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Algorithms for possibility linguistic single-valued neutrosophic decision-making based on COPRAS and aggregation operators with new information measures

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ABSTRACT

The objective of this work is to introduce the concept of the possibility linguistic single-valued neutrosophic set (PLSVNS) for better dealing with the imprecise and uncertain information during the decision-making process. The prominent characteristics of this set are that it considers two distinctive sorts of information such as the membership, indeterminacy, non-membership degrees, and their corresponding possibility degree. In it, first, we stated some operational laws, score and accuracy functions, comparison laws between the pairs of the set. Then, we define weighted averaging and geometric aggregation operators (AOs) to collaborate the PLSVNSs into a single one. Further, we present two algorithms based on a complex proportional assessment (COPRAS) method and AOs based method under PLSVNS information to solve the decision-making problems. In these methods, the information related to weights of decision makers and criteria is determined with the help of a distance and entropy measures. Finally, a practical real-life example is provided to expose the materialness and the viability of our work.

1. Introduction

Multiple criteria group decision-making (MCGDM) problems are the prominent part of the research in decision theory and seek incredible consideration in commonsense fields. The fundamental target of such problems is to choose the foremost desirable alternative among limited options concurring to the preference values of the criteria given by distinctive choice makers. In real situations, we encounter many decision-making (DM) problems, involving the uncertainty or vagueness and hence such problems may not be modeled by the existing classical theories. So, in order to process such information, an intuitionistic fuzzy (IF) set (IFS) [1] theory, which is an effective extension of the fuzzy set (FS) theory [2], gives better way out to manage the inaccuracy, dubiousness, and vulnerabilities in the information and in tackling the DM issues. IFS is way better equipped to speak to the genuine world situation more really because it also factors in the hesitancy of the decision maker, a feature that is not possible in the fuzzy sets. Since its existence, many researchers have utilized this theory to solve the various DM problems under different environment [3–13]. However, it is remarked that neither the FS nor IFS theory is able to deal with indeterminate and inconsistent data. To deal with it, Smarandache [14] created a neutrosophic set (NS) which portray the uncertain information by taking the functions of truth, indeterminacy, and falsity, all are independent to each other and are the subsets of $]0^-, 1^+[$. As the NS theory is more in line with human instinctive feelings and judgment, at the same time it moreover has the degree of indeterminacy, subsequently, it can depict the unclear information more helpfully than the FS and uncertainties speculations. NS theory handles the indeterminate information, but this theory is hard to implement on the practical problems, therefore, Wang et al. [15] presented the concept of the single-valued neutrosophic set (SVNS), a special case of NS. Due to its importance, several researchers have made their efforts to enrich the concept of neutrosophic sets in the decision-making process. For instance, Garg and Nancy [16] presented some hybrid weighted AOs under NS environment to solve the decision-making problems. Ye [17] presented an AO for the simplified NS. Nancy and Garg [18] presented some weighted averaging and geometric AOs by using Frank norm operations by using single-valued neutrosophic (SVN) information. Abdel-Basset et al. [19] solved the supplier selection problem using the neutrosophic set environment. Garg and Nancy [20] presented some new logarithm operational laws and their corresponding AOs for SVN numbers (SVNNs) and

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applied them to solve the decision-making problems. Peng and Liu [21] presented an algorithm for solving DM problem under neutrosophic soft set environment. Peng and Dai [22] presented an approach based on similarity measures and Technique for order preference by similarity to ideal solution (TOPSIS) to solve the DM problem under the SVNS environment. Garg and Nancy [23] presented a nonlinear methodology for solving the DM problem by using TOPSIS approach under interval neutrosophic set environment. Yang and Li [24] extends the power operator to NS domain which can consider the importance of attributes by considering the support over each other and relieve the influence of unreasonable attribute values of different alternatives given by decision-makers. Garg and Nancy [25] presented some bi-parametric distance Measures on SVNSs and applied them to solve the pattern recognition problems. A bibliometric analysis of NS is presented by Peng and Dai [26]. However, apart from them, several other approaches had presented by the various researchers under the NS environment to solve the decision-making problems [27–33].

In the neutrosophic environment, the information which is evaluated is quantitative in nature and is expressed by the means of numeric numbers. But in a real scenario, most of the times the uncertain or imprecise data evaluated by the decision maker has the qualitative aspects. Like, while figure out the level of 'performance' of any distributor company, decision maker wants to convey his/her assessment by using the labels such as "extremely poor", "poor", "fair", "slightly good", "very good", "extremely good", etc. In these situations, the linguistic variables [34] are used to access the information and deal with the qualitative data. In the field of NS environment, Li et al. [35] introduce the concept of linguistic neutrosophic sets (LNSs) in which membership, indeterminacy, and non-membership are expressed as a linguistic variable instead of real numbers. Since the LNSs are exceptionally appropriate for portraying more complicated linguistic data of human prediction under linguistic DM environment, therefore, researchers have been willing to give their great potential in the advancement of AOs within the linguistic neutrosophic environment. Fang and Ye [36] gave the weighted arithmetic and geometric averaging operators under LNS environment. Liang et al. [37] presented an extended TOPSIS approach with linguistic neutrosophic numbers and applied them to analyze the risks of metallic mines. Garg and Nancy [38] introduced some linguistic prioritized AOs which simultaneously considers the priority among the attributes and the uncertainty in linguistic terms under linguistic SVN (LSVN) domain. Liu et al. [39] presented power Heronian AOs for solving group DM problem using LSVN information.

In recent years, multicriteria methods such as Analytic hierarchy process (AHP), VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) method, TOPSIS and COPRAS method have been increasingly used for quantitative and qualitatively evaluation of complicated economic or social processes. The aim of the evaluation is to choose the best alternatives, ranking the alternatives in the order of their significance. Among these, COPRAS method was firstly introduced by Zavadskas et al. [40] in 1994. This method compares the alternatives and determines their priorities under the conflicting criteria by taking into account the criteria weights. Chatterjee et al. [41] have done comparative analysis on different methods such as AHP, VIKOR, TOPSIS, COPRAS with regards to a computational procedure, effortlessness, probability of visual understanding and kind of the data and concluded that COPRAS strategy shows outperform among them. In the literature, there are many applications of COPRAS method. For instance, Razavi Hajiagha et al. [42] presented the COPRAS method for the information in the intuitionistic terms. Rathi and Balamohan [43] used the COPRAS method to solve the group decision-making problem under fuzzy environment. Zolfani et al. [44] applies the complex assessment strategy to the environmental issues by taking the decision problem. Bausys et al. [45] gives its contribution to the COPRAS method in a neutrosophic domain.

Since all the above-stated studies are widely used into the different environment, but under some certain cases, these existing approaches fail to be utilized for the issues where the linguistic neutrosophic data is given by the experts over various criteria. Furthermore, in the definition of the linguistic single-valued neutrosophic sets (LSVNSs), the possibility of each element of universal set related to each criterion is considered as 1. This poses a limitation in the modeling of some problems. However, in some practical situations, the possibility of each element related to each object may be different from 1. For instance, consider the linguistic term "intelligence" and three experts evaluate the candidate. The possibility of the intelligence of a candidate by the first expert can be 0.8. However, the linguistic rating values in terms of SVNSs corresponding to a candidate is (s_3, s_2, s_4) where s_3 represent the degree of agreement towards the statement, s_2 represent the degree of indeterminacy and s_4 represent a degree of falsity towards the statement. Based on it, the other experts can be expressed with some different possibility values. Thus, to represent such an information more clearly, there is a need to introduce a set which represents all of the corresponding possibility neutrosophic values.

