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Research Article

A New Single-Valued Neutrosophic Rough Sets and Related Topology

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(Fuzzy) rough sets are closely related to (fuzzy) topologies. Neutrosophic rough sets and neutrosophic topologies are extensions of (fuzzy) rough sets and (fuzzy) topologies, respectively. In this paper, a new type of neutrosophic rough sets is presented, and the basic properties and the relationships to neutrosophic topology are discussed. The main results include the following: (1) For a single-valued neutrosophic approximation space (U, R), a pair of approximation operators called the upper and lower ordinary single-valued neutrosophic approximation operators are defined and their properties are discussed. Then the further properties of the proposed approximation operators corresponding to reflexive (transitive) single-valued neutrosophic approximation space are explored. (2) It is verified that the single-valued neutrosophic approximation spaces and the ordinary single-valued neutrosophic topological spaces can be interrelated to each other through our defined lower approximation operator. Particularly, there is a one-to-one correspondence between reflexive, transitive single-valued neutrosophic approximation spaces and quasidiscrete ordinary single-valued neutrosophic topological spaces.

1. Introduction

The original notion of neutrosophic set was proposed by Smarandache [1]. For the convenience of application, Wang et al. [2] investigated the single-valued neutrosophic set (**Svns**). In **Svns**, three independent membership functions (truth, indeterminacy, and falsity) are considered; hence, it can be regarded as extensions of fuzzy set [3] and intuitionistic fuzzy set [4]. There are many works on the theory and application of **Svns** (see Abdel-Basset [5], Ye [6, 7], Samant [8], Yang [9, 10], Zhang [11–13], Zavadskas [14], and Xu [15] as well as Peng's review paper [16]).

The fusion of neutrosophic sets with rough sets theory [17] is an important research direction. According to Li's review paper [18], there exists two fundamental combinations of rough sets and neutrosophic sets: Broumi's rough neutrosophic sets [19] and Sweety's neutrosophic rough sets [20]. Many other models can be regarded as their extensions [12, 21–24].

(i) Broumi's rough neutrosophic sets [19]: let R be an equivalent relation (can be easily extended for an arbitrary binary relation) on U. Then, for each neutrosophic set A on U, a pair of neutrosophic sets $\underline{R}(A)$ and $\overline{R}(A)$ on U are defined as the lower and upper approximations of A w.r.t. (U, R).

(ii) Sweety's neutrosophic rough sets [20]: let R be a neutrosophic relation on U. Then, for each neutrosophic set A on U, a pair of neutrosophic sets $\underline{R}(A)$ and $\overline{R}(A)$ on U are defined as the lower and upper approximations of A w.r.t. (U, R). Yang [10] defined a similar model by considering the single-valued neutrosophic relation and single-valued neutrosophic set on U.

In this paper, we shall introduce a new model of rough sets fusion with neutrosophic sets under the framework of single-valued neutrosophic approximation space (U, R) (i.e., a nonempty set U together with a single-valued neutrosophic relation R on U). For each ordinary subset A of U, we shall define a pair of single-valued neutrosophic sets $\underline{R}(A)$ and $\overline{R}(A)$ on U as the lower and upper approximations of A with respect to (U, R). Obviously, our model is different from Broumi–Sweety–Yang's models, since, in our model, the original sets are ordinary subsets of U and their approximations are single-valued neutrosophic sets, but, in Broumi–Sweety–Yang's models, the original sets and their approximations are all (single-valued) neutrosophic sets. Hence, our rough sets will be called ordinary single-valued neutrosophic rough sets.

(Fuzzy) rough sets are closely related to (fuzzy) topology [25–42]. The well-known result may be that there is a one-to-one correspondence between reflexive and transitive (fuzzy) approximation spaces and quasidiscrete (fuzzy) topological spaces [26, 37, 38]. Under the framework of single-valued neutrosophic sets, two kinds of neutrosophic topological spaces are discussed (for more general neutrosophic topology, refer to Al-Omeri [43] and Lupianez [44]).

(i) Yang's single-valued neutrosophic topological spaces [45]: for a nonempty set U, Yang defined the single-valued neutrosophic topology on U as a subset τ of $\mathrm{Svns}(U)$ (the set of all single-valued neutrosophic sets on U) with some conditions. Yang's space can be regarded as an extension of Lowen's fuzzy topological space [46]. Yang also proved that there is a one-to-one correspondence between reflexive and transitive single-valued neutrosophic approximation spaces and his single-valued neutrosophic rough topological spaces.

(ii) Kim's ordinary single-valued neutrosophic topological spaces [47]: for a nonempty set U, Kim defined the ordinary single-valued neutrosophic topology on U as a neutrosophic set τ on $P(U)$ (the power set of U) with some conditions. Kim's space can be regarded as an extension of Šostak's fuzzy topology [48] (or Ying's fuzzifying topology [49]).

In this paper, we shall prove that there are close relationships between our ordinary single-valued neutrosophic rough sets and Kim's ordinary single-valued neutrosophic topological spaces. The close relationships exhibit that it is meaningful to investigate the new rough sets model.

The method of this paper and the comparison with related literature can be summarized in Table 1.

The remainder of this paper is organized as follows. In Section 2, we will recall some knowledge about neutrosophic sets and rough sets. In Section 3, we shall give the notion of ordinary single-valued neutrosophic upper and lower approximation operators and discuss their properties. Then we will explore the further properties of the proposed approximations corresponding to reflexive (transitive) single-valued neutrosophic approximation space. In Section 4, we will prove that each single-valued neutrosophic approximation space induces an ordinary single valued neutrosophic topological space via our defined lower approximation. In Section 5, we shall verify that each ordinary single-valued neutrosophic topological space induces a single-valued neutrosophic approximation space. In Section 6, we will show that there is a one-to-one correspondence between reflexive and transitive single-valued neutrosophic approximation spaces and quasidiscrete ordinary single-valued neutrosophic topological spaces.

2. Preliminaries

In this section, we recall some knowledge about neutrosophic rough sets and neutrosophic topologies used in this paper.

Unless otherwise stated, we always assume that U is a nonempty infinite set. We denote $P(U)$ as the power set of U and define $A^c = U - A$ for $A \in P(U)$.

Definition 1 (see [2]). An **Svns** $A = (A_T, A_I, A_F)$ on U is defined as three membership functions $A_T, A_I, A_F\colon U \longrightarrow [0,1]$, which are interpreted as truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively. All **Svnss** are denoted by $\mathrm{Svns}(U)$.

Each $\alpha = (\alpha_1, \alpha_2, \alpha_3) \in [0,1]^3$ is called a single-valued neutrosophic number, and its complement is defined as $\alpha^c = (\alpha_3, 1-\alpha_2, \alpha_1)$. We denote the single-valued neutrosophic numbers $\top = (1,0,0)$ and $\bot = (0,1,1)$. Obviously, $\top^c = \bot$ and $\bot^c = \top$.

Remark 1. Pythagorean fuzzy set [50] is also an important extension of intuitionistic fuzzy set. We can observe that when restricting $0 \le (A_T(x))^2 + (A_F(x))^2 \le 1$ and $A_I(x) = \sqrt{1 - (A_T(x))^2 - (A_F(x))^2}$, an **Svns** becomes a Pythagorean fuzzy set.

For $A \in P(U)$, we define $\top_A \in \mathrm{Svns}(U)$ as follows: $\forall x \in U, \top_A(x) = \top$ if $x \in A$ and $\top_A(x) = \bot$ if $x \notin A$.

Definition 2 (see [2, 6, 10]). Let $A, B, A_j\ (j \in J) \in \mathrm{Svns}(U)$.

(1) We denote $A \sqsubseteq B$ if, for any $x \in U$, $A_T(x) \le B_T(x)$, $A_I(x) \ge B_I(x)$, and $A_F(x) \ge B_F(x)$. By $A = B$, we mean $A \sqsubseteq B$ and $B \sqsubseteq A$

(2) We define $A^c \in \mathrm{Svns}(U)$ as $\forall x \in U, A^c(x) = (A(x))^c = (A_F(x), 1 - A_I(x), A_T(x))$

(3) We define $\sqcup_{j\in J} A_j, \sqcap_{j\in J} A_j \in \mathrm{Svns}(U)$ by $\forall x \in U$,

$$\left(\sqcup_{j\in J} A_j\right)(x) = \left(\vee_{j\in J}\left(A_j\right)_T(x), \wedge_{j\in J}\left(A_j\right)_I(x), \wedge_{j\in J}\left(A_j\right)_F(x)\right),$$
$$\left(\sqcap_{j\in J} A_j\right)(x) = \left(\wedge_{j\in J}\left(A_j\right)_T(x), \vee_{j\in J}\left(A_j\right)_I(x), \vee_{j\in J}\left(A_j\right)_F(x)\right). \tag{1}$$

Definition 3 (see [10]). An **Svns** R on $U \times U$ is referred to a single-valued neutrosophic relation (**Svnr**) on U. Then the pair (U, R) is said to be a single-valued neutrosophic approximation space (**Svnas**). Furthermore, R is called

Table 1: Method and comparison.

