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## A New Method for Solving Interval Neutrosophic Linear Programming Problems

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### Highlights

- Present a technique to convert every Interval Neutrosophic problem into the crisp model.
- A powerful tool for dealing with indeterminate and inconsistent information.
- A technique for solving and optimizing the problems based on uncertainty.

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### Abstract

Neutrosophic set theory is a generalization of the intuitionistic fuzzy set which can be considered as a powerful tool to express the indeterminacy and inconsistent information that exist commonly in engineering applications and real meaningful science activities. In this paper an interval neutrosophic linear programming (INLP) model will be presented, where its parameters are represented by triangular interval neutrosophic numbers (TINNs) and call it INLP problem. Afterward, by using a ranking function we present a technique to convert the INLP problem into a crisp model and then solve it by standard methods.

## 1. INTRODUCTION

Linear programming (LP) is an applied technique for solving optimization problems [1-4] that can be used in different aspects of studies [5-8]. Uncertainty is an information deficit that is one of the inseparable components of real-world problems. In the past few decades, various types of sets have been introduced. The fuzzy set (FS) theory which is proposed by Zadeh [9] is a practical approach that is widely used to capture linguistic uncertainty in optimization problems such that it assigns to each element a degree of membership in  $[0,1]$ . As a natural extension of FSs Zadeh [10] proposed the concept of interval-valued fuzzy sets (IVFSs) to express the uncertainty in the membership function. The membership of an element belonging to an IVFS is represented by interval values in  $[0,1]$ . Atanasov in [11] has presented an extension of classical fuzzy sets that is the so-called intuitionistic fuzzy sets (IFSs) such it assigns the degrees of membership (truth-membership) and non-membership (falsity-membership) to each element. In this respect Atanasov and Gragov in [12] by extending the membership (truth-membership) and non-membership (falsity-membership) functions to the interval numbers in  $[0,1]$  proposed the interval-valued intuitionistic fuzzy set (IVIFS). For dealing with the indeterminacy Smarandach in [13, 14] introduced the neutrosophic sets (NSs) as a new generalization of IFSs. This approach added the indeterminacy membership as an independent factor to the basis of intuitionistic fuzzy sets. A neutrosophic set  $N$  in  $X$  can be characterized by three membership functions such as truth, indeterminacy, and falsity membership functions that are completely independent of each other. Wang et al. [15] introduced the concept of single-valued

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neutrosophic sets (SVNS), instead of the neutrosophic set. Also, Wang et al. in [16] proposed interval neutrosophic sets (INSs) where the degrees of truth, indeterminacy, and falsity memberships were extended to a subinterval of  $[0,1]$ . This research, as a first time, presents an LP problem with interval neutrosophic numbers (INNs) that is the so-called interval neutrosophic linear programming (INLP) problem. As a fact, an INLP problem is a problem with one or more coefficients that are represented by INNs. In the same way, a ranking function is introduced to convert each INLP problem into a crisp problem. The rest of this paper is marshaled as follows: in the next section, we will briefly review the basic concept of neutrosophic sets. In section 3 the formularization of our proposed method is presented. In section 4 the numerical examples are presented. In section 5 the method is compared with some other existing methods and the conclusions are discussed in section 6.

## 2. PRELIMINARIES

This section recalls the necessary notions and definitions of NS theory on the real numbers line that can be used in this research.

**Definition 1.** [17] A Neutrosophic Set (NS)  $N$  in a domain  $X$  (finite universe of objectives) can be represented by  $T_N : X \rightarrow ]0^-, 1^+[, I_N : X \rightarrow ]0^-, 1^+[,$  and  $F_N : X \rightarrow ]0^-, 1^+[,$  such that  $0^- \leq T_N(x) + I_N(x) + F_N(x) \leq 3^+ \forall x \in X$ . Where  $T_N(x)$ ,  $I_N(x)$  and  $F_N(x)$  denote the truth, indeterminacy, and falsity membership functions, respectively.

**Definition 2.** [18] A single-valued neutrosophic set (SVNS)  $N$  in a domain  $X$  (finite universe of objectives) can be denoted as  $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$ , where  $T_N : X \rightarrow [0,1]$ ,  $I_N : X \rightarrow [0,1]$  and  $F_N : X \rightarrow [0,1]$  are three maps in  $X$  that satisfy the condition  $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3 \forall x \in X$ . The numbers  $T_N(x)$ ,  $I_N(x)$  and  $F_N(x)$ , are respectively the degrees of truth, indeterminacy and falsity membership of element  $x$  to  $N$ .

