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A NOVEL RISK RANKING METHOD BASED ON THE SINGLE VALUED NEUTROSOPHIC SET

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Abstract. Risk assessment is a key issue in the process of product design and manufacturing. Traditionally risk assessment uses the risk priority number (RPN) method to rank the extent of a threat. However, this simultaneously includes quantitative and qualitative evaluation factors in the process of risk assessment. Moreover, the information provided by different experts for evaluation factors contain ambiguous, incomplete and inconsistent information. These problems lead to more difficulty for risk assessment, and cannot be effectively solved by the traditional RPN method. To solve some limits of the traditional risk analysis method, this paper integrates the single valued neutrosophic set and subsethood measure method to rank the extent of the threat. For missing or incomplete information in the information aggregation process, the minimum, averaging and maximum operators are used to perform data imputation to avoid the distortion of decision results. Finally, a numerical example of high-dose-rate (HDR) brachytherapy treatments is provided to demonstrate the effectiveness and feasibility of the proposed method, and a comparative analysis with some other existing methods is given.

1. **Introduction.** Risk assessment is a multi-criteria decision-making (MCDM) problem that simultaneously comprises quantitative and qualitative evaluation factors. Most current risk assessment methods use the risk priority number (RPN) value to rank the extent of the threat [4]. Extending from the traditional RPN method, Safari et al. [20] proposed the fuzzy VIKOR method to rank enterprise architecture risk factors. Khorshidi et al. [15] applied the universal generating function to overcome the drawbacks of high duplication rate for the RPN method. This RPN method has been applied successfully to semiconductor fabrication [8], the operating procedures of an emergency department [9], thin film transistor liquid crystal display manufacture [10], the supplier selection problem [17] and geothermal power plant management [19]. However, the information that given by experts for evaluation factors will exist subjectivity, hence to become ambiguous, incomplete, missing or inconsistent information. In some real-world applications, the traditional RPN method cannot deal with MCDM problems with ambiguous, incomplete and inconsistent information.

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For fuzzy phenomena in MCDM problems, Zadeh [29] proposed the concept of fuzzy set to deal with the problem of uncertainties that exists in the real world. A fuzzy set A of the universe of discourse X, assigns the grade of membership $\mu(x)$ to every phenomenon, where $\mu(x) \in [0, 1]$. However, fuzzy set cannot deal with the degree of nonmembership and the degree of indeterminacy in some real applications. Extending the concept of fuzzy set, Atanassov [1] presented the intuitionistic fuzzy sets that used the degree of membership $\mu(x)$ and the degree of nonmembership $\nu(x)$ simultaneously to express fuzzy phenomena. However, fuzzy sets and intuitionistic fuzzy sets cannot simultaneously handle indeterminate information and inconsistent information in some real applications [5]. In order to solve this issue, Smarandache [23] proposed the concept of neutrosophic set, which consists of three components, a truth-membership function, an indeterminacy-membership function and a falsity-membership function. The truth-membership, indeterminacy-membership and falsity-membership functions are independent, and lie within $]0^-, 1^+[$ of nonstandard subsets, an extension of the standard interval $[0, 1]$. The neutrosophic set is an expanded concept of crisp sets, fuzzy sets, and intuitionistic fuzzy sets. Therefore, crisp sets, fuzzy sets and intuitionistic fuzzy sets can be viewed as special cases of the neutrosophic set.

Since Smarandache [23] presented the neutrosophic set, many new extension methods have been proposed based on the neutrosophic set to deal with ambiguous and inconsistent information. Based on the interval neutrosophic set operations, Zhang et al. [31] developed the interval neutrosophic number weighted averaging operator and interval neutrosophic number weighted geometric operator for MCDM problems. To date, the neutrosophic set has been widely applied in various fields, including binary classification [16], supplier selection [21], image segmentation [32] and 3D skeleton detection [13], and investment appraisal project [24].

Recently, Sahin and Kucuk [22] proposed the subsethood measure for single valued neutrosophic sets to deal with MCDM problems in neutrosophic information. However, the subsethood measure method cannot handle missing information provided by experts in the information aggregation process. In order to deal with this issue, this paper applies the concept of the subsethood measure method to propose a novel risk ranking method in solving MCDM problems with missing, ambiguous, incomplete and inconsistent information in real world situations.

The remainder of this paper is arranged as follows. Section 2 briefly introduces the basic concepts and definitions of the risk analysis method and single valued neutrosophic set. Section 3 proposed integration of single valued neutrosophic set and subsethood measure method. In Section 4, a numerical example is applied to test and demonstrate the effectiveness of the proposed method. Finally, conclusions and further research suggestions are given in Section 5.

2. Preliminaries.

In this section, some fundamental concepts, definitions and operations of the risk analysis method and single valued neutrosophic set are introduced.

2.1. Risk analysis method.

Risk analysis is a major issue in product design, manufacturing and production processes. Risk analysis results directly influence a company's policies and the development of future operations. Most enterprise risk assessment applies the RPN to rank the cause of potential failures for accident prevention. The traditional RPN method uses three factors, namely severity (S), occurrence (O) and detection (D), to assess the cause of potential failures on a rating score from 1 to 10. The RPN value represents the level of risk, which is the multiplication product of the S, O and D factors. Therefore, RPN $= S \times O \times D$. Because the RPN method is simple to compute and easy to operate, it has been widely applied in several different international standards [6, 7, 19]. These international standards include MIL-STD-1629A [11], IEC 60812 [14], BS 5760-5 [3], ISO-9000, ISO/TS 16949, and QS-9000. Tables 1, 2 and 3 list the traditional RPN method scale for measuring the three factors [18, 19].

