A Study On Directed Neutrosophic Social Networks

Yunming Hui, Arindam Dey

***Abstract*—Popularity of social networks like Facebook, LinkedIn, Twitter, Instagram, WhatsApp is increasingly increasing day by day. Social network scholars are very much interested to capture the uncertainties of social network. The neutrosophic set is a well known tool to handle and represent uncertainties in information due to indeterminateness or incompleteness. The main approach of our study is to investigate the neutrosophic approach to deal the uncertainties that may exist in social network. In this work, we introduce a neutrosophic model to present the social network using directed neutrosophic graph. We call this directed neutrosophic social network (DNSN). The centrality of actors play a very important theatrical role in social network analysis. In this paper, we introduce some centrality measures for DNSN, because DNSN where arcs are associated with directed neutrosophic relation would consist many information. We describe some new centrality measures such as neutrosophic out degree centrality, neutrosophic in degree centrality, neutrosophic out closeness centrality and neutrosophic in closeness centrality DFSNs. We also investigate about directed neutrosophic relation and connectivity for DNSN. We also present the robustness and validness of our proposed centrality measurement for DNSN by describing this technique to some directed neutrosophic graph and determine satisfactory results.**

***Index Terms*—visual-servoing, tracking, biomimetic, redundancy, degrees-of-freedom.**

I. Introduction

In twenty first century, people are now connected more even though online social networks with the smart mobile phones in our daily life. It is basically a platform for interconnecting with huge number of people in everywhere in the world. We exchange information of several issues and topics in social network which helps for e-commerce and e-business, political and social campaigns, influential players (researchers, engineers, employees, organizations, etc.), future events, alumni, etc.

A simple social network consists of a collection of social units (social nodes or vertices) describing individual, groups, companies, etc, which are interconnected by arcs (links or edges) describing relationship between two social units. The structure of social network with social units (actors) and their relations are generally modeled as simple crisp graphs. We usually refer this graph as sociogram in sociology. In a sociogram, social units (nodes/actors) and relations respectively are represented by nodes and arcs of a graph. Graphs are naturally used to model the social network as it is efficient and useful graphical method to describe how objects (items or things) are either logically or physically interconnected together. However, uncertainties may exist in description of any social unit and their relationship. Classical graph is unable to model the uncertainties of the social network properly. The fuzzy set is an useful technique to the handle the uncertainties in information of any real life problem.

In many cases, the social network can be represented by fuzzy graph defined as fuzzy social network (FSN) usages type-1 fuzzy set as edge weight. The degree of membership of a type-1 fuzzy set [1], [2] is crisp (real number), which is determined by human perception. There are several types of uncertainties in the membership grade. It is very hard to evaluate the proper membership grade of a fuzzy set. Neutrosophic set is able to handle and represent this uncertainty.

Smarandache [3] have described the idea of neutrosophic set (NS). NS is used to deal and handle the uncertain information in the real world problems due to incompleteness, indeterminacy, impreciseness and inconsistency. It is described by three member function: truth membership grade ($T(B)$), an indeterminate membership grade ($I(B)$) and a false membership degree ($F(B)$). The values of $T(B)$, $I(B)$ and $F(B)$ are independent and within the nonstandard unit interval. The NS modeling is a well known method for handing the uncertainties in real life scenarios because NS can work not only inconsistent information and it can also handle the incompleteness and indeterminacy of an information [4], [5], [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16], [17].

The analysis of social network is an important study for mapping and determining of relations and position among the connected social units. It helps to determine the significance of different social units (actors) in their network or communications and it also describes their opportunities of relationship. The less constraints of a social unit (person), the social node will be in more favorable position to exchange the information.

In the study of social network, a problem of finding the significant social units of that social network have been researched for a several years. For that purpose, the idea of the centrality measure of a node appeared in a network. The social network analysis assumes very closely related to the ideas of centrality and ability as person in a social network. It describes us that who is in the network, who works as a leader in the network, who works as a mediator in the network, who works almost isolated in the network, or who appears central in the network.

Many social scientists have introduced many types of centrality measurements in social network. Degree, betweenness and closeness of a social unit are the most important issues in the area of social network analysis. The degree centrality of a social unit/individual/person is used to measure the level of popularity and communicating activity, distinguishing the centrality of a social unit based on the degree. The closeness

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centrality of a social unit is the sum of the lengths of the shortest path between a social unit to other rest of the social units. The lower value of closeness centrality is, the higher the centrality value. Closeness centrality measures the independency in communication and relationship or bargaining between the social units. It is basically used to measure the possibility of communicating with other social units calculating on a minimal number of intermediate social units. The betweenness centrality of social unit is centrality measurement that finds the number of times that specific social unit lies in the shortest path from the rest of the social units. It measures the level of control of communication of social unit. It is used to find the social unit who influences the others units.

Centrality measurement is applied mainly in social network analysis and also in behavioral sciences. However, it is used in computer science, biology, politics, management, chemistry, economics and so on. Stephenson and Zelen [8] have introduced the the concept of centrality of a node based on the shortest path between the nodes. They have also introduced a measurement using the idea of data as it is applied in the statistical estimation. This have used the length of path among two the social units in their defined measurement and the length of each and every path based on the data consisted in it.