From this point of view, we introduce the concept of possibility linguistic single-valued neutrosophic set (PLSVNS) based on an idea that each of the elements of the universe has got a possibility degree related to each element of the neutrosophic set. Also, we define some basic operational laws, score and accuracy functions, comparison laws to rank the different PLSVNSs. Based on the operational laws of possibility linguistic single-valued numbers (PLSVNs), we stated some weighted averaging and geometric aggregation operators. Furthermore, this paper extends the COPRAS strategy to the PLSVNS environment. The important highlights of the COPRAS method are: (1) it consider both the angles of the criteria, namely benefit and cost ones, based on the complex proportional assessment; (2) this strategy is compelled to get the DM outcomes in a more convenient way; (3) this strategy makes conceivable to figure out the gap between each alternative and the best one by evaluating the utility degree. Owing to the advantages of both the AOs and COPRAS method, the aim of this paper is to tackle the challenges under the PLSVNS by developing two MCGDM approaches to manage the information for PLSVNSs, which not only have a great power in distinguishing the optimal alternative, but also can met the optimal conditions as per our real-life situations. Therefore, motivated from the features of AOs, COPRAS method and LSVNS, the following are the fundamental targets for this paper:

(i) to present a new concept of possibility LSVNS and their associated score/accuracy function, comparison laws and basic operational laws;
(ii) to propose different weighted averaging and geometric AOs under PLSVNS environment where the information related to each object is represented in terms of possibility linguistic single-valued neutrosophic numbers (PLSVNNs);
(iii) to develop some new distance measures for PLSVNSs to catch the closeness and the discrimination among the sets;
(iv) to establish the COPRAS method to rank the PLSVNSs;
(v) to create two different algorithms based on AOs and COPRAS method to illuminate group DM issues;
(vi) to exhibit an illustration where significance of preferences based on PLSVNS decision problems has been clarified.

To facilitate our discussion, the remainder of this paper is organized as follows: In Section 2, some fundamental concepts of SVNS and LSVNS are briefly reviewed. In Section 3, a new concept of PLSVNS, a score and accuracy function, and some operational laws are presented along with distance measures and weighted averaging and geometric AOs. In Section 4, we present a new COPRAS method for the PLSVNS environment. In Section 5, we propose two novel possibilities linguistic single-valued neutrosophic MCGDM approaches based on proposed AOs and COPRAS method in PLSVNS domain. Section 6 give a numerical example to validate the proposed approaches along with their discussion and comparative study. Finally, the paper ends with a concluding remark in Section 7.

2. Basic concepts

In the following, we discusses the basic concepts associated with neutrosophic theory in universal set X.

Definition 2.1. [14] A neutrosophic set (NS) α in X is defined as

$$\alpha = \{(x, \mu_\alpha(x), \rho_\alpha(x), \nu_\alpha(x)) \mid x \in X\} \tag{1}$$

which assigns to each element $x \in X$, a acceptance degree $\mu_\alpha(x)$, indeterminacy degree $\rho_\alpha(x)$ and the non-acceptance degree $\nu_\alpha(x)$ with the condition that $0^- \leqslant \sup \mu_\alpha(x) + \sup \rho_\alpha(x) + \sup \nu_\alpha(x) \leqslant 3^+$, where sup denotes 'supremum'.

Definition 2.2. [15] A SVNS α in X is stated as

$$\alpha = \{(x, \mu_\alpha(x), \rho_\alpha(x), \nu_\alpha(x)) \mid x \in X\} \tag{2}$$

where $\mu_\alpha(x), \rho_\alpha(x), \nu_\alpha(x) \in [0,1]$ and $0 \leqslant \mu_\alpha(x) + \rho_\alpha(x) + \nu_\alpha(x) \leqslant 3$. For convenience the pair is denoted as $\alpha = (\mu_\alpha, \rho_\alpha, \nu_\alpha)$ and called as SVN number (SVNN).

In real situations, we get into the problems in which the data cannot be communicated by numerical numbers, means the data is qualitative in nature. For representation of such kind of data we have linguistic variables. In order to portray them, it is essential to characterize the linguistic term set (LTS).

Definition 2.3. [36] Let $Q = \{s_0, s_1, \ldots, s_t\}$ be a LTS with odd cardinality $t+1$ and $\overline{Q} = \{s_h | s_0 \leqslant s_h \leqslant s_t, h \in [0,t]\}$ Then, a LSVNS A in X is stated as

$$A = \{(x, s_\theta(x), s_\psi(x), s_\sigma(x)) \mid x \in X\}, \tag{3}$$

where $s_\theta, s_\psi, s_\sigma$ expresses the linguistic acceptance, indeterminacy and non-acceptance degrees, respectively, with $0 \leqslant \theta + \psi + \sigma \leqslant 3t$. The triple $(s_\theta, s_\psi, s_\sigma)$ is called linguistic SVN (LSVN) number (LSVNN). In addition, if $s_\theta, s_\psi, s_\sigma \in Q$, we call $(s_\theta, s_\psi, s_\sigma)$, the original LSVNN, otherwise the virtual.

Definition 2.4. [36] To compare the LSVNNs, a score (S) and an accuracy function (Ac) for a LSVNN $\alpha = (s_\theta, s_\psi, s_\sigma)$, can be represented as

$$S(\alpha) = (2t + \theta - \psi - \sigma)/3 \in [0,t] \tag{4}$$

and

$$\mathrm{Ac}(\alpha) = (\theta + \psi + \sigma)/3 \in [0,t]. \tag{5}$$

By using these functions, we define an order relation for two different LSVNNs α and β as, if $S(\alpha) > S(\beta)$ then $\alpha \succ \beta$, where "$\succ$" means "preferred to" and if $S(\alpha)$ is same as that of $S(\beta)$ then calculate the accuracy function. If $\mathrm{Ac}(\alpha)$ is greater than $\mathrm{Ac}(\beta)$ then $\alpha \succ \beta$.

Definition 2.5. [38] For three LSVNNs $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1})$ and $\alpha_2 = (s_{\theta_2}, s_{\psi_2}, s_{\sigma_2})$, the following operations are defined as:

(i) $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;
(ii) $\alpha_1 \leqslant \alpha_2$ if $s_{\theta_1} \leqslant s_{\theta_2}$ i.e. $\theta_1 \leqslant \theta_2, s_{\psi_1} \geqslant s_{\psi_2}$ i.e. $\psi_1 \geqslant \psi_2$ and $s_{\sigma_1} \geqslant s_{\sigma_2}$ i.e. $\sigma_1 \geqslant \sigma_2$;
(iii) $\alpha_1 \cup \alpha_2 = (\max\{s_{\theta_1}, s_{\theta_2}\}, \min\{s_{\psi_1}, s_{\psi_2}\}, \min\{s_{\sigma_1}, s_{\sigma_2}\})$;
(iv) $\alpha_1 \cap \alpha_2 = (\min\{s_{\theta_1}, s_{\theta_2}\}, \max\{s_{\psi_1}, s_{\psi_2}\}, \max\{s_{\sigma_1}, s_{\sigma_2}\})$.

Definition 2.6. [38] Let $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1})$ and $\alpha_2 = (s_{\theta_2}, s_{\psi_2}, s_{\sigma_2})$ be LSVNNs and λ be any positive real number, then

(i) $\alpha_1 \oplus \alpha_2 = \left(s_{t\left(\frac{\theta_1}{t} + \frac{\theta_2}{t} - \frac{\theta_1\theta_2}{t^2}\right)}, s_{t\left(\frac{\psi_1\psi_2}{t^2}\right)}, s_{t\left(\frac{\sigma_1\sigma_2}{t^2}\right)}\right)$;
(ii) $\alpha_1 \otimes \alpha_2 = \left(s_{t\left(\frac{\theta_1\theta_2}{t^2}\right)}, s_{t\left(\frac{\psi_1}{t} + \frac{\psi_2}{t} - \frac{\psi_1\psi_2}{t^2}\right)}, s_{t\left(\frac{\sigma_1}{t} + \frac{\sigma_2}{t} - \frac{\sigma_1\sigma_2}{t^2}\right)}\right)$;
(iii) $\lambda\alpha = \left(s_{t\left(1-\left(1-\frac{\theta}{t}\right)^\lambda\right)}, s_{t\left(\frac{\psi}{t}\right)^\lambda}, s_{t\left(\frac{\sigma}{t}\right)^\lambda}\right)$;
(iv) $\alpha^\lambda = \left(s_{t\left(\frac{\theta}{t}\right)^\lambda}, s_{t\left(1-\left(1-\frac{\psi}{t}\right)^\lambda\right)}, s_{t\left(1-\left(1-\frac{\sigma}{t}\right)^\lambda\right)}\right)$.