Table 1. Traditional RPN method scale for severity [18, 19]

Rating	Effect	Severity of effect
10	Hazardous without warning	Highest severity ranking of a failure mode, occurring without warning and consequence is hazardous
9	Hazardous with warning	Higher severity ranking of a failure mode occurring with warning, consequence is hazardous
8	Extreme	Operation of system or product is broken down without compromising safe
7	Major	Operation of system or product may be continued but performance of system or product is affected
6	Significant	Operation of system or product is continued and performance of system or product is degraded
5	Moderate	Performance of system or product is affected seriously and the maintenance is needed
4	Low	Performance of system or product is small affected and the maintenance may not be needed
3	Minor	System performance and satisfaction with minor effect
2	Very minor	System performance and satisfaction with slight effect
1	None	No effect

Table 2. Traditional RPN method scale for occurrence [18, 19]

Rating	Probability of failure	Possible failure rates
10	Extremely high: failure almost inevitable	$\geq$in 2
9	Very high	1 in 3
8	Repeated failures	1 in 8
7	High	1 in 20
6	Moderately high	1 in 80
5	Moderate	1 in 400
4	Relatively low	1 in 2000
3	Low	1 in 15,000
2	Remote	1 in 150,000
1	Nearly impossible	$\leq$1 in 1,500,000

2.2. **Single valued neutrosophic set.** Smarandache [23] firstly proposed the concept of neutrosophic set from a philosophical point of view. The neutrosophic set domains include a truth-membership function, an indeterminacy-membership function and a falsity-membership function, and are independent.

Definition 1. [2, 23]. Let X be a universal space of points (objects) and $x \in X$. A neutrosophic set N in X is characterized by a truth-membership function $T_N(x)$,

Table 3. Traditional RPN method scale for detection [18, 19]

Rating	Detection	Likelihood of detection by design control
10	Absolute uncertainty	Potential occurring of failure mode cannot be detected in concept, design and process failure mode and effects analysis (FMEA)/mechanism and subsequent failure mode
9	Very remote	The possibility of detecting the potential occurring of failure mode is very remote/mechanism and subsequent failure mode
8	Remote	The possibility of detecting the potential occurring of failure mode is remote/mechanism and subsequent failure mode
7	Very low	The possibility of detecting the potential occurring of failure mode is very low/mechanism and subsequent failure mode
6	Low	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
5	Moderate	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
4	Moderately high	The possibility of detecting the potential occurring of failure mode is moderately high/mechanism and subsequent failure mode
3	High	The possibility of detecting the potential occurring of failure mode is high/mechanism and subsequent failure mode
2	Very high	The possibility of detecting the potential occurring of failure mode is very high/mechanism and subsequent failure mode
1	Almost certain	The potential occurring of failure mode will be detect/ mechanism and subsequent failure mode

an indeterminacy-membership function $I_N(x)$ and a falsity-membership function $F_N(x)$. These functions $T_N(x)$, $I_N(x)$ and $F_N(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_N(x) : X \to]0^-, 1^+[, I_N(x) : X \to]0^-, 1^+[$ and $F_N(x) : X \to]0^-, 1^+[$. The sum of $T_N(x)$, $I_N(x)$ and $F_N(x)$ is not any restriction, therefore $0^- \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$.

Single valued neutrosophic set is a special instance of neutrosophic set, which is extended from the concept of crisp sets, fuzzy sets and intuitionistic fuzzy sets.

Definition 2. [22, 25]. Let X be a universe of discourse, a single valued neutrosophic set N in X can be expressed as follows:

$$N = \{< x, T_N(x), I_N(x), F_N(x) > | \, x \in X\} \tag{1}$$

where $T_N(x) : X \to [0, 1], I_N(x) : X \to [0, 1]$ and $F_N(x) : X \to [0, 1]$ with the condition $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$ for each x in X.

The values$T_N(x)$, $I_N(x)$ and $F_N(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of x to X, respectively. For convenience of calculation, the single valued neutrosophic set can be simplified to single-valued neutrosophic number, expressed as $N_A = (T_A, I_A, F_A)$ where $T_A, I_A, F_A \in [0, 1]$ and $0 \leq T_A + I_A + F_A \leq 3$ [30].

Definition 3. [22]. Let X be a universe of discourse, A be a single valued neutrosophic set in X, then the empty neutrosophic set and absolute neutrosophic set can be defined as follows:

(1) Empty neutrosophic set can expressed as $< 0, 1, 1 >$, if $T_A(x) = 0,\ I_A(x) = 1,$ and $F_A(x) = 1$, $\forall x \in X$.
(2) Absolute neutrosophic set can expressed as $< 1, 0, 0 >$, if $T_A(x) = 1,\ I_A(x) = 0,$ and $F_A(x) = 0$, $\forall x \in X$.

Definition 4. [26, 30]. If A and B are two single valued neutrosophic numbers, then the summation between A and B can be defined as follows:

$$A \oplus B = (T_A + T_B - T_A \cdot T_B, I_A \cdot I_B, F_A \cdot F_B) \tag{2}$$

Definition 5. [26, 30]. If A and B are two single valued neutrosophic numbers, then the multiplication between A and B can be defined as follows:

$$A \otimes B = (T_A \cdot T_B, I_A + I_B - I_A \cdot I_B, F_A + F_B - F_A \cdot F_B) \tag{3}$$

Definition 6. [26, 30]. If A is a single valued neutrosophic number and is an arbitrary positive real number then:

$$\lambda A = \left(1 - (1 - T_A)^{\lambda}, I_A^{\lambda}, F_A^{\lambda}\right), \lambda > 0 \tag{4}$$

$$A^{\lambda} = \left(T_A^{\lambda}, 1 - (1 - I_A)^{\lambda}, 1 - (1 - F_A)^{\lambda}\right), \lambda > 0 \tag{5}$$

Definition 7. [22]. If A and B are two single valued neutrosophic sets, then the union and intersection of two sets A and B can be defined as follows:

$$\begin{aligned} A \cup B = \{< x, \max\{T_A(x), T_B(x)\}, \min\{I_A(x), I_B(x)\},\\ \min\{F_A(x), F_B(x)\} >| \, x \in X\} \end{aligned} \tag{6}$$

$$\begin{aligned} A \cap B = \{< x, \min\{T_A(x), T_B(x)\}, \max\{I_A(x), I_B(x)\},\\ \max\{F_A(x), F_B(x)\} >| \, x \in X\} \end{aligned} \tag{7}$$

Definition 8. [22]. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a universal space of points (objects). If A and B are two single valued neutrosophic sets, then the normalized Hamming distance based on the Hausdorff metric between A and B can be defined as follows:

$$d(A, B) = \frac{1}{n} \sum_{i=1}^{n} w_i \max\{|T_A(x) - T_B(x)|, |I_A(x) - I_B(x)|, |F_A(x) - F_B(x)\,|\} \tag{8}$$

Definition 9. [22]. If A and B are two single valued neutrosophic sets, then the subsethood measure $S_d(A, B)$, expressing the degree to which A belongs to B, based on distance measure can be defined as follows:

$$S_d(A, B) = 1 - d(A, A \cap B) \tag{9}$$

Example. Let $S = (0.20, 0.85, 0.80), O = (0.10, 0.90, 0.90)$, and $D = (1.00, 0.00, 0.00)$ are three single valued neutrosophic sets. The ideal alternative A^* for single valued neutrosophic set is defined as $A^* = (1, 0, 0)$ [22]. Find $S_d(A^*, A^* \cap A_i)$.

