde{w}_j (j = 1, 2, \ldots, n), \omega_k \in [0, 1]$, and $\sum_{k=1}^{l} \omega_k = 1$.

Step 3: Determine objectives' weights. Because of the importance of objective weights, OWA operator introduced by Xu [36] is used to reduce the effect of human biased judgments on results. If $n = 3$ by equations (21) and (23), then $\mu_3 = 2$ and $\sigma_3 = \sqrt{2/3}$. From equation (13) objective weights vector is obtained $\omega_3 = (0.243, 0.514, 0.243)^T$.

Step 4: Aggregate the subjective IVIF-weights.

In order to use subjective IVIF-weights in total distance formula, there is a need to aggregate relative factors weights. Hence, in this study, the numeral values of subjective weights can be obtained by calculating weights' distances from original point defined as $O = \langle[0, 0], [0, 0]\rangle$ according to equation (9).

Because the weights' results do not satisfy the condition $\sum_{j=1}^{3} w_j = 1$, it is necessary to normalize the subjective weights by the following equation:

$$w_j^{'} = \frac{w_j}{\sum_{j=1}^{3} w_j} \tag{22}$$

Step 5: Determine the entropy measure of decision matrix. It is possible to determine the risk factors' relative weights through the failure modes' performance matrix. This method is useful when the factors' weights are entirely unknown. In this study, factors' weights are determined by experts and hence, it is likely that their assessments result in biased judgment. Therefore, entropy method is used to obtain risk factors' weights as well as direct experts' assessments. Since these weights are obtained based on assessment matrix, they are subjective and can be calculated by the following relation:

$$\begin{aligned} I(A) =&\{\sin \frac{\pi \times [1 + a_{\tilde{A}}(x) + pW_\mu A(x) - c_{\tilde{A}}(x) - qW_v A(x)]}{4} \\ &+ \sin \frac{\pi \times [1 - a_{\tilde{A}}(x) - pW_\mu A(x) + c_{\tilde{A}}(x) + qW_v A(x)]}{4} - 1\} \\ &\times \frac{1}{\sqrt{2} - 1} \end{aligned} \tag{23}$$

where $W_\mu A(x) = b_{\tilde{A}}(x) - a_{\tilde{A}}(x)$ and $W_v A(x) = d_{\tilde{A}}(x) - c_{\tilde{A}}(x)$.

Step 6. Define IVIF-positive ideal solution (IVIF-PIS) and IVIF-negative ideal solution (IVIF-NIS).

$$\begin{aligned} IVIF - PIS = \tilde{R}^+ &= (\tilde{r}_1^+, \tilde{r}_2^+, \ldots, \tilde{r}_n^+) \\ &=(\langle[(\tilde{a}_1^+, \tilde{b}_1^+], [\tilde{c}_1^+, \tilde{d}_1^+]\rangle, \langle[(\tilde{a}_2^+, \tilde{b}_2^+], [\tilde{c}_2^+, \tilde{d}_2^+]\rangle, \ldots, \langle[(\tilde{a}_n^+, \tilde{b}_n^+], [\tilde{c}_n^+, \tilde{d}_n^+]\rangle) \end{aligned} \tag{24}$$

$$\begin{aligned} IVIF - NIS = \tilde{R}^- &= (\tilde{r}_1^-, \tilde{r}_2^-, \ldots, \tilde{r}_n^-) \\ &=(\langle[(\tilde{a}_1^-, \tilde{b}_1^-], [\tilde{c}_1^-, \tilde{d}_1^-]\rangle, \langle[(\tilde{a}_2^-, \tilde{b}_2^-], [\tilde{c}_2^-, \tilde{d}_2^-]\rangle, \ldots, \langle[(\tilde{a}_n^-, \tilde{b}_n^-], l[\tilde{c}_n^-, \tilde{d}_n^-]\rangle) \end{aligned} \tag{25}$$

The more the RPN, the more serious the level is; and therefore, $\tilde{r}_j^+$ and $\tilde{r}_j^-$ are determined as follows:

$$\tilde{r}_j^+ = \langle[1, 1], [0, 0]\rangle$$
$$\tilde{r}_j^- = \langle[0, 0], [1, 1]\rangle$$

Step 7. Calculate distances of each element in decision matrix from its IVIF-PIS and IVIF-NIS.

