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Research paper

A New Perspective on Neutrosophic Differential Equation

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Abstract

In this paper the Neutrosophic ordinary differential equation of first order via neutrosophic numbers is epitomized. We also intend to define the neutrosophic numbers and their (α, β, γ)-cut. Finally a numerical example is given to demonstrate its practicality and effectiveness of the differential equation involving neutrosophic numbers.

Keywords: *Neutrosophic set; Neutrosophic number; Neutrosophic Differential equation*

1. Introduction

Smarandache [14]-[16] initiated the notion of neutrosophic set which is generalization of classical set, fuzzy set , intuitionistic fuzzy set, and so on. In the neutrosophic set, for an element x of the universe, the functions independently expresses the truth-membership degree, indeterminacy-membership degree, and false membership degree of the element x. This indeterminacy imports more information than fuzzy and intuitionistic fuzzy logic. Hence the application of neutrosophic logic would lead to better performance than fuzzy logic. In modeling science and engineering problems there arise some parameters which is uncertain or imprecise. When the model is uncertainty with differential equations then the concepts of imprecise differential equations emerge. Many researchers [1] - [13] have solved in fuzzy and intuitionistic fuzzy sense. But these two logic does not have the term indeterminacy. To handle such situation Neutrosophic set were developed. The multifaceted factors of neutrosophic sets have been applied in the differential equations. In this paper the first order homogeneous ordinary differential equation via neutrosophic numbers have been proposed. The solution of the equation is discussed and applied in bacteria culture model.

2. Preliminaries

Definition 1. [14]
Let X be a universe set. A neutrosophic set A on X is defined as $A = \{<T_A(x), I_A(x), F_A(x)> : x \in X\}$ where $T_A(x), I_A(x), F_A(x) : X \to {}^-]0,1[^+$ represents the degree of membership, degree of indeterministic and degree of non-membership respectively of the element $x \in X$ such that ${}^-0 \le T_A(x), I_A(x), F_A(x) \le 3^+$.

Definition 2. [18]
Let X be a universe set. A single valued neutrosophic set A on X is defined as $A = \{<T_A(x), I_A(x), F_A(x)> : x \in X\}$ where $T_A(x), I_A(x), F_A(x) : X \to [0,1]$ represents the degree of membership, degree of indeterministic and degree of non-membership respectively of the element $x \in X$ such that $0 \le T_A(x), I_A(x), F_A(x) \le 3$.

Definition 3.
(α,β,γ)-cut: The (α,β,γ)-cut Neutrosophic set is denoted by $F_{(\alpha,\beta,\gamma)}$, where $\alpha, \beta, \gamma \in [0,1]$ and are fixed numbers such that $\alpha + \beta + \gamma \le 3$ is defined as by $F_{(\alpha,\beta,\gamma)} = \{<T_A(x), I_A(x), F_A(x)> : x \in X, T_A(x) \ge \alpha , I_A(x) \le \beta, F_A(x) \le \gamma \}$.

Definition 4.
A neutrosophic set A defined on the universal set of real numbers R is said to be neutrosophic number if it has the following properties.
(i) A is normal it there exist $x_0 \in R$ such that $T_A(x_0) = 1$. ($I_A(x_0) = F_A(x_0) = 0$).
(ii) A is convex for the truth function $T_A(x)$ ie.,
$T_A(\mu x_1 + (1-\mu)x_2) \ge \min (T_A(x_1), T_A(x_2))\ \forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.
(iii) A is concave set for the indeterministic function $I_A(x)$ and false function $F_A(x)$ ie.,
$I_A(\mu x_1 + (1-\mu)x_2) \ge \max (I_A(x_1), I_A(x_2))\ \forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.
$T_A(\mu x_1 + (1-\mu)x_2) \ge \max (F_A(x_1), F_A(x_2))\ \forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.

Definition 5. [6]
A triangular neutrosophic number A is a subset of neutrosophic number in R with the following truth function, indeterministic function and falsity function which is given by:

$$T_A(x) = \begin{cases} \left(\dfrac{x-a}{b-a}\right)\vartheta_A & for \quad a \le x \le b \\ \vartheta_A & for \quad x = b \\ \left(\dfrac{c-x}{c-b}\right)\vartheta_A & for \quad b \le x \le c \\ 0 & otherwise \end{cases}$$

$$I_A(x) = \begin{cases} \left(\dfrac{b-x}{b-a}\right)\nu_A & for \quad a \le x \le b \\ \nu_A & for \quad x = b \\ \left(\dfrac{x-c}{c-b}\right)\nu_A & for \quad b \le x \le c \\ 1 & otherwise \end{cases}$$

$$F_A(x)=\begin{cases}\left(\frac{b-x}{b-a}\right)\kappa_A & for \quad a\le x\le b\\ \kappa_A & for \quad x=b\\ \left(\frac{x-c}{c-b}\right)\kappa_A & for \quad b\le x\le c\\ 1 & otherwise\end{cases}$$

Where $a \le b \le c$ and a triangular neutrosophic number is denoted by $A_{TN}\langle(a,b,c);\vartheta_A,\nu_A,\kappa_A\rangle$.

