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Abstract. We answer the following question: Are neutrosophic $\underline{\mu}$-compactness and neutrosophic $\underline{\mu}$-countably compactness equivalent? which posted in [10]. Since every neutrosophic topology is neutrosophic $\underline{\mu}$-topology, we answer the question for neutrosophic topological spaces, more precisely, we give an example of neutrosophic topology which is neutrosophic countably comapact but not neutrosophic compact

Keywords: Neutrosophic topological spaces; Neutrosophic compact; Neutrosophic Lindelöf; Neutrosophic countably compact space

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1. Introduction

Neutrosophic sets first introduced in [25, 27] as a generalization of intuitionistic fuzzy sets [14], where each element $x \in X$ has a degree of indeterminacy with the degree of membership and the degree of non-membership . Operations on neutrosophic sets are investigated after that. Neutrosophic topological spaces are studied by Smarandache [27], Lupianez [19,20] and Salama [23]. The interior , closure, exterior and boundary of neutrosophic sets can be found in [26]. Neutrosophic sets applied to generalize many notaions about soft topology and applications [18], [22], [15], generalized open and closed sets [28] , fixed point theorems [18] , graph theory [17]and rough topology and applications [21]. Neutrosophy has many applications specially in decision making, for more details about new trends of neutrosophic applications one can consult [1]- [7].

Generalized topology and continuity introduced in 2002 in [13], where many generalized open sets in general topology become examples in generalized topological spaces, and it become one of the most important generalization in topology which has different properties than general topology, see for example [9], [11] and [12]. There are a lot of studies about neutrosophic topological spaces that shows the importance of studying neutrosophic topology where it has possible applications, see for example [24], Neutrosophic μ-topological spaces first introduced in [10], and since Neutrosophic μ-topological space is a generalization of neutrosophic topological space it guarantees generalized results that are still true for neutrosophic topological spaces, see for example Theorem 2.30 in [10] which shows that neutrosophic μ-compactness and neutrosophic μ-countably compactness are equivalent, and this is not true in crisp topology, but it becomes true for neutrosophic topological spaces since every neutrosophic μ-topological space is neutrosophic topological space, another thing about the importance of neutrosophic μ-topological space is that some existing notations about neutrosophic topology can be considered as examples of neurosophic μ-topological spaces, see for example Theorem 2.9 in [10] which shows the relationship between μ-topological space and previous studies where we can consider all neutrosophic α-open sets over (X; τ) and all neutrosophic pre-closed sets in (X; τ) (introduced in [8]) as examples of strong neutrosophic μ-topology over X. The following question appeared in [10].

Definition 1.1. [25]: A set A is said *neutrosophic* on X if $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle; x \in X\}$; $\mu, \sigma, \nu : X \to]^-0, 1^+[$ and $^-0 \leq \mu(x) + \sigma(x) + \nu(x) \leq 3^+$.

The class of all neutrosophic set on the universe X is by $\mathcal{N}(X)$. We will exhibit the basic neutrosophic operations definitions (union, intersection and complement. Since there are different definitions of neutrosophic operations, we will organize the existing definitions into two types, in each type these operation will be consistent and functional.

Definition 1.2. [24][*Neutrosophic sets operations*] Let $A, A_\alpha, B \in \mathcal{N}(X)$ such that $\alpha \in \Delta$. Then we define the neutrsophic:

(1) (*Inclusion*): $A \sqsubseteq B$ If $\mu_A(x) \leq \mu_B(x)$, $\sigma_A(x) \geq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.
(2) (*Equality*): $A = B \Leftrightarrow A \sqsubseteq B \wedge B \sqsubseteq A$.
(3) (*Intersection*) $\underset{\alpha\in\Delta}{\sqcap} A_\alpha(x) = \{\langle x, \underset{\alpha\in\Delta}{\wedge} \mu_{A_\alpha}(x), \underset{\alpha\in\Delta}{\vee} \sigma_A(x), \underset{\alpha\in\Delta}{\vee} \nu_A(x)\rangle; x \in X\}$.
(4) (*Union*) $\underset{\alpha\in\Delta}{\sqcup} A_\alpha(x) = \{\langle x, \underset{\alpha\in\Delta}{\vee} \mu_{A_\alpha}(x), \underset{\alpha\in\Delta}{\wedge} \sigma_A(x), \underset{\alpha\in\Delta}{\wedge} \nu_A(x)\rangle; x \in X\}$.
(5) (*Complement*) $A^c = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle; x \in X\}$
(6) (*Universal set*) $1_X = \{\langle x, 1, 0, 0\rangle; x \in X\}$; will be called the *neutrosophic universal set*.
(7) (*Empty set*) $0_X = \{\langle x, 0, 1, 1\rangle; x \in X\}$; will be called the *neutrosophic empty set*.

