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A novel multi-criteria group decision-making method for heterogeneous and dynamic contexts using multi-granular fuzzy linguistic modelling and consensus measures

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ABSTRACT

This paper presents a novel multi-criteria group decision-making method that is capable of working in heterogeneous and dynamic environments. It is applicable in non-static frameworks where the decision context can vary at any time during the process. It also makes experts comfortable by allowing them to provide information using their most preferred means. By using multi-granular fuzzy linguistic modelling, the experts can provide preferences using their preferred linguistic label set. Furthermore, they also can choose the criteria values that they want to provide preferences for. Also, experts, alternatives and criteria can be added at any time during the decision process. Finally, consensus measures are applied in order to promote further debate and to help the experts reach an agreement.

1. Introduction

Multi-criteria group decision-making methods have become quite popular in recent literature [2,15,24]. Its main purpose is to allow a set of experts to rank a set of alternatives according to a certain set of criteria. This way, experts can use a comfortable and organized framework to make rational decisions.

The appearance of Web 2.0 technologies [11,12] has modified the environment that experts used to employ in order to carry out decision making processes. Nowadays, decisions are performed in heterogeneous and dynamic environments [18,26,38]. In these kinds of environments, experts can join or leave the discussion at any time and participate from anywhere by using the Internet and smart devices. They should also be able to provide their preferences using the preference providing method that they feel most comfortable with. This is quite important since, in real life problems, the experts have different necessities, each one wanting to use a different means for providing each of their preferences. Also, in dynamic and heterogeneous contexts, it is quite common for alternatives and criteria values sets to be modified anytime during the process [36]. This is because new solutions or criteria values can appear at any time during the discussion. For instance, the participation of new experts in the process allows for more information to be available. This will help generate more alternatives and criteria values.

In multi-criteria group decision-making environments not all the experts are experienced on all the criteria values that must be taken into account in the process. For instance, if some sets of experts are trying to elucidate which computer equipment they should buy for their employees, it is possible that they have experience on different criteria. For example, there may be experts that know better about the available money and benefits of the company and have a clear idea of how much money the company can invest. In turn, there may be other experts that know what computer features are really important for employees. Also, there may be experts that focus on the computer market and are aware of better price-quality options. Therefore, it would be desirable to design multi-criteria group decision-making methods that allow the experts to provide information only for those criteria values that they are familiar with.

Recent real multi-criteria group decision-making contexts are therefore heterogeneous, since each expert wants to provide the information about the specific part of the discussion that they feel comfortable with, using their preferred means. They are also dynamic, since experts, alternatives and criteria are not fixed during the whole process. On the contrary, they can be modified at any time. Furthermore, experts could

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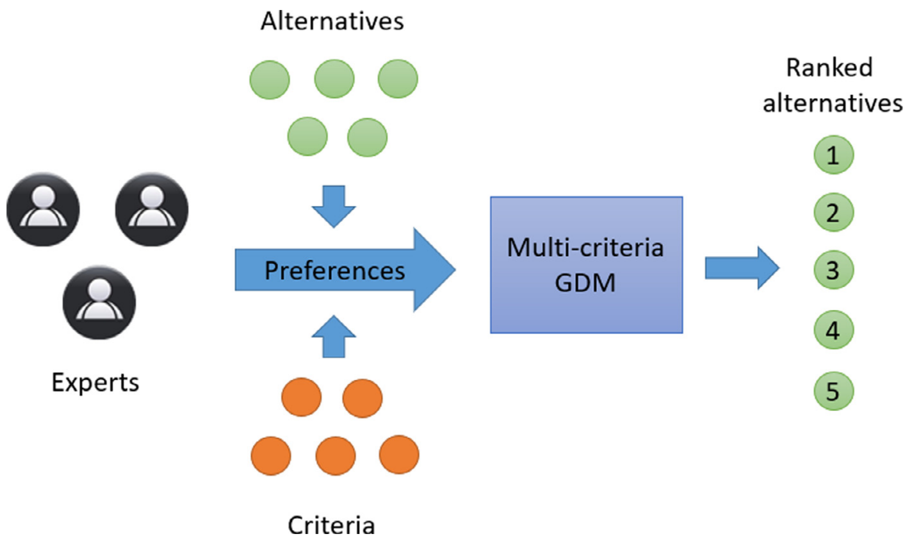

Fig. 1. Multi-criteria group decision-making scheme.

also want to modify the criteria values that they provide information for. For instance, there might be an expert that may not prefer to provide information about a certain criterion in the first multi-criteria group decision-making round. They may prefer to wait until the discussion is more advanced and the points of views are more defined before they provide an opinion. For instance, in the previous computer equipment selection example, there may be experts that might not want to provide their opinion about the quality-price relation of specific computer equipment until the experts that manage that information have provided clear evidence or until another expert experienced on that criterion joins the discussion.

Recent multi-criteria group decision-making methods research is centred on proposing novel representation methods for the preferences [13,35], novel selection processes [2,46] and applications to specific problems [1,48]. Since all these methods consider that the sets of experts, alternatives and criteria are fixed and do not vary over time, they cannot be adapted to heterogeneous and dynamic environments [50]. Therefore, there is a need for methods that can manage these kinds of environments which are typical in real life scenarios. The presented model tries to fill this gap presenting a novel multi-criteria group decision-making method that deals with heterogeneous and dynamic contexts. Our method allows the experts, alternatives and criteria to be added or removed at any time in the process. Experts can provide the information using the linguistic label set they prefer. That information will be uniformed using multi-granular fuzzy linguistic modelling methods [24,27] in order for the system to be able to handle it. Also, experts can provide information only for that criteria values that they feel comfortable with. Dynamic aggregation operators are implemented in order to reduce the amount of necessary computations. Finally, consensus measures [3,22,44,45] are applied in order to promote debate when experts have not reached an agreement.

This paper is organized as follows. In Section 2, The basic concepts needed to understand the presented method are explained. In Section 3, the proposed novel method is detailed. In Section 4, an application example is shown. In Section 5, advantages, drawbacks and related literature are discussed. Finally, some conclusions are pointed out.

2. Preliminaries

In this section, the basic concepts needed to understand the developed method are presented. In Section 2.1, the basic concepts of multi-criteria group decision-making methods are introduced. In Section 2.2, the basic concepts of multi-granular fuzzy linguistic modelling methods are described.

2.1. Multi-criteria group decision making

Multi-criteria group decision-making methods can be considered to be an extension of traditional group decision-making methods [5,19]. In a traditional group decision-making method, experts need to rank a set of alternatives based on the preferences they provide. In multi-criteria group decision-making methods, a new concept is introduced, the criteria. This way, experts are asked to rank the alternatives considering a pre-specified set of standards. Introducing criteria to group decision-making processes helps experts carry out the decision in a less subjective way. This is because they must focus on judging how each alternative fulfils the required set of criteria instead of using their own personal feelings.

Formally, it is possible to define a multi-criteria group decision-making problem as stated below:

Definition 2.1. Let $E = \{e_1, \dots, e_n\}$ be a set of experts, $X = \{x_1, \dots, x_m\}$ a set of alternatives or solutions and $C = \{c_1, \dots, c_l\}$ a set of criteria values that refer to the elements in X. A multi-criteria group decision-making process uses the preferences provided by the experts, P, to sort X according to how the elements in it fulfil the criteria specified in C.