Bonacich [18], [19] have proposed an another idea of centrality measurement of a social node. He has introduced the centrality measurement of different social nodes by applying the eigenvector. The eigenvector is linked with the highest eigenvalue in the adjacent matrix. Brunelli et al. [20] proposed an efficient and robust measurement of centrality. Their defined centrality is calculated based on the level influence of other social units according to their value of eigenvector centrality. Sohn and Kim [21] have introduced a robust technique to determine the zone of centrality measure in an urbanized place. Kennarrec et al. [22] have proposed a new type of centrality measurement. They have introduced the the idea of the centrality of second order. It can be calculated in a distributed way. Qi et al. [23] have introduced a new type of centrality measure for weighted networks. They have defined this centrality as Laplacian centrality measure. It measures the importance between local and global characterization of a social node. The main disadvantage of Laplacian centrality is that it cannot be used in directed social networks. Pozo et al. [24] have proposed an another type of centrality measures of a directed social network based on the game theory. Landherr et a1. [25] have described the 5 types of centrality measurements. Those centrality measures are based of three necessities for the centrality measures.

Since the social networks in real life scenarios are often uncertain, the model of real life social network is very difficult and always challenging. Fuzzy set is an useful tool to deal the uncertainties in real world problems. The idea of a fuzziness in social network and the techniques to analysis those social networks have attracted many researchers in last few years. Nair and Sarasarnma [26] used the fuzzy set theory to analysis the social network. They have modeled the undirected social network in fuzzy environment using fuzzy graph where the social units are represented as actors or nodes or vertices and the relationship between the social units are represented as the edges or arcs or links. Liao and Hu [27] have introduced the idea of undirected FSN and described some properties of it. They have described some fundamental definition and relevant theory for future study of the FSN. Samanta et al. [28], [29], [30], [31], [32] have done lots of researches in the domain of fuzzy social network. Fan et al. [33] have presented the regular and structural equivalence in an undirected fuzzy social networks. Liao et a1. [34] have modified the idea of centrality measure to the fuzzy environment. They have introduced the Definition of three types of fuzzy centrality: degree, betweenness and closeness in an undirected fuzzy social networks. Kundu and Pal [35] have represented the social network with a set of granules. They have used the fuzzy set for describing the granular. They have defined this model as fuzzy granular social network. Hu et a1. [36] have modified the conception of centralization and centrality to the fuzzy environment. They have described the closeness centrality and group closeness centrality in an undirected FSNs. Hu et al. [37] have extended the idea of fuzzy centrality in directed fuzzy social network. They have introduced some new Definition of fuzzy centrality measurement defined as in degree, out degree centrality, in closeness and out closeness centrality in fuzzy environment.

Neutrosophic set [38], [39], [40] is a well known and popular theory which one can deal the natural phenomenon of imprecision and uncertainty in real world problem. Neutrosophicness is extended version of fuzziness to handle the uncertainty. The main objective of this work is to introduce a model of social network using directed weighted neutrosophic graph which will be very simple to model and analysis in real life scenarios.

To the best our information, there exists no study on social network using neutrosophic graph. In this study, we propose a model to express the social network based on directed neutrosophic graph. We define this social network as directed neutrosophic social network (DNSN). The centrality of social unit acts a significant role to analysis in social network. Some new centrality measures are introduced for DNSN, because DNSN where link between social units are joined with directed neutrosophic relation would consist several information. We describe some new centrality measures such as neutrosophic out degree centrality, neutrosophic in degree centrality, neutrosophic out closeness centrality and neutrosophic in closeness centrality in DFSNs. We also investigate about directed neutrosophic relation and connectivity for DNSN. We also present the validness and robustness of our proposed centrality measure for DNSN by describing this technique to some directed neutrosophic graph and determine satisfactory results.

II. Preliminaries

Definition 1: The neutrosophic set is described by three membership functions;: $T_B(m)$, $I_B(m)$ and $F_B(m)$. Here, $T_B(m)$, $I_B(m)$ and $F_B(m)$ are true, indeterminate and false membership functions which are always in the interval $]^{-}0, 1^{+}[$ respectively.

$$^{-}0 \leq \sup\ T_B(m)) + \sup\ I_B(m) + \sup\ F_B(m) \leq 3^{+} \quad (1)$$

Here, ξ represents an universal set and B represents a neutrosophic set [3] on the universal set ξ.

Definition 2: Let B represents the single valued neutrosophic sets (SVNs) [41] B on the ξ is described as following

$$B = \{\langle m : T_B(m), I_B(x), F_B(m) | m \in \xi \rangle\} \tag{2}$$

The functions $T_B(m) \in [0,1]$, $I_B(m) \in [0,1]$ and $F_B(m) \in [0,1]$ are defined as degree of truth membership, degree of indeterminacy membership and degree of falsity membership of x in A, satisfy the following condition:

$$^{-}0 \leq \sup\ T_B(m) + \sup\ I_B(m) + \sup\ F_B(m) \leq 3^{+} \tag{3}$$

Definition 3: Let $B = \{\langle T_B(m), I_B(x), F_B(m) | \rangle\}$ is a SVN, The score function S [42] of SVN B is computed using the value of truth membership ($T_B(m)$), indeterminacy membership ($I_B(x)$) and falsity membership ($F_B(m)$) and it is calculated as follow:

$$\begin{aligned} S(B) &= \frac{1 + p \cdot q}{2} \\ p &= 1 + (T_B(m) - 2I_B(m) - F_B(m)) \\ q &= (2 - T_B(m) - F_B(m)) \end{aligned} \tag{4}$$

III. Problem formulation

In simple social networking analysis, we have used for binary relations in which a two social units is either connected or disconnected. However, the relationship between two social units is generally vague in nature. Many researchers have used fuzzy set and fuzzy graph to describe this vagueness. However, we cannot model the uncertainties due inconsistent information and indeterminate information about any real life problem using fuzzy graph. The neutrosophic set theory, proposed by Smarandache [43] is desirable for handling this type of uncertainties and imprecision's related with information relating several parameters.