**Definition 3.** A neutrosophic number (NN)  $N$  is an extension of the fuzzy set on  $\mathbb{R}$  such that the truth, indeterminacy and falsity membership functions could be defined as follows:

$$T_N(x) = \begin{cases} \frac{x - a^l}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{a^u - x}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 0, & \text{O.W,} \end{cases} \quad (1)$$

$$I_N(x) = \begin{cases} \frac{a^m - x}{a^m - a^l}, & \delta a^l + (1 - \delta)a^m \leq x \leq a^m, \\ \frac{x - a^m}{a^u - a^m}, & a^m \leq x \leq (1 - \delta)a^m + \delta a^u, \\ \delta, & \text{O.W,} \end{cases} \quad (2)$$

$$F_N(x) = \begin{cases} \frac{a^m - x}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{x - a^m}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 1, & \text{O.W,} \end{cases} \tag{3}$$

where  $\delta \in (0,1)$  is the maximum degree of indeterminacy and  $a^l \leq a^m \leq a^u$ .

The various functions of the single-valued neutrosophic number (SVNN)  $N$  are shown in Figure 1.

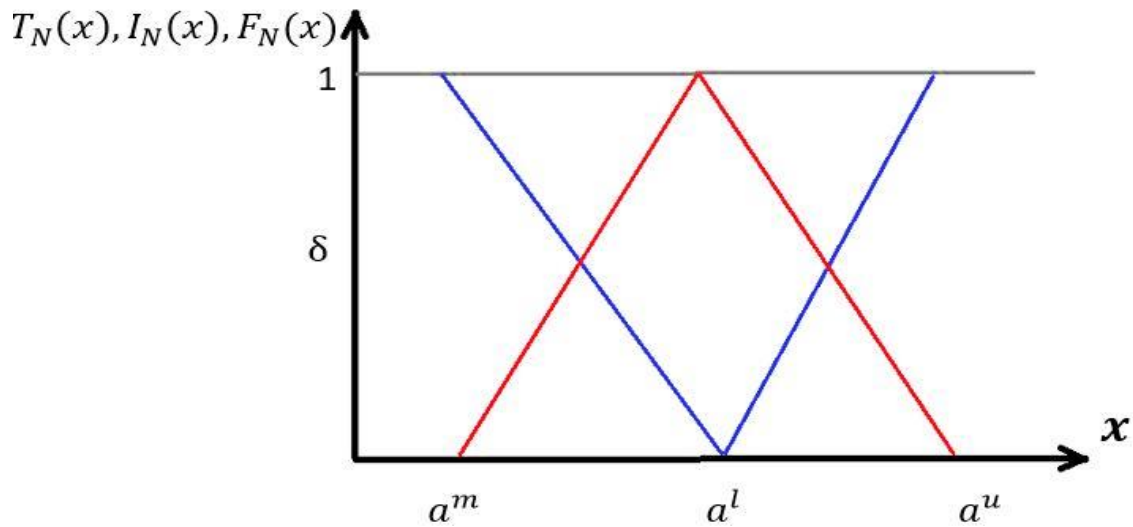


Figure 1. Truth, indeterminacy and falsity membership functions of  $N$ .

**Definition 4.** [19] The addition and subtraction operations between two SVNNs such as  $N = [(a^l, a^m, a^u); \alpha_N, \delta_N, \beta_N]$  and  $M = [(b^l, b^m, b^u); \alpha_M, \delta_M, \beta_M]$  could be defined as:

$$N + M = [(a^l + b^l, a^m + b^m, a^u + b^u); \alpha_N \wedge \alpha_M, \delta_N \vee \delta_M, \beta_N \vee \beta_M], \tag{4}$$

$$N - M = [(a^l - b^l, a^m - b^m, a^u - b^u); \alpha_N \wedge \alpha_M, \delta_N \vee \delta_M, \beta_N \vee \beta_M], \tag{5}$$

furthermore, the scalar multiplication is defined as:

$$kN = \begin{cases} [(ka^l, ka^m, ka^u); \alpha_N, \delta_N, \beta_N], & k > 0, \\ [(ka^u, ka^m, ka^l); \alpha_N, \delta_N, \beta_N], & k < 0. \end{cases} \tag{6}$$

**Definition 5.** [20, 21] Let  $N$  and  $M$  are two NNs The ranking orders of these two numbers will be as:

- If  $L(N) > L(M)$  then  $N$  is bigger than  $M$ ,
- If  $L(N) < L(M)$  then  $N$  is smaller than  $M$ ,
- If  $L(N) = L(M)$  then  $N$  is equal to  $M$ .