In most cases, the criteria will not have the same importance. Therefore, it is possible to associate a weighting vector $W = \{w_1, \dots, w_l\}$ that indicates the weight that should be assigned to each criteria when calculating the final results. A brief graphical scheme of this process can be seen in Fig. 1.

Multi-criteria group decision-making methods are quite popular in recent literature. For instance, in [43], neutrosophic linguistic sets are used for multi-criteria group decision-making environments. In [40], the hesitant fuzzy VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method [28,29] is applied to carry out multi-criteria group decision-making processes. In [7], a multi-criteria group decision-making method designed to improve the expressive richness of the managed information is proposed. Finally, in [30–32], Parreiras et al. present several consensus models for multi-criteria group decision-making methods.

In the literature, different aggregation operators can be found. Their main purpose and the type of data that they dealt with varies. For instance, in [34], a novel dynamic aggregation operator for multi-period decision making algorithms is designed. The information is represented using hesitant fuzzy sets. The designed operator avoids the impact of harsh changes of opinions by taking those preferences provided in previous rounds into account. This way, opinions provided by experts who completely changed their mind from one round to the next have less impact on the final aggregated value. Although it may sound logical to punish experts that do not have clear criteria, there are some cases where this scheme may not be adequate. For instance, imagine a situation where an expert is always voting a certain preference. Suddenly, they notice that their vision of the problem is totally wrong and change their opinion. In this case, their previous preference values are included in the results even if the expert does not agree for a good reason. One way to solve this could be by counting the number of times that a specific expert changes their mind throughout the process. This will give a more realistic measure of the reliability of the expert. The authors have proposed two different aggregation operators: dynamic weighted averaging and dynamic weighted geometric mean. The main difference between them is that the weighted geometric mean punishes the infrequent values more. That is, outlier opinions have less impact in the final results. In [49], ordinal information is employed to carry out the preference providing step. This method includes a data cleansing process that tries to reduce the effect of conflicting opinions. Moreover, it employs the power average operator, which increases the influence of the argument with the highest degree of support. This approach tries to emphasize the arguments of the decision making process that are supported by most of the experts in the aggregation process. This way, it is understood that the more the experts agree on a specific argument, the more reliability that argument has. Therefore, when experts do not agree on an argument, its importance is devaluated. Although it is logical to give more credit to the information that is ratified by all the experts, some procedures that try to help the experts to reach consensus on the conflictive aspects of the decision could have been included.

In [4], a novel framework for dynamic multi-criteria decision making contexts is presented. The main purpose of this method is to manage situations where alternatives are added or removed during the process. The aggregation operator uses the associativity property for minimizing the number of computations that must be performed. It also implements a feature called retention policy that allows the detection of past and current alternatives that should be included in the aggregation process. This way, not all the alternatives that have appeared in the process are considered in the aggregation. The alternatives that are above a certain threshold are the ones maintained. The main drawback of the method is that it does not take the temporal performance of the alternatives into account. Therefore, this information is not included in the final calculations.

In order to solve this issue, in [52], the authors propose a dynamic aggregation operator that takes the evolution of the alternatives rating throughout the decision making process into account. Like the previous method, this method focuses on an environment where the aggregation process takes preferences from all the decision making rounds into account. Bipolar linguistic term sets are used to represent the preferences provided by the experts. The main purpose of the aggregation operator is to include information about the rating evolution of the alternatives. This is an important issue to take into account since it is not the same if an alternative has gained popularity throughout the process than if it has lost it. If an alternative has gained popularity, it is possible that some new argument has appeared in the discussion process that has caused other experts to support it. Consequently, developing methods that focus on this specific issue is of great interest. Due to the fact that the proposed method continues to fulfil the associativity property, the amount of computations that must be performed are the same in each round. This is an important issue to take into account, since if a lot of information has to be aggregated, the time required to do so may not be considered acceptable.

In [17], the aggregation operator presented in [52] is improved. Authors present a decision making method that allows the modification of criteria, alternatives and assessments during the process. Experts' preferences use a bipolar scale and information is represented using the 2-tuple linguistic modelling representation which allows the use of an uninorm in order to carry out the aggregation process. As in the previous method, assessments provided in different decision making rounds influence the results. Therefore, if an alternative got a good result in round $t-1$ and its popularity decreases in round t, then the rating given to the preference decreases less than if a static aggregation operator were used. Although this can be considered good behaviour in most situations, the aggregation method is based exclusively on the preferences of the experts. Therefore, it does not take the reason for loss of popularity of the alternative into account. There may be cases where a certain argument that has appeared in round t makes the loss of the popularity of a specific alternative seem logical. Since that argument was not available in round $t-1$, taking that information into account in the aggregation process may not be suitable.

2.2. Multi-granular fuzzy linguistic modelling

Human-computer communication is a critical issue in multi-criteria group decision-making environments. Since all the computational systems are accustomed to work using numerical information and humans are more used to providing information in a linguistic manner, there is a communication gap that must be overcome. As the system must work with human-provided information, there is a need for tools that allow the experts to provide information in a reliable manner.

One way of solving this communication gap is by using linguistic modelling. Linguistic modelling allows the experts to express themselves using labels belonging to a specific linguistic label set. For instance, if the following linguistic label set is used for providing preferences:

$$S^5 = \{Very\ Low, Low, Medium, High, Very\ High\}$$

The main problem that this approach entails is that the number of labels that the experts can use is fixed. This way, if an expert wants to provide a more concrete preference to the system, they will find that they cannot. On the other hand, there may be cases where an expert would prefer to provide their preferences by using a linguistic label set with a lower granularity. For instance by using $S^3 = \{Low, Medium, High\}$. In conclusion, each expert wants to provide information using the means they feel more comfortable with.

In order to allow the experts to use the linguistic label sets they prefer, it is possible to use multi-granular fuzzy linguistic modelling methods [27]. A multi-granular fuzzy linguistic modelling approach follows the scheme below:

1. *Providing information using the source linguistic label sets:* Each of the experts provide information using different linguistic label sets. Each linguistic label set can have different granularity values. This way, each expert can select the accuracy that they prefer when providing information to the system.
2. *Transforming the information into the target linguistic label set*: The system collects all the provided heterogeneous information and transforms it into information belonging to the same linguistic label set. For this purpose, a transformation function is used. This is the label set that will be used by the system to carry out all the required computations.
3. *Carrying out the required computations*: The system uses all the information that is now represented using the same linguistic label set and carries out all the required computations using the linguistic modelling computational framework. If the information must be presented using a different linguistic label set than the one used for computations, or if each expert requires a different linguistic label set for consulting the final results, the information will be transformed into the desired set again.

This process can be seen graphically in Fig. 2.