Definition 4: An undirected neutrosophic social network is described as an undirected neutrosophic relationship structure $\tilde{G}_{un} = \left(X, \tilde{Y}_{un}\right)$, where $X = \{x_1, x_2, ..., x_n\}$ is a nonempty set of social units, and $\tilde{Y}_{un} = \begin{pmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{y}_{n1} & \cdots & \tilde{y}_{nn} \end{pmatrix}$ is a undirected neutrosophic relationship on X.

Several neutrosophic relationships are directional. A neutrosophic relationship is directional if there exists some ties from one social actor to mother node.

Definition 5: A directed neutrosophic social network is described as a neutrosophic relationship structure $\tilde{G}_{dn} = \left(X, \tilde{Y}_{dn}\right)$, where $X = \{x_1, x_2, ..., x_n\}$ is a non-empty set of social units, and $\tilde{Y}_{dn} = \begin{pmatrix} \tilde{y}_{11} & \cdots & \tilde{y}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{y}_{n1} & \cdots & \tilde{y}_{nn} \end{pmatrix}$ is an undirected neutrosophic relationship on X.

We have described two types of neutrosophic social network: undirected and directed. The main conceptual difference between two type of neutrosophic social networks is considered in directed neutrosophic social network and undirected neutrosophic relation is considered in undirected neutrosophic social network. Due to this reason, $\tilde{e}_{xy}$ is and $\tilde{e}_{yx}$ are equal in undirected neutrosophic social network. However, $\tilde{e}_{xy}$ is and $\tilde{e}_{yx}$ are not always equal in directed neutrosophic social network.

IV. The neutrosophic degree Centrality Analysis of directed neutrosophic Social Network

In this section, we present measurement of neutrosophic in degree centrality, neutrosophic out-degree centrality and neutrosophic degree centrality, respectively, in DNSN.

Definition 6: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DNSN and the single valued neutrosophic set (SVNS) is used to represent the arc lengths of $\tilde{G}_{dn}$. The sum of the lengths of the arcs that are adjacent to a social node v_x is calculated which is noting but a SVNS. The neutrosophic value of node v_x, $\tilde{d}_I(v_x)$, is calculated as follows.

$$\tilde{d}_I(v_x) = \sum_{y=1, y\neq x}^{n} \tilde{e}_{yx} \tag{5}$$

The symbol $\sum$ refers to an addition operation of SVNS and $\tilde{e}_{yx}$ denotes a SVNS associated with the arc (i,j). $\tilde{d}_I(v_x) = \left(d_I^T(v_x) d_I^I(v_x), d_I^F(v_x)\right)$ is an another SVNS which represents the neutrosophic in degree centrality (NIDC) of node v_x. The score value of the corresponding SVNS is determined and this score value is called as the NIDC of the node v_x.

$$\begin{aligned} \tilde{d}_I^s(v_x) &= \frac{1 + p \cdot q}{2} \\ p &= \left(d_I^T(v_x) - 2d_I^I(v_x) - d_I^F(v_x)\right) \\ q &= \left(2 - d_I^T(v_x) - d_I^F(v_x)\right) \end{aligned} \tag{6}$$

Here, $\tilde{d}_I^s(v_x)$ represents the NIDC of node v_x.

Definition 7: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DNSN and the SVNS is used to represent the arc lengths of $\tilde{G}_{dn}$. The sum of the lengths of the arcs that are adjacent from a social node v_x is calculated which is noting but a SVNS. The neutrosophic value of node v_x, $\tilde{d}_O(v_x)$, is calculated as follows.

$$\tilde{d}_O(v_x) = \sum_{j=1, j\neq i}^{n} \tilde{e}_{yx} \tag{7}$$

The symbol $\sum$ refers to an addition operation of SVNS and $\tilde{e}_{yx}$ denotes a SVNS associated with the arc (i,j). $\tilde{d}_O(v_x) = \left(d_O^T(v_x) d_O^I(v_x), d_O^F(v_x)\right)$ is an another SVNS which represents the NIDC of node v_x. The score value of the corresponding SVNS is determined and this score value is called as the NIDC of the node v_x.

$$\begin{aligned} \tilde{d}_O^s(v_x) &= \frac{1 + p \cdot q}{2} \\ p &= \left(d_O^T(v_x) - 2d_O^I(v_x) - d_O^F(v_x)\right) \\ q &= \left(2 - d_O^T(v_x) - d_O^F(v_x)\right) \end{aligned} \tag{8}$$

Here, $\tilde{d}_O^s(v_x)$ represents the neutrosophic out degree centrality of node v_x.

Definition 8: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DFSN and the SVNS is applied to describe the edge weights of $\tilde{G}_{dn}$. The sum of NIDC and NODC of node v_x is called neutrosophic degree centrality (NDC) of v_x. The NDC of node v_x, $\tilde{d}(v_x)$, is described as follows.

$$\tilde{d}(v_x) = \tilde{d}_I^s(v_x) + \tilde{d}_O^s(v_x) \tag{9}$$

Here, $\tilde{d}(v_x)$ is the NDC of node v_x.

These three types of degree can be used to reflect the communication ability of a node in DNSN. The NIDC of any node can be applied to describe the receptivity or popularity and the NODC of any node can be applied to measure of expansiveness.