According to Eq.7 to calculate the $A^* \cap A_i$ as follows.

$$\begin{aligned} A^* \cap A_S &= (\min(T_{A^*}(x), T_S(x)), \max(I_{A^*}(x), I_0(x)), \max(F_{A^*}(x), F_D(x))) \\ &= (\min(1, 0.20), \max(0, 0.85), \max(0, 0.80)) \\ &= (0.20, 0.85, 0.80) \\ \Rightarrow \quad A^* \cap A_0 &= (0.10, 0.90, 0.90) \\ \Rightarrow \quad A^* \cap A_D &= (1.00, 0.00, 0.00) \end{aligned}$$

Used Eq. 8 to calculate the value of $d\left(A^{*}, A^{*} \cap A_{i}\right)$.

$$
\begin{aligned}
d\left(A^{*}, A^{*} \cap A_{i}\right) = & \frac{1}{n} \sum_{i=1}^{n} w_{i} \max \left\{\left|T_{A^{*}}(x)-T_{A^{*} \cap A_{i}}(x)\right|,\left|I_{A^{*}}(x)-I_{A^{*} \cap A_{i}}(x)\right|\right. \\
& \left.\left|F_{A^{*}}(x)-F_{A^{*} \cap A_{j}}(x)\right|\right\} \\
& =\frac{1}{3}\left(\frac{1}{3} \max \{|1-0.20|,|0-0.85|,|0-0.80|\}+\frac{1}{3} \max \{|1-0.10|\right. \\
& \left.|0-0.90|,|\ 0-0.90\ |\}+\frac{1}{3} \max \{|1-1|,|0-0|,|0-0|\}\right) \\
& =\frac{1}{3}\left(\frac{1}{3} \times 0.85+\frac{1}{3} \times 0.90+\frac{1}{3} \times 0\right)=0.194
\end{aligned}
$$

$$
\begin{aligned}
S_{d}\left(A^{*}, A^{*} \cap A_{i}\right) & =1-d\left(A^{*}, A^{*} \cap A_{i}\right) \\
& =1-0.194 \\
& =0.806
\end{aligned}
$$

3. **Proposed integration of single valued neutrosophic set and subsethood measure method.** Risk assessment is a critical issue in the process of production design and manufacture. It directly influences the market competitiveness of a company. However, risk assessment is an MCDM problem that simultaneously includes quantitative and qualitative evaluation factors in the process of risk assessment. Moreover, sometimes the information is ambiguous, missing and inconsistent in the information aggregation process. These problems increase the difficulty of risk assessment, which then cannot be effectively solved by the traditional RPN method. In order to effectively solve the above issues, this paper applies the single valued neutrosophic set method to handle indeterminate and inconsistent information in the information aggregation process. For missing or incomplete information, this paper applies minimum, averaging and maximum operators to perform data imputation. The major advantage of neutrosophic sets, which are generalized from crisp sets, fuzzy sets and intuitionistic fuzzy sets. Thus, using the single value neutrosophic sets, not the crisp sets, fuzzy sets, and intuitionistic fuzzy sets, for solving the risk assessment problems is more suitable.

The procedure of the proposed novel risk ranking method can be outlined as follows:

Step 1: Determine the component, failure mode and failure effect of the evaluation item.

Step 2: Determine the possible range of the S, O and D factors by single valued neutrosophic number.

Step 3: Input data using the minimum, averaging and maximum operators. If A and B are two single valued neutrosophic sets, then the minimum, maximum, and averaging operator of two sets A and B can be defined as follows:

$$
\begin{aligned}
& \operatorname{Max}(A, B)=\left(\max \left(T_{A}(x), T_{B}(x)\right), \min \left(I_{A}(x), I_{B}(x)\right), \min \left(F_{A}(x), F_{B}(x)\right)\right) \\
& \operatorname{Min}(A, B)=\left(\min \left(T_{A}(x), T_{B}(x)\right), \max \left(I_{A}(x), I_{B}(x)\right), \max \left(F_{A}(x), F_{B}(x)\right)\right)
\end{aligned}
$$

$$
\text{Averaging }(A, B)=\left(T_{A}(x)+T_{B}(x)-T_{A}(x) \times T_{B}(x), I_{A}(x) \times I_{B}(x), F_{A}(x) \times F_{B}(x)\right)
$$

Step 4: For information provided by different experts, use single valued neutrosophic numbers to aggregate S, O and D factors.

Step 5: According to Eq. 8 and the results of Step 4, used the minimum, averaging and maximum operators to calculate the normalized Hamming distance $d(A^*, A^* \cap A_i)$.

Step 6: According to the results of Step 5, use Eq. 9 to calculate the subsethood measure based on distance measure.

Step 7: Rank all the subsethood measures for the evaluation item according to the $S(A^*, A_i)$ value.

4. Numerical example.

4.1. **Overview.** In this section, an illustrative example of high-dose-rate (HDR) brachytherapy treatments is applied to demonstrate the rationality and correctness of the proposed method. The risk ranking problem of safety analysis for HDR brachytherapy is adapted from Giardina et al. [12]. The failure mode and failure effect of the HDR brachytherapy treatments is shown in Table 4. All linguistic terms for S, O and D factors are converted into the single valued neutrosophic set, as shown in Table 5. Suppose that the S, O and D factors are of equal weight. This risk assessment team members are four experts (TM1, TM2, TM3 and TM4), chosen for their different experiences and backgrounds. Different experts evaluate the possible range of the S, O and D factors by single valued neutrosophic number, respectively, as shown in Table 6.