Multi-granular fuzzy linguistic modelling methods have been used quite often in recent literature. For instance, in [51], 2-tuple linguistic modelling is used in a multi-criteria group decision-making method in a multi-granular fuzzy linguistic modelling environment. In [41], incomplete multi-granular probabilistic linguistic term sets are used in group decision-making environments. Finally, in [25], multi-granular

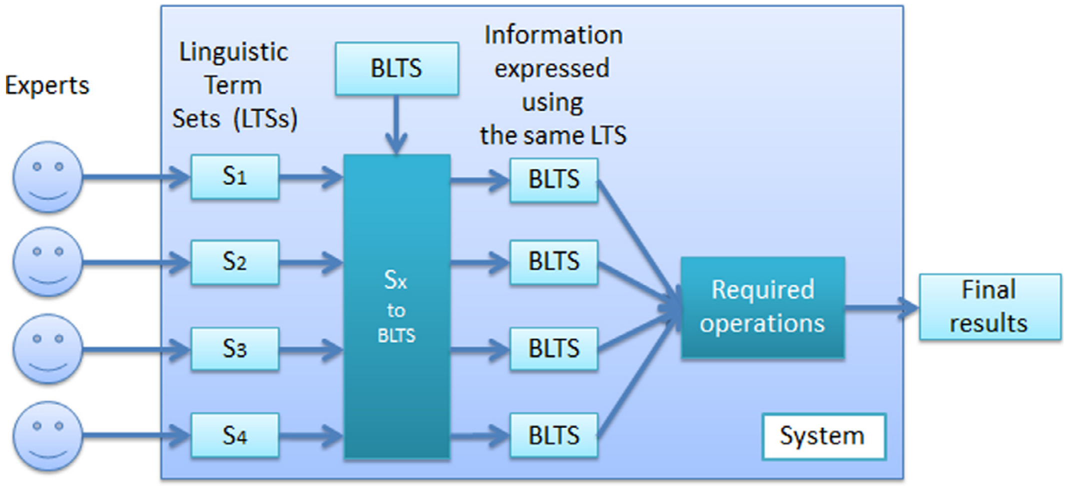

Fig. 2. Multi-granular fuzzy linguistic modelling scheme.

fuzzy linguistic modelling methods are applied over supervised learning classification methods in order to improve the obtained results of the algorithms.

3. A novel multi-criteria GDM for heterogeneous and dynamic contexts

In this section, the novel method is presented. In order to carry out the necessary computations, the following steps are carried out:

- *Defining the initial parameters*: The initial parameters of the multi-criteria group decision-making framework are established.
- *Providing preferences*: Experts provide their preferences for the alternatives according to the defined criteria values. They can do this by using the linguistic label set they prefer and selecting the criteria values that they feel more comfortable with.
- *Standardizing the information*: The information that has been provided by the experts is standardized in order to be represented using the same mean. Thus, the system can operate with the information and generate the ranking classification results.
- *Aggregating results*: All the experts' provided preferences are aggregated in order to generate a single collective preference matrix that contains the overall opinion of all the experts.
- *Calculating decision results*: Selection procedures are applied in order to generate the temporary ranking results. This information gives the experts some idea about which the current most promising alternatives are.
- *Establishing the consensus*: Consensus is calculated using the preferences that have been provided by the experts. If the consensus is high enough, then the experts are considered to have reached an agreement and the decision process is finished. Otherwise, the experts should debate more in order to bring their opinions closer. If too many rounds have passed and no improvement is achieved, it is considered that experts have exposed and discussed all their points of view and, therefore, another decision round is not required.
- *Modifying alternatives, experts and criteria*: At any time in the process, the set of alternatives, experts and criteria can be modified. It is only natural that, for instance, when experts are debating, new ideas or criteria arise. Also, new experts can be invited to the process if the current set of experts feel that they might benefit from their opinions. If a modification of any of these sets is performed and new information is needed, then the experts must provide it.

The following subsections will explain in detail how these steps are carried out.

3.1. Defining initial parameters

Before starting the process, the initial set of parameters for carrying out the multi-criteria group decision-making process are defined. They are detailed below:

- *Initial alternatives set*: The initial set of alternatives that the experts have to discuss, $X = \{x_1 \ldots x_n\}$, is defined. In order to fill the set, information can be extracted from the Web or obtained directly from the experts using a brainstorming process.
- *Initial criteria set*: The initial set of criteria values, $C = \{c_1, \ldots, c_m\}$ is defined. The same processes that were used to obtain the alternatives can be used to fill the set with the required values.
- *Initial set of experts*: The initial set of experts, $E = \{e_1 \ldots, e_l\}$ is established. These are the experts that will participate in the discussion in the first round.
- *Consensus threshold*: The consensus threshold is set in a way that if reached consensus is higher, then the experts' opinions are considered close enough to end the discussion process. If the value is lower, then another decision round is performed. The adequateness of the threshold depends on the necessity of the experts to reach an agreement and the importance or impact of the decision. If the decision is quite critical, a higher value can be set in order to promote a thorough debate. On the contrary, a lower value may also be sufficient.
- *Maximum number of decision rounds*: Since there are cases where the experts have great difficulty reaching an agreement, a maximum number of rounds can be established. To this end, all the decisions will come to an end after several debate iterations. The process has, therefore, a time limit and does not extend indefinitely.

Once all these parameters have been established, experts can start the debate process.

3.2. Providing preferences

Once the initial multi-criteria group decision-making framework has been defined, experts carry out a discussion about the alternatives and how they fulfil each criteria value. At some point, they can provide them system with their preferences. For this purpose, the following steps are used:

- *Selecting the representation mean:* Each expert chooses the linguistic label set that they want to use.
- *Selecting the target criteria values*: Experts select the criteria that they want to provide preferences for. For each expert e_i, there is a set $C^i = \{c^i_1, \ldots, c^i_o\}$ that indicates the criteria values that the expert has selected.

- *Providing preferences*: Each of the experts, e_i, provides a set of preference values in the actual round using a set of preference relation matrices, $\mathfrak{P}_{e_i}$, such as:

$$\mathfrak{P}_{e_i} = \{P^k_{e_i}\} \mid k = 1, \ldots, o \tag{1}$$

where each P^k is a preference relation matrix that has been provided considering the criteria value k. The preference relation matrix, P^k, is defined as follows:

$$P^k_{e_i} = \begin{pmatrix} p^k_{11} & \ldots & p^k_{1n} \\ \ldots & \ldots & \ldots \\ p^k_{n1} & \ldots & p^k_{nn} \end{pmatrix} \tag{2}$$

where k is the selected criteria value c^i_k. Each p^k_{ij} is a label belonging to the linguistic label set selected by the expert.

Once all the experts have provided the required information, it must be standardized so that the system is be able to work with it. In the following subsection, this process is explained in detail.

3.3. Standardizing the information

When providing their preferences, the experts have selected different linguistic label sets. The system must select a target linguistic label set to carry out the required computations. Therefore, all the information that is not represented using the system selected linguistic label set must be converted into it. Although a fixed linguistic label set can be selected for system computations, it could be better to work with the linguistic label set that most of the experts have selected. This way, the number of required transformations is reduced. Multi-granular fuzzy linguistic modelling methods can be used to make the information uniform.

There are a high number of multi-granular fuzzy linguistic modelling methods available in the literature [27]. If they can support the linguistic label sets used, all are then considered suitable and applicable to the method. For exemplary purposes, the method shown in [23] is used. We have selected this model because, thanks to the use of 2-tuple linguistic representation [9,10,20,21], it works with any linguistic label set granularity used. Also, it is an efficient option since it does not require too many computations. Considering that each expert provides a preference relation matrix for each of the criteria values, efficiency is an issue that should be considered in this choice.