Note 1:
Here $T_A(x)$ increases with constant rate for $x \in[a,b]$ and decreases for $x \in [b,c]$$ but $I_A(x)$ and $F_A(x)$ decreases with constant rate for $x \in [a,b]$ and increases for $x \in[b,c]$.

Definition 6. [6]
A trapezoidal neutrosophic number A is a subset of neutrosophic number in R with the following truth function, indeterministic function and falsity function which is given by:

$$T_A(x)=\begin{cases}\left(\frac{x-a}{b-a}\right)\vartheta_A & for \quad a\le x\le b\\ \vartheta_A & for \quad b\le x\le c\\ \left(\frac{d-x}{d-c}\right)\vartheta_A & for \quad c\le x\le d\\ 0 & otherwise\end{cases}$$

$$I_A(x)=\begin{cases}\left(\frac{b-x}{b-a}\right)\nu_A & for \quad a\le x\le b\\ \nu_A & for \quad b\le x\le c\\ \left(\frac{d-x}{d-c}\right)\nu_A & for \quad c\le x\le d\\ 1 & otherwise\end{cases}$$

$$F_A(x)=\begin{cases}\left(\frac{b-x}{b-a}\right)\kappa_A & for \quad a\le x\le b\\ \kappa_A & for \quad b\le x\le c\\ \left(\frac{d-x}{d-c}\right)\kappa_A & for \quad c\le x\le d\\ 1 & otherwise\end{cases}$$

where $a \le b \le c \le d$ and a trapezoidal neutrosophic number is denoted by $A_{TRN}\langle(a,b,c,d);\vartheta_A,\nu_A,\kappa_A\rangle$.

Note 1:
Here $T_A(x)$ increases with constant rate for $x \in[a,b]$ and decreases for $x \in [c,d]$ but $I_A(x)$ and $F_A(x)$ decreases with constant rate for $x \in [a,b]$ and increases for $x \in[c,d]$.

3. First Order Neutrosophic Ordinary Differential Equation

Definition 7.
If A is neutrosophic number then (α,β,γ)-cut is given by

$$A_{(\alpha,\beta,\gamma)}=\begin{cases}[A_1(\alpha),A_2(\alpha)] & for \quad \alpha\in[0,1]\\ [A_1'(\beta),A_2'(\beta)] & for \quad \beta\in[0,1]\\ [A_1''(\gamma),A_2''(\gamma)] & for \quad \gamma\in[0,1]\end{cases}$$

With $\alpha + \beta + \gamma \le 3$.
Here

(i) $\frac{dA_1(\alpha)}{d\alpha}>0,\frac{dA_2(\alpha)}{d\alpha}<0,\ \forall\alpha\in[0,1], A_1(1)\le A_2(1)$

(ii) $\frac{dA_1'(\beta)}{d\beta}<0,\frac{dA_2'(\beta)}{d\beta}>0,\ \forall\beta\in[0,1], A_1'(0)\le A_2'(0)$

(iii) $\frac{dA_1''(\gamma)}{d\gamma}<0,\frac{dA_2''(\gamma)}{d\gamma}>0,\ \forall\gamma\in[0,1], A_1''(0)\le A_2''(0)$

It is given as

$$A_{(\alpha,\beta,\gamma)}=\{[A_1(\alpha),A_2(\alpha)],[A_1'(\beta),A_2'(\beta)],[A_1''(\gamma),A_2''(\gamma)]\}$$

If $A=(a,b,c;\vartheta_A,\nu_A,\kappa_A)$ then (α,β,γ)-cut is given by

$$\begin{aligned}A_{(\alpha,\beta,\gamma)}=\{&[(a+\alpha(b-a))\vartheta_A,(c-\alpha(c-b))\vartheta_A],\\&[(b-\beta(b-a))\nu_A,(b+\beta(c-b))\nu_A],\\&[(b-\gamma(b-a))\kappa_A,(b+\gamma(c-b))\kappa_A]\}\end{aligned}$$