$$d(\tilde{a}_1,\tilde{a}_2)=\sqrt{(\frac{1}{4}((a_1-a_2)^2+(b_1-b_2)^2+(c_1-c_2)^2+(d_1-d_2)^2)} \tag{26}$$

$$\begin{aligned}\tilde{D}_{ij}^{\varepsilon} =& IVIFHWED(\tilde{A},\tilde{A}^{*})\\ =&\varphi_1(\sum_{j=1}^{l} d(a_{ij},a_j^{\varepsilon}).w_j^1)+\varphi_2(\sum_{j=1}^{l} d(a_{ij},a_j^{\varepsilon}).w_j^2)+\varphi_3(\sum_{j=1}^{l} d(a_{ij},a_j^{\varepsilon}).w_j^3)\\ =&\varphi_1\left(w_j^1.\tilde{A}^{\varepsilon}\sum_{j=1}^{l}\left(\sqrt{\frac{1}{4}((a_{ij}-a_j^{\varepsilon})^2+(b_{ij}-b_j^{\varepsilon})^2+(c_{ij}-c_j^{\varepsilon})^2+(d_{ij}-d_j^{\varepsilon})^2)}\right)\right)\\ &+\varphi_2\left(w_j^2.\sum_{j=1}^{l}\left(\sqrt{\frac{1}{4}((a_{ij}-a_j^{\varepsilon})^2+(b_{ij}-b_j^{\varepsilon})^2+(c_{ij}-c_j^{\varepsilon})^2+(d_{ij}-d_j^{\varepsilon})^2)}\right)\right)\\ &+\varphi_3\left(w_j^3.\sum_{j=1}^{l}\left(\sqrt{(\frac{1}{4}((a_{ij}-a_j^{\varepsilon})^2+(b_{ij}-b_j^{\varepsilon})^2+(c_{ij}-c_j^{\varepsilon})^2+(d_{ij}-d_j^{\varepsilon})^2)}\right)\right)\end{aligned} \tag{27}$$

where ε is * or $-$; $\varphi_1,\varphi_2,\varphi_3$ are relative weights used for aggregating subjective and objective results satisfying $\varphi_i\in[0,1],\sum_{i=1}^{3}\varphi_i=1$; and w_j^1,w_j^2,w_j^3 are relative subjective and objective weights.

Step 8. Determine the proposed collective index of elements.

The collective index of failure mode $\tilde{A}_i$ under IVIF-PIS, $\tilde{A}^*$, is defined below:

$$\tau=\tau_1+\tau_2 \tag{28}$$

where

$$\tau_1=\left(\frac{D_i^*}{D_i^-}\right)^{1/n} \tag{29}$$

and

$$\tau_2=(D_i^*)^{1/n}+(1/D_i^-)^{1/n} \tag{30}$$

Step 9. Prioritize the failure modes and determine corrective actions. The smaller the τ, the more serious the overall risk is, and therefore, the higher the priority is. On this basis, the ranking of failure modes can be obtained by the increasing order of relative closeness coefficients.

4. **Practical Examples in the Manufacturing Industry**

4.1. **The First Practical Example.** In this section, in order to demonstrate the proposed new IVIF-decision approach in FMEA based on TOPSIS method, an applicable example adapted from Liu et al. [21] will be resolved. This example involves developing new horizontal directional drilling (HDD) machine that is a complex product with several multidisciplinary sub-systems. This complex machine can be employed for installing underground pipe, conduit, or cable in a shallow arc along a prescribed bore path by using a surface-launched drilling rig, with minimal impact on the surrounding area. Complex sub-systems of machine as well as serious applications of this machine demonstrate the importance of risk assessment regarding the machine's failures before reaching to customers. Hence, conducting FMEA can improve reliability and safety of this machine.

A FMEA team consists of five cross-functional team members that have found nine potential failure modes in the above product. There is a need to rank failure modes in terms of their risk factors that is O, S and D, so that the most important failure modes can be distinguished for corrective actions.