Definition 9: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be DNSN. The mean of NIDC is the average of neutrosophic in-degree centrality of all the nodes of $\tilde{G}_{dn}$ and it is called as neutrosophic in-degree mean centrality of neutrosophic social network. The neutrosophic in-degree mean centrality is calculated as follows.

$$\bar{d}_I = \frac{\sum\limits_{x=1}^{n} \tilde{d}_I^s\left(v_x\right)}{n} \tag{10}$$

Definition 10: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be DNSN. The mean of NODC of graph $\tilde{G}_{dn}$ is the average of neutrosophic out-degree centrality of all the nodes of $\tilde{G}_{dn}$ and it is called as neutrosophic out-degree mean centrality of the social network. The neutrosophic out-degree mean centrality is determined as follows.

$$\bar{d}_O = \frac{\sum\limits_{x=1}^{n} \tilde{d}_O^s\left(v_x\right)}{n} \tag{11}$$

Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DSFN. The $\bar{d}_I^s$ and $\bar{d}_O^s$ are neutrosophic in-degree mean centrality and neutrosophic out-degree mean centrality of $\tilde{G}_{dn}$ respectively, then $\bar{d}_I^s = \bar{d}_O^s$.

One might also be interested in variability of the fuzzy in-degree centralities and fuzzy out-degree centralitics. Unlike the mean fuzzy in-degree centralities and the mean fuzzy out-degree centralities, the variance of the fuzzy in-degree centralities is not necessarily the same as the variance of the fuzzy out-degree centralities.

Definition 11: The average of the squared differences from neutrosophic in-degree mean centrality is called as the neutrosophic in degree variance centrality. The neutrosophic in degree variance centrality, $\sigma^2_{\tilde{d}_I}$, is determined as follows.

$$\sigma^2_{\tilde{d}_I} = \frac{\sum\limits_{i=1}^{n} \left[\tilde{d}_I\left(v_x\right) - \bar{d}_I\right]^2}{n} \tag{12}$$

The square root of the neutrosophic in degree variance centrality is called the neutrosophic in degree standard deviation centrality. The neutrosophic in degree variance centrality provides a rough concept of variability of the neutrosophic in-degree centrality, while the standard deviation can give more concrete to provide the exact from the the neutrosophic in-degree centrality.

Definition 12: The average of the squared differences from neutrosophic out-degree mean centrality is called as the neutrosophic out degree variance centrality. The neutrosophic out degree variance centrality, $\sigma^2_{\tilde{d}_O}$, is determined as follows.

$$\sigma^2_{\tilde{d}_O} = \frac{\sum\limits_{i=1}^{n} \left[\tilde{d}_O\left(v_x\right) - \bar{d}_O\right]^2}{n} \tag{13}$$

The square root of the neutrosophic out degree variance centrality is called the neutrosophic in degree standard deviation centrality.

$\sigma^2_{\tilde{d}_I}$ and $\sigma^2_{\tilde{d}_O}$ measures quantify how unequal the actors in a DFSN are with respect to initiating gr receiving fuzzy relations. These measurements are simple efficient statistical method for describing how centralized a social unit is in DNSN.

In the DNSN, NIDC is used to measure of the popularity of a social unit, and NODC is measurement of ability influence of a social unit. Based on the value of NIDC and NODC, there are 4 types of social units in a DNSN.

1. The social unit is an isolate social unit, if $\tilde{d}_I\left(v_x\right) = \tilde{d}_O\left(v_x\right) = 0$.
2. The social unit only has neutrosophic relation starting from v_x, then it is called as transmitter social unit if $\tilde{d}_I\left(v_x\right) = 0, \tilde{d}_O\left(v_x\right) > 0$.
3. The social unit only has neutrosophic relation terminating at v_x, then it is called as receiver social unit if and only if $\tilde{d}_I\left(v_x\right) > 0, \tilde{d}_O\left(v_x\right) = 0$.
4. The social unit has fuzzy relation both to and from it, then it is called as carrier of ordinary social unit if $\tilde{d}_I\left(v_x\right) > 0, \tilde{d}_O\left(v_x\right) > 0$.

V. The neutrosophic closeness centrality of DNSN

Definition 13: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DNSN. The neutrosophic in-closeness centrality (NICC) of a node v_x is a measurement of the closeness from all the node (excluding the node v_x) to the node v_x in a neutrosophic graph. The NICC of a node is determined as the inverse of the sum of the neutrosophic weight of the shortest routes between the social unit and rest of the social units in the neutrosophic graph. If $\tilde{d}(v_x, v_y)$ is the neutrosophic length from the node v_y to node v_x in a neutrosophic graph, we determine the sum (neutrosophic addition operation) of the neutrosophic lengths between the node v_x and the rest of all the nodes that are in same row(component). It is calculated as follows.

$$\tilde{D}\left(v_y, v_x\right) = \sum_{j=1, y\neq x}^{n} \tilde{d}\left(v_y, v_x\right) \tag{14}$$

$\tilde{D}\left(v_y, v_x\right)$ is a SVNS. The score of $\tilde{D}\left(v_y, v_x\right)$ is computed by using (). The NICC is the inverse of the sum of the neutrosophic connected intensity from all the nodes (excluding v_x) to v_x. The NICC of v_x is computed as follows.

$$\tilde{C}_{CI}\left(v_x\right) = \frac{1}{\tilde{D}\left(v_y, v_x\right)} \tag{15}$$

Definition 14: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be a DNSN. The neutrosophic out-closeness centrality (NOCC) of a node v_x is a measurement of the closeness from v_x to all the nodes in a neutrosophic graph. The NOCC of a node is determined as the inverse of the sum of the neutrosophic cost of the shortest paths between the social unit and rest of the social units in the neutrosophic graph. If $\tilde{d}(v_x, v_y)$ is the neutrosophic distance from the node v_x to node v_y in a neutrosophic graph, we determine the sum (neutrosophic addition operation) of the neutrosophic lengths between the node v_x and the rest of all the nodes that are in same row(component). It is calculated as follows.