The chosen multi-granular fuzzy linguistic methodology uses the concept of 2-tuple fuzzy linguistic representation. A linguistic 2-tuple value can be represented using a pair of values (s_i, α) where s_i is a linguistic label from an specific linguistic label set and α is a number called *symbolic translation* that belongs to the interval $[-0.5, 0.5[$. Using the index i and α, it is possible to generate a numerical value, β, such that $\alpha = \beta - i$. The transformation function that converts β into the (s, α) form is shown below:

$$\begin{aligned} &\Delta : [0, g] \rightarrow S \times [-0.5, 0.5) \\ &\Delta(\beta) = (s_i, \alpha)\ with \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \end{aligned} \tag{3}$$

In order to convert any 2-tuple linguistic value into β form, the following transformation function is applied:

$$\begin{aligned} &\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g] \\ &\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta \end{aligned} \tag{4}$$

Using Δ and Δ^{-1}, a transformation function that converts a label s_i to a label t_i belonging to another linguistic label set can be defined below:

$$\mathcal{T}^{t_i}_{s_i}(s_i, \alpha) = \Delta\left(\frac{(\Delta^{-1}(s_i, \alpha) - 1) \cdot (g_t - 1)}{g_s - 1}\right) + 1 \tag{5}$$

where s_i is the source label, g_s is the granularity of the label set to which s belongs, t is a label that belongs to the target linguistic label set, g_t is the granularity of that linguistic label set. Due to the use of the 2-tuple representation model, the loss of precision that is usually present in this type of operation is avoided. Also, the comprehensibility of the information given is not put at risk.

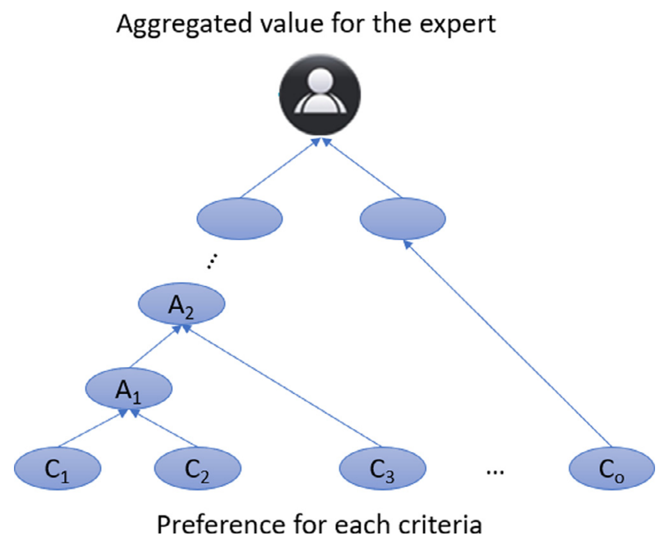

Fig. 3. Distributed aggregation scheme.

3.4. Aggregating results

nce the experts have provided their preferences, information must be aggregated in order to calculate the global collective preference matrix. This process is carried out in two steps:

- *Aggregating individual preferences based on the criteria values*: For each of the experts, the information provided is aggregated. For this purpose, the weighted mean operator can be used. Each expert defines the weighing vector $\Omega^{ij} = \{\omega_1, \ldots, \omega_o\}$ indicating the importance that they give to each selected criteria value. The use of criteria weights and how to assign them is totally dependent on the problem that is tackled. Usually, higher weights are assigned to the more important criteria values in order for them to have a better impact on the decision. Desired but not necessary criteria can have lower weight values.

In order to calculate the collective preference matrix, Γ_{e_i}, for the expert, e_i, all the preference matrices provided by the expert are aggregated as follows:

$$\Gamma_{e_i} = \omega_1 \cdot P^1 + \ldots + \omega_k \cdot P^k + \ldots + \omega_o \cdot P^o \tag{6}$$

In order to save computation time and carry out the process in an organized way, efficient dynamic aggregation techniques can be used. When the preference values for each of the experts are calculated, the scheme in Fig. 3 can be employed. Nodes, $\{C_1, \ldots, C_o\}$ indicate the preferences provided by the expert for their chosen set of criteria. Nodes A_i indicate aggregated values. Each node takes their input values and generate an output value using the mean aggregation operator. By following this scheme, it is possible to parallelise the process and carry out the required computations in a distributed architecture. This will drastically reduce the time required for computations.

In cases where the expert does not aggregate or eliminate criteria and the weights are not modified from one round to the next, the aggregation of the expert preference values can be performed as follows:

$$\Gamma^t_{e_i} = \Gamma^{t-1}_{e_i} - \omega_k \cdot P^k(t-1) \ldots + \omega_k \cdot P^k(t) \ldots \tag{7}$$

where $\Gamma^{t-1}_{e_i}$ refers to the aggregated value from the previous round, $P^k(t-1)$ is the preference matrix or matrices that has been modified and $P^k(t)$ is the new provided preference matrix/matrices. Thanks to this process, the amount of required computations is reduced. It should be noted that $\Gamma^t_{e_i} = \Gamma^{t-1}_{e_i}$ when experts do not modify their preferences.

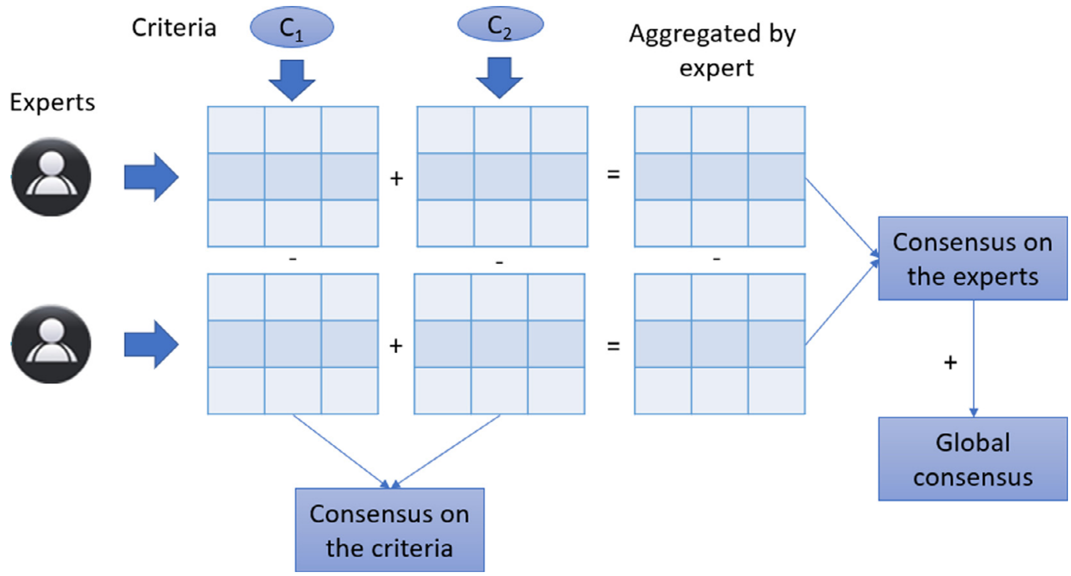

Fig. 4. Graphical scheme of the consensus process carried out in the method.

- *Aggregating collective preferences based on the experts*: Once that the collective matrices of all the experts have been calculated, they are aggregated in order to generate a global collective matrix containing all the preferences from all the experts and all the criteria values. It is possible to assign weights to the experts. If the weighting vector $W = \{w_1, \dots, w_l\}$ is used to establish the importance of the experts, the following expression can be used to calculate the global collective preference matrix:

$$\Gamma = w_1 \cdot \Gamma_{e_1} \dots + w_i \cdot \Gamma_{e_i} + \dots + w_l \cdot \Gamma_{e_l} \tag{8}$$

The weights assigned to the experts are also a problem dependent issue. Usually, experts that have better knowledge or occupy a higher position in the environment where the decision process is being performed are assigned higher weight values. The same process described for aggregating the preferences related to the criteria values can be employed in order to calculate the Γ value.