As precise assessment is usually not possible in real-world, the FMEA team members evaluate failure modes and risk factors' importance by using linguistic variables expressed in IVIF-numbers as shown in Tables 1 and 2. The evaluation matrix assessed by team members' assessments can be seen in [21].

$$X = (\tilde{x}_{ij})_{9\times 15} = \begin{bmatrix} \langle[0.5,0.55],[0.35,0.4]\rangle & \langle[0.5,0.55],[0.35,0.4]\rangle & \cdots & \langle[0.4,0.45],[0.45,0.5]\rangle & \langle[0.4,0.45],[0.45,0.5]\rangle \\ \langle[0.5,0.55],[0.35,0.4]\rangle & \langle[0.6,0.65],[0.25,0.3]\rangle & \cdots & \langle[0.6,0.65],[0.25,0.3]\rangle & \langle[0.5,0.55],[0.35,0.4]\rangle \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \langle[0.5,0.55],[0.35,0.4]\rangle & \langle[0.5,0.55],[0.35,0.4]\rangle & \cdots & \langle[0.4,0.45],[0.45,0.5]\rangle & \langle[0.4,0.45],[0.45,0.5]\rangle \end{bmatrix}$$

Since the FMEA team consists of different members with different expertise, it is better to differentiate in their opinion importance in reality. In order to consider these differences, several relative weights are used for each member with respect to her/his expertise, role and influence in FMEA team. As mentioned, it is possible to express these weights in linguistic terms and convert to IVIF-numbers; but, in this example these weights are assumed to be crisp; i.e., $0.15, 0.20, 0.25, 0.10$ and 0.30.

After establishing IVIF-matrix of FMEA team members' assessments as illustrated in the above, these opinions are aggregated into unique assessment by using equation (18). The aggregated assessments are provided as below:

$$X = (\tilde{x}_{ij})_{9\times 3} = \begin{bmatrix} \langle[0.493,0.543],[0.356,0.406]\rangle & \langle[0.486,0.536],[0.362,0.407]\rangle & \langle[0.390,0.440],[0.451,0.501]\rangle \\ \langle[0.615,0.666],[0.231,0.283]\rangle & \langle[0.651,0.702],[0.195,0.244]\rangle & \langle[0.527,0.577],[0.320,0.372]\rangle \\ \vdots & \ddots & \ddots \\ \langle[0.447,0.497],[0.402,0.452]\rangle & \langle[0.452,0.503],[0.397,0.443]\rangle & \langle[0.462,0.512],[0.387,0.437]\rangle] \end{bmatrix}$$

Then, it is necessary to determine subjective and objective weights. As discussed earlier, in this study three different weights are taken into account: weights expressed by FMEA team members, entropy weights, and objective weights. Since the two formers are obtained by human judgments, they are classified as subjective weights and the latter is objective and calculated by the normal distribution-based method suggested by [36]. These three weights are calculated according to equations (13), (19), and (23). The resulting weights are presented in Table 3.

Subjective Weights	O	S	D
Aggregated sub-weights	0.30	0.39	0.31
Entropy weights	0.34	0.34	0.32
Objective Weights	Rank 1	Rank 2	Rank 3
Objective Weights	0.24	0.51	0.24

Table 3. Subjective and Objective Weights for the First Practical Example

Finally, by employing subjective weights (i.e., weights expressed by FMEA team members and entropy weights), objective weights, and aggregated assessment matrix, the IVIFHWED for each failure mode is obtained by equation (27). In this example, the parameters $\varphi_1, \varphi_2, \varphi_3$ are defined with values of $0.2, 0.3$ and 0.5. Two former weights are subjective and get the sum of values to be 0.5 and; the later has a value of 0.5.

Moreover, the IVIF-PIS and IVIF-NIS are assumed to be

$$\tilde{A}_j^* = [\langle [1,1,],[0,0]\rangle, \langle [1,1,],[0,0]\rangle, \ldots, \langle [1,1,],[0,0]\rangle]$$

and

$$\tilde{A}_j^- = [\langle [0,0],[1,1]\rangle, \langle [0,0],[1,1]\rangle, \ldots, \langle [0,0],[1,1]\rangle],$$

respectively. The results are illustrated in Table 4.

4.2. **The Second Practical Example.** To evaluate the applicability and validity of proposed new IVIF-decision approach, an application example adapted from Zhou and Thai [45] is also provided to compare the results of proposed approach with fuzzy RPN and grey RPN methods. This practical example evaluates 17 tanker equipment failures according to 5 experts' judgments. The linguistic fuzzy variables are adapted to linguistic IVIF-variables proposed in this study. The decision matrix containing the experts' opinions about 17 failure modes with respect to risk factors provides from Zhou and Thai [45].

After translating experts' opinions into IVIFNs, the opinions of different experts are aggregated by employing equation (18). The relative weights of five experts are: 0.15, 0.25, 0.25, 0.20 and 0.15. Then, the objective weights of risk factors should be calculated. Using equations (13) and (23), the objective weights are determined and illustrated in Table 6. The subjective weights of the factors O, S and D are also determined to be 0.40, 0.35 and 0.25, respectively. Finally, by applying the IVIFHWED operator obtained by equation (27), the final ranking of failures is calculated. The final failures' prioritization is reported in Table 7. In addition, the results of proposed IVIF method are compared with fuzzy RPN and grey RPN methods.