$$\tilde{D}_O\left(v_x, v_y\right) = \sum_{j=1, y\neq x}^{n} \tilde{d}\left(v_x, v_y\right) \tag{16}$$

$\tilde{D}_O\left(v_y, v_x\right)$ is a SVNS. The score of $\tilde{D}_O\left(v_x, v_y\right)$ is computed by using (). The NOCC is the inverse of the sum of the neutrosophic connected intensity from the node v_x to the rest of all other nodes. The NOCC of v_x is computed as follows.

$$\tilde{C}_{CO}\left(v_x\right) = \frac{1}{\tilde{D}_O\left(v_y, v_x\right)} \tag{17}$$

Definition 15: The sum of value of NOCC and NICC of node v_x is called neutrosophic closeness centrality of v_x. The neutrosophic closeness centrality of v_x can be calculated

$$\tilde{C}_{DC}\left(v_x\right) = \tilde{C}_{CI}\left(v_x\right) + \tilde{C}_{CO}\left(v_x\right) \tag{18}$$

Definition 16: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be DNSN. The mean of NICC is the average of neutrosophic in-degree centrality of all the nodes of $\tilde{G}_{dn}$ and it is called as neutrosophic in-closeness mean centrality of neutrosophic social network. The neutrosophic in-closeness mean centrality is calculated as follows.

$$\bar{C}_{CI} = \frac{\sum\limits_{x=1}^{n} \tilde{C}_{CI}\left(v_x\right)}{n} \tag{19}$$

Definition 17: Let $\tilde{G}_{dn} = \left(V, \tilde{E}_{dn}\right)$ be DNSN. The mean of NOCC is the average of neutrosophic out-degree centrality of all the nodes of $\tilde{G}_{dn}$ and it is called as neutrosophic in-closeness mean centrality of neutrosophic social network. The neutrosophic out-closeness mean centrality is calculated as follows.

$$\bar{C}_{CO} = \frac{\sum\limits_{x=1}^{n} \tilde{C}_{CO}\left(v_x\right)}{n} \tag{20}$$

In some real life scenarios, we have to determine the NICC or NOCC of a social network. The mean value of NICC and NOCC are always same, however the deviation of the NICC and NOCC may not be same.

Definition 18: The standard deviation of the NICC, which we represent by $\sigma_{\tilde{C}_{CI}}$ is determined as

$$\sigma_{\tilde{C}_{CI}} = \sqrt{\frac{\sum\limits_{i=1}^{n} \left[\tilde{C}_{CI}\left(v_x\right) - \bar{d}_{CI}\right]^2}{n}} \tag{21}$$

The square of the standard deviation of the NICC is called the neutrosophic in degree variance centrality.

Definition 19: The standard deviation of the NICC, which we represent by $\sigma_{\tilde{C}_{CI}}$ is determined as

$$\sigma_{\tilde{C}_{CI}} = \sqrt{\frac{\sum\limits_{i=1}^{n} \left[\tilde{C}_{CI}\left(v_x\right) - \bar{d}_{CI}\right]^2}{n}} \tag{22}$$

The square of the standard deviation of the NICC is called the neutrosophic in degree variance centrality.

$\sigma_{\tilde{C}_{CI}}$ and $\sigma_{\tilde{C}_{CO}}$ describe the measurement how unequal the nodes in a DNSN are with respect to starting or obtaining indirect and direct neutrosophic relationship. The NICC and NOCC are simple but also very efficient statistics for describing how centralized a DNSN is.

The value of NICC describes not only the popularity of a social unit in DNSN, but also measures the popularity of the social unit by indirectly neutrosophic relation. The value of NOCC measures not only a social unit directly influence but it can also describe indirect influence of the social node.

VI. Neutrosophic betweenness centrality

Betweenness centrality is an important measurement in social network analysis, network data model and computer network. In many real life scenarios, the distance between the social units not only play significant property in social network analysis. It is also very useful to find the social units with higher number of times works as a simple bridge in a shortest path from a social unit to an another unit. Those types of social unit can control the flow of data in a social network. Betweenness centrality is used to determine the potential of a social unit for control of data communication in social network. Here, we introduce the concept of betweenness centrality of a node in DNSN.

Definition 20: Betweenness centrality is a measure the betweenness of a social unit in a DNSN and it determines of centrality in a DNSN based on shortest paths. Betweenness centrality is the total number of times a social unit works as a bridge of a neutrosophic shortest path between two social units. We define this betweenness centrality as neutrosophic betweenness centrality (NBC). The NBC of social unit v_x is computed as follows.

$$C_{NBC}(v_x) = \sum_{s\neq v_x\neq d} \frac{\sigma_{sd}(v_x)}{\sigma_{sd}} \tag{23}$$

$\sigma_{st}(v_x)$ represents the total number of those neutrosophic shortest paths that pass node v_x and σ_{st} is used to represent the number of neutrosophic shortest paths between source node s and destination node d.