Rough set	R	A	$\overline{R}(A), \underline{R}(A)$	Topology τ_R	Bijection
Rough neutrosophic sets [19]	−	+	+	Yang's topology [45]	×
Neutrosophic rough sets [10, 20]	+	+	+	Yang's topology [45]	√
Our neutrosophic rough sets	+	−	+	Kim's topology [47]	√

Notes: "+" represents that the set is a single-valued neutrosophic set, and "−" represents that it is not; "√" represents that there is a bijection between the considered rough sets and topologies, and "×" represents that there is no bijection.

(i) Reflexive if $\forall x \in U$, $R(x,x) = \top$, i.e., $R_T(x,x) = 1, R_I(x,x) = 0, R_F(x,x) = 0$

(ii) Transitive if R_T, R_I^c, R_F^c are all transitive fuzzy relations, that is, $\forall x, y, z \in U$

$$R_T(x,y) \wedge R_T(y,z) \leq R_T(x,z), R_I(x,y) \vee R_I(y,z) \geq R_I(x,z), R_F(x,y) \vee R_F(y,z) \geq R_F(x,z). \tag{2}$$

Definition 4 (see Definition 3.1 in [10]). Let (U,R) be an **Svnas**. For $A \in \mathrm{Svns}(U)$, the upper and lower approximations of A, denoted by $\overline{YR}(A), \underline{YR}(A) \in \mathrm{Svns}(U)$, are defined as follows: $\forall x \in U$,

$$\begin{aligned}
\overline{YR}(A)_T(x) &= \bigvee_{y\in U} (R_T(x,y) \wedge A_T(y)),\\
\overline{YR}(A)_I(x) &= \bigwedge_{y\in U} (R_I(x,y) \vee A_I(y)),\\
\overline{YR}(A)_F(x) &= \bigwedge_{y\in U} (R_F(x,y) \vee A_F(y)),\\
\underline{YR}(A)_T(x) &= \bigwedge_{y\in U} (R_F(x,y) \vee A_T(y)),\\
\underline{YR}(A)_I(x) &= \bigvee_{y\in U} ((1 - R_I(x,y)) \wedge A_I(y)),\\
\underline{YR}(A)_F(x) &= \bigvee_{y\in U} (R_T(x,y) \wedge A_F(y)).
\end{aligned} \tag{3}$$

The pair $(\overline{YR}(A), \underline{YR}(A))$ is referred to the single-valued neutrosophic rough sets of A. $\overline{YR}$ and $\underline{YR}$ are said to be the single-valued neutrosophic upper and lower approximation operators, respectively.

Definition 5 (see Definition 8 in [47]). An **Svns** τ on $P(U)$, that is, $\tau = (\tau_T, \tau_I, \tau_F)$ with $\tau_T, \tau_I, \tau_F\colon P(U) \longrightarrow [0,1]$, is referred to an ordinary single-valued neutrosophic topology (**OSvnt**) on U if τ fulfills the following conditions:

$$\begin{aligned}
&(\mathrm{OSvnt1})\ \tau(\emptyset) = \tau(U) = \top,\\
&(\mathrm{OSvnt2})\ \text{For any } A, B \in P(U),\\
&\tau_T(A\cap B) \geq \tau_T(A) \wedge \tau_T(B),\\
&\tau_I(A\cap B) \leq \tau_I(A) \vee \tau_I(B),\\
&\tau_F(A\cap B) \leq \tau_F(A) \vee \tau_F(B),\\
&(\mathrm{OSvnt3})\ \text{For any } A_j\ (j\in J) \in P(U),\\
&\tau_T\left(\cup_{j\in J} A_j\right) \geq \bigwedge\nolimits_{j\in J} \tau_T\left(A_j\right),\\
&\tau_I\left(\cup_{j\in J} A_j\right) \leq \bigvee\nolimits_{j\in J} \tau_I\left(A_j\right),\\
&\tau_F\left(\cup_{j\in J} A_j\right) \leq \bigvee\nolimits_{j\in J} \tau_F\left(A_j\right).
\end{aligned} \tag{4}$$

The pair (U,τ) is said to be an ordinary single-valued neutrosophic topological space (**OSvnts**).

For examples and more results about **OSvnts**, refer to [47].

The following lemma can be easily observed. We will use it without mentioning again.

Lemma 1. *Let $\alpha, \beta \in [0,1]$. Then the following conditions are equivalent:*

(1) $\alpha \leq \beta$

(2) For all $\gamma \in [0,1)$, $\gamma < \alpha \Rightarrow \gamma < \beta$

(3) For all $\gamma \in [0,1)$, $\gamma < \alpha \Rightarrow \gamma \leq \beta$

(4) For all $\gamma \in (0,1]$, $\gamma \leq \alpha \Rightarrow \gamma \leq \beta$

(5) For all $\gamma \in (0,1]$, $\gamma > \beta \Rightarrow \gamma \geq \alpha$

3. Ordinary Single-Valued Neutrosophic Rough Sets for Svnas

In this section, we present the notions and properties of ordinary single-valued neutrosophic upper and lower approximation operators.

Definition 6. Let (U,R) be an **Svnas**. For $A \in P(U)$, the upper and lower approximations of A, denoted by $\overline{R}(A), \underline{R}(A) \in \mathrm{Svns}(U)$, are defined as follows: $\forall x \in U$,

$$\begin{aligned}
\overline{R}(A)_T(x) &= \bigvee_{y\in A} R_T(x,y),\\
\overline{R}(A)_I(x) &= \bigwedge_{y\in A} R_I(x,y),\\
\overline{R}(A)_F(x) &= \bigwedge_{y\in A} R_F(x,y),\\
\underline{R}(A)_T(x) &= \bigwedge_{y\notin A} R_F(x,y),\\
\underline{R}(A)_I(x) &= \bigvee_{y\notin A} (1 - R_I(x,y)),\\
\underline{R}(A)_F(x) &= \bigvee_{y\notin A} R_T(x,y).
\end{aligned} \tag{5}$$

The pair $(\overline{R}(A), \underline{R}(A))$ is referred to the ordinary single-valued neutrosophic rough sets of A. $\overline{R}$ and $\underline{R}$ are said to be the ordinary single-valued neutrosophic upper and lower approximation operators, respectively.

Remark 2

(1) The definition of $\overline{R}(A)_T(x)$ is an interpretation of the fact that "the join of $R_T(x)$ and A is not empty," and the definition of $\underline{R}(A)_T(x)$ is an interpretation of the fact that "$R_T(x)$ is contained in A (or equivalent, A^c is contained in $R_F(x)$)."

(2) For a fuzzy relation r on U, it is easily observed that r induces an **Svnr** R_r on U defined as follows: $\forall (x, y) \in U \times U$, $(R_r)_T(x, y) = r(x, y)$, $(R_r)_I(x, y) = 0$, $(R_r)_F(x, y) = 1 - r(x, y)$. For $A \in P(U)$, we have $\overline{R_r}(A)_T = \vee_{y \in A} r(x, y) = \overline{r}(A)$, $\underline{R_r}(A)_T = \vee_{y \notin A}(1 - r(x, y)) = \underline{r}(A)$, where $\overline{r}(A)$, $\underline{r}(A)$ are the fuzzy approximations of ordinary subset w.r.t. fuzzy relation in the work of Yao [51]. Therefore, the single-valued neutrosophic approximations in this paper are a generalization of Yao's fuzzy approximations.

(3) Obviously, the single-valued neutrosophic approximation operators in this paper are different from the single-valued neutrosophic approximation operators in the work of Yang [10], since our operators are defined from $P(U)$ to $\mathrm{Svns}(U)$ and Yang's operators are defined from $\mathrm{Svns}(U)$ to $\mathrm{Svns}(U)$.

Example 1. Let (U, R) be an **Svnas** with $U = \{x_1, x_2, x_3\}$ and let R be defined as in Table 2.