Failure modes	D^+	D^-	τ_1	τ_2	T	Ranking
1	0.465	0.541	0.951	2.002	2.953	6
2	0.327	0.680	0.784	1.826	2.610	3
3	0.369	0.639	0.833	1.879	2.712	4
4	0.544	0.465	1.054	2.107	3.161	9
5	0.487	0.521	0.978	2.030	3.008	8
6	0.416	0.593	0.888	1.936	2.825	5
7	0.298	0.711	0.748	1.788	2.537	1
8	0.319	0.691	0.773	1.814	2.586	2
9	0.474	0.533	0.962	2.013	2.975	7

Table 4. Distance Measures and Closeness Coefficient for the First Practical Example

Failure modes	Proposed IVIF-decision approach	IFHWED-based FMEA [22]	Fuzzy FMEA [33]
1	6	7	6
2	3	2	3
3	4	5	5
4	9	9	9
5	8	6	7
6	5	4	4
7	1	1	1
8	2	3	2
9	7	8	8

Table 5. Ranking Comparisons on the HDD Machine for the First Practical Example

Subjective Weights	O	S	D
Aggregated sub weights	0.40	0.35	0.25
Entropy weights	0.36	0.36	0.29
Objective Weights	Rank 1	Rank 2	Rank 3
Objective Weights	0.24	0.51	0.24

Table 6. Subjective and Objective Weights for the Second Practical Example

Failure modes	D^+	D^-	τ_1	τ_2	τ	Proposed IVIF decision method	Fuzzy RPN	Grey method
1	0.369	0.638	0.833	1.879	2.712	8	8	8
2	0.364	0.647	0.825	1.870	2.695	9	6	6
3	0.391	0.616	0.860	1.907	2.766	7	7	7
4	0.296	0.712	0.746	1.786	2.532	13	14	14
5	0.165	0.844	0.580	1.607	2.187	17	17	17
6	0.491	0.518	0.982	2.034	3.016	3	3	3
7	0.353	0.655	0.814	1.858	2.672	10	10	11
8	0.276	0.732	0.723	1.761	2.483	14	12	12
9	0.244	0.765	0.683	1.718	2.401	15	15	15
10	0.307	0.700	0.760	1.801	2.561	12	13	13
11	0.344	0.664	0.803	1.847	2.650	11	11	10
12	0.761	0.252	1.446	2.497	3.943	1	1	1
13	0.416	0.592	0.889	1.937	2.826	5	4	4
14	0.405	0.604	0.875	1.923	2.798	6	9	9
15	0.625	0.384	1.176	2.231	3.407	2	2	2
16	0.417	0.591	0.891	1.939	2.830	4	5	5
17	0.174	0.836	0.592	1.619	2.211	16	16	16

Table 7. Distance Measures, Closeness Coefficient and Ranking Comparisons for the Second Practical Example

5. **Discussion and Results**

In this section, a sensitivity analysis on the impact of FMEA factors' weights is reported to further study on the collective index and ranking for the first application example. Because, as in equations (27) - (30) the collective index of each failure mode (alternative) is provided by a closeness to the IVIF-PIS and IVIF-NIS, the values of final ranking may depend on weights of the FMEA factors including objective and subjective importance. Several values of factors' weights are taken into consideration for the analysis of the collective index.

The idea of this sensitivity analysis is to exchange each FMEA factor's weight with another factor's weight. Thus, three combinations of these FMEA factors are assessed with each combination presented as a condition. The main condition (condition 1) proposes the original results of the first application example. Computational results of this sensitivity analysis are presented in Table 8 and depicted in Figure 2.