VII. Neutrosophic decay centrality

Neutrosophic decay centrality (NDC) is a measurement of the closeness of a social unit to the other rest of the social units in a DNSN. The NDC of social unit is calculated based on the distance and a new parameter called the neutrosophic decay parameter $\delta(0 < \delta < 1)$. The NDC of a node v_x for a specific value of the δ is computed as follows.

$$NDC_{\delta}(v_x) = \sum_{v_x\neq v_y} \delta^{\tilde{d}(v_x, v_y)} \tag{24}$$

VIII. Case Study

Let $\tilde{G}_{d7} = \left(V, \tilde{E}_{d7}\right)$ is a directed neutrosophic graph. It is used to model a Whatsapp group of family members, where $V = \{v_1, v_2, ..., v_7\}$ denotes a set of 7 family members, $\tilde{E}_{d7}$ denotes directed neutrosophic relation between the 7 members. We have got the neutrosophic communication relations among 7 members and this social network is shown in Fig. 2. An undirected neutrosophic graph can be used to represent this social network. It is shown in in Fig. 1. In an undirected neutrosophic social network, edges are generally an absent or present in an undirected neutrosophic relation with no another data attached.

We compute the NIDC, NODC and NDC of the member using (6), (8) and (9). We have shown those three NDC in Table II.

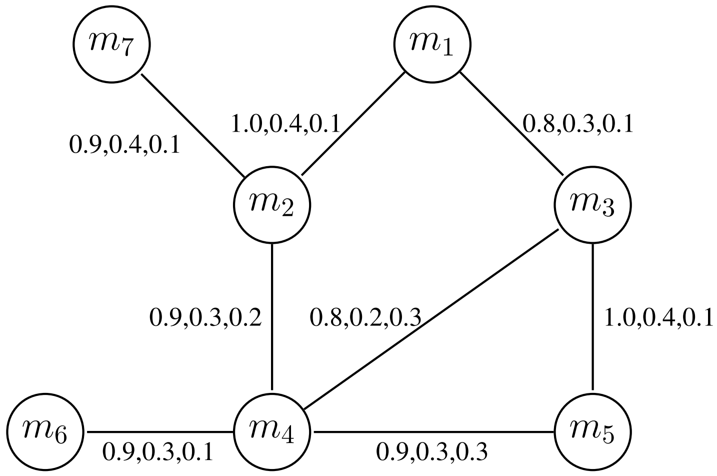

Fig. 1. Undirected Neutrosophic social network

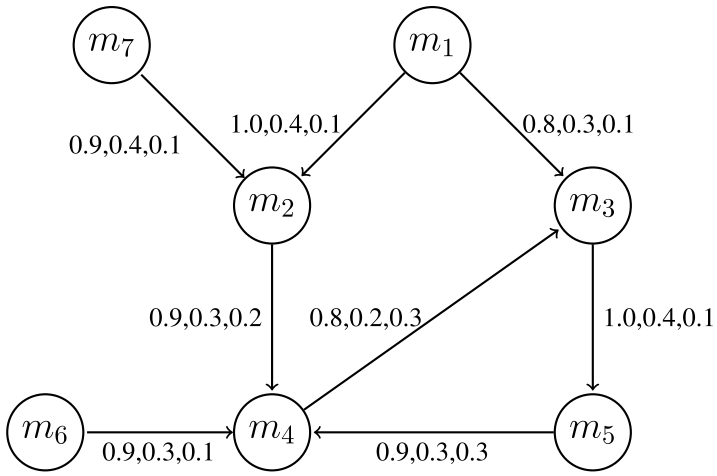

Fig. 2. Directed Neutrosophic social network

TABLE I
THE NEUTROSOPHIC DEGREE CENTRALITY OF RESEARCH TEAM

Members	NIDC	NODC	NDC
m_1	0.00	1.100	1.100
m_2	0.545	0.555	1.100
m_3	0.555	0.535	1.090
m_4	0.555	0.500	1.055
m_5	0.545	0.500	1.045
m_6	0.000	0.600	0.600
m_7	0.000	0.500	0.500

TABLE II
THE NEUTROSOPHIC DEGREE CENTRALITY OF RESEARCH TEAM

Members	NICC	NOCC	NCC
m_1	0.0000	0.9569	0.9569
m_2	0.9569	1.8181	2.7750
m_3	0.9569	1.8300	2.7869
m_4	0.9478	1.8300	2.7750
m_5	1.8300	2.0000	3.8300
m_6	0.0000	1.6600	1.6600
m_7	0.0000	2.0000	2.000

m_1 and m_2 are the top two social units of this DNSN. The total information of these seven members based on 3 centrality measurements are shown in Table II. From the Table II, we find that m_3 and m_4 obtain the two maximum values of NIDC. It indicates that several other members consider m_3 and m_4 as a friend. While m_1, m_6 and m_7 have got the three lowest values (=0.00) of neutrosophic in-degree centrality. It describes that members (m_1, m_6 and m_7) are not received friendship by other members. Based on , The member m_1 has got the highest value of neutrosophic out-degree centrality and m_1, m_6 and m_7 have got the three lowest values of neutrosophic out-degree centrality. It indicates that m_1 appoints lots of others as members, but m_1, m_6 and m_7 have less influence on other members. The m_1 and m_2 have got the highest value of NDC which indicates that m_1 and m_2 are joined with other social units with very high influence.

Using the 15 and 17, the NICC and NOCC of this DNSN are computed. We have listed the values based on all closeness centrality technique in Table 2. From the Table 2, we find that m_5 obtains the highest value of the NICC. The member m_5 has very well social relation and maximum acceptance presents in the fact that indirect friendly relationship is taken into circumstance. The member m_5 has got the maximum value of NOCC. It indicates that the member m_5 nominates many others as members.