Taking $A = \{x_1, x_2\}$, we have

$$\begin{aligned}
\underline{R}(A)_T(x_1) &= R_F(x_1, x_3) = 0.4,\\
\underline{R}(A)_I(x_1) &= 1 - R_I(x_1, x_3) = 1,\\
\underline{R}(A)_F(x_1) &= R_T(x_1, x_3) = 1,\\
\underline{R}(A)_T(x_2) &= R_F(x_2, x_3) = 1,\\
\underline{R}(A)_I(x_2) &= 1 - R_I(x_2, x_3) = 1,\\
\underline{R}(A)_F(x_2) &= R_T(x_2, x_3) = 0.6,\\
\underline{R}(A)_T(x_3) &= R_F(x_3, x_3) = 0,\\
\underline{R}(A)_I(x_3) &= 1 - R_I(x_3, x_3) = 1,\\
\underline{R}(A)_F(x_1) &= R_T(x_3, x_3) = 1,\\
\overline{R}(A)_T(x_1) &= R_T(x_1, x_1) \vee R_T(x_1, x_2) = 0 \vee 0.3 = 0.3,\\
\overline{R}(A)_I(x_1) &= R_I(x_1, x_1) \wedge R_I(x_1, x_2) = 0 \wedge 0.1 = 0,\\
\overline{R}(A)_F(x_1) &= R_F(x_1, x_1) \wedge R_F(x_1, x_2) = 1 \wedge 0.6 = 0.6,\\
\overline{R}(A)_T(x_2) &= R_T(x_2, x_1) \vee R_T(x_2, x_2) = 0 \vee 0.6 = 0.6,\\
\overline{R}(A)_I(x_2) &= R_I(x_2, x_1) \wedge R_I(x_2, x_2) = 0.2 \wedge 0.5 = 0.2,\\
\overline{R}(A)_F(x_2) &= R_F(x_2, x_1) \wedge R_F(x_2, x_2) = 0.4 \wedge 1 = 0.4,\\
\overline{R}(A)_T(x_3) &= R_T(x_3, x_1) \vee R_T(x_3, x_2) = 1 \vee 1 = 1,\\
\overline{R}(A)_I(x_3) &= R_I(x_3, x_1) \wedge R_I(x_3, x_2) = 0 \wedge 0.5 = 0,\\
\overline{R}(A)_F(x_3) &= R_F(x_3, x_1) \wedge R_F(x_3, x_2) = 1 \wedge 1 = 1.
\end{aligned} \tag{6}$$

Hence, we obtain $\underline{R}(A)$ and $\overline{R}(A)$ as in Table 3.

Theorem 1. *Let (U, R) be an **Svnas**. Then we have the following:*

(1) $\underline{R}(U) = \top_U$; $\overline{R}(\varnothing) = \top_\varnothing$

(2) If $A \subseteq B$, then $\underline{R}(A) \sqsubseteq \underline{R}(B)$ and $\overline{R}(A) \sqsubseteq \overline{R}(B)$

(3) For all $A_j\,(j \in J) \in P(U)$, $\underline{R}(\cap_{j \in J} A_j) = \sqcap_{j \in J} \underline{R}(A_j)$ and $\overline{R}(\cup_{j \in J} A_j) = \sqcup_{j \in J} \overline{R}(A_j)$

(4) For $A \in P(U)$, $\overline{R}(A) = (\underline{R}(A^c))^c$ and $\underline{R}(A) = (\overline{R}(A^c))^c$

Proof. For (1)–(3), we prove only the results for lower approximation. The proofs for upper approximation are similar and hence are omitted.

(1) For any $x \in U$, we have $\underline{R}(U)_T(x) = \wedge_{y \notin U} R_F(x, y) = 1$, $\underline{R}(U)_I(x) = \vee_{y \notin U}(1 - R_I(x, y)) = 0$, $\underline{R}(U)_F(x) = \vee_{y \notin U} R_T(x, y) = 0$. Hence, $\underline{R}(U) = \top_U$.

(2) For any $x \in U$ and $A \subseteq B$, we obtain

$$\begin{aligned}
\underline{R}(A)_T(x) &= \bigwedge_{y \notin A} R_F(x, y) \le \bigwedge_{y \notin B} R_F(x, y) = \underline{R}(B)_T(x),\\
\underline{R}(A)_I(x) &= \bigvee_{y \notin A}(1 - R_I(x, y)) \ge \bigvee_{y \notin B}(1 - R_I(x, y)) = \underline{R}(B)_I(x),\\
\underline{R}(A)_F(x) &= \bigvee_{y \notin A} R_T(x, y) \ge \bigvee_{y \notin B} R_T(x, y) = \underline{R}(B)_F(x).
\end{aligned} \tag{7}$$

Hence, $\underline{R}(A) \sqsubseteq \underline{R}(B)$.

(3) For any $x \in U$,

$$\begin{aligned}
\left(\sqcap_{j\in J}\underline{R}(A_j)\right)_T(x) &= \bigwedge_{j\in J}\underline{R}(A_j)_T(x)\\
&= \bigwedge_{j\in J}\bigwedge_{y\notin A_j} R_F(x,y) = \bigwedge_{y\notin \cap_{j\in J}A_j} R_F(x) = \underline{R}\left(\cap_{j\in J}A_j\right)_T(x),\\
\left(\sqcap_{j\in J}\underline{R}(A_j)\right)_I(x) &= \bigvee_{j\in J}\underline{R}(A_j)_I(x)\\
&= \bigvee_{j\in J}\bigvee_{y\notin A_j}(1-R_I(x,y)) = \bigvee_{y\notin \cap_{j\in J}A_j}(1-R_I(x,y)) = \underline{R}\left(\cap_{j\in J}A_j\right)_I(x),\\
\left(\sqcap_{j\in J}\underline{R}(A_j)\right)_F(x) &= \bigvee_{j\in J}\underline{R}(A_j)_F(x)\\
&= \bigvee_{j\in J}\bigvee_{y\notin A_j} R_T(x,y) = \bigvee_{y\notin \cap_{j\in J}A_j} R_T(x,y) = \underline{R}\left(\cap_{j\in J}A_j\right)_F(x).
\end{aligned} \tag{8}$$

Hence, $\underline{R}(\cap_{j\in J}A_j) = \sqcap_{j\in J}\underline{R}(A_j)$.

(4) For any $x \in U$,

$$\begin{aligned}
(\underline{R}(A^c))^c_T(x) &= (\underline{R}(A^c))_F(x) = \bigvee_{y\in A} R_T(x,y) = \overline{R}(A)_T(x),\\
(\underline{R}(A^c))^c_I(x) &= 1-(\underline{R}(A^c))_I(x) = 1-\bigvee_{y\in A}(1-R_I(x,y)) = \bigwedge_{y\in A} R_I(x,y) = \overline{R}(A)_I(x),\\
(\underline{R}(A^c))^c_F(x) &= (\underline{R}(A^c))_T(x) = \bigwedge_{y\in A} R_F(x,y) = \overline{R}(A)_F(x).
\end{aligned} \tag{9}$$

Hence, $\overline{R}(A) = (\underline{R}(A^c))^c$. That is, $\underline{R}(A) = (\overline{R}(A^c))^c$ can be proved similarly.

The following theorem gives a characterization on the approximation operators generated by reflexive **Svnas**. ☐

Theorem 2. *Let (U, R) be an* **Svnas**. *Then the following three are equivalent:*

(1) R is reflexive

(2) $\underline{R}(A) \sqsubseteq \top_A$ for each $A \in P(U)$

(3) $\top_A \sqsubseteq \overline{R}(A)$ for each $A \in P(U)$

Proof. (1) ⇒ (2). If $x \in A$, then

$$\begin{aligned}
\underline{R_T}(A)(x) &\le (\top_A)_T(x) = 1,\\
\underline{R_I}(A)(x) &\ge (\top_A)_I(x) = 0,\\
\underline{R_F}(A)(x) &\ge (\top_A)_F(x) = 0.
\end{aligned} \tag{10}$$

If $x \notin A$, then

$$\begin{aligned}
\underline{R_T}(A)(x) &= \bigwedge_{y\notin A} R_F(x,y) \le R_F(x,x) \overset{(1)}{=} 0 = (\top_A)_T(x),\\
\underline{R_I}(A)(x) &= \bigvee_{y\notin A}(1-R_I(x,y)) \ge 1-R_I(x,x) \overset{(1)}{=} 1-0 = 1 = (\top_A)_I(x),\\
\underline{R_F}(A)(x) &= \bigvee_{y\notin A}(R_T(x,y)) \ge R_T(x,x) \overset{(1)}{=} 1 = (\top_A)_F(x).
\end{aligned} \tag{11}$$

Hence, $\underline{R}(A) \sqsubseteq \top_A$.

(2) ⇒ (1). For any $x \in U$, by (2), we have

Table 2: Svnas.

R	x_1	x_2	x_3
x_1	(0, 0, 1)	(0.3, 0.1, 0.6)	(1, 0, 0.4)
x_2	(0, 0.2, 0.4)	(0.6, 0.5, 1)	(0.6, 0,1)
x_3	(1, 0, 1)	(1, 0.5, 1)	(1, 0, 0)

Table 3: The upper and lower approximation.