Proposition 1.3. *[24] For $A, A_\alpha \in \mathcal{N}(X)$ for every $\alpha \in \Delta$ we have:*

(1) $A \sqcap (\underset{\alpha\in\Delta}{\sqcup} A_\alpha) = \underset{\alpha\in\Delta}{\sqcup} (A \sqcap A_\alpha)$.
(2) $A \sqcup (\underset{\alpha\in\Delta}{\sqcap} A_\alpha) = \underset{\alpha\in\Delta}{\sqcap} (A \sqcup A_\alpha)$.

Definition 1.4. [24] [*Neutrosophic Topology*] $\tau \subset \mathcal{N}(X)$ is called a *neutrosophic topology* for X if

(1) $0_X, 1_X \in \tau$.
(2) If $A_\alpha \in \tau$ for every $\alpha \in \Delta$, then $\underset{\alpha\in\Delta}{\sqcup} A_\alpha \in \tau$,
(3) For every $A, B \in \tau$, we have $A, B \in \tau$.

The ordered pair (X, τ) will be said a *neutrosophic space* over X. The elements of τ will be called *neutrosophic open sets*. For any $A \in \mathcal{N}(X)$, If $A^c \in \tau$, then we say A is *neutrosophic closed*.

2. Neutrosophic Countably Compact Spaces

Definition 2.1. [10] Let X be nonempty, $0 < \alpha, \beta, \gamma < 1$. Then $A \in \mathcal{N}(X)$ is said a *neutrosophic point* iff there exists $x \in X$ such that $A = \{\langle x, \alpha, \beta, \gamma\rangle\} \cup \{\langle \acute{x}, 0, 1, 1\rangle; \acute{x} \neq x\}$. Neutrosophic points will be denoted by $x_{\alpha,\beta,\gamma}$.

Definition 2.2. [10] We say $x_{\alpha,\beta,\gamma}$ in the neutrosophic set A -in symbols $x_{\alpha,\beta,\gamma} \in A$- iff $\alpha < \mu_A(x), \beta > \sigma_A(x)$ and $\gamma > \nu_A(x)$.

Lemma 2.3. *[10] Let $A \in \mathcal{N}(X)$ and suppose that for every $x_{\alpha,\beta,\gamma} \in A$ there exists $B(x_{\alpha,\beta,\gamma}) \in \mathcal{N}(X)$ such that $x_{\alpha,\beta,\gamma} \in B(x_{\alpha,\beta,\gamma}) \sqsubseteq A$. Then $A = \sqcup\{B(x_{\alpha,\beta,\gamma}); x_{\alpha,\beta,\gamma} \in A\}$.*

Corollary 2.4. *[10] $A \in \mathcal{N}(X)$ is neutrosophic open in (X, τ) iff for every $x_{\alpha,\beta,\gamma} \in A$ there exists a neutrosophic set $B(x_{\alpha,\beta,\gamma}) \in \tau$; $x_{\alpha,\beta,\gamma} \in B(x_{\alpha,\beta,\gamma}) \sqsubseteq A$.*

Definition 2.5. [10] Let (X, τ) be a neutrosophic topology on X. A sub-collection $\mathcal{B} \subseteq \tau$ is called a neutrosophic base for τ if for any $U \in \tau$ there exists $\acute{\mathcal{B}} \subseteq \mathcal{B}$ such that $U = \sqcup\{B; B \in \acute{\mathcal{B}}\}$.

Definition 2.6. [10] Consider the neutrosophic space (X, τ) . We say the collection $\mathcal{U}$ from τ is a *neutrosophic open cover* of X, if $1_X = \sqcup\{U; U \in \mathcal{U}\}$.

Definition 2.7. [10] Consider the space (X, τ) and the neutrosophic open cover $\mathcal{U}$ of X. Then we say the sub-collection $\acute{\mathcal{U}} \subseteq \mathcal{N}(X)$ is a *neutrosophic subcover* of X from $\mathcal{U}$, if $\acute{\mathcal{U}}$ is neutrosophic covers X and $\acute{\mathcal{U}} \subseteq \mathcal{U}$.

The following is an immediate result of Corollary2.4.