IX. Conclusion

In this work, we introduce the idea of undirected and directed neutrosophic graph. We propose a method to model the social network using directed neutrosophic graph. We define this social network as DNSN. We have modified the centrality theory to DNSN to analysis to this social network. First, we introduce some new centrality measurement for DNSN: NIDC, NODC and NDC. The NODC is used to measure of expansiveness of a social unit and the NIDC is used to measure of popularity of a social unit. In this work, we assume the sociometric relationship between the members, a member with a high NODC is one unit who has linked with other units as friends. A social unit with a low NODC nominates very less social units as friends. A social unit with a high NIDC is a social unit whom many others member (social units) nominate this social unit as a friend and a social unit with a low value of NIDC is preferred by few others social units. In this work, we introduce some other new types of centrality measurement for DNSN: NICC, NOCC and NCC. The main difference between NCC and NDC presents in the fact that undirected neutrosophic relation is chosen into consideration. We have also discussed the mean of NIDC and mean NODC for the DNSN. In DNSN, the mean NIDC and NODC are equal. We have introduced the idea of the neutrosophic betweenness centrality of social unit of a DNSN. We have defined the NDC of a social unit in a DNSN. The idea of variance of the NIDC, NODC, NICC and NOCC of DNSN are also proposed in this study. These measurement are very simple and efficient statistical analysis for describing how centralized a social unit is in a DNSN.

Centrality analysis is a well known and efficient method for social network analysis. This measurement concept is used to find the central position in the social network. We extend the idea of centrality analysis for DNSN. One simple numerical example of directed as well as undirected weighted neutrosophic graph to model one small social networks. We call it as DNSN. Due to the small size of the social network, it is very easy and useful to realize the significant of the DNSN. Therefore, in future work, we have to represent a high density social network using the directed weighted neutrosophic graph. We need to analysis this social network. Despite the requirement for future work, the introduced model described in this work is an significant initial contributions to neutrosophic graph and social network under neutrosophic environment.

References

[1] L. A. Zadeh, "Fuzzy sets," *Information and control*, vol. 8, no. 3, pp. 338–353, 1965.

[2] R. Prasertpong and M. Siripitukdet, "Rough set models induced by serial fuzzy relations approach in semigroups." *Engineering Letters*, vol. 27, no. 1, pp. 216–225, 2019.

[3] F. Smarandache, "Invisible paradox," *Neutrosophy/Neutrosophic Probability, Set, and Logic", Am. Res. Press, Rehoboth*, vol. 1, no. 2, pp. 22–23, 1998.

[4] J. Ye, "Single-valued neutrosophic minimum spanning tree and its clustering method," *Journal of intelligent Systems*, vol. 23, no. 3, pp. 311–324, 2014.

[5] H. L. Yang, Z. L. Guo, Y. She, and X. Liao, "On single valued neutrosophic relations," *Journal of Intelligent & Fuzzy Systems*, vol. 30, no. 2, pp. 1045–1056, 2016.

[6] S. Naz, H. Rashmanlou, and M. A. Malik, "Operations on single valued neutrosophic graphs with application," *Journal of Intelligent & Fuzzy Systems*, vol. 32, no. 3, pp. 2137–2151, 2017.

[7] D. Xu, Y. Hong, and H. Qiao, "Multiple attribute decision making based on neutrosophic sets in venture capital." *Engineering Letters*, vol. 26, no. 4, pp. 441–446, 2018.

[8] F. Karaaslan and B. Davvaz, "Properties of single-valued neutrosophic graphs," *Journal of Intelligent & Fuzzy Systems*, vol. 34, no. 1, pp. 57–79, 2018.

[9] R. Dhavaseelan, R. Vikramaprasad, and V. Krishnaraj, "Certain types of neutrosophic graphs," *Int. J. Math. Sci. Appl*, vol. 5, no. 2, pp. 333–339, 2015.

[10] S. Broumi, M. Talea, A. Bakali, and F. Smarandache, "On bipolar single valued neutrosophic graphs," *Journal of New Theory*, vol. 11, pp. 84–102, 2016.

[11] L. Yang, B. Li, and H. Xu, "Novel power aggregation operators based on einstein operations for interval neutrosophic linguistic sets," *IAENG International Journal of Applied Mathematics*, vol. 48, no. 4, pp. 475–484, 2018.

[12] L. Yang and B. Li, "An extended single-valued neutrosophic normalized weighted bonferroni mean einstein aggregation operator." *IAENG International Journal of Applied Mathematics*, vol. 48, no. 4, pp. 373–380, 2018.

[13] A. Dey, A. Pal, and T. Pal, "Interval type 2 fuzzy set in fuzzy shortest path problem," *Mathematics*, vol. 4, no. 4, p. 62, 2016.

[14] R. Intan and S. H. G. Chan, "Extended generalization of fuzzy rough sets." *IAENG International Journal of Computer Science*, vol. 46, no. 2, pp. 257–263, 2019.

[15] A. Dey, R. Pradhan, A. Pal, and T. Pal, "A genetic algorithm for solving fuzzy shortest path problems with interval type-2 fuzzy arc lengths," *Malaysian Journal of Computer Science*, vol. 31, no. 4, pp. 255–270, 2018.

[16] W. Zheng, J. Qiao, L. Feng, and P. Fu, "Optimal cooperative virtual multi-input and multi-output network communication by double improved ant colony system and genetic algorithm," *IAENG International Journal of Computer Science*, vol. 45, no. 1, pp. 89–96, 2018.

[17] L. Yang and B. Li, "Multi-valued neutrosophic linguistic power operators and their applications." *Engineering Letters*, vol. 26, no. 4, pp. 518–525, 2018.

[18] P. Bonacich, "Factoring and weighting approaches to status scores and clique identification," *Journal of mathematical sociology*, vol. 2, no. 1, pp. 113–120, 1972.