	$\underline{R}(A)$	$\overline{R}(A)$
x_1	(0.4, 1, 1)	(0.3, 0, 0.6)
x_2	(1, 1, 0.6)	(0.6, 0.2, 0.4)
x_3	(0, 1, 1)	(1, 0, 1)

$$\begin{aligned} R_T(x,x) &= \bigvee_{y\notin U-\{x\}} R_T(x,y) \\ &= \underline{R}(U-\{x\})_F(x) \geq \left(\top_{U-\{x\}}\right)_F(x) = 1, \\ 1-R_I(x,x) &= \bigvee_{y\notin U-\{x\}} (1-R_I(x,y)) \\ &= \underline{R}(U-\{x\})_I(x) \geq \left(\top_{U-\{x\}}\right)_I(x) = 1 \\ &\Rightarrow R_I(x,x) = 0, \\ R_F(x,x) &= \bigwedge_{y\notin U-\{x\}} R_F(x,y) \\ &= \underline{R}(U-\{x\})_T(x) \leq \left(\top_{U-\{x\}}\right)_T(x) = 0. \end{aligned} \tag{12}$$

Hence, R is reflexive.

(2) ⇔ (3). It can be concluded from Theorem 1 (4).

The following theorem presents a characterization on the approximation operators generated by transitive **Svnas**. □

Theorem 3. *Let (U,R) be an **Svnas**. Then the following three are equivalent:*

(1) R is transitive.

(2) For each $A\in P(U)$ and $x\in U$,

$$\begin{aligned} \underline{R}(A)_T(x) &= \bigvee_{B\subseteq A}\left(\underline{R}(B)_T(x)\wedge\bigwedge_{y\in B}\underline{R}(B)_T(y)\right), \\ \underline{R}(A)_I(x) &= \bigwedge_{B\subseteq A}\left(\underline{R}(B)_I(x) scale190\%\vee\bigvee_{y\in B}\underline{R}(B)_I(y)\right), \\ \underline{R}(A)_F(x) &= \bigwedge_{B\subseteq A}\left(\underline{R}(B)_F(x) scale190\%\vee\bigvee_{y\in B}\underline{R}(B)_F(y)\right). \end{aligned} \tag{13}$$

(3) For each $A\in P(U)$ and $x\in U$,

$$\begin{aligned} \overline{R}(A)_T(x) &= \bigwedge_{A\subseteq B}\left(\overline{R}(B)_T(x) scale190\%\vee\bigvee_{y\notin B}\overline{R}(B)_T(y)\right), \\ \overline{R}(A)_I(x) &= \bigvee_{A\subseteq B}\left(\overline{R}(B)_I(x)\wedge\bigwedge_{y\notin B}\overline{R}(B)_I(y)\right), \\ \overline{R}(A)_F(x) &= \bigvee_{A\subseteq B}\left(\overline{R}(B)_F(x)\wedge\bigwedge_{y\notin B}\overline{R}(B)_F(y)\right). \end{aligned} \tag{14}$$

Proof. (1) ⇒ (2). Let $A\in P(U)$ and $x\in U$.

(i) For any $B\subseteq A$, we have $\underline{R}(B)_T(x)\leq\underline{R}(A)_T(x)$ and so

$$\begin{aligned} \bigvee_{B\subseteq A}\left(\underline{R}(B)_T(x)\wedge\bigwedge_{y\in B}\underline{R}(B)_T(y)\right) &\leq \bigvee_{B\subseteq A}\left(\underline{R}(B)_T(x)\right) \\ &= \underline{R}(A)_T(x). \end{aligned} \tag{15}$$

Conversely, let $\alpha=\underline{R}(A)_T(x)=\vee_{y\notin A}R_F(x,y)$; then $R_F(x,y)\geq\alpha$ for any $y\notin A$. Take $B_x=\{z\in U| R_F(x,z)<\alpha\}$; then $B_x\subseteq A$. It follows that

$$\begin{aligned} \underline{R}(B_x)_T(x) &= \bigwedge_{y\notin B_x} R_F(x,y)\geq\alpha, \\ \bigwedge_{y\in B_x}\underline{R}(B_x)_T(y) &= \bigwedge_{y\in B_x}\bigwedge_{z\notin B_x} R_F(y,z). \end{aligned} \tag{16}$$

Note that, for any $y\in B_x, z\notin B_x$, we have $R_F(x,y)<\alpha, R_F(x,z)\geq\alpha$. Since R is transitive, we have $R_F(x,y)\vee R_F(y,z)\geq R_F(x,z)\geq\alpha$, which means that $R_F(y,z)\geq\alpha$. So,

$$\bigwedge_{y\in B_x}\underline{R}(B_x)_T(y) = \bigwedge_{y\in B_x}\bigwedge_{z\notin B_x} R_F(y,z)\geq\alpha, \tag{17}$$

and then

$$\begin{aligned} &\bigvee_{B\subseteq A}\left(\underline{R}(B)_T(x)\wedge\bigwedge_{y\in B}\underline{R}(B)_T(y)\right) \\ &\geq \underline{R}(B_x)_T(x)\wedge\bigwedge_{y\in B_x}\underline{R}(B_x)_T(y)\geq\alpha=\underline{R}(A)_T(x). \end{aligned} \tag{18}$$

Hence,

$$\bigvee_{B\subseteq A}\left(\underline{R}(B)_T(x)\wedge\bigwedge_{y\in B}\underline{R}(B)_T(y)\right) = \underline{R}(A)_T(x). \tag{19}$$

(ii) For any $B\subseteq A$, we have $\underline{R}(B)_I(x)\geq\underline{R}(A)_I(x)$ and so

$$\begin{aligned} &\bigwedge_{B\subseteq A}\left(\underline{R}(B)_I(x) scale190\%\vee\bigvee_{y\in B}\underline{R}(B)_I(y)\right) \\ &\geq \bigwedge_{B\subseteq A}\left(\underline{R}(B)_I(x)\right) = \underline{R}(A)_I(x). \end{aligned} \tag{20}$$

Conversely, let $\alpha=\underline{R}(A)_I(x)=\vee_{y\notin A}(1-R_I(x, y))$; then $1-R_I(x,y)\leq\alpha$ for any $y\notin A$. Take $B_x=\{z\in U|1-R_I(x,z)>\alpha\}$; then $B_x\subseteq A$. It follows that

$$\begin{aligned} \underline{R}(B_x)_I(x) &= \bigvee_{y\notin B_x}(1-R_I(x,y))\leq\alpha, \\ \bigvee_{y\in B_x}\underline{R}(B_x)_I(y) &= \bigvee_{y\in B_x}\bigvee_{z\notin B_x}(1-R_I(y,z)). \end{aligned} \tag{21}$$

Note that, for any $y\in B_x, z\notin B_x$, we have $1-R_I(x,y)>\alpha, 1-R_I(x,z)\leq\alpha$. Since R is transitive, we have $(1-R_I(x,y))\wedge(1-R_I(y,z))\leq(1-R_I(x,z))\leq\alpha$, which means that $1-R_I(y,z)\leq\alpha$. So,

$$\bigvee_{y\in B_x}\underline{R}(B_x)_I(y)=\bigvee_{y\in B_x}\bigvee_{z\notin B_x}(1-R_I(y,z))\leq\alpha, \quad (22)$$

and then

$$\begin{aligned}&\bigwedge_{B\subseteq A}\left(\underline{R}(B)_I(x)\bigvee\bigvee_{y\in B}\underline{R}(B)_I(y)\right)\\&\leq\underline{R}(B_x)_I(x)\bigvee\bigvee_{y\in B_x}\underline{R}(B_x)_I(y)\leq\alpha=\underline{R}(A)_I(x).\end{aligned} \quad (23)$$

Hence,

$$\bigwedge_{B\subseteq A}\left(\underline{R}(B)_I(x)\bigvee\bigvee_{y\in B}\underline{R}(B)_I(y)\right)=\underline{R}(A)_I(x). \quad (24)$$

(iii) For any $B\subseteq A$, we have $\underline{R}(B)_F(x)\geq\underline{R}(A)_F(x)$ and so

$$\begin{aligned}\bigwedge_{B\subseteq A}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{y\in B}\underline{R}(B)_F(y)\right)&\geq\bigwedge_{B\subseteq A}(\underline{R}(B)_F(x))\\&=\underline{R}(A)_F(x).\end{aligned} \quad (25)$$

Conversely, let $\alpha=\underline{R}(A)_F(x)=\vee_{y\notin A}R_T(x,y)$; then $R_T(x,y)\leq\alpha$ for any $y\notin A$. Take $B_x=\{z\in U|R_T(x,z)>\alpha\}$; then $B_x\subseteq A$. It follows that

$$\begin{aligned}\underline{R}(B_x)_F(x)&=\bigvee_{y\notin B_x}R_T(x,y)\leq\alpha,\\\bigvee_{y\in B_x}\underline{R}(B_x)_F(y)&=\bigvee_{y\in B_x}\bigvee_{z\notin B_x}R_T(y,z).\end{aligned} \quad (26)$$

Note that, for any $y\in B_x, z\notin B_x$, we have $R_T(x,y)>\alpha, R_T(x,z)\leq\alpha$. Since R is transitive, we have $R_T(x,y)\wedge R_T(y,z)\leq R_T(x,z)\leq\alpha$, which means that $R_T(y,z)\leq\alpha$. So,

$$\bigvee_{y\in B_x}\underline{R}(B_x)_F(y)=\bigvee_{y\in B_x}\bigvee_{z\notin B_x}R_T(y,z)\leq\alpha, \quad (27)$$

and then

$$\begin{aligned}\bigwedge_{B\subseteq A}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{y\in B}\underline{R}(B)_F(y)\right)&\leq\underline{R}(B_x)_F(x)\bigvee\bigvee_{y\in B_x}\underline{R}(B_x)_F(y)\\&\leq\alpha=\underline{R}(A)_F(x).\end{aligned} \quad (28)$$

Hence,

$$\bigwedge_{B\subseteq A}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{y\in B}\underline{R}(B)_F(y)\right)=\underline{R}(A)_F(x). \quad (29)$$

(2) $\Rightarrow$ (1). Let $x,y,z\in U$.