Corollary 2.8. *[10] A sub-collection $\mathcal{U}$ from the neutrosophic space (X, τ) is an open cover of X iff for every $x_{\alpha,\beta,\gamma}$ in X there exists $U \in \mathcal{U}$ such that $x_{\alpha,\beta,\gamma} \in U$.*

Theorem 2.9. *Consider the collection $\mathcal{B}$ of neutrosophic sets on the universe X. Then $\mathcal{B}$ is a neutrosophic base for some neutrosophic topology on X iff*

(1) *For every $U \in \tau$ and every $x_{\alpha,\beta,\gamma} \in U$ there exists $B \in \mathcal{B}$ such that $x_{\alpha,\beta,\gamma} \in B \sqsubseteq U$.*
(2) *For every $A, B \in \mathcal{B}$ we have $A \sqcap B$ is a union of elements from $\mathcal{B}$.*

Proof. $\rightarrow$) Obvious!
$\leftarrow$) Suppose $\mathcal{B}$ satisfies the two conditions in the theorem. Let $\tau(\mathcal{B})$ be all possible neutrosophic unions of elements from $\mathcal{B}$ with 0_X. It suffices to show that $\tau(\mathcal{B})$ is a neutrosophic topology on X. From the first condition and the construction of $\tau(\mathcal{B})$ we have $0_X, 1_X \in \tau(\mathcal{B})$. Now let $H, K \in \tau(\mathcal{B})$. Then $H = \bigsqcup_i H_i$ and $K = \bigsqcup_j K_j$ where $H_i, K_j \in \mathcal{B}$ for every i, j. So we have (by parts (3) and (4) of Proposition1.3)
$H \sqcap K = (\bigsqcup_i H_i) \sqcap (\bigsqcup_j K_j) = \bigsqcup_j ((\bigsqcup_i H_i) \sqcap K_j) = \bigsqcup_j \bigsqcup_i (H_i \sqcap K_j)$
Since $H_i, K_j \in \mathcal{B}$ for every i, j, we have $H \sqcap K \in \tau(\mathcal{B})$. The proof that the union of elements from $\tau(\mathcal{B})$ is an element from $\tau(\mathcal{B})$ is easy! And we done. □

$\tau(\mathcal{B})$ will be called the neutrosophic topology generated by the neutrosophic base $\mathcal{B}$ on X .

Definition 2.10. [10] (X, τ) is said to be *neutrosophic compact* if each neutrosophic open (in τ) cover of X has a finite neutrosophic subcover.

Theorem 2.11. *[10] Consider the space (X, τ), and let $\mathcal{B}$ be a neutrosophic base for τ. Then (X, τ) is a neutrosophic compact space iff every neutrosophic open cover of X from $\mathcal{B}$ has a finite neutrosophic subcover.*

Definition 2.12. [10] A neutrosophic space (X, τ) is said:

(1) A *neutrosophic* Lindelöf space if each neutrosophic open cover of X from τ has a countable neutrosophic subcover of X.
(2) A *neutrosophic* countably compact space if each neutrosophic open countable cover of X from τ has a finite neutrosophic subcover of X.

The following thee results have proofs similar to their correspondings about neutrosophic $\underline{\mu}$-topological spaces in [10].

Theorem 2.13. *Every neutrosophic space with a countable neutrosophic base is neutrosophic Lindelöf.*

Theorem 2.14. *Every neutrosophic Lindelöf and* countably compact space *is compact.*

Corollary 2.15. *Every neutrosophic* countably compact space *with a neutrosophic countable base is neutrosophic compact.*

The following example show that neutrosophic Lindelöf spaces are not neutrosophic countably compact.

Example 2.16. Let $Y = \{a, b\}$ and let $\mathcal{B} = \{A_n; n = 1, 2, 3, ...\}$ where $A_n = \{\langle y, 1 - \frac{1}{2n}, \frac{1}{2n}, \frac{1}{2n}\rangle; y \in X\}$. We will show that $\mathcal{B}$ is a base for some neutrosophic topology on Y ;

i.e. we want to show $\mathcal{B}$ satisfies (1) and (2) in Theorem2.9.

First condtion: $\mathcal{B}$ neutrosophic covers Y, actually:

$\sqcup\mathcal{B} = \sqcup\{A_n; n = 1,2,3,...\} = \{\langle y, \vee_1^\infty 1 - \frac{1}{2n}, \wedge_1^\infty \frac{1}{2n}, \wedge_1^\infty \frac{1}{2n}\rangle; y \in Y\} = \{\langle y,1,0,0\rangle; y \in Y\} = 1_Y$.