[19] ——, "Power and centrality: A family of measures," *American journal of sociology*, vol. 92, no. 5, pp. 1170–1182, 1987.

[20] M. Brunelli, M. Fedrizzi, and M. Fedrizzi, "Fuzzy m-ary adjacency relations in social network analysis: Optimization and consensus evaluation," *Information Fusion*, vol. 17, pp. 36–45, 2014.

[21] K. Sohn and D. Kim, "Zonal centrality measures and the neighborhood effect," *Transportation Research Part A: Policy and Practice*, vol. 44, no. 9, pp. 733–743, 2010.

[22] A. M. Kermarrec, E. Le Merrer, B. Sericola, and G. Trédan, "Second order centrality: Distributed assessment of nodes criticity in complex networks," *Computer Communications*, vol. 34, no. 5, pp. 619–628, 2011.

[23] X. Qi, E. Fuller, Q. Wu, Y. Wu, and C. Q. Zhang, "Laplacian centrality: A new centrality measure for weighted networks," *Information Sciences*, vol. 194, pp. 240–253, 2012.

[24] M. del Pozo, C. Manuel, E. González-Arangüena, and G. Owen, "Centrality in directed social networks. a game theoretic approach," *Social Networks*, vol. 33, no. 3, pp. 191–200, 2011.

[25] A. Landherr, B. Friedl, and J. Heidemann, "A critical review of centrality measures in social networks," *Business & Information Systems Engineering*, vol. 2, no. 6, pp. 371–385, 2010.

[26] P. S. Nair and S. T. Sarasamma, "Data mining through fuzzy social network analysis," in *NAFIPS 2007-2007 Annual Meeting of the North American Fuzzy Information Processing Society*. IEEE, 2007, pp. 251–255.

[27] L. p. Liao and R. j. Hu, "On the definition and property analysis of fuzzy social network based on fuzzy graph," *Journal of Guangdong University of Technology (Social Science Edition)*, vol. 12, no. 3, pp. 46–51, 2012.

[28] S. Samanta and M. Pal, "A new approach to social networks based on fuzzy graphs," *Turkish Journal of Fuzzy Systems*, vol. 5, no. 2, pp. 078–099, 2014.

[29] R. Mahapatra, S. Samanta, M. Pal, and Q. Xin, "Rsm index: a new way of link prediction in social networks," *Journal of Intelligent & Fuzzy Systems*, vol. 37, no. 2, pp. 2137–2151, 2019.

[30] A. Lakdashti, H. Rashmanlou, R. Borzooei, S. Samanta, and M. Pal, "New concepts of bipolar fuzzy graphs," *Journal of Multiple-Valued Logic & Soft Computing*, vol. 33, no. 1/2, pp. 117–133, 2019.

[31] R. Mahapatra, S. Samanta, T. Allahviranloo, and M. Pal, "Radio fuzzy graphs and assignment of frequency in radio stations," *Computational and Applied Mathematics*, vol. 38, no. 3, pp. 117–134, 2019.

[32] K. Das, S. Samanta, and M. Pal, "Study on centrality measures in social networks: a survey," *Social Network Analysis and Mining*, vol. 8, no. 1, p. 13, 2018.

[33] T. F. Fan, C. J. Liau, and T. Y. Lin, “A theoretical investigation of regular equivalences for fuzzy graphs,” *International Journal of Approximate Reasoning*, vol. 49, no. 3, pp. 678–688, 2008.
[34] L. p. Liao, R. j. Hu, and G. y. Zhang, “The centrality analysis of fuzzy social networks,” *Fuzzy Systems and Mathematics*, vol. 27, no. 2, pp. 169–173, 2012.
[35] S. Kundu and S. K. Pal, “Fgsn: Fuzzy granular social networks–model and applications,” *Information Sciences*, vol. 314, pp. 100–117, 2015.
[36] R.-j. Hu, G. y. Zhang, and L. p. Liao, “The closeness centrality analysis of fuzzy social network based on inversely attenuation factor,” in *Fuzzy Information & Engineering and Operations Research & Management*. Springer, 2014, pp. 457–465.
[37] R.-J. Hu, Q. Li, G.-Y. Zhang, and W.-C. Ma, “Centrality measures in directed fuzzy social networks,” *Fuzzy Information and Engineering*, vol. 7, no. 1, pp. 115–128, 2015.
[38] S. Broumi, A. Bakali, M. Talea, F. Smarandache, and G. Selvachandran, “Computing operational matrices in neutrosophic environments: A matlab toolbox,” *Neutrosophic Sets Syst*, vol. 18, pp. 58–66, 2017.
[39] S. Broumi, P. K. Singh, M. Talea, A. Bakali, F. Smarandache, and V. V. Rao, “Single-valued neutrosophic techniques for analysis of wifi connection,” in *International Conference on Advanced Intelligent Systems for Sustainable Development*. Springer, 2018, pp. 405–412.
[40] S. Broumi, A. Bakali, M. Talea, and F. Smarandache, “A matlab toolbox for interval valued neutrosophic matrices for computer applications,” *Uluslararası Yönetim Bilişim Sistemleri ve Bilgisayar Bilimleri Dergisi*, vol. 1, no. 1, pp. 1–21, 2017.
[41] H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, “Single valued neutrosophic sets,” *Review of the Air Force Academy*, vol. 1, no. 1, pp. 10–18, 2010.
[42] H. Garg *et al.*, “An improved score function for ranking neutrosophic sets and its application to decision-making process,” *International Journal for Uncertainty Quantification*, vol. 6, no. 5, pp. 23–28, 2016.
[43] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability: Neutrsophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability*. Infinite Study, 2005.