(i) Note that

$$\begin{aligned}&R_T(x,z)=\underline{R}(U-\{z\})_F(x)\\&\overset{(2)}{=}\bigwedge_{A\subseteq U-\{z\}}\left(\underline{R}(A)_F(x)\bigvee\bigvee_{u\in A}\underline{R}(A)_F(u)\right),\\&R_T(x,y)=\underline{R}(U-\{y\})_F(x)\\&\overset{(2)}{=}\bigwedge_{B\subseteq U-\{y\}}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{v\in B}\underline{R}(B)_F(v)\right),\\&R_T(y,z)=\underline{R}(U-\{z\})_F(y)\\&\overset{(2)}{=}\bigwedge_{C\subseteq U-\{z\}}\left(\underline{R}(C)_F(y)\bigvee\bigvee_{w\in C}\underline{R}(C)_F(w)\right).\end{aligned} \quad (30)$$

Take any $A\subseteq U-\{z\}$; then $y\in A$ or $y\notin A$.

Case 1: if $y\in A$, then

$$\begin{aligned}&\underline{R}(A)_F(x)\bigvee\bigvee_{u\in A}\underline{R}(A)_F(u)\\&\geq\bigvee_{u\in A}\underline{R}(A)_F(u),\text{by } y\in A\\&=\underline{R}(A)_F(y)\bigvee\bigvee_{u\in A}\underline{R}(A)_F(u),\text{by } A\subseteq U-\{z\}\\&\geq\bigwedge_{C\subseteq U-\{z\}}\left(\underline{R}(C)_F(y)\bigvee\bigvee_{w\in C}\underline{R}(C)_F(w)\right)\\&\geq\bigwedge_{B\subseteq U-\{y\}}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{v\in B}\underline{R}(B)_F(v)\right)\\&\bigwedge\bigwedge_{C\subseteq U-\{z\}}\left(\underline{R}(C)_F(y)\bigvee\bigvee_{w\in C}\underline{R}(C)_F(w)\right).\end{aligned} \quad (31)$$

Case 2: if $y\notin A$, then $A\subseteq U-\{y\}$ and so

$$\begin{aligned}&\underline{R}(A)_F(x)\bigvee\bigvee_{u\in A}\underline{R}(A)_F(u),\text{by } A\subseteq U-\{y\}\\&\geq\bigwedge_{B\subseteq U-\{y\}}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{v\in B}\underline{R}(B)_F(v)\right)\\&\geq\bigwedge_{B\subseteq U-\{y\}}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{v\in B}\underline{R}(B)_F(v)\right)\\&\bigwedge\bigwedge_{C\subseteq U-\{z\}}\left(\underline{R}(C)_F(y)\bigvee\bigvee_{w\in C}\underline{R}(C)_F(w)\right).\end{aligned} \quad (32)$$

By a combination of Cases 1 and 2, we obtain

$$\begin{aligned}&\bigwedge_{A\subseteq U-\{z\}}\left(\underline{R}(A)_F(x)\bigvee\bigvee_{u\in A}\underline{R}(A)_F(u)\right)\\&\geq\bigwedge_{B\subseteq U-\{y\}}\left(\underline{R}(B)_F(x)\bigvee\bigvee_{v\in B}\underline{R}(B)_F(v)\right)\\&\bigwedge\bigwedge_{C\subseteq U-\{z\}}\left(\underline{R}(C)_F(y)\bigvee\bigvee_{w\in C}\underline{R}(C)_F(w)\right),\end{aligned} \quad (33)$$

that is, $R_T(x,z)\geq R_T(x,y)\wedge R_T(y,z)$, as desired.

(ii) Note that

$$
\begin{aligned}
1 - R_I(x,z) &= \underline{R}(U - \{z\})_I(x) \\
&\overset{(2)}{=} \bigwedge_{A \subseteq U - \{z\}} \left(\underline{R}(A)_I(x) \mathit{scale}190\% \vee \bigvee_{u \in A} \underline{R}(A)_I(u) \right), \\
1 - R_I(x,y) &= \underline{R}(U - \{y\})_I(x) \\
&\overset{(2)}{=} \bigwedge_{B \subseteq U - \{y\}} \left(\underline{R}(B)_I(x) \mathit{scale}190\% \vee \bigvee_{v \in B} \underline{R}(B)_I(v) \right), \\
1 - R_I(y,z) &= \underline{R}(U - \{z\})_I(y) \\
&\overset{(2)}{=} \bigwedge_{C \subseteq U - \{z\}} \left(\underline{R}(C)_I(y) \mathit{scale}190\% \vee \bigvee_{w \in C} \underline{R}(C)_I(w) \right).
\end{aligned} \tag{34}
$$

Similar to (i), we can prove that $1 - R_I(x,z) \geq (1 - R_I(x,y)) \wedge (1 - R_I(y,z))$; that is, $R_I(x,z) \leq R_I(x,y) \vee R_I(y,z)$, as desired.

(iii) Note that

$$
\begin{aligned}
R_F(x,z) &= \underline{R}(U - \{z\})_T(x) \\
&\overset{(2)}{=} \bigvee_{A \subseteq U - \{z\}} \left(\underline{R}(A)_T(x) \bigwedge \bigwedge_{u \in A} \underline{R}(A)_T(u) \right), \\
R_F(x,y) &= \underline{R}(U - \{y\})_T(x) \\
&\overset{(2)}{=} \bigvee_{B \subseteq U - \{y\}} \left(\underline{R}(B)_T(x) \bigwedge \bigwedge_{v \in B} \underline{R}(B)_T(v) \right), \\
R_F(y,z) &= \underline{R}(U - \{z\})_T(y) \\
&\overset{(2)}{=} \bigvee_{C \subseteq U - \{z\}} \left(\underline{R}(C)_T(y) \bigwedge \bigwedge_{w \in C} \underline{R}(C)_T(w) \right).
\end{aligned} \tag{35}
$$

Take any $A \subseteq U - \{z\}$; then $y \in A$ or $y \notin A$.

Case 1: if $y \in A$, then

$$
\begin{aligned}
&\underline{R}(A)_T(x) \bigwedge \bigwedge_{u \in A} \underline{R}(A)_T(u) \\
&\leq \bigwedge_{u \in A} \underline{R}(A)_T(u), \text{by } y \in A \\
&= \underline{R}(A)_T(y) \bigwedge \bigwedge_{u \in A} \underline{R}(A)_T(u), \text{by } A \subseteq U - \{z\} \\
&\leq \bigvee_{C \subseteq U - \{z\}} \left(\underline{R}(C)_T(y) \bigwedge \bigwedge_{w \in C} \underline{R}(C)_T(w) \right) \\
&\leq \bigvee_{B \subseteq U - \{y\}} \left(\underline{R}(B)_T(x) \bigwedge \bigwedge_{v \in B} \underline{R}(B)_T(v) \right) \\
&\mathit{scale}190\% \vee \bigvee_{C \subseteq U - \{z\}} \left(\underline{R}(C)_T(y) \bigwedge \bigwedge_{w \in C} \underline{R}(C)_T(w) \right).
\end{aligned} \tag{36}
$$