Second condition: The neutrosophic intersection of two elements from $\mathcal{B}$ is the neutrosophic union of elements from $\mathcal{B}$, but is clear that for any A_n and A_m in $\mathcal{B}$ we have $A_n \sqcap A_m = A_t$ where $t = \max\{n,m\}$ which an element of $\mathcal{B}$, so that $\mathcal{B}$ is a neutrosophic base form some neutrosophic topology $\tau(\mathcal{B})$ on Y. Since $\tau(\mathcal{B})$ has a countable base, $\tau(\mathcal{B})$ is neutrosophic Lindelöf. Now, we will show that $\tau(\mathcal{B})$ is not neutrosophic countably paracompact (which implies it is not neutrosophic compact). By contrapositive, suppose Y is neutrosophic countably paracompact. Then $\mathcal{U} = \mathcal{B}$ is a countable neutrosophic open cover of Y. But Y is a neutrosophic countably paracompact space, so that we have $\mathcal{U}$ has a neutrosophic finite subcover, say $\mathcal{U}^* = \{A_{n1}, A_{n2}, ..., A_{nk}\}$. But $A_{n1} \sqcup A_{n2} \sqcup ... \sqcup A_{nk} = A_t$ where $t = \max\{n_1, n_2, ..., n_k\}$, and $A_t = \{\langle y, 1 - \frac{1}{2t}, \frac{1}{2t}, \frac{1}{2t}\rangle; y \in Y\} \neq 1_Y$, a contradiction. So Y is not neutrosophic countably paracompact and hence it is not neutrosophic compact.

The following theorem shows that neutrosophic compact spaces and neutrusophic countably compact spaces are equivalent if the universe of discourse is countable, which is not true for topological spaces.

Theorem 2.17. *For every countable neutrosophic topological space Y, the following two statements are equivalent :*

(1) *Y is neutrosophic compact.*
(2) *Y is neutrosophic countably compact.*

Proof. $\Rightarrow$) Obvious!

$\Leftarrow$) Suppose that Y is a countable neutrosophic countably compact space, and let $\mathcal{U}$ be a neutrosophic open cover of Y. For every $y \in Y$ we define the following three subsets of $[0,1]$.

(1) $D_\mu^y = \{\mu_A(y); A \in \mathcal{U}\}$.
(2) $D_\sigma^y = \{\sigma_A(y); A \in \mathcal{U}\}$.
(3) $D_\nu^y = \{\nu_A(y); A \in \mathcal{U}\}$.

Let D_1^y, D_2^y and D_3^y be three countable dense subsets of D_μ^y, D_σ^y and D_ν^y respectively in the usual sense (the usual topology on the unit interval). Since $\mathcal{U}$ is a neutrosophic μ-open cover of Y, we have $\sup D_1^y = \sup D_\mu^y = 1$,$\inf D_2^y = \inf D_\sigma^y = 0$ and $\inf D_3^y = \inf D_\nu^y = 0$. Let $\mathcal{U}(y) = \{A \in \mathcal{U}; \mu_A(y) \in D_1^y, \sigma_A(y) \in D_2^y \text{ or } \nu_A(y) \in D_3^y\}$. It is clear that $\mathcal{U}(y)$ is countable. Let $\mathcal{U}^* = \cup\{\mathcal{U}(y); y \in Y\}$. Since Y is countable, $\mathcal{U}^*$ is a countable sub-collection from $\mathcal{U}$. We will show that $\mathcal{U}^*$ is a neutrosophic cover of Y. Set $B = \sqcup\mathcal{U}^*$. For every $y \in Y$ we have:

(1) $\mu_B(y) = \vee\{\mu_A(y); A \in B\} \geq \vee\{\mu_A(y); A \in D_1^y\} = \sup D_1^y = 1$.

(2) $\sigma_B(y) = \wedge\{\sigma_A(y); A \in B\} \geq \wedge\{\sigma_A(y); A \in D_1^y\} = \inf D_2^y = 0$.

(3) $\nu_B(y) = \wedge\{\nu_A(y); A \in B\} \geq \wedge\{\vee_A(y); A \in D_1^y\} = \inf D_3^y = 0$.

Which implies that $B = 1_Y$ and $\mathcal{U}^*$ is a neutrosophic countable open cover. Since Y is a neutrosophic μ-countably compact space, $\mathcal{U}^*$ has a finite subcover , that is Y is compact. □

The following example shows that neutrosophic compactness and neutrosophic countably compactness are not equivalent.