Once the global Γ matrix has been calculated, it is possible to generate the temporary rank of the alternatives.

3.5. Calculating decision results

Using the Γ matrix, it is possible to calculate the ranking of alternatives. Any selection operator is applicable. For exemplary purposes, the existing guided dominance degree quantifier (QGDD) operator [8] is be applied. The chosen selection process can be applied using the following steps:

1. *Expressing the collective preference matrix using β values*: The information obtained in the collective matrix is transformed into numbers using the β form of the 2-tuple representation. For this purpose, the expression (4) is applied.
2. *Applying QGDD operator*: QGDD operator is used to calculate the ranking. For each alternative x_i, its associated value is computed as follows:

$$\mathcal{D}_i = \phi(\Gamma_{ik}), k = 1 \dots n \tag{9}$$

where ϕ is the mean operator and Γ_{ik} indicates the value located in the position (*i, k*) of the matrix Γ.

3.6. Establishing the consensus

Consensus measures are an interesting way of getting an idea of how the multi-criteria group decision process is going. They tell us if the experts have reached an agreement or if more debate must be carried out.

The similarity between two preference relation matrices, P^i and P^j, can be calculated by applying the following expression:

$$Sim_{P^i}^{P^j} = \phi(abs(P^i - P^j)) \tag{10}$$

where *abs* is the absolute value operator and ϕ is the mean operator. It should be noted that the mean operator is applied to the obtained distances values in order to generate a unique value determining the similarity. Using this expression, there are several interesting consensus values that can be calculated:

- *Consensus among experts*: Using the expression (10), it is possible to calculate the consensus between two specific experts, e_i and e_j, in a certain moment of the multi-criteria group decision-making process. The value is computed as follows:

$$SimE_{e_i}^{e_j} = Sim_{\Gamma_{e_i}}^{\Gamma_{e_j}} \tag{11}$$

- *Global consensus value*: If all the *SimE* values are aggregated, it is possible to obtain a global consensus value establishing the consensus level in an specific multi-criteria group decision-making round. This is done by applying the following expression:

$$\mathcal{G} = SimE_{e_i}^{e_j} \mid i, j = 1 \dots l, i < j, i \neq j \tag{12}$$

- *Consensus on a criteria value*: It is obtained by calculating the similarity of the preferences that experts have provided for that criteria value. This is done by applying the following expression:

$$SimC_{c_k} = \phi(abs(P_i^k - P_j^k)) \mid, i, j = 1 \dots l, i < j, i \neq j \tag{13}$$

A scheme of the proposed calculation can be seen In Fig. 4.

If the obtained consensus is low, there are several actions that can be carried out:

- *The consensus on a criteria value is low*: In this case, the system can encourage the experts to discuss the alternatives focusing on that criteria value. This way, it may be possible for them to reach an agreement and improve the overall consensus value.
- *The consensus on experts is low*: The experts that are far away from the main opinion flow are identified. This way, in the next discussion round, the other experts can focus on trying to convince them. The system can make suggestions to the experts several times but since it should make sure the final decision is neutral, no direct action

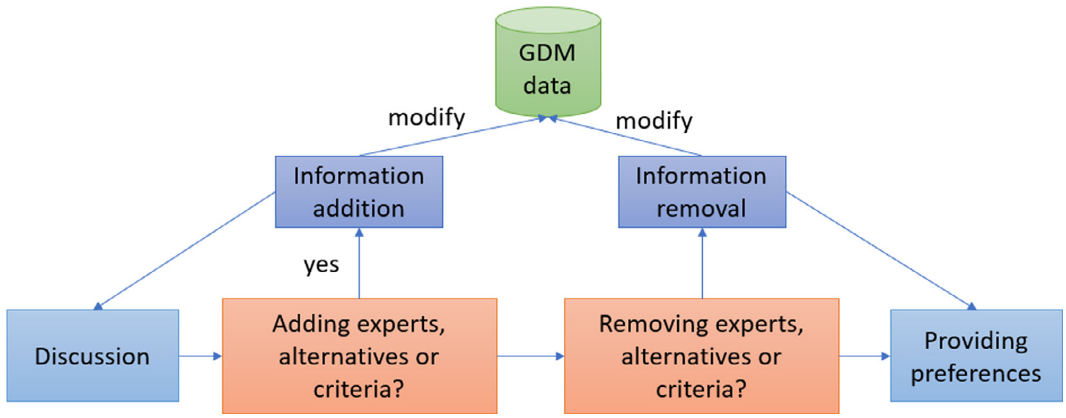

Fig. 5. Graphical scheme of the group decision-making parameters modification process carried out in the method.

is performed on these outlier experts. If the maximum number of rounds is reached, the final decision will be the one that the selection operators pinpoint, independent of the final consensus value.

3.7. Modifying alternatives, criteria values and experts

The described multi-criteria group decision-making process is carried out several times until one of the following two events occur:

- *The consensus is high enough*: If the obtained global consensus value is above the established threshold, then it is understood that the experts have reached a solution that they are all comfortable with. Therefore, the process can end.
- *The maximum number of rounds is reached*: Since it is not always possible to reach a high enough consensus, the decision process will not end until an acceptable number of rounds have been performed.

At any point in these decision-making rounds, it is possible that the initial parameters of the multi-criteria group decision-making framework defined may change. The following typical multi-criteria group decision-making dynamic events are handled by the developed method:

- *New experts are invited to the discussion*: It is possible that, at some point in the discussion, a new expert is invited to the process. For instance, this can occur because the expert themselves decides to join because the discussion is starting to catch their attention. Also, it is possible that at some point in the discussion, the actual set of experts decides that a specific expert should join due to their knowledge on the topic or because the decision affects them.
- *An expert wants to abandon the discussion*: It is possible that, due to the topic that is being discussed or, for some personal reasons, an expert decides that they do not want to participate in the decision process.
- *Some alternatives are added*: During the discussion, it is possible that new possible solutions appear. Debates typically make experts think and come up with new ideas that should be considered in the process.
- *Criteria values are modified*: During the discussion, it is possible that experts realize that new criteria values that were not considered at the beginning of the process should be added. Also, it is possible that some criteria values that were considered to be necessary turn out to be irrelevant.

If any of these events occur, the affected sets must be modified in order to include or exclude the required information. If new information is added, experts are asked to provide the required preferences values. For instance, if a new criteria value is included in the process, experts are asked to provide the associated preference matrix. If new alternatives are added, then experts must compare the new alternative with the others and provide the required preference values. If a new expert joins the discussion, after some debate, they are asked to provide their preference values.

On the contrary, if information is excluded from the process, the related information must be removed from the system. For instance, if an expert is excluded, their preferences values are removed. If an alternative or a criteria value is removed, the expert's related information is erased from the system. After that, decision results are recalculated based on the remaining information. A scheme of this process can be seen in Fig. 5.