3. Possibility linguistic SVN sets

In the following, we introduce the concept of PLSVNSs and its based some laws.

Definition 3.1. A possibility linguistic SVN set (PLSVNS) in universal set X is defined as follows:

$$\alpha = \{(s_\theta(x), s_\psi(x), s_\sigma(x); p(x)) | x \in X\} \tag{6}$$

which consists of two kind of information: one is LSVN value $(s_\theta(x), s_\psi(x), s_\sigma(x))$ and other one is possibility degree, $p(x)$, of existence of this LSVN value for any $x \in X$ and $p(x) \in (0,1]$. For convenience, we denote α as $(s_\theta, s_\psi, s_\sigma; p)$ and named as possibility LSVN number (PLSVNN).

To compare two PLSVNNs, the score and the accuracy functions are stated as below:

Definition 3.2. Let $\alpha = (s_\theta, s_\psi, s_\sigma; p)$ is PLSVNN. Then the score function (Sc) is defined as

$$\mathrm{Sc}(\alpha) = \frac{p(2t + \theta - \psi - \sigma)}{3t} \in [0,1] \tag{7}$$

and the accuracy function (H) is defined as

$$H(\alpha) = \frac{p(\theta + \psi + \sigma)}{3t} \in [0,1] \tag{8}$$

Definition 3.3. For two PLSVNNs α and β, an order relation for ranking them is defined as

(i) if $\mathrm{Sc}(\alpha) > \mathrm{Sc}(\beta)$, then $\alpha \succ \beta$, where "$\succ$" means "preferred to";
(ii) if $\mathrm{Sc}(\alpha) = \mathrm{Sc}(\beta)$, and $H(\alpha) > H(\beta)$, then $\alpha \succ \beta$.

Definition 3.4. For two PLSVNNs $\alpha_1 = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1}; p_1)$ and $\alpha_2 = (s_{\theta_2}, s_{\psi_2}, s_{\sigma_2}; p_2)$, we have

(i) $\alpha_1^c = (s_{\theta_1}, s_{\psi_1}, s_{\sigma_1}; 1 - p_1)$;
(ii) $\alpha_1 \leqslant \alpha_2$ if $s_{\theta_1} \leqslant s_{\theta_2}$ i.e. $\theta_1 \leqslant \theta_2, s_{\psi_1} \geqslant s_{\psi_2}$ i.e. $\psi_1 \geqslant \psi_2, s_{\sigma_1} \geqslant s_{\sigma_2}$ i.e. $\sigma_1 \geqslant \sigma_2$ and $p_1 \leqslant p_2$;

(iii) $\alpha_1 \cup \alpha_2 = (\max\{s_{\theta_1}, s_{\theta_2}\}, \min\{s_{\psi_1}, s_{\psi_2}\}, \min\{s_{\sigma_1}, s_{\sigma_2}\}; \max\{p_1, p_2\})$;
(iv) $\alpha_1 \cap \alpha_2 = (\min\{s_{\theta_1}, s_{\theta_2}\}, \max\{s_{\psi_1}, s_{\psi_2}\}, \max\{s_{\sigma_1}, s_{\sigma_2}\}; \min\{p_1, p_2\})$.

Definition 3.5. Let $\alpha_j = \left(s_{\theta_j}, s_{\psi_j}, s_{\sigma_j}; p_j\right), j = 1, 2$ be two PLSVNNs and $\lambda > 0$, then

(i) $\alpha_1 \oplus \alpha_2 = \left(s_{t\left(1-\left(1-\frac{\theta_1}{t}\right)\left(1-\frac{\theta_2}{t}\right)\right)}, s_{t\left(\frac{\psi_1\psi_2}{t^2}\right)}, s_{t\left(\frac{\sigma_1\sigma_2}{t^2}\right)}; 1-(1-p_1)(1-p_2)\right)$;

(ii) $\alpha_1 \otimes \alpha_2 = \left(s_{t\left(\frac{\theta_1\theta_2}{t^2}\right)}, s_{t\left(1-\left(1-\frac{\psi_1}{t}\right)\left(1-\frac{\psi_2}{t}\right)\right)}, s_{t\left(1-\left(1-\frac{\sigma_1}{t}\right)\left(1-\frac{\sigma_2}{t}\right)\right)}; p_1 p_2\right)$;

(iii) $\lambda\alpha_1 = \left(s_{t\left(1-\left(1-\frac{\theta_1}{t}\right)^{\lambda}\right)}, s_{t\left(\frac{\psi_1}{t}\right)^{\lambda}}, s_{t\left(\frac{\sigma_1}{t}\right)^{\lambda}}; 1-(1-p_1)^{\lambda}\right)$;

(iv) $\alpha_1^{\lambda} = \left(s_{t\left(\frac{\theta_1}{t}\right)^{\lambda}}, s_{t\left(1-\left(1-\frac{\psi_1}{t}\right)^{\lambda}\right)}, s_{t\left(1-\left(1-\frac{\sigma_1}{t}\right)^{\lambda}\right)}; (p_1)^{\lambda}\right)$.

Definition 3.6. Let α_1 and α_2 be two PLSVNSs and $\tau > 1$, then the generalized normalized distance between them is stated as

$$d_\tau(\alpha_1, \alpha_2) = \left\{ \frac{1}{3n} \sum_{j=1}^{n} \left(\left| p_1(x_j)\frac{\theta_1(x_j)}{t} - p_2(x_j)\frac{\theta_2(x_j)}{t} \right|^{\tau} + \left| p_1(x_j)\frac{\psi_1(x_j)}{t} - p_2(x_j)\frac{\psi_2(x_j)}{t} \right|^{\tau} + \left| p_1(x_j)\frac{\sigma_1(x_j)}{t} - p_2(x_j)\frac{\sigma_2(x_j)}{t} \right|^{\tau} \right) \right\}^{\frac{1}{\tau}} \tag{9}$$

Remark 3.1. The following observations are noted from the Definition 3.6:

1) The measure defined in Eq. (9) satisfies the properties of the distance measure.
2) For $\tau = 1, 2$, Eq. (9) reduces to normalized Hamming and Euclidean distances, respectively.

Definition 3.7. For the PLSVNNs $\alpha_j; j = 1, 2, \ldots, n$, the PLSVN weighted averaging and geometric operators, denoted by PLSVNWA and PLSVNWG respectively, are defined as:

$$\text{PLSVNWA}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \mathop{\oplus}_{j=1}^{n} \omega_j \alpha_j = \left(s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\theta_j}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{n}\left(\frac{\psi_j}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{n}\left(\frac{\sigma_j}{t}\right)^{\omega_j}\right)}; 1-\prod_{j=1}^{n}(1-p_j)^{\omega_j} \right) \tag{10}$$

and

$$\text{PLSVNWG}(\alpha_1, \alpha_2, \ldots, \alpha_n) = \mathop{\otimes}_{j=1}^{n} \alpha_j^{\omega_j} = \left(s_{t\left(\prod_{j=1}^{n}\left(\frac{\theta_j}{t}\right)^{\omega_j}\right)}, s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\psi_j}{t}\right)^{\omega_j}\right)}, s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\sigma_j}{t}\right)^{\omega_j}\right)}; \prod_{j=1}^{n} p_j^{\omega_j} \right) \tag{11}$$

where ω_j is the associated weight of α_j with $\omega_j > 0$ and $\sum_{j=1}^{n}\omega_j = 1$.