Case 2: if $y \notin A$, then $A \subseteq U - \{y\}$ and so

$$
\begin{aligned}
&\underline{R}(A)_T(x) \bigwedge \bigwedge_{u \in A} \underline{R}(A)_T(u), \text{by } A \subseteq U - \{y\} \\
&\leq \bigvee_{B \subseteq U - \{y\}} \left(\underline{R}(B)_T(x) \bigwedge \bigwedge_{v \in B} \underline{R}(B)_T(v) \right) \\
&\leq \bigvee_{B \subseteq U - \{y\}} \left(\underline{R}(B)_T(x) \bigwedge \bigwedge_{v \in B} \underline{R}(B)_T(v) \right) \\
&\mathit{scale}190\% \vee \bigvee_{C \subseteq U - \{z\}} \left(\underline{R}(C)_T(y) \bigwedge \bigwedge_{w \in C} \underline{R}(C)_T(w) \right).
\end{aligned} \tag{37}
$$

By a combination of Cases 1 and 2, we obtain

$$
\begin{aligned}
&\bigvee_{A \subseteq U - \{z\}} \left(\underline{R}(A)_T(x) \bigwedge \bigwedge_{u \in A} \underline{R}(A)_T(u) \right) \\
&\leq \bigvee_{B \subseteq U - \{y\}} \left(\underline{R}(B)_T(x) \bigwedge \bigwedge_{v \in B} \underline{R}(B)_T(v) \right) \\
&\mathit{scale}190\% \vee \bigvee_{C \subseteq U - \{z\}} \left(\underline{R}(C)_T(y) \bigwedge \bigwedge_{w \in C} \underline{R}(C)_T(w) \right),
\end{aligned} \tag{38}
$$

that is, $R_F(x,z) \leq R_F(x,y) \vee R_F(y,z)$, as desired.

From (i)–(iii), we know that R is transitive.

(2) ⇔ (3). It can be concluded from Theorem 1 (4). □

4. Ordinary Single-Valued Neutrosophic Topological Space Induced by Single-Valued Neutrosophic Approximation Space

In this section, we shall consider the **OSvnt** induced by **Svnas** through the ordinary single-valued neutrosophic lower approximation operator.

At first, we fix a subclass of ordinary single-valued neutrosophic topological spaces.

Definition 7. An **OSvnts** (U, τ) is said to be quasidiscrete if it fulfills the following:

$$
\begin{aligned}
&(\text{OSvnt}2s) \text{for any } A_j \in P(U)\,(j \in J), \\
&\tau_T\left(\cap_{j \in J} A_j \right) \geq \wedge_{j \in J} \tau_T\left(A_j\right), \\
&\tau_I\left(\cap_{j \in J} A_j \right) \leq \vee_{j \in J} \tau_I\left(A_j\right), \\
&\tau_F\left(\cap_{j \in J} A_j \right) \leq \vee_{j \in J} \tau_F\left(A_j\right).
\end{aligned} \tag{39}
$$

It is not difficult to see that quasidiscrete **OSvnts** is an extension of quasidiscrete topological space [10].

Theorem 4. *Let (U, R) be an **Svnas**. Then the **Svns** τ_R on $P(U)$ is defined as follows: for any $A \in P(U)$,*

$$
\begin{aligned}
(\tau_R)_T(A) &= \bigwedge_{x \in A} \underline{R}(A)_T(x), \\
(\tau_R)_I(A) &= \bigvee_{x \in A} \underline{R}(A)_I(x), \\
(\tau_R)_F(A) &= \bigvee_{x \in A} \underline{R}(A)_F(x),
\end{aligned} \tag{40}
$$

*is a quasidiscrete **OSvnt** on U.*

Proof. OSvnt1: it follows that

$$
\begin{aligned}
(\tau_R)_T(\varnothing) &= \bigwedge_{x \in \varnothing} \underline{R}(\varnothing)_T(x) = 1, \\
(\tau_R)_I(\varnothing) &= \bigvee_{x \in \varnothing} \underline{R}(\varnothing)_I(x) = 0, \\
(\tau_R)_F(\varnothing) &= \bigvee_{x \in \varnothing} \underline{R}(\varnothing)_F(x) = 0, \\
(\tau_R)_T(U) &= \bigwedge_{x \in U} \underline{R}(U)_T(x) = 1, \\
(\tau_R)_I(U) &= \bigvee_{x \in U} \underline{R}(U)_I(x) = 0, \\
(\tau_R)_F(U) &= \bigvee_{x \in U} \underline{R}(U)_F(x) = 0.
\end{aligned} \tag{41}
$$

OSvnt2s: let $A_j \in P(U)\,(j \in J)$. Then

$$\begin{aligned}\bigwedge_{j\in J}(\tau_R)_T(A_j) &= \bigwedge_{j\in J}\bigwedge_{x_j\in A_j}\underline{R}(A_j)_T(x_j)\\ &\le \bigwedge_{j\in J}\bigwedge_{x\in\cap_{j\in J}A_j}\underline{R}(A_j)_T(x), \text{by Theorem 1 (3)}\\ &= \bigwedge_{x\in\cap_{j\in J}A_j}\underline{R}(\cap_{j\in J}A_j)_T(x) = (\tau_R)_T(\cap_{j\in J}A_j),\\ \bigvee_{j\in J}(\tau_R)_I(A_j) &= \bigvee_{j\in J}\bigvee_{x_j\in A_j}\underline{R}(A_j)_I(x_j)\\ &\ge \bigvee_{j\in J}\bigvee_{x\in\cap_{j\in J}A_j}\underline{R}(A_j)_I(x), \text{by Theorem 1 (3)}\\ &= \bigvee_{x\in\cap_{j\in J}A_j}\underline{R}(\cap_{j\in J}A_j)_I(x) = (\tau_R)_I(\cap_{j\in J}A_j).\end{aligned} \tag{42}$$

Similarly, we can prove that $\vee_{j\in J}(\tau_R)_F(A_j) \ge (\tau_R)_F(\cap_{j\in J}A_j)$.

OSvnt3: let $A_j \in P(U)\,(j \in J)$. Then it follows by Theorem 1 (2) that

$$\begin{aligned}&\bigwedge_{j\in J}(\tau_R)_T(A_j) = \bigwedge_{j\in J}\bigwedge_{x_j\in A_j}\underline{R}(A_j)_T(x_j),\\ &\le \bigwedge_{j\in J}\bigwedge_{x_j\in A_j}\underline{R}(\cup_{j\in J}A_j)_T(x_j)\\ &= \bigwedge_{x\in\cup_{j\in J}A_j}\underline{R}(\cup_{j\in J}A_j)_T(x) = (\tau_R)_T(\cup_{j\in J}A_j),\\ &\bigvee_{j\in J}(\tau_R)_I(A_j) = \bigvee_{j\in J}\bigvee_{x_j\in A_j}\underline{R}(A_j)_I(x_j),\\ &\ge \bigvee_{j\in J}\bigvee_{x_j\in A_j}\underline{R}(\cup_{j\in J}A_j)_I(x_j)\\ &= \bigvee_{x\in\cup_{j\in J}A_j}\underline{R}(\cup_{j\in J}A_j)_I(x) = (\tau_R)_I(\cup_{j\in J}A_j).\end{aligned} \tag{43}$$

Similarly, we can prove that $\vee_{j\in J}(\tau_R)_F(A_j) = (\tau_R)_F(\cup_{j\in J}A_j)$. □

Remark 3. The definition of $\tau_R(A)$ is an interpretation of the fact that "A is contained in its lower approximation."

5. Single-Valued Neutrosophic Approximation Space Induced by Ordinary Single-Valued Neutrosophic Topological Space

In this section, we shall consider the **Svnas** induced by **OSvnt**.

Theorem 5. *Let (U,τ) be an **OSvnts**. Then the **Svnr** R_τ on U is defined as follows: for any $(x,y)\in U\times U$,*

$$\begin{aligned}(R_\tau)_T(x,y) &= \bigwedge_{(x,y)\in A\times A^c}\tau_F(A),\\ (R_\tau)_I(x,y) &= \bigvee_{(x,y)\in A\times A^c}(1-\tau_I(A)),\\ (R_\tau)_F(x,y) &= \bigvee_{(x,y)\in A\times A^c}\tau_T(A),\end{aligned} \tag{44}$$

is reflexive and transitive.

Proof. Reflexivity: it follows that

$$\begin{aligned}(R_\tau)_T(x,x) &= \bigwedge_{(x,x)\in A\times A^c}\tau_F(A) = 1,\\ (R_\tau)_I(x,x) &= \bigvee_{(x,x)\in A\times A^c}(1-\tau_I(A)) = 0,\\ (R_\tau)_F(x,x) &= \bigvee_{(x,x)\in A\times A^c}\tau_T(A) = 0.\end{aligned} \tag{45}$$

Transitivity: let $x,y,z\in U$.

(i) Note that

$$\begin{aligned}(R_\tau)_T(x,y) &= \bigwedge_{(x,y)\in A\times A^c}\tau_F(A),\\ (R_\tau)_T(y,z) &= \bigwedge_{(y,z)\in B\times B^c}\tau_F(B),\\ (R_\tau)_T(x,z) &= \bigwedge_{(x,z)\in D\times D^c}\tau_F(D).\end{aligned} \tag{46}$$

Take any $D\in P(U)$ with $(x,z)\in D\times D^c$; then $y\in D$ or $y\in D^c$.