4. Illustrative example

In this section, in order to clarify how this method works, an application example is shown. Imagine that a set of three experts $E = \{e_1, e_2, e_3\}$ need to decide which smartphones should they buy their employees. They want to make the decision based on three different criteria values: screen quality (c_1), CPU included (c_2) and quality-price relation (c_3). They are going to discuss 5 different solutions, $X = \{x_1, \ldots, x_5\}$. To provide their preferences, e_1 and e_3 decide to use the linguistic label set $S^5 = \{s_1^5, \ldots, s_5^5\}$ while e_2 wants to use the linguistic label set $S^9 = \{s_1^9, \ldots, s_9^9\}$ which has more labels. e_1 and e_3 decide to provide preferences for all the criteria values while e_2 does not feel sure about c_3 and they decide to provide preferences only for c_1 and c_2. Preferences provided by the experts are specified below:

$$P_{e_1}^1 = \begin{pmatrix} - & s_2^5 & s_3^5 & s_2^5 & s_3^5 \\ s_1^5 & - & s_3^5 & s_1^5 & s_2^5 \\ s_5^5 & s_4^5 & - & s_5^5 & s_4^5 \\ s_2^5 & s_1^5 & s_1^5 & - & s_1^5 \\ s_3^5 & s_2^5 & s_1^5 & s_3^5 & - \end{pmatrix} P_{e_1}^2 = \begin{pmatrix} - & s_2^5 & s_1^5 & s_2^5 & s_1^5 \\ s_2^5 & - & s_2^5 & s_2^5 & s_2^5 \\ s_4^5 & s_4^5 & - & s_4^5 & s_4^5 \\ s_2^5 & s_1^5 & s_2^5 & - & s_2^5 \\ s_2^5 & s_3^5 & s_2^5 & s_2^5 & - \end{pmatrix}$$

$$P_{e_1}^3 = \begin{pmatrix} - & s_2^5 & s_1^5 & s_1^5 & s_2^5 \\ s_2^5 & - & s_2^5 & s_3^5 & s_2^5 \\ s_5^5 & s_5^5 & - & s_5^5 & s_4^5 \\ s_1^5 & s_2^5 & s_3^5 & - & s_3^5 \\ s_1^5 & s_1^5 & s_2^5 & s_1^5 & - \end{pmatrix} P_{e_2}^1 = \begin{pmatrix} - & s_3^9 & s_4^9 & s_5^9 & s_4^9 \\ s_4^9 & - & s_6^9 & s_3^9 & s_2^9 \\ s_9^9 & s_8^9 & - & s_7^9 & s_8^9 \\ s_2^9 & s_1^9 & s_3^9 & - & s_4^9 \\ s_5^9 & s_4^9 & s_2^9 & s_3^9 & - \end{pmatrix}$$

$$P_{e_2}^2 = \begin{pmatrix} - & s_3^9 & s_2^9 & s_1^9 & s_4^9 \\ s_5^9 & - & s_2^9 & s_1^9 & s_1^9 \\ s_9^9 & s_8^9 & - & s_6^9 & s_9^9 \\ s_3^9 & s_2^9 & s_1^9 & - & s_3^9 \\ s_1^9 & s_1^9 & s_2^9 & s_1^9 & - \end{pmatrix} P_{e_3}^1 = \begin{pmatrix} - & s_2^5 & s_1^5 & s_2^5 & s_3^5 \\ s_2^5 & - & s_3^5 & s_2^5 & s_2^5 \\ s_4^5 & s_5^5 & - & s_4^5 & s_5^5 \\ s_3^5 & s_2^5 & s_1^5 & - & s_1^5 \\ s_1^5 & s_1^5 & s_2^5 & s_3^5 & - \end{pmatrix}$$

$$P^2_{e_3} = \begin{pmatrix} - & s^5_3 & s^5_2 & s^5_2 & s^5_1 \\ s^5_1 & - & s^5_3 & s^5_2 & s^5_1 \\ s^5_5 & s^5_5 & - & s^5_5 & s^5_5 \\ s^5_4 & s^5_3 & s^5_2 & - & s^5_1 \\ s^5_1 & s^5_1 & s^5_2 & s^5_3 & - \end{pmatrix} P^3_{e_3} = \begin{pmatrix} - & s^5_2 & s^5_2 & s^5_2 & s^5_1 \\ s^5_2 & - & s^5_3 & s^5_1 & s^5_1 \\ s^5_4 & s^5_4 & - & s^5_4 & s^5_4 \\ s^5_3 & s^5_2 & s^5_1 & - & s^5_1 \\ s^5_2 & s^5_1 & s^5_2 & s^5_1 & - \end{pmatrix}$$

It should be noted that preference matrices do not have to be symmetrical. This is because experts' opinions do not have to be totally coherent. This occurs because of the experts 'human-nature'. It should be noted that they can react differently to the questions *How much do you prefer x_1 to x_2?* and *How much do you prefer x_2 to x_1?* even if both questions are asking for the same information. Thanks to preference relations matrices, it is possible to represent and study the consistency of the preference values. There are several studies in the literature that perform this task [33,47].

The consensus threshold for this process has been set to 0.2. Therefore, if the consensus value is below 0.2, the obtained decision results can be considered to be the final ones without any more debate sessions. First, all the information must be expressed using the same representation mean. e_2 is the only expert that has provided information using a different linguistic label set. Therefore, the most efficient way of homogenizing the preferences is to apply the multi-granular fuzzy linguistic 2-tuple transformation that has been shown in expression (5) to the preferences provided by e_2. After that, all the information is represented using S^5. After applying the required computations, the following results are obtained:

$$P^1_{e_2} = \begin{pmatrix} - & s^5_2 & (s^5_2, 0.5) & s^5_3 & (s^5_2, 0.5) \\ (s^5_2, 0.5) & - & (s^5_3, 0.5) & s^5_2 & (s^5_1, 0.5) \\ s^5_5 & (s^5_4, 0.5) & - & s^5_4 & (s^5_4, 0.5) \\ (s^5_1, 0.5) & s^5_1 & s^5_2 & - & (s^5_2, 0.5) \\ s^5_3 & (s^5_2, 0.5) & (s^5_1, 0.5) & s^5_2 & - \end{pmatrix}$$

$$P^2_{e_2} = \begin{pmatrix} - & s^5_2 & (s^5_1, 0.5) & s^5_1 & (s^5_2, 0.5) \\ s^5_3 & - & (s^5_1, 0.5) & s^5_1 & s^5_1 \\ s^5_5 & (s^5_4, 0.5) & - & (s^5_3, 0.5) & s^5_5 \\ s^5_2 & (s^5_1, 0.5) & s^5_1 & - & s^5_2 \\ s^5_1 & s^5_1 & (s^5_1, 0.5) & s^5_1 & - \end{pmatrix}$$

Once the information has been homogenized, the collective preferences for each expert are calculated. Each expert can decide individually on the level of importance that should be given to each criteria value. After some meditation, they decide to use the following weighting vectors:

$$\Omega^1 = \{0.5, 0.25, 0.25\}$$

$$\Omega^2 = \{0.34, 0.66\}$$

$$\Omega^3 = \{0.25, 0.25, 0.5\}$$

Preferences matrices referring to different criteria provided by each expert are aggregated using the weighted mean and the Ω vectors provided by the experts. The resulting value represents the overall preference for each expert considering all the criteria values. The resulting expert collective matrices are given below:

$$\Gamma_{e_1} = \begin{pmatrix} - & 2 & 2 & 1.75 & 2.25 \\ 1.5 & - & 2.5 & 1.75 & 2 \\ 4.75 & 4.25 & - & 4.75 & 4 \\ 1.75 & 1.25 & 1.75 & - & 1.75 \\ 2.25 & 2 & 1.5 & 2.25 & - \end{pmatrix}$$

$$\Gamma_{e_2} = \begin{pmatrix} - & 2. & 1.84 & 1.68 & 2.5 \\ 2.83 & - & 2.18 & 1.34 & 1.17 \\ 5. & 4.5 & - & 3.67 & 4.83 \\ 1.83 & 1.33 & 1.34 & - & 2.17 \\ 1.68 & 1.51 & 1.5 & 1.34 & - \end{pmatrix}$$

$$\Gamma_{e_3} = \begin{pmatrix} - & 2.25 & 1.75 & 2 & 1.5 \\ 1.75 & 3 & - & 1.5 & 1.25 \\ 4.25 & 4.5 & - & 4.25 & 4.5 \\ 3.25 & 2.25 & 1.25 & - & 1 \\ 1.5 & 1 & 2 & 2 & - \end{pmatrix}$$

As shown above, the β value of the 2-tuple representation has been used for representing the preference information.