Example 2.18. Consider the set of all countable ordinals W_0 with the usual ordering. Let $\beta = \{[s,t), s,t < \omega_1(\text{the first uncountable ordinal})\}$. We know that β is a base for some topology τ on $Y = W_0$. For every $[s,t) \in \beta$ define the neutrosophic set

$$A_{[s,t)} = \begin{cases} (y,1,0,0) & \text{if } y \in [s,t) \\ (y,0,1,1) & \text{if } y \notin [s,t) \end{cases}$$

Set $\acute{\beta} = \{A_{[s,t)}; [s,t) \in \beta\}$. We will show that $\acute{\beta}$ is a base for some neutrosophic topology on Y. First we show it is a neutrosophic cover for Y. Let $A = \sqcup\beta$; it suffices to show that $A = 1_Y$. But for every $y \in Y$, we have $y \in [s,y)$ for some $s < y$, so that $\mu_A(y) = \vee\{\mu_C(y); C \in \acute{\beta}\} \geq \mu_{[s,y)} = 1$, $\sigma_A(y) = \wedge\{\sigma_C(y); C \in \acute{\beta}\} \leq \sigma_{[s,y)} = 0$, and $\nu_A(y) = \wedge\{\nu_C(y); C \in \acute{\beta}\} \leq \nu_{[s,y)} = 0$, that means $A = 1_Y$ and β covers Y. Now, we will show that the intersection of any two elements from β is empty or an element of β. Let $A_{[s_1,t_1)}$ and $A_{[s_2,t_2)}$ be two neutrosophic sets in β and set $C = A_{[s_1,t_1)} \sqcap A_{[s_2,t_2)}$, if $[s_1,t_1) \cap [s_2,t_2) = \emptyset$, then for every $y \in Y$ we have $y \notin [s_1,t_1)$ or $y \notin [s_2,t_2)$, which implies $\mu_C = \mu_{[s_1,t_1)} \wedge \mu_{[s_2,t_2)} = 0$, $\sigma_C = \sigma_{[s_1,t_1)} \vee \sigma_{[s_2,t_2)} = 1$ and $\nu_C = \nu_{[s_1,t_1)} \vee \nu_{[s_2,t_2)} = 1$ and that means $A_{[s_1,t_1)} \sqcap A_{[s_2,t_2)} = 0_Y$. Now, suppose that $[s_1,t_1) \cap [s_2,t_2) \neq \emptyset$. Then for every $y < \max\{s_1,s_2\}$ or $y \geq \min\{t_1,t_2\}$ we have $y \notin [s_1,t_1)$ or $y \notin [s_2,t_2)$, which means $\mu_C = 0$, $\sigma_C = 1$ and $\nu_C = 1$, and if $\max\{s_1,s_2\} \leq y < \min\{t_1,t_2\}$, then $y \in [s_1,t_1)$ and $y \in [s_2,t_2)$, that is $\mu_C = 1$, $\sigma_C = 0$ and $\nu_C = 0$, so that we have

$$A_{[s_1,t_1)} \sqcap A_{[s_2,t_2)} = A_{[s,t)} = \begin{cases} (y,1,0,0) & \text{if } y \in [s,t) \\ (y,0,1,1) & \text{if } y \notin [s,t) \end{cases} \in \beta$$

where $s = \max\{s_1,s_2\}$ and $t = \max\{t_1,t_2\}$. Let $\tau(\beta)$ be the neutrosophic topology generated on Y by β. Then $\tau(\beta)$ is a neutrosophic countably compact space: We will prove this by showing $\tau(\beta)$ has no countable cover form β. Let $\mathcal{C} = \{A_n = [s_n,t_n); n = 1,2,3,...\}$ be any countable subset from β, it suffices to show that $\mathcal{C}$ does not cover Y; by contapositive, suppose $\mathcal{C}$ covers Y, then $D = \sqcup\mathcal{C} = \bigsqcup_{i=1}^{\infty} A_n = 1_Y$. So that for every $y \in Y$ we have

$\mu_C = \overset{\infty}{\underset{i=1}{\vee}} \mu_{A_n} = 1$; since $\mu_{A_n} = 1$ or 0 for every $n = 1, 2, 3, ...$, there exist i such that $\mu_{A_i} = 1$, that is $y \in A_i = [s_i, t_i)$, which implies $Y = \overset{\infty}{\underset{i=1}{\cup}} [s_n, t_n)$, a contradiction, since Y is uncountable and $\overset{\infty}{\underset{i=1}{\cup}} [s_n, t_n)$ is countable, so β has no countable cover for Y, and so Y is neutrosophic countably compact. Now, to show that Y is not neutrosophic compact. But β is a neutrosophic open cover of Y and has no countable, and hence no finite, subcover, that means Y is not neutrosophic compact.