Case 1: if $y\in D$, then $(y,z)\in D\times D^c$. So,

$$\begin{aligned}&(R_\tau)_T(x,y)\wedge(R_\tau)_T(y,z)\\ &\le (R_\tau)_T(y,z) = \bigwedge_{(y,z)\in B\times B^c}\tau_F(B)\le\tau_F(D).\end{aligned} \tag{47}$$

Case 2: if $y\in D^c$, then $(x,y)\in D\times D^c$. So,

$$\begin{aligned}&(R_\tau)_T(x,y)\wedge(R_\tau)_T(y,z)\\ &\le (R_\tau)_T(x,y) = \bigwedge_{(x,y)\in A\times A^c}\tau_F(A)\le\tau_F(D).\end{aligned} \tag{48}$$

By a combination of Cases 1 and 2, we obtain that

$$\begin{aligned}&(R_\tau)_T(x,y)\wedge(R_\tau)_T(y,z)\\ &\le \bigwedge_{(x,z)\in D\times D^c}\tau_F(D) = (R_\tau)_T(x,z).\end{aligned} \tag{49}$$

(ii) Note that

$$\begin{aligned}(R_\tau)_I(x,y) &= \bigvee_{(x,y)\in A\times A^c}(1-\tau_I(A)),\\ (R_\tau)_I(y,z) &= \bigvee_{(y,z)\in B\times B^c}(1-\tau_I(B)),\\ (R_\tau)_I(x,z) &= \bigvee_{(x,z)\in D\times D^c}(1-\tau_I(D)).\end{aligned} \tag{50}$$

Take any $D\in P(U)$ with $(x,z)\in D\times D^c$; then $y\in D$ or $y\in D^c$.

Case 1: if $y\in D$, then $(y,z)\in D\times D^c$. So,

$$\begin{aligned}&(R_\tau)_I(x,y)\vee(R_\tau)_I(y,z)\\ &\ge (R_\tau)_I(y,z) = \bigvee_{(y,z)\in B\times B^c}(1-\tau_I(B))\\ &\ge (1-\tau_I(D)).\end{aligned} \tag{51}$$

Case 2: if $y\in D^c$, then $(x,y)\in D\times D^c$. So,

$$\begin{aligned}&(R_\tau)_I(x,y)\vee(R_\tau)_I(y,z)\\ &\ge (R_\tau)_I(x,y) = \bigvee_{(x,y)\in A\times A^c}(1-\tau_I(A))\\ &\ge (1-\tau_I(D)).\end{aligned} \tag{52}$$

By a combination of Cases 1 and 2, we obtain that

$$(R_\tau)_I(x,y)\vee(R_\tau)_I(y,z) \geq \bigvee_{(x,z)\in D\times D^c}(1-\tau_I(D)) = (R_\tau)_I(x,z). \quad (53)$$

(iii) Similar to (ii), one can prove that $(R_\tau)_F(x,y)\vee(R_\tau)_F(y,z)\geq(R_\tau)_F(x,z)$. □

Remark 4. Note that neither of the topological conditions (OSvnt1)-(OSvnt3) is used in the above theorem. Hence, it can be extended to any single-valued neutrosophic relation on $P(U)$.

6. One-to-One Correspondence between Reflexive and Transitive Single-Valued Neutrosophic Approximation Spaces and Quasidiscrete Ordinary Single-Valued Neutrosophic Topological Spaces

In this section, we prove that there is a one-to-one correspondence between reflexive and transitive **Svnas** and quasidiscrete **OSvnts**.

Theorem 6. *Let* (U,R) *be an* **Svnas**. *Then* $R_{\tau_R}\sqsupseteq R$, *and* $R_{\tau_R}=R$ *if* R *is reflexive and transitive.*

Proof. (1) For $x,y\in U$,

$$\begin{aligned}
\left(R_{\tau_R}\right)_T(x,y) &= \bigwedge_{(x,y)\in A\times A^c}(\tau_R)_F(A)\\
&= \bigwedge_{(x,y)\in A\times A^c}\bigvee_{z\in A}\underline{R}(A)_F(z)\\
&= \bigwedge_{(x,y)\in A\times A^c}\bigvee_{z\in A}\bigvee_{w\notin A}R_T(z,w)\\
&= \bigwedge_{(x,y)\in A\times A^c}\bigvee_{(z,w)\in A\times A^c}R_T(z,w), \text{taking } z=x, w=y\\
&\geq \bigwedge_{(x,y)\in A\times A^c}R_T(x,y)=R_T(x,y),\\
\left(R_{\tau_R}\right)_I(x,y) &= \bigvee_{(x,y)\in A\times A^c}\left(1-(\tau_R)_I(A)\right)\\
&= \bigvee_{(x,y)\in A\times A^c}\left(1-\bigvee_{z\in A}\underline{R}(A)_I(z)\right)\\
&= \bigvee_{(x,y)\in A\times A^c}\left(1-\bigvee_{z\in A}\bigvee_{w\notin A}(1-R_I(z,w))\right)\\
&= \bigvee_{(x,y)\in A\times A^c}\bigwedge_{(z,w)\in A\times A^c}R_I(z,w), \text{taking } z=x, w=y\\
&\leq \bigvee_{(x,y)\in A\times A^c}R_I(x,y)=R_I(x,y),\\
\left(R_{\tau_R}\right)_F(x,y) &= \bigvee_{(x,y)\in A\times A^c}(\tau_R)_T(A)\\
&= \bigvee_{(x,y)\in A\times A^c}\bigwedge_{z\in A}\underline{R}(A)_T(z)\\
&= \bigvee_{(x,y)\in A\times A^c}\bigwedge_{z\in A}\bigwedge_{w\notin A}R_F(z,w)\\
&= \bigvee_{(x,y)\in A\times A^c}\bigwedge_{(z,w)\in A\times A^c}R_F(z,w), \text{taking } z=x, w=y\\
&\leq \bigvee_{(x,y)\in A\times A^c}R_F(x,y)=R_F(x,y).
\end{aligned} \quad (54)$$

Hence, $R_{\tau_R}\sqsupseteq R$.

(2) Let R be reflexive and transitive and $x,y\in U$.

(i) Note that

$$\left(R_{\tau_R}\right)_T(x,y)\leq R_T(x,y)\Leftrightarrow\forall\alpha\in[0,1),\quad \alpha<\left(R_{\tau_R}\right)_T(x,y) \text{ implies } \alpha<R_T(x,y). \quad (55)$$

We assume that there is an $\alpha_0\in[0,1)$ such that $\alpha_0<(R_{\tau_R})_T(x,y)$ but $\alpha_0\geq R_T(x,y)$. Putting $A_0=\{z\in U|R_T(x,z)>\alpha_0\}$, by reflexivity of R, we have $R_T(x,x)=1>\alpha_0$, so $x\in A_0$, and by $\alpha_0\geq R_T(x,y)$ we have $y\in(A_0)^c$. This means that $(x,y)\in A_0\times(A_0)^c$. From

$$\alpha_0<\left(R_{\tau_R}\right)_T(x,y)=\bigwedge_{(x,y)\in A\times A^c}\bigvee_{(z,w)\in A\times A^c}R_T(z,w), \quad (56)$$

we know that there exists $(z,w)\in A_0\times(A_0)^c$ such that $R_T(z,w)>\alpha_0$; that is, $R_T(x,z)>\alpha_0$ and $R_T(x,w)\leq\alpha_0$. It follows by the transitivity that

$$\alpha_0<R_T(x,z)\wedge R_T(z,w)\leq R_T(x,w)\leq\alpha_0, \quad (57)$$

a contradiction! Therefore, $\alpha<(R_{\tau_R})_T(x,y)$ always implies that $\alpha<R_T(x,y)$. Hence, $(R_{\tau_R})_T(x,y)\leq R_T(x,y)$.