Definition 8.
Consider the first order linear homogeneous Neutrosophic Ordinary Differential Equation: $\frac{dy}{dx}=ky$ with the initial condition $y(x_0) = y_0$ where k and y_0 are triangular neutrosophic numbers. Let the solution of the above neutrosophic differential equation be y(x) and its (α,β,γ)-cut be

$$\begin{aligned}y(x,\alpha,\beta,\gamma)=(&[y_1(x,\alpha),y_2(x,\alpha)],\\&[y_1'(x,\beta),y_2'(x,\beta)],\\&[y_1''(x,\gamma),y_2''(x,\gamma)])\end{aligned}$$

The solution is strong if

(i) $\frac{dy_1(x,\alpha)}{d\alpha}>0,\frac{dy_2(x,\alpha)}{d\alpha}<0,\ \forall\alpha\in[0,1], y_1(x,1)\le y_2(x,1)$

(ii) $\frac{dy_1'(x,\beta)}{d\beta}>0,\frac{dy_2'(x,\beta)}{d\beta}<0,\ \forall\beta\in[0,1], y_1'(x,0)\le y_2'(x,0)$

(iii) $\frac{dy_1''(x,\gamma)}{d\gamma}>0,\frac{dy_2''(x,\gamma)}{d\gamma}<0,\ \forall\gamma\in[0,1], y_1''(x,0)\le y_2''(x,0)$

Otherwise the solution is weak solution.

4. Solution of Differential Equation with Triangular Neutrosophic Number

Consider the first order linear homogeneous Neutrosophic Ordinary Differential Equation: $\frac{dy}{dx}=ky$ with the initial condition

$y(x_0) = A_{TN}\langle(a,b,c);\vartheta_A,\nu_A,\kappa_A\rangle$. (1)

Case I:
If $k > 0$
Taking (α,β,γ)-cut of the equation (1) we get

$$\frac{dy}{dx}([y_1(x,\alpha),y_2(x,\alpha)],[y_1'(x,\beta),y_2'(x,\beta)],[y_1''(x,\gamma),y_2''(x,\gamma)])$$
$$=k([y_1(x,\alpha),y_2(x,\alpha)],[y_1'(x,\beta),y_2'(x,\beta)],[y_1''(x,\gamma),y_2''(x,\gamma)])$$

with the initial condition

$$y(x_0;\alpha,\beta,\gamma)=([a_1\alpha,a_2\alpha],[a_1'\beta,a_2'\beta],[a_1''\gamma,a_2''\gamma]),$$

$\alpha + \beta + \gamma \le 3$, $\alpha, \beta, \gamma \in [0,1]$.
ie.,

$$\dot{y}_1(x,\alpha)=ky_1(x,\alpha) \quad \dot{y}_2(x,\alpha)=ky_2(x,\alpha)$$
$$\dot{y}_1'(x,\beta)=ky_1'(x,\beta) \quad \dot{y}_2'(x,\beta)=ky_2'(x,\beta)$$
$$\dot{y}_1''(x,\gamma)=ky_1''(x,\gamma) \quad \dot{y}_2''(x,\gamma)=ky_2''(x,\gamma)$$

With the initial condition

$$y_1(x_0,\alpha)=a_1(\alpha) \quad y_2(x_0,\alpha)=a_2(\alpha)$$
$$y_1'(x_0,\beta)=a_1'(\beta) \quad y_2'(x_0,\beta)=a_2'(\beta)$$
$$y_1''(x_0,\gamma)=a_1''(\gamma) \quad y_2''(x_0,\gamma)=a_2''(\gamma)$$

Then the solution of the above equation is given by

$$y_1(x,\alpha)=a_1(\alpha)e^{k(x-x_0)} \quad y_2(x,\alpha)=a_2(\alpha)e^{k(x-x_0)}$$
$$y_1'(x,\beta)=a_1'(\beta)e^{k(x-x_0)} \quad y_2'(x,\beta)=a_2'(\beta)e^{k(x-x_0)}$$
$$y_1''(x,\gamma)=a_1''(\gamma)e^{k(x-x_0)} \quad y_2''(x,\gamma)=a_2''(\gamma)e^{k(x-x_0)}$$

Case II:
If $k<0$
Let k = -p. Taking (α,β,γ)-cut of the equation (1) we get

$$\frac{dy}{dx}([y_1(x,\alpha),y_2(x,\alpha)],[y_1'(x,\beta),y_2'(x,\beta)],[y_1''(x,\gamma),y_2''(x,\gamma)])$$
$$=-p([y_1(x,\alpha),y_2(x,\alpha)],[y_1'(x,\beta),y_2'(x,\beta)],[y_1''(x,\gamma),y_2''(x,\gamma)])$$

with the initial condition

$$y(x_0;\alpha,\beta,\gamma)=([a_1\alpha,a_2\alpha],[a_1'\beta,a_2'\beta],[a_1''\gamma,a_2''\gamma]),$$