According to Figure 2, failure mode 7 has the highest rank and failure mode 4 has the lowest rank when the FMEA factors' weights are exchanged in three conditions. The analysis results show that the change of factors' weights does not remarkably change the ranking of nine failure modes in the first application example. It concedes with what this study has expected and illustrates that the ranking of nine failure modes remains stable in this application throughout the changes of weights. To show the performance of suggested IVIF-decision approach

Conditions	Weights			Values of collective index								
	O	*S*	*D*	FM1	FM2	FM3	FM4	FM5	FM6	FM7	FM8	FM9
Main	0.301	0.390	0.309	2.953	2.610	2.712	3.161	3.008	2.825	2.537	2.586	2.975
(1)	0.338	0.338	0.324									
	0.243	0.514	0.243									
	0.301	0.309	0.390	2.974	2.620	2.718	3.216	3.061	2.878	2.531	2.623	2.974
(2)	0.338	0.324	0.338									
	0.243	0.243	0.514									
	0.390	0.301	0.309	2.967	2.624	2.740	3.180	3.033	2.865	2.539	2.639	2.975
(3)	0.338	0.338	0.324									
	0.514	0.243	0.243									
	0.309	0.390	0.301	2.953	2.610	2.710	3.162	3.009	2.827	2.538	2.589	2.975
(4)	0.324	0.338	0.338									
	0.243	0.514	0.243									

TABLE 8. Sensitivity Analysis on FMEA Factors' Weights

in the FMEA based on compromise solution concept, a practical example adopted from Liu et al. [21] (i.e., the first practical example) has been recalculated. To do this, first experts' opinions have been collected by linguistic terms and transferred to IVIF-numbers. Then, the opinions have been aggregated through IVIFWA operator and the weights have been determined. In the proposed approach, both subjective and objective weights have been considered. For subjective weights, aggregated weights assessed by experts have been employed as well as the entropy measure that calculates the weights based on decision matrix. Because of the importance of objective weights, a normal distribution-based method has been used to determine the weights of the OWA operator. Then, IVIFHWED operator has been taken to determine the distance of each element from IVIF-positive ideal solution (IVIF-PIS) and IVIF-negative ideal solution (IVIF-NIS); and finally, the proposed collective index has been calculated. The comparisons of ranking results for the proposed IVIF-decision approach and other decision methods under uncertainty, mentioned in [21], are illustrated in Table 5.

From Table 5, it can be seen that the ranking order of the proposed IVIF-decision approach is consistent with the ranking order of two-mentioned methods. In all of them, the failure mode 7 has the highest priority with the collective index of 2.537 and the failure mode 4 with collective index of 3.161 has the lowest priority.

In comparison with Liu et al. [21], failure mode 1 is more important than failure mode 5. As it can be seen in the decision matrix with respect to occurrence and severity factors, failure mode 1 has a higher degree. Although experts have given a higher score to failure mode 5 with respect to detection factor, this factor is not as important as severity.

As it is shown in Table 7 for the second practical example, the results of proposed IVIF-approach are consistent with fuzzy RPN and grey RPN methods. Only difference between the proposed IVIF-approach and former one is about failure modes 2 and 14. By considering Table 6 for the experts' opinions, failure 14 is more important with respect to occurrence factor, whereas the failure 2 has higher priority with respect to severity factor. The occurrence relative weight is higher than severity's weight; however, the difference between failures scores is not significant; the obtained result may be due to conformity of scales of original example to scales proposed in this study. Therefore, it can be concluded that this approach

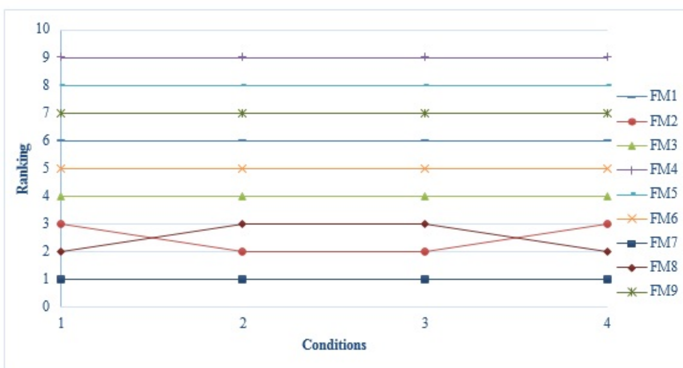

Figure 2. Sensitivity Analysis Under Different FMEA Factors' Weights

has a high performance in comparison with other FMEA methods; there are main advantages in the proposed IVIF-decision approach, unlike the previous studies as follows:

- Dealing with uncertainties in the decision process by interval-valued intuitionistic fuzzy sets: Because it is not very easy to assess the failure modes precisely, many authors have suggested performing the FMEA analysis under uncertain environments [22, 9]. There are some studies that used notions and concepts of the IFSs [22, 7, 8]. The concept of IVIFSs is generalized form of IFSs theory that considers membership and non-membership functions as interval values. Hence, it can have a meaningful role in enhancing decision-making results and handling vagueness better than IFSs [33]. Therefore, this study has used the notions and concepts of IVIFSs that can retain more information about experts' assessments (judgments), and it is less likely to lose information duration of calculations and analysis under uncertainty. Moreover, because of applying the concept of the compromise solution in the proposed IVIF-decision approach under uncertainty, this concept's features help the proposed approach to have a clear computational form, uncomplicated and easy to understand steps, and finally the ability to properly determine the best option.