Table 4. The FMEA of the HDR brachytherapy treatments [12]

Identification number (ID)	Component	Failure mode	Failure effect
1	Stepping motor	Electrical blackout	High-dose-rate (HDR) unit is stopped and dc motor withdraws the source to the safe
2	Direct current safety motor	Loss of power	Operator goes into the treatment room (TR) to manually return the source to the safe
3	Dwell position distance control device	Stepper motor failure	Source position not correct
4	Secondary timer	Electronic fault	Incorrect check of the primary timer
5	Backup battery	Power-off	Direct current motor fault
6	Backup battery	Operator forgets to charge the battery	Direct current motor fault
7	Software	Power-off	Safety and control system fault
8	Stop button on the console	Contact fault	During treatment, the stop button on the console did not retract the wire source
9	Physicist	Dose calculation errors during treatment planning system (TPS)	Incorrect HDR treatment
10	Therapist	Data insertion errors during TPS	Incorrect HDR treatment
11	Medical operator	Incorrect patient identification	Incorrect data are used during treatment control system (TCS)
12	Medical operator	Incorrect medical application of the catheter or applicator	Incorrect HDR treatment
13	Therapist	Error in loading patient information (from the database)	Incorrect data are used during TCS
14	Therapist	Error in the data entry for dwell time or dwell position programming	Incorrect data are used during TCS

4.2. **Risk ranked using the traditional RPN method.** The traditional RPN method applies three risk factors of severity (S), occurrence (O) and detection (D) to calculate the RPN value. The RPN value is the multiplication product of the S, O and D factors. Therefore, RPN $= S\ O\ D$. A higher RPN value expresses more critical and important failure risk, and must receive a higher priority for corrective action. In the traditional RPN method, the S, O and D factors possible range information must be complete information provided by the experts. Based on Table 6, expert TM3 provides S, O and D factor information that is partially incomplete, and only experts (TM1, TM2 and TM4) provide complete information.

TABLE 5. Single valued neutrosophic number conversion for S, O and D factors (adapted from [30])

Level	S	O	D	Single valued neutrosophic numbers
10	Hazardous	Extremely high	Absolute uncertainty	(1.00, 0.00, 0.00)
9	Serious	Very high	Very remote	(0.90, 0.10, 0.10)
8	Extreme	Repeated failures	Remote	(0.80, 0.15, 0.20)
7	Major	High	Very low	(0.70, 0.25, 0.30)
6	Significant	Moderately high	Low	(0.60, 0.35, 0.40)
5	Moderate	Moderate	Moderate	(0.50, 0.50, 0.50)
4	Low	Relatively low	Moderately high	(0.40, 0.65, 0.60)
3	Minor	Low	High	(0.30, 0.75, 0.70)
2	Very minor	Remote	Very high	(0.20, 0.85, 0.80)
1	None	Nearly impossible	Almost certain	(0.10, 0.90, 0.90)

TABLE 6. The S, O and D factors of the possible range of linguistic rating

ID	S				O				D			
	TM1	TM2	TM3	TM4	TM1	TM2	TM3	TM4	TM1	TM2	TM3	TM4
1	2	2	1	2	1	1	2	1	10	10	9	10
2	1	1	1	1	10	10	9	9	2	2	1	2
3	1	2	1	1	8	7	8	8	3	2	3	4
4	3	2	3	3	7	7	8	6	2	2	2	3
5	4	4	3	3	9	8	9	9	2	1	2	2
6	3	2	2	3	9	9	9	9	2	2	2	2
7	1	1	2	1	9	8	8	9	9	8	9	9
8	1	1	1	2	10	9	10	10	9	9	9	10
9	4	3	3	5	9	8	8	9	3	2	3	3
10	5	6	4	5	9	9	9	10	2	3	2	2
11	5	5	*	6	9	8	*	9	3	4	*	3
12	2	3	*	2	1	1	*	2	10	9	*	10
13	5	5	4	6	9	10	10	8	2	2	2	3
14	4	4	5	4	9	9	10	9	2	2	1	2

* Missing or incomplete information

The aggregated RPN values of the HDR brachytherapy treatments are therefore as shown in Table 17.

4.3. **Risk ranked using the subsethood measure method.** Subsethood measure for single valued neutrosophic set was first introduced by Sahin and Kucuk [22] to deal with the MCDM problem in a single-valued neutrosophic environment. Based on Table 6, because some information provided by expert TM3 was incomplete, only the complete information from experts TM1, TM2 and TM4 is considered. According to the information from experts TM1, TM2 and TM4, the aggregated S, O and D factors by single valued neutrosophic numbers are as shown in Table 7.

The ideal alternative A^* for single valued neutrosophic set is defined as $A^* = (1, 0, 0)$ [22].According to the results of Table 7, Definition 8 is used to calculate the normalized Hamming distance based on the Hausdorff metric as shown in Table 8.

According to the results of Table 8, Definition 9 used to calculate the subsethood measures for the HDR brachytherapy treatments, as shown in Table 17.

TABLE 7. The S, O and D factors by single valued neutrosophic numbers

ID	S	O	D
1	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)
2	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(0.20, 0.85, 0.80)
3	(0.10, 0.90, 0.90)	(0.80, 0.15, 0.20)	(0.30, 0.75, 0.70)
4	(0.30, 0.75, 0.70)	(0.70, 0.25, 0.30)	(0.20, 0.85, 0.80)
5	(0.40, 0.65, 0.60)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
6	(0.30, 0.75, 0.70)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
7	(0.10, 0.90, 0.90)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)
8	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)
9	(0.40, 0.65, 0.60)	(0.90, 0.10, 0.10)	(0.30, 0.75, 0.70)
10	(0.50, 0.50, 0.50)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
11	(0.50, 0.50, 0.50)	(0.90, 0.10, 0.10)	(0.30, 0.75, 0.70)
12	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)
13	(0.50, 0.50, 0.50)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
14	(0.40, 0.65, 0.60)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)