$\alpha+\beta+\gamma\le 3$, α, β, γ ∈ [0,1] .
ie.,

$$\dot{y}_1(x,\alpha)=-py_1(x,\alpha) \quad \dot{y}_2(x,\alpha)=-py_2(x,\alpha)$$
$$\dot{y}_1'(x,\beta)=-py_1'(x,\beta) \quad \dot{y}_2'(x,\beta)=-py_2'(x,\beta)$$
$$\dot{y}_1''(x,\gamma)=-py_1''(x,\gamma) \quad \dot{y}_2''(x,\gamma)=-py_2''(x,\gamma)$$

With the initial condition

$$y_1(x_0,\alpha)=a_1(\alpha) \quad y_2(x_0,\alpha)=a_2(\alpha)$$
$$y_1'(x_0,\beta)=a_1'(\beta) \quad y_2'(x_0,\beta)=a_2'(\beta)$$
$$y_1''(x_0,\gamma)=a_1''(\gamma) \quad y_2''(x_0,\gamma)=a_2''(\gamma)$$

Then the solution of the above equation is given by

$$y_1(x,\alpha)=\left(\frac{a_1(\alpha)+a_2(\alpha)}{2}\right)e^{-p(x-x_0)}+\left(\frac{a_1(\alpha)-a_2(\alpha)}{2}\right)e^{p(x-x_0)}$$
$$y_2(x,\alpha)=\left(\frac{a_1(\alpha)+a_2(\alpha)}{2}\right)e^{-p(x-x_0)}-\left(\frac{a_1(\alpha)-a_2(\alpha)}{2}\right)e^{p(x-x_0)}$$
$$y_1'(x,\beta)=\left(\frac{a_1'(\beta)+a_2'(\beta)}{2}\right)e^{-p(x-x_0)}+\left(\frac{a_1'(\beta)-a_2'(\beta)}{2}\right)e^{p(x-x_0)}$$
$$y_2'(x,\beta)=\left(\frac{a_1'(\beta)+a_2'(\beta)}{2}\right)e^{-p(x-x_0)}-\left(\frac{a_1'(\beta)-a_2'(\beta)}{2}\right)e^{p(x-x_0)}$$
$$y_1''(x,\gamma)=\left(\frac{a_1''(\gamma)+a_2''(\gamma)}{2}\right)e^{-p(x-x_0)}+\left(\frac{a_1''(\gamma)-a_2''\gamma)}{2}\right)e^{p(x-x_0)}$$
$$y_2''(x,\gamma)=\left(\frac{a_1''(\gamma)+a_2''(\gamma)}{2}\right)e^{-p(x-x_0)}-\left(\frac{a_1''(\gamma)-a_2''\gamma)}{2}\right)e^{p(x-x_0)}$$

5. Application:

Consider a colony of bacteria in a rich environment. In such an environment, the population P of the bacteria reproduce via binary fission.The rate at which such a population increases will be proportional to the number of bacteria. We can express this rule as a differential equation: $\frac{dP}{dt}=kP$

If the initial population is a neutrosophic number (3, 4, 5; 0.8, 0.2, 0.3). What is the population of bacteria after two days.(The constant proportional k = 1/3).

Solution:

$\frac{dP}{dt}=\frac{1}{3}P$, where P(0) =(3, 4, 5; 0.8, 0.2, 0.3).

The solution is

$$P_1(t,\alpha)=(2.4+0.8\alpha)e^{\frac{t}{3}} \quad ;P_2(t,\alpha)=(4-0.8\alpha)e^{\frac{t}{3}}$$
$$P_1'(t,\beta)=(0.8-0.2\beta)e^{\frac{t}{3}} \quad ;P_2'(t,\alpha)=(0.8+0.2\beta)e^{\frac{t}{3}}$$
$$P_1''(t,\gamma)=(1.2-0.3\gamma)e^{\frac{t}{3}} \quad ;P_2''(t,\gamma)=(1.2+0.3\gamma)e^{\frac{t}{3}}$$