Corollary 2.19. *There is a neutrosophic $\underline{\mu}-$topological spaces which is neutrosophic countably compact but not neutrosophic compact.*

Proof. Since every neutrosophic space is $\underline{\mu}-$topological space, we have Example2.18 is an example of a neutrosophic $\underline{\mu}-$topological spaces which is neutrosophic countably compact but not neutrosophic compact. □

The approach we used in Example2.18 can be generalized to get more counterexample for neutrosophic topological spaces as follows.

Theorem 2.20. *Let (X, τ) be a topological space and for every $U \in \tau$ set*

$$A_U = \begin{cases} (x, 1, 0, 0) & \text{if } x \in U \\ (x, 0, 1, 1) & \text{if } x \notin U \end{cases}$$

and let $Neut(\tau) = \{A_U; U \in \tau\}$. Then $(X, Neut(\tau))$ is a neutrosophic topological space.

Proof. Since $\emptyset, X \in \tau$, we have $A_\emptyset, A_X \in Neut(\tau)$, but

$$A_\emptyset = \begin{cases} (x, 1, 0, 0) & \text{if } x \in \emptyset \\ (x, 0, 1, 1) & \text{if } x \notin \emptyset \end{cases} = \begin{cases} (x, 1, 0, 0) & \text{if } x \in \emptyset \\ (x, 0, 1, 1) & \text{if } x \in X \end{cases} = 0_X$$

$$A_X = \begin{cases} (x, 1, 0, 0) & \text{if } x \in X \\ (x, 0, 1, 1) & \text{if } x \notin X \end{cases} = \begin{cases} (x, 1, 0, 0) & \text{if } x \in X \\ (x, 0, 1, 1) & \text{if } x \in \emptyset \end{cases} = 1_X$$

So we have $0_X, 1_X \in Neut(\tau)$. Now, let $H = A_U \sqcap A_V$ where $A_U, A_V \in Neut(\tau)$. Then

$$\mu_H(x) = \begin{cases} 1 & \text{if } x \in U \\ 0 & \text{if } x \notin U \end{cases} \wedge \begin{cases} 1 & \text{if } x \in V \\ 0 & \text{if } x \notin V \end{cases} = \begin{cases} 1 & \text{if } x \in U \cap V \\ 0 & \text{if } x \notin U \cap V \end{cases} = \mu_{A_{(U \cap V)(x)}}$$

$$\sigma_H(x) = \begin{cases} 0 & \text{if } x \in U \\ 1 & \text{if } x \notin U \end{cases} \vee \begin{cases} 0 & \text{if } x \in V \\ 1 & \text{if } x \notin V \end{cases} = \begin{cases} 0 & \text{if} x \in U \cap V \\ 1 & \text{if} x \notin U \cap V \end{cases} = \sigma_{A_{(U \cap V)}}(x)$$

$$\nu_H(x) = \begin{cases} 0 & \text{if } x \in U \\ 1 & \text{if } x \notin U \end{cases} \vee \begin{cases} 0 & \text{if } x \in V \\ 1 & \text{if } x \notin V \end{cases} = \begin{cases} 0 & \text{if} x \in U \cap V \\ 1 & \text{if} x \notin U \cap V \end{cases} = \nu_{A_{(U \cap V)}}(x)$$

So we have $A_U \sqcap A_V = A_{(U \cap H)} \in Neut(\tau)$. Similarly we show that $\underset{\alpha \in \Delta}{\sqcup} A_\alpha \in Neut(\tau)$ whenever $A_\alpha \in Neut(\tau)$ for every $\alpha \in \Delta$. □

3. Applications and further studies

This paper is a completion part of [10] and gives an answer for the following question: Are neutrosophic $\underline{\mu}$-compactness and neutrosophic $\underline{\mu}$-countably compactness equivalent? which posted in [10]. We give an example to show that the answer is no! the approach is used to give such example can be generalized to give many counter examples in neutrosophic topology using those existing in general topology. This paper, also, studied more advanced notations about neutrosophic topology such as neutrosophic comapactness and neutrosophic Lindelöf, which opens doors for more studies about neutrosophic topology, such as neutrosophic para-compactness, and other covering properties

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