TABLE 8. The value of $d(A^*, A^* \cap A_i)$ by subsethood measure method

ID	S			O			D		
1	0.267	0.283	0.267	0.300	0.300	0.300	0.000	0.000	0.000
2	0.300	0.300	0.300	0.000	0.000	0.000	0.267	0.283	0.267
3	0.300	0.300	0.300	0.067	0.050	0.067	0.233	0.250	0.233
4	0.233	0.250	0.233	0.100	0.083	0.100	0.267	0.283	0.267
5	0.200	0.217	0.200	0.033	0.033	0.033	0.267	0.283	0.267
6	0.233	0.250	0.233	0.033	0.033	0.033	0.267	0.283	0.267
7	0.300	0.300	0.300	0.033	0.033	0.033	0.033	0.033	0.033
8	0.300	0.300	0.300	0.000	0.000	0.000	0.033	0.033	0.033
9	0.200	0.217	0.200	0.033	0.033	0.033	0.233	0.250	0.233
10	0.167	0.167	0.167	0.033	0.033	0.033	0.267	0.283	0.267
11	0.167	0.167	0.167	0.033	0.033	0.033	0.233	0.250	0.233
12	0.267	0.283	0.267	0.300	0.300	0.300	0.000	0.000	0.000
13	0.167	0.167	0.167	0.033	0.033	0.033	0.267	0.283	0.267
14	0.200	0.217	0.200	0.033	0.033	0.033	0.267	0.283	0.267

4.4. **Risk ranked using the different information measures method.** This paper using the different information measures method to compare the risk ranking results of different methods. In the traditional information measures method, the S, O and D factors possible range information must be complete information provided by the experts.

(1) Similarity information measures method.

Ye [27] proposed similarity measures between interval neutrosophic sets to deal with MCDM problems for scientific and engineering applications.

Definition 10. [27]. If A and B are two single valued neutrosophic sets, then the two similarity information measures $S_1(A, B)$ and $S_2(A, B)$ can be defined as follows:

$$S_1(A,B) = 1 - \frac{1}{3}\sum_{i=1}^{n} w_i \left[\left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \left|F_A(x_i) - F_B(x_i)\right|\right] \tag{10}$$

$$S_2(A,B) = 1 - \left\{\frac{1}{3}\sum_{i=1}^{n} w_i \left[\left(T_A(x_i) - T_B(x_i)\right)^2 + \left(I_A(x_i) - I_B(x_i)\right)^2 + \left(F_A(x_i) - F_B(x_i)\right)^2\right]\right\}^{1/2} \tag{11}$$

According to the results of Table 7, Definition 10 is used to calculate the two similarity information measures $S_1(A,B)$ and $S_2(A,B)$ for the HDR brachytherapy treatments, as shown in Table 17.

(2) Distance measures method. Ye [27] proposed the weighted Hamming distance and the weighted Euclidean distance to deal with MCDM problems. According to the results of Table 7, Eqs. 12,13 are used to calculate the weighted Hamming distance ($d_H(A,B)$) and the weighted Euclidean distance ($d_E(A,B)$) or the HDR brachytherapy treatments, as shown in Table 17.

$$d_H(A,B) = \frac{1}{3}\sum_{i=1}^{n} w_i \left[\left|T_A(x_i) - T_B(x_i)\right| + \left|I_A(x_i) - I_B(x_i)\right| + \left|F_A(x_i) - F_B(x_i)\right|\right] \tag{12}$$

$$d_E(A,B) = \left\{\frac{1}{3}\sum_{i=1}^{n} w_i \left[\left(T_A(x_i) - T_B(x_i)\right)^2 + \left(I_A(x_i) - I_B(x_i)\right)^2 + \left(F_A(x_i) - F_B(x_i)\right)^2\right]\right\}^{1/2} \tag{13}$$

(3) Correlation information measures method.

Ye [28] proposed correlation coefficient of single-valued neutrosophic sets to deal with MCDM problems under indeterminate and inconsistent information.

Definition 11. [28]. If A and B are two single valued neutrosophic sets, then the correlation coefficient of A and B can be defined as follows:

$$C(A,B) = \frac{\sum_{i=1}^{n}\left[T_A(x_i) \times T_B(x_i) + I_A(x_i) \times I_B(x_i) + F_A(x_i) \times F_B(x_i)\right]}{\left\{\sum_{i=1}^{n}\left[T_A^2(x_i) + I_A^2(x_i) + F_A^2(x_i)\right]\right\}^{1/2}} \times \frac{1}{\left\{\sum_{i=1}^{n}\left[T_B^2(x_i) + I_B^2(x_i) + F_B^2(x_i)\right]\right\}^{1/2}} \tag{14}$$

4.5. **Risk ranked using the proposed method.** Risk assessment is an MCDM problem involving ambiguous, missing and inconsistent information that can be suitably dealt with by single valued neutrosophic set. The proposed novel risk ranking method uses single valued neutrosophic set to aggregate information, and can effectively deal with ambiguous and inconsistent information provided by different experts. The proposed method is organized into seven steps as follows:

Step 1: Determine the component, failure mode and failure effect of the evaluation item

The risk assessment team members must jointly determine the component, failure mode and failure effect of HDR brachytherapy treatments, as shown in Table 4.

Step 2: Determine the possible range of the S, O and D factors by single valued neutrosophic number

Based on their different experiences and backgrounds, the risk assessment team members must determine the possible range of the S, O and D factors by single valued neutrosophic number, respectively, as shown in Tables 6. According to the results of Table 5 and Table 6, the possible range of linguistic rating for S, O and D factors converted into single valued neutrosophic numbers are as shown in Table 9.