4. COPRAS method with PLSVN information

COPRAS method is mainly developed by Zavadskas et al. [40], which consists the direct and relative dependencies of the importance and the utility degree of the alternatives under the contrary criteria values. This method takes into the consideration of the given choices with regards to the distinct criteria and their corresponding weights. This method has the capability to consider the both beneficial and non-beneficial criteria, which can evaluated independently during the assessment process. The main advantage of this method is that it can be utilized to calculate the utility degree of the given choices showing the degree to which one alternative is way better or more awful than other opinions taken for comparison.

Consider a DM problem which consists of 'm' alternatives $A_1, A_2, \ldots, A_m$ which are evaluated under the set of 'n' criteria $C_1, C_2, \ldots, C_n$. During the evaluation, the preference values of each alternative $A_i; (i = 1, 2, \ldots, m)$ corresponding to criteria $C_j; (j = 1, 2, \ldots, n)$ is given in terms of PLSVNNs among their possibilities. These preference values are denoted as $\alpha_{ij} = \left(s_{\theta_{ij}}, s_{\psi_{ij}}, s_{\sigma_{ij}}; p_{ij}\right)$, where $s_{\theta_{ij}}, s_{\psi_{ij}}, s_{\sigma_{ij}}$ represents the satisfactory, indeterminant, non-satisfactory degree in linguistic term, respectively, and p_{ij} is the corresponding possibility of the LSVNN. Assume that the importance of each $C_j; (j = 1, 2, \ldots, n)$ is given by the weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$, such that $\omega_j > 0$ and $\sum_{j=1}^{n}\omega_j = 1$. In order to access and select the best alternative in terms of their priority and utility degree, the following procedural steps are presented as below.

Step 1: Arrange the information given by the decision maker(s) towards the assessment of each alternative in the form of decision matrix K as

$$K = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{bmatrix} (s_{\theta_{11}}, s_{\psi_{11}}, s_{\sigma_{11}}; p_{11}) & (s_{\theta_{12}}, s_{\psi_{12}}, s_{\sigma_{12}}; p_{12}) & \ldots & (s_{\theta_{1n}}, s_{\psi_{1n}}, s_{\sigma_{1n}}; p_{1n}) \\ (s_{\theta_{21}}, s_{\psi_{21}}, s_{\sigma_{21}}; p_{21}) & (s_{\theta_{22}}, s_{\psi_{22}}, s_{\sigma_{22}}; p_{22}) & \ldots & (s_{\theta_{2n}}, s_{\psi_{2n}}, s_{\sigma_{2n}}; p_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (s_{\theta_{m1}}, s_{\psi_{m1}}, s_{\sigma_{m1}}; p_{m1}) & (s_{\theta_{m2}}, s_{\psi_{m2}}, s_{\sigma_{m2}}; p_{m2}) & \ldots & (s_{\theta_{mn}}, s_{\psi_{mn}}, s_{\sigma_{mn}}; p_{mn}) \end{bmatrix} \end{array}$$

Step 2: Construct the weighted decision matrix $K_\omega = (\hat{\alpha}_{ij})_{m\times n}$ where $\hat{\alpha}_{ij} = \left(s_{\hat{\theta}_{ij}}, s_{\hat{\psi}_{ij}}, s_{\hat{\sigma}_{ij}}; \hat{p}_{ij}\right)$ is given by

$$\hat{\alpha}_{ij} = \omega_j\alpha_{ij} = \left(s_{t\left(1-\left(1-\frac{\theta_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(\frac{\psi_{ij}}{t}\right)^{\omega_j}}, s_{t\left(\frac{\sigma_{ij}}{t}\right)^{\omega_j}}; 1-(1-p_{ij})^{\omega_j}\right) \quad (12)$$

Step 3: Discretion the given criteria into the benefit and non-benefit types. Assume that out of n criteria, k are benefit and the remaining $n-k$ are non-benefit types. In the benefit type criteria, the higher is the rating values, the better is the fulfillment of the goal. Thus, the different rating values of such benefit type corresponding to each alternative are aggregated by utilizing the PLSVNWA operator and is given by:

$$Q_i = \overset{k}{\underset{j=1}{\oplus}}\hat{\alpha}_{ij} = \left(s_{t\left(1-\prod_{j=1}^{k}\left(1-\frac{\hat{\theta}_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{k}\left(\frac{\hat{\psi}_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{k}\left(\frac{\hat{\sigma}_{ij}}{t}\right)^{\omega_j}\right)}; 1-\prod_{j=1}^{k}(1-\hat{p}_{ij})^{\omega_j}\right) \quad (13)$$

where k, is the number of criteria that must be maximized.

Step 4: For non-benefit type criteria, the lower is the values, the better is the fulfillment of our goal. So, for obtaining the overall assessment values of each alternatives corresponding to such $n-k$ criteria, we utilize PLSVNWA operator and their aggregated values are computed as

$$T_i = \overset{n-k}{\underset{j=k+1}{\oplus}}\hat{\alpha}_{ij} = \left(s_{t\left(1-\prod_{j=k+1}^{n-k}\left(1-\frac{\hat{\theta}_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\prod_{j=k+1}^{n-k}\left(\frac{\hat{\psi}_{ij}}{t}\right)^{\omega_j}}, s_{t\prod_{j=k+1}^{n-k}\left(\frac{\hat{\sigma}_{ij}}{t}\right)^{\omega_j}}; 1-\prod_{j=k+1}^{n-k}(1-\hat{p}_{ij})^{\omega_j}\right) \quad (14)$$

where $n-k$, is the number of criteria which have cost optimization direction i.e. the criteria with non-beneficial type.

Step 5: Determine the minimal value of T_i as:

$$T_{\min} = \min_i\{T_i\}$$

where $\min_i\{T_i\} = \min\{T_1, T_2, \ldots, T_m\}$.

Step 6: Calculate the relative significance value or priority value of the alternatives by using Eq. (16) as

$$V_i = \mathrm{Sc}(Q_i) + \frac{\mathrm{Sc}(T_{\min})\sum_{i=1}^{m}\mathrm{Sc}(T_i)}{\mathrm{Sc}(T_i)\sum_{i=1}^{m}\frac{\mathrm{Sc}(T_{\min})}{\mathrm{Sc}(T_i)}}; \quad \text{provided} \quad \mathrm{Sc}(T_i) \neq 0 \quad (16)$$

The value of V_i expresses the degree of satisfaction earned by the alternative. The maximum value of V_i represents the higher priority of that alternative i.e. $V_{\max}$ is the best choice among the given choices.

Step 7: Compute the quantitative utility for each alternative as

$$U_i = \left(\frac{V_i}{V_{\max}}\right) \times 100\% \quad (17)$$

where $V_{\max} = \max_i\{V_i\}$ is non-zero. It can be easily seen that utility degree is directly related with the relative priorities of the alternatives i.e. if the value of the relative priority increase or decrease then the value of utility degree is also going to increase or decrease. The utility value is attained by comparing the priority of alternatives with the most efficient alternative hence it ranges from 0% to 100%.

Hence, this COPRAS method permits for assessing the direct and relative dependence of priorities and utility degree of the considered alternatives in a DM problem including different criteria, their weights and the rating values of alternative with regard to all the criteria.

5. Approaches for solving DM problems

This section offers two different DM approaches which utilize the COPRAS technique and the aggregation operators with some new information measures under the PLSVNNs. The presented approaches are illustrated with a numerical example.