Here

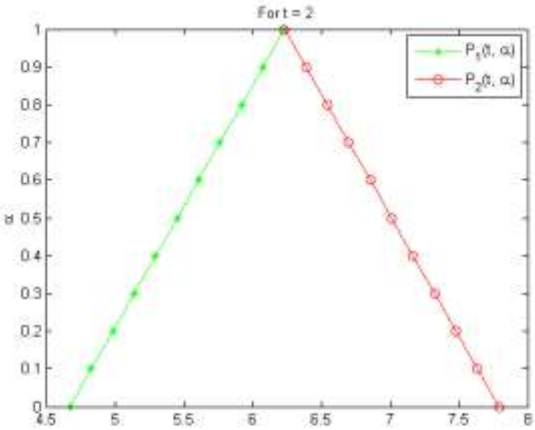
$$\frac{dP_1}{d\alpha}=0.8e^{\frac{t}{3}}>0, \qquad \frac{dP_2}{d\alpha}=-0.8e^{\frac{t}{3}}<0$$
$$\frac{dP_1'}{d\beta}=-0.2e^{\frac{t}{3}}<0, \qquad \frac{dP_2'}{d\beta}=0.2e^{\frac{t}{3}}>0$$
$$\frac{dP_1''}{d\gamma}=-0.3e^{\frac{t}{3}}<0, \qquad \frac{dP_2''}{d\gamma}=0.3e^{\frac{t}{3}}>0$$

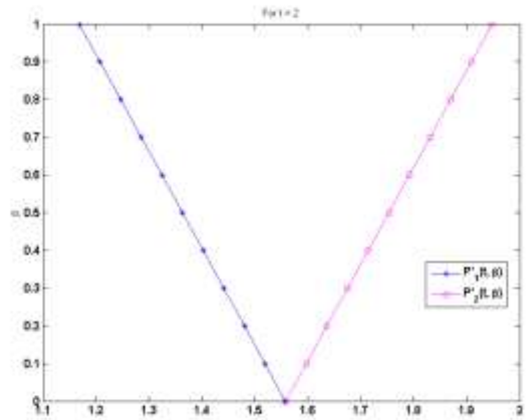
Hence the solution is strong.
The solution for t = 2 and different values for (α,β,γ) is given in the following table.

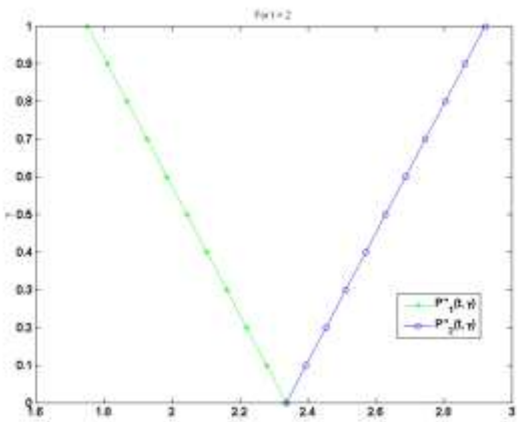
Table 1: Solution for t = 2

α	$P_1(t,\alpha)$	$P_2(t,\alpha)$	β	$P_1'(t,\beta)$	$P_2'(t,\beta)$	γ	$P_1''(t,\gamma)$	$P_2''(t,\gamma)$
0	4.6746	7.7909	0	1.5582	1.5582	0	2.3373	2.3373
0.1	4.8304	7.6351	0.1	1.5192	1.5971	0.1	2.2788	2.3957
0.2	4.9862	7.4793	0.2	1.4803	1.6361	0.2	2.2204	2.4541
0.3	5.1420	7.3235	0.3	1.4413	1.6751	0.3	2.1620	2.5126
0.4	5.2978	7.1677	0.4	1.4024	1.7140	0.4	2.1036	2.5710
0.5	5.4537	7.0118	0.5	1.3634	1.7530	0.5	2.0451	2.6294
0.6	5.6095	6.8560	0.6	1.3245	1.7919	0.6	1.9867	2.6879
0.7	5.7653	6.7002	0.7	1.2855	1.8309	0.7	1.9283	2.7463
0.8	5.9211	6.5444	0.8	1.2465	1.8698	0.8	1.8698	2.8047
0.9	6.0769	6.3886	0.9	1.2076	1.9088	0.9	1.8114	2.8632
1.0	6.2327	6.2327	1.0	1.1686	1.9477	1.0	1.7530	2.9216

The graphical interpretation of the above table is shown below







6. Conclusion

In this paper the first order differential equation involving neutrosophic numbers have been solved. To solve this equation the (α,β,γ) - cut method were used for the neutrosophic numbers. To show the effectiveness of proposed method it has been applied in the field of bacteria culture model and the solution is given for the truth, indeterminacy and falsity function using MATLAB and the graphical representation is also interpreted. This will promote the future study on higher order differential equations with neutrosophic numbers.

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