TABLE 9. The S, O and D factors of the possible range by single valued neutrosophic number

ID	S				O				D			
	TM1	TM2	TM3	TM4	TM1	TM2	TM3	TM4	TM1	TM2	TM3	TM4
1	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)
2	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)
3	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.80, 0.15, 0.20)	(0.70, 0.25, 0.30)	(0.80, 0.15, 0.20)	(0.80, 0.15, 0.20)	(0.30, 0.75, 0.70)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(0.40, 0.65, 0.60)
4	(0.30, 0.75, 0.70)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(0.30, 0.75, 0.70)	(0.70, 0.25, 0.30)	(0.70, 0.25, 0.30)	(0.80, 0.15, 0.20)	(0.60, 0.35, 0.40)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)
5	(0.40, 0.65, 0.60)	(0.40, 0.65, 0.60)	(0.30, 0.75, 0.70)	(0.30, 0.75, 0.70)	(0.90, 0.10, 0.10)	(0.80, 0.15, 0.20)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)
6	(0.30, 0.75, 0.70)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)
7	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.90, 0.10, 0.10)	(0.80, 0.15, 0.20)	(0.80, 0.15, 0.20)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.80, 0.15, 0.20)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)
8	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)
9	(0.40, 0.65, 0.60)	(0.30, 0.75, 0.70)	(0.30, 0.75, 0.70)	(0.50, 0.50, 0.50)	(0.90, 0.10, 0.10)	(0.80, 0.15, 0.20)	(0.80, 0.15, 0.20)	(0.90, 0.10, 0.10)	(0.30, 0.75, 0.70)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(0.30, 0.75, 0.70)
10	(0.50, 0.50, 0.50)	(0.60, 0.35, 0.40)	(0.40, 0.65, 0.60)	(0.50, 0.50, 0.50)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)
11	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)	(*,*,*)	(0.60, 0.35, 0.40)	(0.90, 0.10, 0.10)	(0.80, 0.15, 0.20)	(*,*,*)	(0.90, 0.10, 0.10)	(0.30, 0.75, 0.70)	(0.40, 0.65, 0.60)	(*,*,*)	(0.30, 0.75, 0.70)
12	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)	(*,*,*)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.10, 0.90, 0.90)	(*,*,*)	(0.20, 0.85, 0.80)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(*,*,*)	(1.00, 0.00, 0.00)
13	(0.50, 0.50, 0.50)	(0.50, 0.50, 0.50)	(0.40, 0.65, 0.60)	(0.60, 0.35, 0.40)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)	(0.80, 0.15, 0.20)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.30, 0.75, 0.70)
14	(0.40, 0.65, 0.60)	(0.40, 0.65, 0.60)	(0.50, 0.50, 0.50)	(0.40, 0.65, 0.60)	(0.90, 0.10, 0.10)	(0.90, 0.10, 0.10)	(1.00, 0.00, 0.00)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)	(0.20, 0.85, 0.80)	(0.10, 0.90, 0.90)	(0.20, 0.85, 0.80)

Step 3: Data imputation using the minimum, averaging and maximum operators

For incomplete information, according to results of Table 9, use the minimum, averaging and maximum operators to perform data imputation, and the results are shown in Table 10.

TABLE 10. Data imputation by minimum, averaging and maximum operators

ID	Minimum operator								
	S			O			D		
11	0.50	0.50	0.50	0.80	0.15	0.20	0.30	0.75	0.70
12	0.20	0.85	0.80	0.10	0.90	0.90	0.90	0.10	0.10
	Averaging operator								
11	0.536	0.444	0.464	0.874	0.114	0.126	0.335	0.715	0.665
12	0.235	0.815	0.765	0.135	0.883	0.865	1.000	0.000	0.000
	Maximum Operator								
11	0.60	0.35	0.40	0.90	0.10	0.10	0.40	0.65	0.60
12	0.30	0.75	0.70	0.20	0.85	0.80	1.00	0.00	0.00

Step 4: For different expert-provided information, aggregate S, O and D factors by single valued neutrosophic numbers

According to the results of Table 9 and Table 10, use Eq. 2 and minimum, averaging and maximum operators to aggregate S, O and D factors by single valued neutrosophic numbers, as shown in Tables 11, 12 and 13.

TABLE 11. Aggregated S, O and D factors by minimum operator

ID	S	O	D
1	(0.18, 0.86, 0.82)	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)
2	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)
3	(0.13, 0.89, 0.87)	(0.78, 0.17, 0.22)	(0.30, 0.75, 0.70)
4	(0.28, 0.77, 0.72)	(0.71, 0.24, 0.29)	(0.23, 0.82, 0.77)
5	(0.35, 0.70, 0.65)	(0.88, 0.11, 0.12)	(0.18, 0.86, 0.82)
6	(0.25, 0.80, 0.75)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
7	(0.13, 0.89, 0.87)	(0.86, 0.12, 0.14)	(0.88, 0.11, 0.12)
8	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)
9	(0.38, 0.65, 0.62)	(0.86, 0.12, 0.14)	(0.28, 0.77, 0.72)
10	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
11	(0.53, 0.46, 0.47)	(0.86, 0.12, 0.14)	(0.33, 0.72, 0.67)
12	(0.23, 0.82, 0.77)	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)
13	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
14	(0.43, 0.61, 0.57)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)

TABLE 12. Aggregated S, O and D factors by averaging operator

ID	S	O	D
1	(0.18, 0.86, 0.82)	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)
2	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)
3	(0.13, 0.89, 0.87)	(0.78, 0.17, 0.22)	(0.30, 0.75, 0.70)
4	(0.28, 0.77, 0.72)	(0.71, 0.24, 0.29)	(0.23, 0.82, 0.77)
5	(0.35, 0.70, 0.65)	(0.88, 0.11, 0.12)	(0.18, 0.86, 0.82)
6	(0.25, 0.80, 0.75)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
7	(0.13, 0.89, 0.87)	(0.86, 0.12, 0.14)	(0.88, 0.11, 0.12)
8	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)
9	(0.38, 0.65, 0.62)	(0.86, 0.12, 0.14)	(0.28, 0.77, 0.72)
10	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
11	(0.54, 0.44, 0.46)	(0.87, 0.11, 0.13)	(0.34, 0.72, 0.66)
12	(0.23, 0.82, 0.77)	(0.13, 0.88, 0.87)	(1.00, 0.00, 0.00)
13	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
14	(0.43, 0.61, 0.57)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)

Step 5: Use the minimum, averaging and maximum operators to calculate the normalized Hamming distance $d(A^*, A^* \cap A_i)$.

According to Eq. 8 and the results of Tables 11,12,13 use the minimum, averaging and maximum operators to calculate the normalized Hamming distance, as shown in Tables 14,15,16.

Step 6: Calculate the subsethood measure based on distance measure

According to the results of Tables 14,15,16, use Eq. 9 to calculate the subsethood measure for the HDR brachytherapy treatments, as shown in Table 17.