In order to carry out the alternatives ranking, the global collective preference matrix must be calculated. This is done by aggregating the collective preferences matrices of the experts. In this example, it is assumed that all the experts have the same level of importance in the process. Therefore, the weight vector $W = \{0.33, 0.33, 0.33\}$ is used. The obtained *Gamma* matrix is shown below:

$$\Gamma = \begin{pmatrix} - & 2.083333 & 1.863333 & 1.81 & 2.083333 \\ 2.026667 & - & 2.56 & 1.53 & 1.473333 \\ 4.666667 & 4.416667 & - & 4.223333 & 4.443333 \\ 2.276667 & 1.61 & 1.446667 & - & 1.64 \\ 1.81 & 1.503333 & 1.666667 & 1.863333 & - \end{pmatrix}$$

It should be noted that it is possible to use the expression (3) to express any piece of information using linguistic values. For instance, the global collective preference matrix can be expressed using the 2-tuple form as follows:

$$\Gamma = \begin{pmatrix} - & (s^5_2, 0.083) & (s^5_2, -0.137) & (s^5_2, -0.19) & (s^5_2, 0.083) \\ (s^5_2, 0.026) & - & (s^5_3, -0.44) & (s^5_2, -0.47) & (s^5_1, 0.473) \\ (s^5_5, -0.333) & (s^5_4, 0.416) & - & (s^5_4, 0.223) & (s^5_4, 0.443) \\ (s^5_2, 0.276) & (s^5_2, -0.39) & (s^5_1, 0.446) & - & (s^5_2, -0.36) \\ (s^5_2, -0.19) & (s^5_2, -0.497) & (s^5_2, -0.333) & (s^5_2, -0.137) & - \end{pmatrix}$$

After calculating the global collective matrix, the selection process described in Section 3.5 is applied.

After applying the QGDD operator, the following results are obtained:

$$\mathcal{D} = \{2.168, 2.118, 4.15, 1.995, 1.968\}$$

Imagine that, at this point, the experts decide to welcome a new member to the multi-criteria group decision-making process team. This member, e_4, after some debate, provides the following preferences to the system:

$$P^2_{e_4} = \begin{pmatrix} - & s^5_1 & s^5_1 & s^5_2 & s^5_1 \\ s^5_2 & - & s^5_2 & s^5_1 & s^5_2 \\ s^5_5 & s^5_5 & - & s^5_5 & s^5_5 \\ s^5_2 & s^5_2 & s^5_1 & - & s^5_1 \\ s^5_3 & s^5_2 & s^5_2 & s^5_3 & - \end{pmatrix} P^3_{e_4} = \begin{pmatrix} - & s^5_2 & s^5_1 & s^5_2 & s^5_1 \\ s^5_1 & - & s^5_2 & s^5_2 & s^5_1 \\ s^5_4 & s^5_5 & - & s^5_4 & s^5_5 \\ s^5_2 & s^5_2 & s^5_2 & - & s^5_1 \\ s^5_1 & s^5_1 & s^5_2 & s^5_2 & - \end{pmatrix}$$

Table 1
Consensus calculation.

Experts	Consensus value
e_1, e_2	0.0874
e_1, e_3	0.1125
e_1, e_4	0.0875
e_2, e_3	0.1091
e_2, e_4	0.1057
e_3, e_4	0.085
Global	0.0978

$$P^4_{e_4} = \begin{pmatrix} - & s^5_2 & s^5_2 & s^5_2 & s^5_1 \\ s^5_1 & - & s^5_3 & s^5_1 & s^5_1 \\ s^5_5 & s^5_4 & - & s^5_5 & s^5_5 \\ s^5_2 & s^5_2 & s^5_1 & - & s^5_2 \\ s^5_3 & s^5_1 & s^5_2 & s^5_1 & - \end{pmatrix}$$

As shown above, they have used the linguistic label set S^5 to provide information about criteria values $\{c_2, c_3, c_4\}$. Since this new expert has been introduced into the process, the alternative rankings must be recalculated. First of all, Γ_{e_4} is obtained after aggregating the expertâs preference matrices of each of the criteria values:

$$\Gamma_{e_4} = \begin{pmatrix} - & (s_1, 0.5) & (s_1, 0.25) & s_2 & s_1 \\ (s_1, 0.5) & - & (s_2, 0.25) & (s_1, 0.25) & (s_1, 0.5) \\ (s_5, -0.25) & (s_5, -0.25) & - & (s_5, -0.25) & s_5 \\ s_2 & s_2 & (s_1, 0.25) & - & (s_1, 0.25) \\ (s_2, 0.5) & (s_1, 0.5) & s_2 & (s_2, 0.25) & - \end{pmatrix}$$

Afterwards, the global collective matrix must be calculated again. The updated matrix is shown below:

$$\Gamma = \begin{pmatrix} - & 1.937 & 1.71 & 1.857 & 1.812 \\ 1.895 & - & 2.482 & 1.46 & 1.48 \\ 4.687 & 4.5 & - & 4.355 & 4.582 \\ 2.207 & 1.707 & 1.397 & - & 1.542 \\ 1.982 & 1.502 & 1.75 & 1.96 & - \end{pmatrix}$$

Finally, the selection operator is applied and the ranking results are obtained:

$$\mathcal{D} = \{2.0635, 2.0635, 4.225, 1.9710, 2.0390\}$$

$$Ranking = \{x_3, x_4, x_5, \{x_1, x_2\}\}$$

In order to determine if the experts agree on the results or if they should carry out another discussion round, consensus measures are calculated. Consensus results are given in Table 1. All the presented values have been uniformed to the interval [0,1] 0 being the value that expresses total agreement and 1 the value indicating total discordance. They were originally expressed using the interval [0,4] since 5 is the index of the highest label and 1 is the index of the lowest. With this is mind, the highest possible value was $5 - 1 = 4$.

Since the obtained consensus value 0.0978 is below 0.2, the obtained ranking results can be considered to be final. Therefore, the most voted option, x_3 is considered to be the most voted alternative. Consequently, experts are recommended to buy that smartphone for their employees.

If consensus for specific criteria needs to be calculated, it is possible to apply the weighted mean to the preferences of the experts according to the specific criteria. For instance, to calculate the consensus for c_1, the weighed mean operator can be applied to $P^1_{e_1}$, $P^1_{e_2}$ and $P^1_{e_3}$. The resulting value is 0.1433 and therefore means that the consensus is quite high.