**Definition 6.** [16] Let  $X$  be a space of discourse, an interval neutrosophic set (INS)  $N$  through  $X$  taking the form  $N = \{x, T_N(x), I_N(x), F_N(x); x \in X\}$  where  $T_N(x), I_N(x), F_N(x) \subseteq [0,1]$  and  $0 \leq \text{Sup}T_N(x) + \text{Sup}I_N(x) + \text{Sup}F_N(x) \leq 3$  for all  $x \in X$ .  $T_N(x)$ ,  $I_N(x)$  and  $F_N(x)$  represent truth membership, indeterminacy membership, and falsity membership of  $x$  to  $N$ , respectively.

**Definition 7.** An interval neutrosophic number (INN)  $N$  is an extended version of the fuzzy set on  $/R$  whose the truth, indeterminacy and falsity membership functions are given as follows:

$$T_N^L(x) = \begin{cases} \frac{x - a^l + h_N(a^l - x)}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{a^u - x + h_N(x - a^u)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 0, & \text{O.W,} \end{cases} \quad (8)$$

$$T_N^U(x) = \begin{cases} \frac{x - a^l + h_N(a^m - x)}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{a^u - x + h_N(x - a^m)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ h_N, & \text{O.W,} \end{cases} \quad (9)$$

where  $T_N(x) = [T_N^L(x), T_N^U(x)]$ ,

$$I_N^L(x) = \begin{cases} \frac{a^m - x + h_N(x - a^m)}{a^m - a^l}, & \delta a^l + (1 - \delta)a^m \leq x \leq a^m, \\ \frac{x - a^m + h_N(a^m - x)}{a^u - a^m}, & a^m \leq x \leq (1 - \delta)a^m + \delta a^u, \\ \delta, & \text{O.W,} \end{cases} \quad (10)$$

$$I_N^U(x) = \begin{cases} \frac{a^m - x + h_N(x - a^l)}{a^m - a^l}, & \delta a^l + (1 - \delta)a^m \leq x \leq a^m, \\ \frac{x - a^m + h_N(a^u - x)}{a^u - a^m}, & a^m \leq x \leq (1 - \delta)a^m + \delta a^u, \\ 1 - \delta, & \text{O.W,} \end{cases} \quad (11)$$

where  $I_N(x) = [I_N^L(x), I_N^U(x)]$ ,

$$F_N^L(x) = \begin{cases} \frac{a^m - x + h_N(x - a^l)}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{x - a^m + h_N(a^m - x)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 1 - h_N, & \text{O.W,} \end{cases} \quad (12)$$

$$F_N^U(x) = \begin{cases} \frac{a^m - x + h_N(x - a^l)}{a^m - a^l}, & a^l \leq x \leq a^m, \\ \frac{x - a^m + h_N(a^u - x)}{a^u - a^m}, & a^m \leq x \leq a^u, \\ 1, & \text{O.W,} \end{cases} \quad (13)$$

where  $F_N(x) = [F_N^L(x), F_N^U(x)]$  and  $h_N = T_N^U(x) - T_N^L(x)$  such that  $\delta \in (0, 1)$  and  $h_N \leq \delta$ .

The truth, indeterminacy and falsity membership functions of an interval neutrosophic number with the above properties are shown in Figure 2.