TABLE 13. Aggregated S, O and D factors by maximum operator

ID	S	O	D
1	(0.18, 0.86, 0.82)	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)
2	(0.10, 0.90, 0.90)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)
3	(0.13, 0.89, 0.87)	(0.78, 0.17, 0.22)	(0.30, 0.75, 0.70)
4	(0.28, 0.77, 0.72)	(0.71, 0.24, 0.29)	(0.23, 0.82, 0.77)
5	(0.35, 0.70, 0.65)	(0.88, 0.11, 0.12)	(0.18, 0.86, 0.82)
6	(0.25, 0.80, 0.75)	(0.90, 0.10, 0.10)	(0.20, 0.85, 0.80)
7	(0.13, 0.89, 0.87)	(0.86, 0.12, 0.14)	(0.88, 0.11, 0.12)
8	(0.13, 0.89, 0.87)	(1.00, 0.00, 0.00)	(1.00, 0.00, 0.00)
9	(0.38, 0.65, 0.62)	(0.86, 0.12, 0.14)	(0.28, 0.77, 0.72)
10	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
11	(0.55, 0.42, 0.45)	(0.88, 0.11, 0.12)	(0.35, 0.70, 0.65)
12	(0.25, 0.80, 0.75)	(0.15, 0.87, 0.85)	(1.00, 0.00, 0.00)
13	(0.51, 0.49, 0.49)	(1.00, 0.00, 0.00)	(0.23, 0.82, 0.77)
14	(0.43, 0.61, 0.57)	(1.00, 0.00, 0.00)	(0.18, 0.86, 0.82)

TABLE 14. The value of $d(A^*, A^* \cap A_i)$ by minimum operator

ID	S			O			D		
1	0.275	0.287	0.275	0.291	0.296	0.291	0.000	0.000	0.000
2	0.300	0.300	0.300	0.000	0.000	0.000	0.275	0.287	0.275
3	0.291	0.296	0.291	0.074	0.057	0.074	0.232	0.249	0.232
4	0.241	0.258	0.241	0.097	0.080	0.097	0.258	0.275	0.258
5	0.216	0.233	0.216	0.040	0.037	0.040	0.275	0.287	0.275
6	0.249	0.266	0.249	0.033	0.033	0.033	0.267	0.283	0.267
7	0.291	0.296	0.291	0.047	0.041	0.047	0.040	0.037	0.040
8	0.291	0.296	0.291	0.000	0.000	0.000	0.000	0.000	0.000
9	0.206	0.218	0.206	0.047	0.041	0.047	0.241	0.258	0.241
10	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
11	0.158	0.152	0.158	0.047	0.041	0.047	0.225	0.241	0.225
12	0.258	0.275	0.258	0.291	0.296	0.291	0.000	0.000	0.000
13	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
14	0.191	0.203	0.191	0.000	0.000	0.000	0.275	0.287	0.275

Step 7: Ranking all the subsethood measure for evaluation item according to $S(A^*, A_i)$ value.

According to the results of Step 6, sort the $S(A^*, A_i)$ value from large to small, as shown in Table 17.

4.6. **Comparisons and discussion.** In order to validate the effectiveness and feasibility of the proposed novel risk ranking method, a numerical example verification is performed in Section 4. The results of the proposed method are compared with those of the traditional RPN, subsethood measure, and some other existing information measures methods. The input data of the numerical example are shown in Tables 4,5,6. The final ranking results of the different risk assessment methods are organized in Table 17. From the comparison of Tables 4,5,6 and Table 17, it is found that the proposed novel risk ranking method has some major advantages.

TABLE 15. The value of $d\left(A^{*}, A^{*} \cap A_{i}\right)$ by averaging operator

ID	S			O			D		
1	0.275	0.287	0.275	0.291	0.296	0.291	0.000	0.000	0.000
2	0.300	0.300	0.300	0.000	0.000	0.000	0.275	0.287	0.275
3	0.291	0.296	0.291	0.074	0.057	0.074	0.232	0.249	0.232
4	0.241	0.258	0.241	0.097	0.080	0.097	0.258	0.275	0.258
5	0.216	0.233	0.216	0.040	0.037	0.040	0.275	0.287	0.275
6	0.249	0.266	0.249	0.033	0.033	0.033	0.267	0.283	0.267
7	0.291	0.296	0.291	0.047	0.041	0.047	0.040	0.037	0.040
8	0.291	0.296	0.291	0.000	0.000	0.000	0.000	0.000	0.000
9	0.206	0.218	0.206	0.047	0.041	0.047	0.241	0.258	0.241
10	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
11	0.155	0.148	0.155	0.042	0.038	0.042	0.222	0.238	0.222
12	0.255	0.272	0.255	0.288	0.294	0.288	0.000	0.000	0.000
13	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
14	0.191	0.203	0.191	0.000	0.000	0.000	0.275	0.287	0.275

TABLE 16. The value of $d\left(A^{*}, A^{*} \cap A_{i}\right)$ by maximum operator

ID	S			O			D		
1	0.275	0.287	0.275	0.291	0.296	0.291	0.000	0.000	0.000
2	0.300	0.300	0.300	0.000	0.000	0.000	0.275	0.287	0.275
3	0.291	0.296	0.291	0.074	0.057	0.074	0.232	0.249	0.232
4	0.241	0.258	0.241	0.097	0.080	0.097	0.258	0.275	0.258
5	0.216	0.233	0.216	0.040	0.037	0.040	0.275	0.287	0.275
6	0.249	0.266	0.249	0.033	0.033	0.033	0.267	0.283	0.267
7	0.291	0.296	0.291	0.047	0.041	0.047	0.040	0.037	0.040
8	0.291	0.296	0.291	0.000	0.000	0.000	0.000	0.000	0.000
9	0.206	0.218	0.206	0.047	0.041	0.047	0.241	0.258	0.241
10	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
11	0.149	0.139	0.149	0.040	0.037	0.040	0.216	0.233	0.216
12	0.249	0.266	0.249	0.283	0.292	0.283	0.000	0.000	0.000
13	0.165	0.163	0.165	0.000	0.000	0.000	0.258	0.275	0.258
14	0.191	0.203	0.191	0.000	0.000	0.000	0.275	0.287	0.275

Firstly, it is able to handle ambiguous and inconsistent information in the information aggregation process. The traditional RPN method requires the possible range for S, O and D factors to be single a linguistic term set, and cannot effectively deal with any inconsistent information provided by the experts. The proposed novel risk ranking method and subsethood measure method took into account the ambiguous and inconsistent information for information processing, and the final ranking results of the risk item are clearly different from the results obtained by traditional RPN method.