5.1. Description of problem

Consider a group decision-making problem which consists 'm' alternatives $A_1, A_2, \ldots, A_m$, 'n' criteria $C_1, C_2, \ldots, C_n$ and 'l' decision makers $K^{(1)}, K^{(2)}, \ldots, K^{(l)}$ which are evaluated the given alternatives under PLSVN environment. Each decision maker $K^{(q)}, q = 1, 2, \ldots, l$ represent their preferences in the form of PLSVNN $\alpha_{ij}^{(q)} = \left(s_{\theta_{ij}^{(q)}}, s_{\psi_{ij}^{(q)}}, s_{\sigma_{ij}^{(q)}}; p_{ij}^{(q)}\right)$ corresponding to i^{th} alternative $A_i (i = 1, 2, \ldots, m)$ rating on j^{th} criteria $C_j (j = 1, 2, \ldots, n)$. The complete decision matrix $M^{(q)}$ of the q^{th} decision maker towards the alternatives is summarized as

$$M^{(q)} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array} \\ \begin{pmatrix} \left(s_{\theta_{11}^{(q)}}, s_{\psi_{11}^{(q)}}, s_{\sigma_{11}^{(q)}}; p_{11}^{(q)}\right) & \left(s_{\theta_{12}^{(q)}}, s_{\psi_{12}^{(q)}}, s_{\sigma_{12}^{(q)}}; p_{12}^{(q)}\right) & \ldots & \left(s_{\theta_{1n}^{(q)}}, s_{\psi_{1n}^{(q)}}, s_{\sigma_{1n}^{(q)}}; p_{1n}^{(q)}\right) \\ \left(s_{\theta_{21}^{(q)}}, s_{\psi_{21}^{(q)}}, s_{\sigma_{21}^{(q)}}; p_{21}^{(q)}\right) & \left(s_{\theta_{22}^{(q)}}, s_{\psi_{22}^{(q)}}, s_{\sigma_{22}^{(q)}}; p_{22}^{(q)}\right) & \ldots & \left(s_{\theta_{2n}^{(q)}}, s_{\psi_{2n}^{(q)}}, s_{\sigma_{2n}^{(q)}}; p_{2n}^{(q)}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(s_{\theta_{m1}^{(q)}}, s_{\psi_{m1}^{(q)}}, s_{\sigma_{m1}^{(q)}}; p_{m1}^{(q)}\right) & \left(s_{\theta_{m2}^{(q)}}, s_{\psi_{m2}^{(q)}}, s_{\sigma_{m2}^{(q)}}; p_{m2}^{(q)}\right) & \ldots & \left(s_{\theta_{mn}^{(q)}}, s_{\psi_{mn}^{(q)}}, s_{\sigma_{mn}^{(q)}}; p_{mn}^{(q)}\right) \end{pmatrix} \end{array}$$

Assume that the importance of each decision maker and criterion are given in the form of weights as $\left(\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(l)}\right)$ and $(\omega_1, \omega_2, \ldots, \omega_n)$ respectively such that $0 < \lambda^{(q)}, \omega_j \leqslant 1$ and $\sum_{q=1}^{l}\lambda^{(q)} = 1; \sum_{j=1}^{n}\omega_j = 1$. If the weights of such criteria and decision makers are known a priori then we can easily utilized. On the other hand, if the information about them are completely unknown then we utilize the following entropy measure procedure to compute the weight vector of each decision maker and criteria.

(i) *Weight vector for decision maker* $K^{(q)}, q = 1, 2, \ldots, l$.

For the collection of 'l' decision maker $K^{(1)}, K^{(2)}, \ldots, K^{(l)}$, the entropy $E^{(q)}, (q = 1, 2, \ldots, l)$ is defined as

$$E^{(q)} = \frac{1}{m(\sqrt{(2)}-1)} \sum_{i=1}^{m} \left[\sin\left(\frac{\pi \gamma_i^{(q)}}{2}\right) + \sin\left(\frac{\pi\left(1-\gamma_i^{(q)}\right)}{2}\right) - 1 \right] \tag{18}$$

where $\gamma_i^{(q)} = \frac{1}{3tn}\sum_{j=1}^{n} p_{ij}^{(q)}\left(2t + \theta_{ij}^{(q)} - \psi_{ij}^{(q)} - \sigma_{ij}^{(q)}\right)$.

In DM problems, if decision makers have comparable almost distinctive choices for different alternatives, the entropy value will be bigger. That is, the more prominent value of entropy for decision maker, the smaller is the differences between the choices. According to entropy theory, if the entropy value is smaller over the choices then more valuable information is provided by the decision maker in the DM procedure. Therefore, a decision maker with small entropy value will be given the higher priority than others. Then, by combining the above aspects, the weights for decision maker $K^{(q)}; (q = 1, 2, \ldots, l)$ based on the entropy $E^{(q)}$ is defined as:

$$\lambda^{(q)} = \frac{1 - E^{(q)}}{l - \sum_{q=1}^{l} E^{(q)}} \tag{19}$$

where $\lambda^{(q)} \in (0, 1]$ and $\sum_{q=1}^{l}\lambda^{(q)} = 1$. From the Eq. (19), it can easily conclude that smaller the value of $E^{(q)}$, the greater weight will be assigned to the decision maker $K^{(q)}$.

(ii) *weight vector for each criterion* $C_j (j = 1, 2, \ldots, n)$

For the set of 'n' criteria $C_1, C_2, \ldots, C_n$ of the collective decision matrix, the entropy $F_j, (j = 1, 2, \ldots, n)$ is defined as

$$F_j = \frac{1}{m\left(\sqrt{2}-1\right)} \sum_{i=1}^{m} \left[\sin\left(\frac{\pi \zeta_{ij}}{2}\right) + \sin\left(\frac{\pi(1-\zeta_{ij})}{2}\right) - 1 \right] \tag{20}$$

where

$$\begin{aligned}\zeta_{ij} &= 1 - d_2\left(\alpha_{ij}^{(q)}, \alpha_{ij}^{+}\right) \\ &= \frac{1}{l}\sum_{q=1}^{l}\left[1 - \left(\frac{1}{3}|p_{ij}\frac{\theta_{ij}^{(q)}}{t} - p_{ij}^{+}\frac{\theta_{ij}^{+}}{t}|^2 + |p_{ij}\frac{\psi_{ij}^{(q)}}{t} - p_{ij}^{+}\frac{\psi_{ij}^{-}}{t}|^2 + |p_{ij}\frac{\sigma_{ij}^{(q)}}{t} - p_{ij}^{+}\frac{\sigma_{ij}^{-}}{t}|^2\right)^{\frac{1}{2}}\right]\end{aligned} \tag{21}$$

Here $\alpha_{ij}^{+} = \left(s_{\theta_{ij}^{+}}, s_{\psi_{ij}^{-}}, s_{\sigma_{ij}^{-}}, p_{ij}^{+}\right)$ is a positive ideal solution and $d_2\left(\alpha_{ij}, \alpha_{ij}^{+}\right)$ is the Euclidean distance between the each preference value of decision matrices and the positive ideal preference values α_{ij}^{+}. In order to evaluate the information by considering all the aspects, we define the weights of criteria by using entropy F_j as follows:

$$\omega_j = \frac{1 - F_j}{n - \sum_{j=1}^{n} F_j} \tag{22}$$

where $\omega_j \in (0, 1]$ and $\sum_{j=1}^{n}\omega_j = 1$. From the Eq. (22), it can easily conclude that smaller the value of F_j, the greater weight will be assigned to the criteria ω_j.

Now, based on the collective information under PLSVN environment and the weight vector, we develop two new approaches based on COPRAS and the aggregation operators to find the most accessible alternative(s). The procedure steps falls under these approaches are described as below.

5.2. Approach based on COPRAS method

In this section, we present a decision making approach to rank the given alternative(s) by using proposed COPRAS method under PLSVN environment. The steps involved for solving the group decision making problems under this approach are summarized as below, where the flow chart is presented in Fig. 1.

Step 1: Aggregate the preferences of each decision matrix given by the different decision makers $K^{(q)}$ in the form of the decision matrices $M^{(q)} = \left(\alpha_{ij}^{(q)}\right)$ into a single decision matrix $M = (\alpha_{ij})$ by utilizing the PLSVNWA operator.