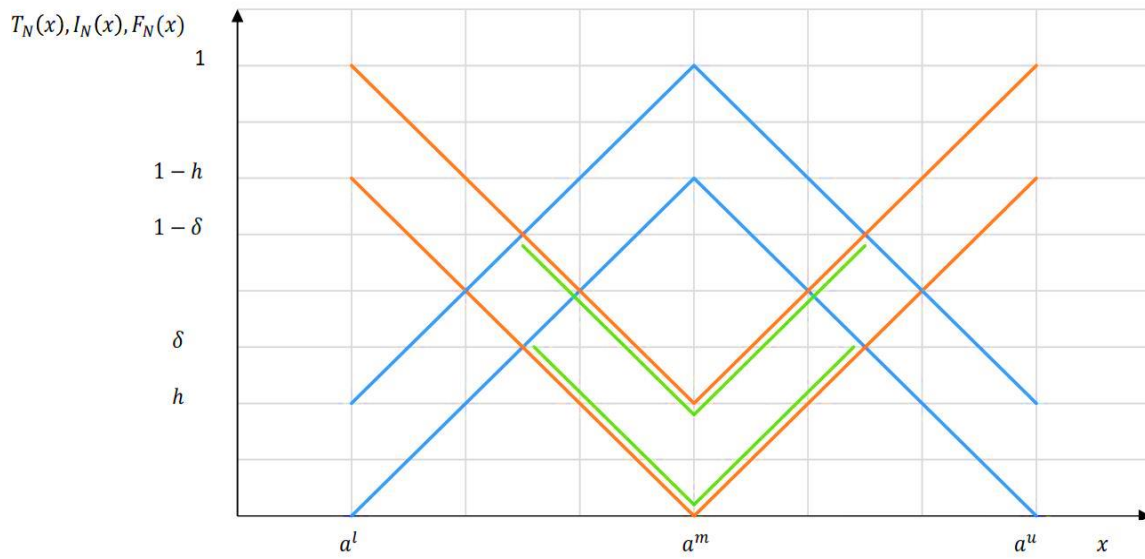


Figure 2. Truth, indeterminacy and falsity membership functions of INN

**Remark 1.** An INS  $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$  will be reduced to the NS if  $\alpha_N^l = \alpha_N^u, \delta_N^l = \delta_N^u$  and  $\beta_N^l = \beta_N^u$ .

**Definition 8.** Let  $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$  and  $M = [(b^l, b^m, b^u); [\alpha_M^l, \alpha_M^u], [\delta_M^l, \delta_M^u], [\beta_M^l, \beta_M^u]]$  are two INNs. The addition and subtraction operations for these two INNs are defined as follows:

$$N + M = [(a^l + b^l, a^m + b^m, a^u + b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l \alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u \alpha_M^u], [\delta_N^l \delta_M^l, \delta_N^u \delta_M^u], [\beta_N^l \beta_M^l, \beta_N^u \beta_M^u]],$$

$$N - M = [(a^l - b^l, a^m - b^m, a^u - b^u); [\alpha_N^l + \alpha_M^l - \alpha_N^l \alpha_M^l, \alpha_N^u + \alpha_M^u - \alpha_N^u \alpha_M^u], [\delta_N^l \delta_M^l, \delta_N^u \delta_M^u], [\beta_N^l \beta_M^l, \beta_N^u \beta_M^u]].$$

### 3. PROPOSED INTERVAL NEUTROSOPHIC LINEAR PROGRAMMING METHOD

In this section, by using a new ranking function for interval neutrosophic numbers we suggest a new method for solving INLP problems. The basis of our work will be presented as follows:

**Step (1).** Insert the INLP problem with triangular interval neutrosophic numbers.

**Step (2).** By using the following method convert the INLP problem to the crisp model.

In order to compare any two triangular INNs based on the proposed ranking function, let  $N = [(a^l, a^m, a^u); [\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u], [\beta_N^l, \beta_N^u]]$  be a symmetric interval neutrosophic number, where  $[\alpha_N^l, \alpha_N^u], [\delta_N^l, \delta_N^u]$  and  $[\beta_N^l, \beta_N^u]$  are respectively the truth, indeterminacy, and falsity membership degrees of  $N$ . Also  $a^l, a^m$  and  $a^u$  are respectively the lower, median, and upper bounds for  $N$ . The ranking function for the interval neutrosophic number  $N$  will be defined as follows:

$$L(N) = \frac{1}{4} [a^l + a^u + 2a^m] + (\bar{\alpha}_N - \bar{\delta}_N - \bar{\beta}_N), \tag{14}$$

where  $\bar{\alpha}_N = \frac{\alpha_N^l + \alpha_N^u}{2}, \bar{\delta}_N = \frac{\delta_N^l + \delta_N^u}{2}$  and  $\bar{\beta}_N = \frac{\beta_N^l + \beta_N^u}{2}$ . Moreover, we have :

$$N \geq \tilde{0} \quad \text{if} \quad \frac{a^l + a^u + 2a^m}{4} \geq 0.$$

**Step (3).** By applying the previous ranking function, convert each triangular INN to a crisp number. This leads to convert the INLP problem to the crisp model.