Secondly, the proposed method is able to handle incomplete and missing information in the information aggregation process. The traditional RPN, subsethood measure, and some other existing information measures methods cannot handle incomplete and missing information. For incomplete and missing information, the traditional RPN, subsethood measure, and some other existing information measures methods will delete incomplete information to facilitate decision-making. This, however, will cause the number of samples to be reduced, and some of the valuable information provided by experts will not be included in the decision-making process. In the numerical example of HDR brachytherapy treatments, because expert TM3 provides partially missing or incomplete information for the S, O and D factors, the traditional RPN, subsethood measure, and some other existing information measures methods only use the complete information from experts TM1, TM2 and TM4 for decision-making. The proposed novel risk ranking method uses the data filling method to fill in missing values. Therefore, the proposed method can fully consider all of the information provided by the experts (TM1, TM2, TM3, and TM4), and is more suitable for solving risk assessment problems.

Finally, the neutrosophic set is a generalization of the crisp sets, fuzzy sets and intuitionistic fuzzy sets. Therefore, the proposed method can simultaneously deal with fuzzy information, intuitionistic fuzzy information, and neutrosophic information to avoid information distortion during the evaluation process for the HDR brachytherapy treatments. When handling indeterminate and inconsistent information, using the single valued neutrosophic sets for the risk ranking of HDR brachytherapy treatments is therefore more suitable, than using crisp set, fuzzy set or intuitionistic fuzzy set.

5. **Conclusions and further research.** Risk assessment determines risk management priorities under limited resources to prevent the occurrence of accidents. However, traditional risk ranking methods cannot effectively deal with ambiguous, incomplete, missing or inconsistent information provided by experts. When they encounter incomplete and missing information, the traditional RPN, subsethood measure, and some other existing information measures methods will directly delete that incomplete information. This will cause the evaluation results to be distorted. In order to effectively solve the limit of traditional risk ranking methods, this paper integrates the single valued neutrosophic set and subsethood measure method to rank the extent of the threat. Linguistic terms are used to express the level of S, O and D factors, and the subsethood measure is used to aggregate all the information provide by experts. In order to fully consider all available information and avoid information loss, this paper used the minimum, averaging and maximum operators to perform data imputation. Risk assessment of HDR brachytherapy treatments is applied as an illustrative example to demonstrate the rationality and correctness of the proposed method. The simulation results demonstrated that the proposed method can provide more effective and correct outcomes than can the traditional RPN and subsethood measure methods. In further work, the proposed novel risk ranking method will be applied to other related areas such as supplier selection, resource allocation, talent selection and decision-making.

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TABLE 17. Comparison of different ranking methods

ID	S	O	D	RPN [4,12]	Ranking RPN [4,12]	$S(A^*,A_i)$ [22]	Ranking subsethood measure [22]	$S_1(A,B)$ [27]	Ranking $S_1(A,B)$ [27]	$S_2(A,B)$ [27]	Ranking $S_2(A,B)$ [27]	$d_H(A,B)$ [27]	Ranking $d_H(A,B)$ [27]	$d_E(A,B)$ [27]	Ranking $d_E(A,B)$ [27]
1	2	1	10	20	12	0.806	10	0.865	10	0.449	13	0.572	10	0.702	12
2	1	10	2	20	12	0.806	10	0.865	10	0.475	12	0.572	10	0.702	12
3	1	8	3	24	11	0.794	13	0.856	13	0.514	9	0.600	13	0.673	11
4	3	7	2	42	10	0.789	14	0.854	14	0.485	11	0.606	14	0.649	10
5	4	9	2	72	7	0.822	7	0.885	7	0.520	7	0.511	7	0.594	7
6	3	9	2	54	9	0.811	9	0.874	9	0.508	10	0.544	9	0.630	9
7	1	9	9	81	6	0.878	2	0.933	2	0.755	2	0.367	2	0.526	3
8	1	10	9	90	3	0.889	1	0.944	1	0.762	1	0.333	1	0.523	2
9	4	9	3	108	2	0.833	6	0.896	6	0.576	4	0.478	6	0.549	4
10	5	9	2	90	3	0.839	4	0.898	4	0.525	5	0.472	4	0.556	5
11	5	9	3	135	1	0.850	3	0.909	3	0.582	3	0.439	3	0.508	1
12	2	1	10	20	12	0.806	10	0.865	10	0.449	13	0.572	10	0.702	12
13	5	9	2	90	3	0.839	4	0.898	4	0.525	5	0.472	4	0.556	5
14	4	9	2	72	7	0.822	7	0.885	7	0.520	7	0.511	7	0.594	7

ID	S	O	D	$C(A,B)$ [28]	Ranking $C(A,B)$ [28]	Proposed method					
						Minimum operator	Ranking minimum operator	Averaging operator	Ranking averaging operators	Maximum Operator	Ranking maximum operator
1	2	1	10	0.86	12	0.806	10	0.806	10	0.806	10
2	1	10	2	0.86	12	0.804	12	0.804	12	0.804	12
3	1	8	3	1.05	11	0.794	13	0.794	13	0.794	13
4	3	7	2	1.18	10	0.790	14	0.790	14	0.790	14
5	4	9	2	1.43	7	0.813	8	0.813	8	0.813	9
6	3	9	2	1.21	9	0.806	10	0.806	10	0.806	10
7	1	9	9	1.79	2	0.872	2	0.872	2	0.872	2
8	1	10	9	1.72	3	0.901	1	0.901	1	0.901	1
9	4	9	3	1.69	6	0.826	7	0.826	7	0.826	7
10	5	9	2	1.71	4	0.853	3	0.853	4	0.853	4
11	5	9	3	2.02	1	0.851	5	0.855	3	0.860	3
12	2	1	10	0.86	12	0.810	9	0.811	9	0.814	8
13	5	9	2	1.71	4	0.853	3	0.853	4	0.853	4
14	4	9	2	1.43	7	0.837	6	0.837	6	0.837	6

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