Step 2: Compute the weighted decision matrix K_ω by using Eq. (12).

Step 3: By using Eq. (13) to compute the aggregated values of the weighted decision matrix for benefit type criteria.

Step 4: Utilize Eq. (14) to aggregate the rating values of the cost type criteria into a single ones.

Step 5: Compute the relative priority values of each alternative by using Eqs. (15) and (16).

Step 6: Determine the utility degree for each alternative by using Eq. (17).

Step 7: Rank the alternative according to decreasing order of the utility degree.

5.3. Approach based on the aggregation operators

In this section, we present a decision making approach to rank the given alternative(s) by using the proposed weighted averaging or geometric aggregation operators. The steps followed under this approach are summarized as below, where the flow chart is presented in Fig. 2.

Step 1: In the decision matrix $M^{(q)}$, transform the rating values $\left(\alpha_{ij}^{(q)}\right)$ of each cost type criteria into the benefit type by using the normalizing formula as

$$r_{ij}^{(q)} = \begin{cases} \left(s_{\theta_{ij}^{(q)}}, s_{\psi_{ij}^{(q)}}, s_{\sigma_{ij}^{(q)}}; p_{ij}^{(q)}\right); & \text{for benefit type criteria} \\ \left(s_{\sigma_{ij}^{(q)}}, s_{\psi_{ij}^{(q)}}, s_{\theta_{ij}^{(q)}}; 1 - p_{ij}^{(q)}\right); & \text{for cost type criteria} \end{cases} \tag{23}$$

and hence obtain the normalized decision matrix $H^{(q)} = \left(r_{ij}^{(q)}\right)$ corresponding to each decision maker.

Step 2: Aggregate the preferences of each decision maker towards the each alternative into the collective one. For it, the preferences values $r_{ij}^{(q)}, q = 1, 2, \ldots, l$ are aggregated either by using PLSVNWA or PLSVNWG operator. For instance, if we utilize PLSVNWA operator and weight vector $\lambda^{(q)}$ to aggregate each decision maker preferences corresponding to each alternative then the overall values of alternative A_i under criteria C_j, denoted by $\alpha_{ij} = \left(s_{\theta_{ij}}, s_{\psi_{ij}}, s_{\sigma_{ij}}; p_{ij}\right)$ is given by

$$\begin{aligned}\alpha_{ij} &= \text{PLSVNWA}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)}\right) \\ &= \left(s_{t\left(1-\prod_{q=1}^{l}\left(1-\frac{\theta_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}, s_{t\left(\prod_{q=1}^{l}\left(\frac{\psi_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}, s_{t\left(\prod_{q=1}^{l}\left(\frac{\sigma_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}; 1 - \prod_{q=1}^{l}\left(1 - p_{ij}^{(q)}\right)^{\lambda^{(q)}}\right)\end{aligned} \tag{24}$$

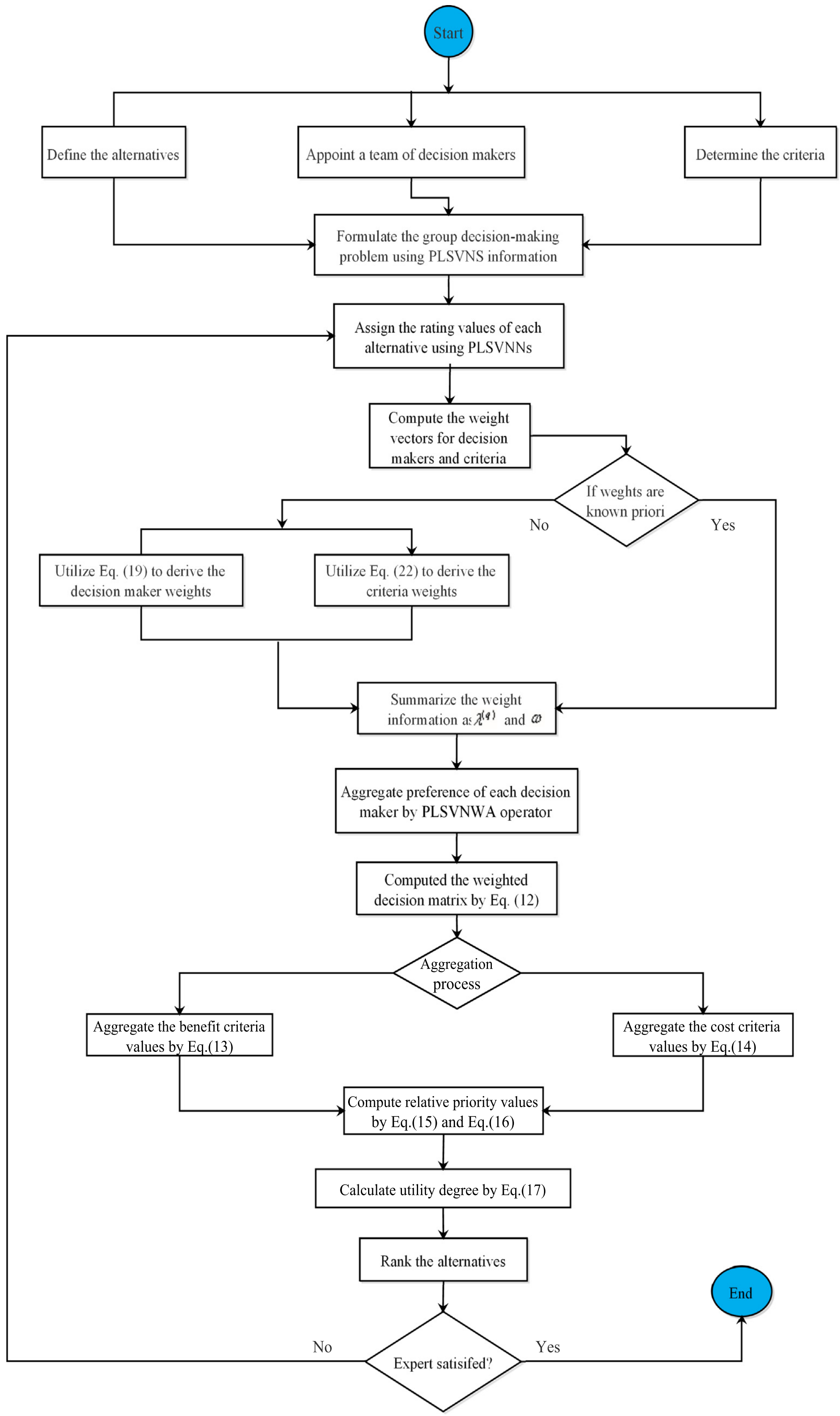

Fig. 1. Flowchart of the proposed approach based on COPRAS method.