**Step (4).** Solve the crisp model using the standard simplex method to achieve the optimal solution.

#### 4. NUMERICAL EXAMPLES

In this section, two examples based on INNs have been presented to illustrate the effectiveness of our proposed model. It should be noted that we also can consider the INLP problems with the variables that are considered as INNs but since we always prefer to obtain a crisp value for the optimal solution, so the values of  $x_j$  are considered as real numbers.

##### 4.1. Example 1

In this example, we consider the fully INLP problem where all parameters except  $x_j$  are considered as interval neutrosophic numbers.

$$\text{Max } \tilde{Z} \approx \tilde{7}x_1 + \tilde{6}x_2 + 1\tilde{4}x_3$$

s.t.

$$1\tilde{5}x_1 + \tilde{1}x_2 \lesssim \tilde{10},$$

$$\tilde{9}x_1 + \tilde{4}x_2 + \tilde{8}x_3 \lesssim \tilde{2},$$

$$1\tilde{9}x_2 + 1\tilde{1}x_3 \lesssim \tilde{4},$$

$$x_1, x_2, x_3 \geq 0.$$

where

$$\tilde{7} = \langle (1, 7, 13), [0.7, 0.9], [0.1, 0.4], [0.2, 0.4] \rangle,$$

$$\tilde{6} = \langle (2, 6, 10), [0.2, 0.6], [0.2, 0.5], [0.1, 0.8] \rangle,$$

$$1\tilde{4} = \langle (7, 14, 21), [0.5, 0.7], [0.4, 0.6], [0.3, 0.4] \rangle,$$

$$1\tilde{5} = \langle (14, 15, 16), [0.6, 0.8], [0.1, 0.4], [0.4, 0.9] \rangle,$$

$$\tilde{1} = \langle (0, 1, 2), [0.2, 0.7], [0.1, 0.6], [0.8, 0.9] \rangle,$$

$$\tilde{10} = \langle (5, 10, 15), [0.1, 0.5], [0.1, 0.4], [0.6, 0.9] \rangle,$$

$$\tilde{9} = \langle (4, 9, 14), [0.4, 0.5], [0.5, 0.8], [0.4, 0.8] \rangle,$$

$$\tilde{4} = \langle (1, 4, 7), [0.1, 0.9], [0.4, 0.5], [0.3, 0.4] \rangle,$$

$$\tilde{8} = \langle (6, 8, 10), [0.7, 0.8], [0.5, 0.6], [0.1, 0.6] \rangle,$$

$$\tilde{2} = \langle (1, 2, 3), [0.3, 0.6], [0.1, 0.9], [0.4, 0.6] \rangle,$$

$$\tilde{19} = \langle (5, 19, 33), [0.5, 0.9], [0.3, 0.5], [0.7, 0.8] \rangle,$$

$$\tilde{11} = \langle (8, 11, 14), [0.6, 0.8], [0.4, 0.9], [0.6, 0.9] \rangle,$$

$$\tilde{4} = \langle (0, 4, 8), [0.3, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle.$$

By applying the proposed ranking function in Equation (14) we have:

$$\text{Max } Z = 6.75x_1 + 5.6x_2 + 13.75x_3$$

s.t.

$$14.8x_1 + 0.25x_2 \leq 9.3,$$

$$8.2x_1 + 3.7x_2 + 7.85x_3 \leq 1.45,$$

$$18.55x_2 + 10.3x_3 \leq 4.15,$$

$$x_1, x_2, x_3 \geq 0.$$

The standard form of the above INLP problem will be constructed as follows:

$$\text{Max } Z = 6.75x_1 + 5.6x_2 + 13.75x_3$$

s.t.

$$14.8x_1 + 0.25x_2 + s_1 = 9.3,$$

$$8.2x_1 + 3.7x_2 + 7.85x_3 + s_2 = 1.45,$$

$$18.55x_2 + 10.3x_3 + s_3 = 4.15,$$

$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0,$$

where  $s_1, s_2$  and  $s_3$  are the slack variables. By using the simplex method we have the first simplex tableau as shown in Table 1.