(ii) Note that

$$\left(R_{\tau_R}\right)_I(x,y)\geq R_I(x,y)\Leftrightarrow\forall\alpha\in(0,1],\quad \alpha\leq R_I(x,y) \text{ implies } \alpha\leq\left(R_{\tau_R}\right)_I(x,y). \quad (58)$$

We assume that there is an $\alpha_0 \in (0,1]$ such that $\alpha_0 \le R_I(x,y)$ but $\alpha_0 > (R_{\tau_R})_I(x,y)$. Putting $A_0 = \{z \in U | R_I(x,z) < \alpha_0\}$, by reflexivity of R, we have $R_I(x,x) = 0 < \alpha_0$, so $x \in A_0$, and by $\alpha_0 \le R_I(x,y)$ we have $y \in (A_0)^c$. This means that $(x,y) \in A_0 \times (A_0)^c$. From

$$\alpha_0 > \left(R_{\tau_R}\right)_I(x,y) = \bigvee_{(x,y)\in A\times A^c} \bigwedge_{(z,w)\in A\times A^c} R_I(z,w), \tag{59}$$

we know that there exists $(z,w) \in A_0 \times (A_0)^c$ such that $R_I(z,w) < \alpha_0$; that is, $R_I(x,z) < \alpha_0$ and $R_I(x,w) \ge \alpha_0$. It follows by the transitivity that

$$\alpha_0 > R_I(x,z) \vee R_I(z,w) \ge R_I(x,w) \ge \alpha_0, \tag{60}$$

a contradiction! Therefore, $\alpha \le R_I(x,y)$ always implies that $\alpha \le (R_{\tau_R})_I(x,y)$. Hence, $(R_{\tau_R})_I(x,y) \ge R_I(x,y)$.

(iii) Similar to (ii), we can prove that $(R_{\tau_R})_F(x,y) \ge R_F(x,y)$.

(i)–(iii) show that $R \sqsupseteq R_{\tau_R}$, and so $R_{\tau_R} = R$ by (1).

(3) If $R_{\tau_R} = R$, then it follows by Theorems 4 and 5 that R is reflexive and transitive. □

Theorem 7. *Let (U,R) be an* ***OSvnts****. Then $\tau_{R_\tau} \sqsupseteq \tau$, and $\tau_{R_\tau} = \tau$ if τ is quasidiscrete.*

Proof

(1) Let $A \in P(U)$. Then

$$\begin{aligned}
\left(\tau_{R_\tau}\right)_T(A) &= \bigwedge_{x\in A} \underline{R_\tau}(A)_T(x) \\
&= \bigwedge_{x\in A} \bigwedge_{y\in A^c} (R_\tau)_F(x,y) \\
&= \bigwedge_{(x,y)\in A\times A^c} \bigvee_{(x,y)\in B\times B^c} \tau_T(B), \text{taking } B = A \\
&\ge \bigwedge_{(x,y)\in A\times A^c} \tau_T(A) = \tau_T(A), \\
\left(\tau_{R_\tau}\right)_I(A) &= \bigvee_{x\in A} \underline{R_\tau}(A)_I(x) \\
&= \bigvee_{x\in A} \bigvee_{y\in A^c} \left(1 - (R_\tau)_I(x,y)\right) \\
&= \bigvee_{(x,y)\in A\times A^c} \left(1 - \bigvee_{(x,y)\in B\times B^c} \left(1 - \tau_I(B)\right)\right) \\
&= \bigvee_{(x,y)\in A\times A^c} \bigwedge_{(x,y)\in B\times B^c} \tau_I(B), \text{taking } B = A \\
&\le \bigvee_{(x,y)\in A\times A^c} \tau_I(A) = \tau_I(A).
\end{aligned} \tag{61}$$

Similarly, we can prove that $(\tau_{R_\tau})_F(A) \le \tau_F(A)$.

(2) Let $A \in P(U)$.

(i) Note that

$$\left(\tau_{R_\tau}\right)_T(A) \le \tau_T(A) \Leftrightarrow \forall \alpha \in [0,1), \quad \alpha < \left(\tau_{R_\tau}\right)_T(A) \text{ implies } \alpha \le \tau_T(A). \tag{62}$$

We assume that

$$\alpha < \left(\tau_{R_\tau}\right)_T(A) = \bigwedge_{(x,y)\in A\times A^c} \bigvee_{(x,y)\in B\times B^c} \tau_T(B). \tag{63}$$

Then, for any $(x,y) \in A \times A^c$, there is $B_{xy} \in P(U)$ such that $(x,y) \in B_{xy} \times (B_{xy})^c$ and $\alpha < \tau_T(B_{xy})$. Putting $B_y = \cup_{x\in A} B_{xy}$, by (OSvnt3), we have

$$\tau_T\left(B_y\right) = \tau_T\left(\cup_{x\in A} B_{xy}\right) \ge \bigwedge_{x\in A} \tau_T\left(B_{xy}\right) \ge \alpha. \tag{64}$$

Note that $A = \cap_{y\in A^c} B_y$ (indeed, if $z \in A$, then, for any $y \in A^c, z \in B_{zy} \subseteq B_y$, and so $z \in \cap_{y\in A^c} B_y$; hence, $A \subseteq \cap_{y\in A^c} B_y$; if $z \notin A$, then, for any $x \in A$, we have $(x,z) \in A \times A^c$, and then $z \notin B_{xz}$ so $z \notin B_z$, which means that $z \notin \cap_{y\in A^c} B_y$; hence, $\cap_{y\in A^c} B_y \subseteq A$); then it follows by OSvnt2s that

$$\tau_T(A) = \tau_T\left(\cap_{y\in A^c} B_y\right) \ge \bigwedge_{y\in A^c} \tau_T\left(B_y\right) \ge \alpha. \tag{65}$$

Therefore, $(\tau_{R_\tau})_T(A) \le \tau_T(A)$.

(ii) Note that

$$\left(\tau_{R_\tau}\right)_I(A) \ge \tau_I(A) \Leftrightarrow \forall \alpha \in (0,1], \quad \alpha > \left(\tau_{R_\tau}\right)_I(A) \text{ implies } \alpha \ge \tau_I(A). \tag{66}$$

We assume that

$$\left(\tau_{R_\tau}\right)_I(A) = \bigvee_{(x,y)\in A\times A^c} \bigwedge_{(x,y)\in B\times B^c} \tau_I(B) < \alpha. \tag{67}$$

Then, for any $(x,y) \in A \times A^c$, there is $B_{xy} \in P(U)$ such that $(x,y) \in B_{xy} \times (B_{xy})^c$ and $\alpha > \tau_I(B_{xy})$. Putting $B_y = \cup_{x\in A} B_{xy}$, by OSvnt3, we have

$$\tau_I\left(B_y\right) = \tau_I\left(\cup_{x\in A} B_{xy}\right) \leq \bigvee_{x\in A} \tau_I\left(B_{xy}\right) \leq \alpha. \quad (68)$$

Note that $A = \cap_{y\in A^c} B_y$; then it follows by OSvnt2s that

$$\tau_I(A) = \tau_I\left(\cap_{y\in A^c} B_y\right) \leq \bigvee_{y\in A^c} \tau_I\left(B_y\right) \leq \alpha. \quad (69)$$

Therefore, $(\tau_{R_\tau})_I(A) \geq \tau_I(A)$.

(iii) Similar to (ii), we can prove that $(\tau_{R_\tau})_F(A) \geq \tau_F(A)$.

(i)–(iii) show that $\tau_{R_\tau} \sqsubseteq \tau$, and so $\tau_{R_\tau} = \tau$ by (1).

(3) If $\tau_{R_\tau} = \tau$, then it follows by Theorems 4 and 5 that τ is quasidiscrete.

From Theorems 6 and 7, we obtain the following corollary. □

Corollary 1. *There is a one-to-one correspondence between reflexive and transitive* ***Svnas*** *and quasidiscrete* ***OSvnts*** *with the same underlying set.*

Remark 5. We can give a similar discussion on **Svnas** and ordinary single-valued neutrosophic cotopology in [47] via the ordinary single-valued neutrosophic upper approximation operator.

7. Conclusions

In this paper, we presented a new model of neutrosophic rough sets. The difference between this model and the existing models is that, in our model, the original sets are ordinary subsets of U and their approximations are single-valued neutrosophic sets; however, in the existing models, the original sets and their approximations are all (single-valued) neutrosophic sets. We also discussed the basic properties of the proposed rough sets and gave their relationships with Kim's ordinary single-valued neutrosophic topology. Particularly, we proved by our lower approximation operator that there is a one-to-one correspondence between reflexive and transitive single-valued neutrosophic approximation spaces and quasidiscrete ordinary single-valued neutrosophic topological spaces. In the future work, we shall present a more general single-valued neutrosophic topology such that it can be regarded as an extension of bifuzzy topology in [49]. We will also consider the corresponding single-valued neutrosophic rough sets related to the new single-valued neutrosophic topology. Furthermore, from Remark 1, we know that when restricting single-valued neutrosophic sets to Pythagorean fuzzy sets, we can define a model of Pythagorean fuzzy rough sets. It is well known that Pythagorean fuzzy sets and (fuzzy) rough sets have been applied in many fields, particularly in multiple attribute decision-making [9, 16, 52–55]. Therefore, in the future, we will also consider the potential application of Pythagorean fuzzy rough sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflicts of interest.

Authors' Contributions

Qiu Jin and Kai Hu contributed the central idea, and all authors contributed to the writing and revisions.

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