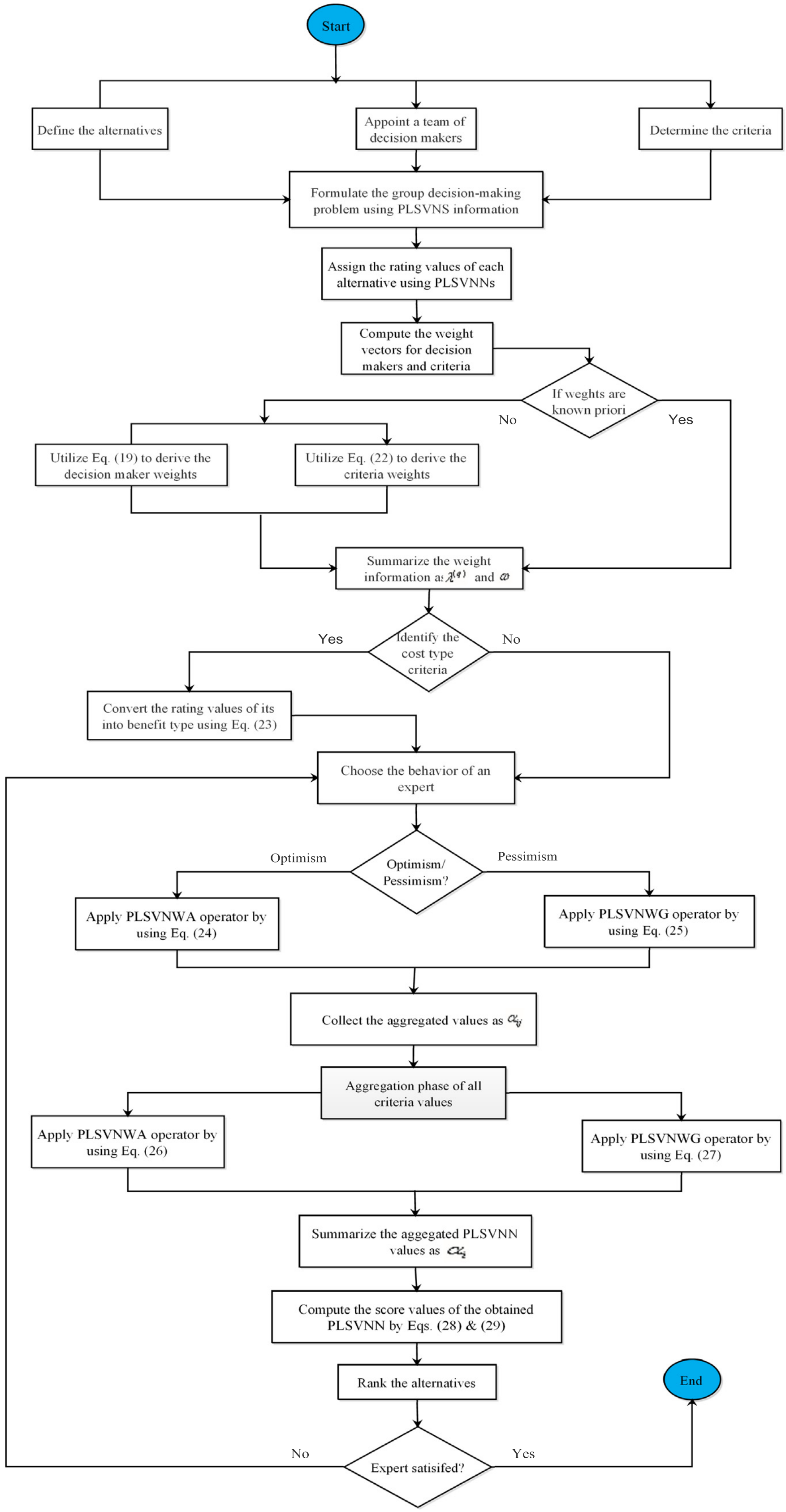

Fig. 2. Flowchart of the proposed approach based on aggregation operators.

On the other hand, if we utilize PLSVNWG operator to aggregate the preference values of each decision maker then the aggregated values are computed as

$$\begin{aligned}\alpha_{ij} =& \mathrm{PLSVNWG}\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)}\right)\\ =& \left(s_{t\left(\prod_{q=1}^{l}\left(\frac{\theta_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}, s_{t\left(1-\prod_{q=1}^{l}\left(1-\frac{\psi_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}, s_{t\left(1-\prod_{q=1}^{l}\left(1-\frac{\sigma_{ij}^{(q)}}{t}\right)^{\lambda^{(q)}}\right)}, \prod_{q=1}^{l}\left(p_{ij}^{(q)}\right)^{\lambda^{(q)}}\right)\end{aligned} \tag{25}$$

Step 3: Aggregate the values $\alpha_{ij}(j = 1, 2, \ldots, n)$ by utilizing the weight vector of criteria $\omega_j(j = 1, 2, \ldots, n)$ and the appropriate averaging or geometric aggregation operator to get the collective values of each alternative $\alpha_i(i = 1, 2, \ldots, m)$. For instance, if we take PLSVNWA operator to aggregate each value then we get the collective value $\alpha_i = \left(s_{\theta_i}, s_{\psi_i}, s_{\sigma_i}; p_i\right)$ as

$$\begin{aligned}\alpha_i =& \mathrm{PLSVNWA}(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in})\\ =& \left(s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\theta_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{n}\left(\frac{\psi_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(\prod_{j=1}^{n}\left(\frac{\sigma_{ij}}{t}\right)^{\omega_j}\right)}; 1-\prod_{j=1}^{n}(1-p_{ij})^{\omega_j}\right)\end{aligned} \tag{26}$$

On the other hand, by taking PLSVNWG operator to aggregate the values, we get

$$\begin{aligned}\alpha_i =& \mathrm{PLSVNWG}(\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in})\\ =& \left(s_{t\left(\prod_{j=1}^{n}\left(\frac{\theta_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\psi_{ij}}{t}\right)^{\omega_j}\right)}, s_{t\left(1-\prod_{j=1}^{n}\left(1-\frac{\sigma_{ij}}{t}\right)^{\omega_j}\right)}; \prod_{j=1}^{n}(p_{ij})^{\omega_j}\right)\end{aligned} \tag{27}$$

Step 4: Compute the score value of the aggregate number $\alpha_i = \left(s_{\theta_i}, s_{\psi_i}, s_{\sigma_i}; p_i\right), i = 1, 2, \ldots, n$ by using Eq. (7) as

$$\mathrm{Sc}(\alpha_i) = \frac{p_i(2t + \theta_i - \psi_i - \sigma_i)}{3t}$$

However, if the score values of two aggregated PLSVNNs are equal then, calculate their accuracy values using the Eq. (8):

$$H(\alpha_i) = \frac{p_i(\theta_i + \psi_i + \sigma_i)}{3t} \tag{29}$$

Step 5: Obtain the ranking order of alternatives using Definition 3.3 and hence select the most desirable one(s).

6. Numerical example

The above presented approaches are illustrated with a numerical example related to outsourcing supplier selection problem and compared their results with some of the existing approaches.

In the following case study, we discuss about the IT Outsourcing Selection problem. Millennium semiconductors (MS), established in October 1995, is an ISO 9001 − 2015 organization with distribution of electronic components as its core expertise. MS is the leading distributor of electronic components in India is synonymous with innovation and today it is one of the most reputed name in market. MS has established roots in almost every region of India with catering more than 1500 customers all segments from last two decades. It participates in innovative work, creation and showcasing of items, for example, full shading ultra brilliance LED epitaxial items, chips, compound sun powered cells and high power concentrating sun oriented items. The branch workplaces of MS is situated in Delhi, Bangalore, Hyderabad, Ahmedabad, Chennai and Mumbai in India and abroad workplaces in Singapore and Shenzhen (China). MS contributes the extraordinary layer part of labour and financial resources to its competition rather than IT. The outsourcing of IT is a better option for MS as of its lack of ability to do it efficiently. Therefore, MS selects the following outsourcing providers: Tata Consultancy Services (A_1), Infosys (A_2), Wipro (A_3), HCl (A_4) and TatvaSoft (A_5) under the four criteria namely, Design development (C_1), Quality product (C_2), Delivery time (C_3) and Cost (C_4). Now to find the more suitable or best outsourcing provider among the above choices, MS hires the three decision makers $K^{(1)}, K^{(2)}$ and $K^{(3)}$ who have the responsibilities to evaluate the given alternatives and rate their preferences in terms of PLSVNN on the basis of linguistic term set $Q = \{s_0 = \text{“extremelypoor”}, s_1 = \text{“verypoor”}, s_2 = \text{“poor”}, s_3 = \text{“slightlypoor”}, s_4 = \text{“fair”}, s_5 = \text{“slightlygood”}, s_6 = \text{“good”}, s_7 = \text{“verygood”}, s_8 = \text{“extremelygood”}\}$. The rating values of these decision makers are summarized in the form of the matrices in Tables 1–3 respectively.