**Table 1.** Initial simplex form

Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	R.H.S
$s_1$	14.8	0.25	0	1	0	0	9.3
$s_2$	8.2	3.7	7.85	0	1	0	1.45
$s_3$	0	18.55	10.3	0	0	1	4.15
Z	-6.75	-5.6	-13.75	0	0	0	0

In the previous table  $x_3$  is the coming variable and  $s_2$  is the leaving variable. The second simplex tableau presented in Table 2 as follows:



**Table 2.** Optimal simplex form

Basis	$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	R.H.S
$s_1$	14.8	0.25	0	1	0	0	9.3
$x_3$	1.04	0.47	1	0	0.13	0	0.18
$s_3$	-10.76	13.7	0	0	-1.31	1	2.25
Z	7.61	0.88	0	0	1.75	0	2.54

where  $Z^* = 2.54$  and Table 2 is the optimal simplex for our example.

#### 4.2. Example 2 (Case Study)

In a computer manufacturing plant, we need to produce four basic units, such as RAMs, graphic cards, hard drives, and CPUs, to produce each computer. All productions have to get through four parts. These four parts include "Design, Fabrication, Probe, and Assembly". The favorable time for each unit manufactured and its profit is presented in Table 3. The minimum production amount for supplementing monthly products is presented in Table 4. The purpose of the company is to produce products in this limit for maximizing the general profits.

**Table 3.** Departments and profits

Products	Design	Fabrication	Probe	Assembly	Unit profit
$P_1$	0.2	0.5	0.1	0.1	14\$
$P_2$	0.5	3	2	0.6	7\$
$P_3$	0.4	4	4	0.8	5\$
$P_4$	1	2	0.2	0.2	8\$

**Table 4.** Time capacity and minimum production level

Sector	Capacity (in hours)	Products	Minimum production level
Design	1300	$P_1$	100
Fabrication	3340	$P_2$	280
Probe	1800	$P_3$	194
Assembly	2100	$P_4$	400

The neutrosophic values for each INN in the previous tables are represented as follows:

$$\begin{aligned}\tilde{14} &= \langle (12, 14, 16), [0.3, 0.7], [0.2, 0.8], [0.2, 0.9] \rangle, \\ \tilde{7} &= \langle (2, 7, 12), [0.1, 0.6], [0.4, 0.7], [0.6, 0.8] \rangle, \\ \tilde{5} &= \langle (4, 5, 6), [0.2, 0.5], [0.4, 0.9], [0.3, 0.4] \rangle, \\ \tilde{8} &= \langle (3, 8, 13), [0.2, 0.5], [0.3, 0.8], [0.6, 0.9] \rangle, \\ \tilde{1300} &= \langle (1000, 1300, 1600), [0.1, 0.6], [0.2, 0.7], [0.3, 0.8] \rangle, \\ \tilde{3340} &= \langle (3215, 3340, 3465), [0.7, 0.9], [0.2, 0.7], [0.4, 0.9] \rangle, \\ \tilde{1800} &= \langle (1390, 1800, 2210), [0.4, 1], [0.2, 0.6], [0.1, 0.2] \rangle, \\ \tilde{2100} &= \langle (1818, 2100, 2510), [0.3, 0.7], [0.1, 0.6], [0.4, 0.8] \rangle, \\ \tilde{100} &= \langle (99, 100, 101), [0.1, 0.7], [0.2, 0.6], [0.3, 0.4] \rangle, \\ \tilde{280} &= \langle (230, 280, 330), [0.7, 0.9], [0.1, 0.2], [0.2, 0.5] \rangle, \\ \tilde{194} &= \langle (184, 194, 204), [0.1, 0.6], [0.3, 0.7], [0.1, 0.7] \rangle, \\ \tilde{400} &= \langle (200, 400, 600), [0.1, 0.4], [0.2, 0.6], [0.4, 0.8] \rangle.\end{aligned}$$