5. Discussion

In this paper, a novel multi-criteria group decision-making method that works in dynamic and heterogeneous environments is presented. While other methods in the literature focus on the way that the information is represented and worked with, the method presented in this paper has been designed to improve the following aspects:

- *Improving the expert's experience*: Our method allows each expert to select the linguistic label set that they prefer. This way, experts can provide their preferences using the granularity value that they feel more comfortable with. Using multi-granular fuzzy linguistic methods, the system can homogenize all the provided information and compute the required results. Furthermore, preference relation matrices are selected for the providing preferences step. By carrying out pairwise comparisons, experts can provide a reliable measure of how they feel about the alternatives. Other multi-criteria group decision-making methods make the experts provide information about the alternatives separately. This approach does not allow the experts to compare the alternatives properly. Finally, experts can select the criteria values that they want to provide preferences for. In such a way, if an expert does not feel comfortable with one of the criteria, they can leave it to rest of the experts to measure it.
- *Modelling real work decision frameworks*: Most of the multi-criteria group decision-making methods that have been published in the recent literature work with a fixed number of alternatives, criteria and experts. Nevertheless, in real world discussions, criteria, alternatives and experts' sets can be modified at any time during the process. This mainly occurs because new ideas arise during the discussion process. The appearance of Web 2.0 technologies [11] have also increased the amount of information that the experts must deal with, making the decision making process more dynamic than ever. Therefore, there is a real need for methods that can work in this kind of environment.

The developed method design has the following weak points that will be addressed in the future:

- *The use of linguistic label sets in the process*: Although linguistic label sets are a comfortable way of providing information to a computational system, they force the experts to use a restricted set of words. This has the advantage of forcing the experts to follow an organized structure when providing their preferences. In such a way, the provided information is reliable, and the system can perfectly understand what the experts are trying to communicate. Nevertheless, it would be desirable to design a preference providing system that allows the experts to provide information without any representation restriction. That is, by using free text. It should be noted that it would be necessary to provide the computational system with means that allow it to clearly understand the expert. Due to the complexity of human language, this is clearly not an easy task, making it an interesting line of research in the future.
- *Working with a high number of criteria and alternative values*: Although preference relation matrices are an interesting method experts can use to provide their preferences, they can become a troublesome tool when dealing with a high number of criteria or alternatives. This is because they may force the experts to provide the system with too much information. For instance, in a framework with 5 alternatives and 4 criteria values, experts need to provide $(5 \cdot 5 - 5) \cdot 4 = 80$ preference values. Although the amount of preferences values is high, the reliability and accuracy of the information is higher than providing one value per alternative. It should be noted that the presented method could be easily adapted in order for the experts to provide information about alternatives instead of using pairs of alternatives. Therefore, if it is necessary, the reliability of preference relations can be sacrificed in favour of reducing the number of preference values that should be provided to the system.

There is a lot of research in recent literature concerning multi-criteria group decision-making methods that are applicable in different decision frameworks. Nevertheless, there is little research available on heterogeneous and dynamic contexts. For instance, in [15,42], both methods focus on using neutrosophic fuzzy sets to represent information. In [15], authors must provide information for all of the criteria and they must all use the same representation mean. In [42], the proposed method is designed to solve a specific problem. Furthermore, authors do not develop means that dynamise how preferences are provided to the system. Also, they are not defining a multi-purpose multi-criteria group decision-making method. In [24], a multi-criteria group decision-making method that is applicable to frameworks that have a high number of alternatives is presented. This method uses fuzzy ontologies in order to store the high number of available alternatives. Nevertheless, it does not introduce any means to allow experts to provide flexible preferences to the system. Also, dynamic scenarios where experts and alternatives can appear at any time are not covered. In [39], authors present a multi-criteria group decision-making method that works using interval type-2 fuzzy sets. Again, this method focuses on establishing a framework for using a specific preference representation mean. It does not model any kind of dynamic and heterogeneous environment. In [14], a review of methods that employ multi-criteria group decision-making methods to solve the renewable energy development problem is presented. Again, the methods reviewed by the paper focus on solving a specific problem. Furthermore, none of them are designed to deal with dynamic and heterogeneous contexts. Finally, in [16], interval valued linguistic values are used in an ORESTE method to solve multi-criteria group decision-making processes. Experts provide their preferences using interval linguistic values. This method creates groups of experts that evaluate the alternatives according to a set of criteria. Therefore, it does not provide each expert with their most preferred mean to provide preferences to the system. Also, the experts, criteria and alternatives sets are the same during the whole process.

As shown above, most of multi-criteria group decision-making methods focus on using different preferences representation methods and selection operators. Nevertheless, there are no multi-criteria group decision approaches like the one developed in this paper that focus on dealing with heterogeneous information and dynamic contexts. It is possible to find some solutions for group decision-making environments, like the ones proposed in [6,18,23,37]. In [6], authors review methods that allow each expert to provide preferences using their preferred means of representation. The reviewed methods work only in group decision-making environments. Therefore, the way to deal with criteria in dynamic and heterogeneous contexts has not been studied. For instance, they do not take situations into account where the experts do not want to provide information for all the criteria values. In [23], multi-granular fuzzy linguistic modelling is used so that the experts can select the linguistic label that they prefer to provide their preferences. In [37], a group decision-making method that is capable of introducing new experts during the decision process is presented. It is clear that there is a need to extend both methods in order to deal with multi-criteria group decision-making contexts. Finally, in [18], experts can select from three different types of preference relations: a multiplicative preference relation, an additive preference relation and an ordinal 2-tuple relation matrix. The method is built in order to work in an environment with a fixed number of experts and alternatives. Again, this method does not consider criteria values.

In conclusion, methods like the one developed here are needed. Plenty of multi-criteria group decision-making methods have been developed that use different preference representations and that are able to solve specific situations. They all consider fixed scenarios where the sets of experts, criteria and alternatives are not altered. Thus, there has been little research done on methods that model dynamic contexts and allow the experts to select their preferred means of representation. Furthermore, other methods do not consider situations in which experts are not fond of specific criteria values. The proposed method tries to solve all these issues presenting a multi-purpose multi-criteria group decision-making scheme that works in heterogeneous and dynamic environments.

6. Conclusions

It is quite common for real multi-criteria group decision-making processes to not be as static as most of the literature methods suggest. In this paper, a novel method that works with heterogeneous and dynamic environments is proposed. Our method deals with situations such as experts joining or leaving the multi-criteria group decision-making process at any time. Also, it allows alternatives and criteria sets to be modified during the process. In conclusion, it should be noted that the discussion that the experts carry out to make the decision is an open debate where new ideas and modifications to the decision parameters can occur at any time. Therefore, there is a need for methods like the one presented that can handle these modifications in an organized way.

In other multi-criteria group decision-making methods that are available in the recent literature, experts are asked to provide information considering all the criteria values. Nevertheless, in real life, experts may not have all the answers and they probably do not feel comfortable providing information related to criteria values that they do not know about or do not feel comfortable with. In the method proposed, they can select the criteria values that they want to provide information for.

Moreover, it is important to note that experts have different needs when providing their preferences. In order to include a comfortable user-computer communication system, our method allows them to provide information using the linguistic label set that they feel more comfortable with. The heterogeneous information that the system receives is uniformed using the multi-granular fuzzy linguistic modelling method.

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Supplementary material

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