- Considering objective weights as well as subjective weights simultaneously in the decision process: There are many studies that have considered only one of subjective or objective weights in performing FMEA analysis. However, none of them is complete in the use of proper weights in reality. Subjective weights are criticized for the probability of considering unfair judgments of experts. Thus, they may result in biased ranking orders. Objective weights can reduce the effects of biased assessments and mitigate them in order to achieve better result. Nevertheless, it is considerable that none of them is complete without the other. Hence, in this study, both subjective and objective weights are performed in the proposed IVIF-decision approach unlike the previous studies.
- Providing two kinds of subjective weights in the decision process: This study has considered two kinds of subjective weights: direct assessment as well as entropy measure, and therefore, evaluation results of the FMEA is more trustworthy. Considering entropy measure as subjective weight ensures that the decision matrix is conforming to direct weights' assessments and thus less probability to have biased ranking orders.
- Presenting a new collective index to rank failure mode alternatives: it is very important for decision makers to prioritize failure mode alternatives accurately; hence, choosing an appropriate ranking index that discriminates among failure mode alternatives properly and recognizes the most serious ones is a critical part for the evaluation of FMEA. This index is based on weighted Euclidean distances of alternatives to positive and negative ideal points. Computational results indicate that this new index has an acceptance performance and ensure that the failure mode alternatives are ranked properly. Providing capacity of considering new risk factors with different scales in calculation and failures ranking is another advantage of this new collective index.

6. Conclusions and Further Research

FMEA is a well-known method as a preventive risk assessment approach that aims to find, analyze and mitigate serious failure modes. It is proved to be effective, easy to understand and well-documented method; however, it has some weak points and has been criticized by numerous authors. For instance, firstly, it does not contemplate relative importance among O, S and D. Secondly, different combinations of O, S and D may make exactly the same value of RPN, but their risk implications may be completely different. Thirdly, the three risk factors are difficult to be explicitly evaluated. This paper introduced a novel IVIF-decision approach based on a compromise solution concept that can overcome the above weak points and can revise the traditional FMEA. To contemplate the relative weights among the risk factors, both subjective and objective weights consist of aggregated weights obtained by experts' assessments, entropy and OWA operators have been employed in the proposed approach. Furthermore, this approach has been defined under an interval-valued intuitionistic fuzzy (IVIF)-environment to guarantee that the assessments information would be retained during the calculations. Hence, it considers uncertainty in experts' judgments as well as reduces the probability of obtaining two ranking orders with the same value. Finally, a new solution index based on alternatives' Euclidean distance to their positive and negative ideal points has been introduced to ensure that failure mode alternatives are ranked properly. To present the effectiveness of suggested approach, two practical examples have been illustrated in the manufacturing industry. Because of the product's complexity, FMEA can be useful to identify and eliminate the most serious risks under uncertainty. As it has been shown and discussed, this approach has an acceptable performance, as well as it ensures that relative weights are trustworthy and mitigates biased aspect of human judgments under the IVIF-environment. Further research can concentrate on some interesting aspects: Firstly, some well-known decision-making methods, such as VIKOR, AHP, and ELECTRE, can be developed under an IVIF-environment based on the proposed objective and subjective weights. Considering IVIF-environment with different weights in the decision process can improve retaining information during computations. Secondly, it is considerable that there are some famous theories for representing uncertainty and vagueness hidden in experts' judgments, such as D-S evidence theory, Z-numbers and D-numbers. Thirdly, social and culture indicators can be regarded in the FMEA analysis with different phases. Fourthly, applying new extensions of fuzzy sets and decision support systems (DSSs) concurrently for computerizing the proposed decision approach is recommended to handle uncertainty in order to decrease the needed effort and time for computations.

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Z. Hajighasemi*, Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran
E-mail address: Z.hajighasemi99@gmail.com

S. Meysam Mousavi, Department of Industrial Engineering, Faculty of Engineering, Shahed University, Tehran, Iran
E-mail address: sm.mousavi@shahed.ac.ir

*Corresponding author