Let  $x_1, x_2, x_3$  and  $x_4$  represent the number of produced RAMs, graphics cards, hard drives, and CPUs, respectively. The above problem can be formulated as follows:

$$\begin{aligned}\text{Max } \tilde{Z} &\approx 14\tilde{x}_1 + 7\tilde{x}_2 + 5\tilde{x}_3 + 8\tilde{x}_4 \\ \text{s.t.} \\ 0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 &\lesssim \tilde{1300}, \\ 0.5x_1 + 3x_2 + 4x_3 + 2x_4 &\lesssim \tilde{3340}, \\ 0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 &\lesssim \tilde{1800}, \\ 0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 &\lesssim \tilde{2100}, \\ x_1 &\lesssim \tilde{100}, \\ x_2 &\lesssim \tilde{280}, \\ x_3 &\lesssim \tilde{194}, \\ x_4 &\lesssim \tilde{400}, \\ x_1, x_2, x_3 &\geq 0.\end{aligned}$$

By applying the proposed ranking function in Equation (14) the following crisp model can be obtained:

$$\begin{aligned}\text{Max } Z &= 13.45x_1 + 6.1x_2 + 4.4x_3 + 7.05x_4 \\ \text{s.t.} \\ 0.2x_1 + 0.5x_2 + 0.4x_3 + 1x_4 &\leq 1299.35, \\ 0.5x_1 + 3x_2 + 4x_3 + 2x_4 &\leq 3339.7, \\ 0.1x_1 + 2x_2 + 4x_3 + 0.2x_4 &\leq 1800.15, \\ 0.1x_1 + 0.6x_2 + 0.8x_3 + 0.2x_4 &\leq 2099.55,\end{aligned}$$

$$\begin{aligned}
 x_1 &\geq 99.65, \\
 x_2 &\geq 280.3, \\
 x_3 &\geq 193.45, \\
 x_4 &\geq 399.25, \\
 x_1, x_2, x_3, x_4 &\geq 0.
 \end{aligned}$$

By using the primal simplex method the results of the above problem will be obtained as follows:

$$\begin{aligned}
 x_1 &= 1853, \\
 x_2 &= 280.3, \\
 x_3 &= 193.45, \\
 x_4 &= 399.25, \\
 Z^* &= 30298.57.
 \end{aligned}$$

## 5. COMPARISON ANALYSIS AND DISCUSSION

In order to illustrate the efficiency and eventuality of our model, a comparative study with other existing methods that were outlined in [22-24] is conducted.

By comparing our model with Deli and Şubaş in [23] we are noted that:

1. Their model is very complex and too time-consuming in calculations, but our model is so simple.
2. Their model is only able to solve the problem with single-valued neutrosophic numbers but by using the proposed model in addition to single-valued neutrosophic numbers we can handle interval-valued neutrosophic numbers.
3. Due to the lack of necessity in determining the truth, falsity, and indeterminacy-membership degrees as crisp values, the proposed model is more effective and feasible than their model.

By comparing the present method with Akyar et al. in [22] we also noted that:

1. By considering all aspects of the decision-making such as truthiness, indeterminacy, and falsity the proposed model can exhibit reality efficiently than their model.
2. Although in their model only the fuzzy linear programming problems are converted to crisp models, their model has more variables and constraints than our model.

Also by comparing our method with Dubei and Mehra in [24] we also founded that:

1. In their model, they only consider truthiness and falsity while in the real circumstances the decision-making process has the form “agree, not sure and disagree”, where this drawback can be treated by NSs.
2. In their model, the degrees of membership and non-membership are considered as single-valued numbers, while by considering these degrees as interval numbers in our model, we can overcome the uncertainty in determining the membership and non-membership degrees.
3. Their model can only handle incomplete data but by using the proposed method we can handle both incomplete and indeterminate data.

## 6. CONCLUSION

In this research, by considering an LP problem based on INNs we have presented a new linear programming model. In this model in view of considering the truthiness, indeterminacy, and falsity degrees we can cover all aspects of real daily life circumstances. It should be noticed there is no necessity the values of these degrees be crisp values. In this respect, we proposed a ranking function that is capable of converting every

triangular interval neutrosophic number to its equivalent crisp value. Subsequently, every INLP problem could be converted to the crisp model where can be solved by standard methods easily. The proposed model indicates more simplicity applicability and more efficiency in comparison with other existing models.

### CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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