In order to compute the importance of each decision maker and criterion, we find the weight vector associated with them by using rating values of each decision maker summarized in Tables 1–3. For it, by using Eqs. (18) and (19), we compute the weight vector to the decision maker $K^{(q)}, q = 1, 2, 3$ as $\lambda^{(1)} = 0.3333, \lambda^{(2)} = 0.3337$ and $\lambda^{(3)} = 0.3330$. Similarly, the weight vectors corresponding to each criteria are computed by utilizing Eqs. (20) and (22) and hence we get $\omega_1 = 0.3188, \omega_2 = 0.2345, \omega_3 = 0.1666$ and $\omega_4 = 0.2801$. Then, based on these information, we applied the above developed two approaches to find the most desirable alternative(s).

Table 1
PLSVN decision matrix given by decision maker $K^{(1)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_6, s_2, s_1;\ 0.3)$	$(s_6, s_2, s_6;\ 0.4)$	$(s_6, s_2, s_4;\ 0.7)$	$(s_3, s_2, s_2;\ 0.6)$
A_2	$(s_5, s_1, s_2;\ 0.4)$	$(s_5, s_1, s_1;\ 0.6)$	$(s_4, s_2, s_2;\ 0.7)$	$(s_6, s_1, s_2;\ 0.5)$
A_3	$(s_4, s_2, s_3;\ 0.5)$	$(s_5, s_2, s_3;\ 0.7)$	$(s_4, s_4, s_2;\ 0.8)$	$(s_5, s_2, s_4;\ 0.4)$
A_4	$(s_6, s_3, s_1;\ 0.6)$	$(s_3, s_2, s_5;\ 0.7)$	$(s_3, s_2, s_2;\ 0.6)$	$(s_6, s_3, s_3;\ 0.5)$
A_5	$(s_4, s_2, s_4;\ 0.4)$	$(s_5, s_3, s_2;\ 0.3)$	$(s_5, s_2, s_1;\ 0.5)$	$(s_4, s_1, s_1;\ 0.6)$

Table 2
PLSVN decision matrix given by decision maker $K^{(2)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_6, s_2, s_2;\ 0.4)$	$(s_6, s_1, s_2;\ 0.8)$	$(s_6, s_2, s_3;\ 0.6)$	$(s_6, s_4, s_3;\ 0.5)$
A_2	$(s_5, s_1, s_2;\ 0.4)$	$(s_6, s_2, s_3;\ 0.2)$	$(s_4, s_3, s_2;\ 0.3)$	$(s_5, s_1, s_2;\ 0.4)$
A_3	$(s_4, s_2, s_2;\ 0.8)$	$(s_3, s_2, s_2;\ 0.9)$	$(s_4, s_2, s_2;\ 0.6)$	$(s_5, s_2, s_3;\ 0.7)$
A_4	$(s_6, s_2, s_1;\ 0.7)$	$(s_3, s_1, s_2;\ 0.6)$	$(s_5, s_4, s_2;\ 0.2)$	$(s_6, s_2, s_2;\ 0.5)$
A_5	$(s_4, s_4, s_1;\ 0.4)$	$(s_3, s_3, s_3;\ 0.3)$	$(s_5, s_2, s_1;\ 0.2)$	$(s_4, s_2, s_1;\ 0.5)$

Table 3
PLSVN decision matrix given by decision maker $K^{(3)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_5, s_1, s_2;\ 0.5)$	$(s_5, s_4, s_2;\ 0.7)$	$(s_2, s_1, s_2;\ 0.6)$	$(s_5, s_2, s_2;\ 0.2)$
A_2	$(s_6, s_2, s_2;\ 0.2)$	$(s_6, s_3, s_2;\ 0.5)$	$(s_3, s_4, s_2;\ 0.5)$	$(s_4, s_2, s_1;\ 0.5)$
A_3	$(s_5, s_3, s_2;\ 0.6)$	$(s_4, s_3, s_2;\ 0.4)$	$(s_4, s_3, s_2;\ 0.7)$	$(s_3, s_1, s_2;\ 0.7)$
A_4	$(s_4, s_3, s_2;\ 0.7)$	$(s_3, s_2, s_2;\ 0.7)$	$(s_6, s_4, s_2;\ 0.6)$	$(s_6, s_3, s_3;\ 0.6)$
A_5	$(s_4, s_2, s_1;\ 0.8)$	$(s_4, s_2, s_4;\ 0.6)$	$(s_5, s_2, s_2;\ 0.5)$	$(s_6, s_3, s_1;\ 0.5)$

6.1. Results based on COPRAS method

The following steps of the proposed approach are executed on to the considered data to find the most suitable alternative(s).

Step 1: Aggregate the given preferences values summarized in Tables 1–3 by using PLSVNWA operator. The resultant matrix is represented in Table 4.
Step 2: By using Eq. (12), the weighted decision matrix K_ω is computed and their values are summarized in Table 5.
Step 3: By using Eq. (13), the aggregated values of all benefit type criteria corresponding to each alternative $A_i (i = 1, 2, \ldots, 5)$ are computed and get $Q_1 = (s_{3.9969}, s_{3.4511}, s_{3.7607}; 0.2726)$, $Q_2 = (s_{3.8201}, s_{3.1345}, s_{3.5190}; 0.2320)$, $Q_3 = (s_{2.7389}, s_{4.0031}, s_{4.0034}; 0.3567)$, $Q_4 = (s_{3.0427}, s_{3.8353}, s_{3.4440}; 0.3466)$ and $Q_5 = (s_{2.5758}, s_{4.2611}, s_{3.7606}; 0.2721)$.
Step 4: The aggregated values of the alternatives corresponding to the cost type criteria are computed by using Eq. (14) and we get $T_1 = (s_{2.8214}, s_{3.8850}, s_{4.7547}; 0.2262)$, $T_2 = (s_{2.5772}, s_{5.0721}, s_{4.0373}; 0.2125)$, $T_3 = (s_{2.3203}, s_{5.0720}, s_{4.7723}; 0.2804)$, $T_4 = (s_{3.3650}, s_{5.2943}, s_{4.6455}; 0.2256)$ and $T_5 = (s_{2.7551}, s_{4.3069}, s_{3.2846}; 0.2137)$.
Step 5: By using Eq. (16), the relative priority values of each alternative are computed and get $V_1 = 0.2357$, $V_2 = 0.2308$, $V_3 = 0.2472$, $V_4 = 0.2678$ and $V_5 = 0.2070$.
Step 6: The utility degrees for each alternative $A_i (i = 1, 2, \ldots, 5)$ are computed by using Eq. (17) and hence we get $U_1 = 87.9963$, $U_2 = 86.1720$, $U_3 = 92.2850$, $U_4 = 100$ and $U_5 = 77.2776$.
Step 7: Based on the utility degree the ranking order of the given set of alternative is $A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$ and hence the best one is A_4.

6.2. Results based on aggregation operators

The following steps of the proposed approach based on the aggregation operators are executed to find the most desirable alternative(s).

Step 1: Since the criteria C_3 and C_4 are the cost types while other are the benefit types, so by utilizing Eq. (23), we normalize the rating values of each decision maker. The updated values are summarized in decision matrices given in Tables 6–8 respectively.
Step 2: By taking weights of decision makers $\lambda^{(q)}, q = 1, 2, 3$, we aggregate preference values corresponding to each decision maker by using PLSVNWA and PLSVNWG operators. The aggregated values corresponding to them are summarized in Table 9 and 10 respectively.
Step 3a: By taking weight $\omega_j (j = 1, 2, 3, 4)$ of the criteria and preference value summarized in Table 9, we compute the final aggregated values of each alterative $A_i (i = 1, 2, \ldots, 5)$ by using PLSVNWA operator. Their corresponding values are obtained as $\alpha_1 = (s_{4.6502}, s_{1.6759}, s_{2.8675}; 0.5346)$, $\alpha_2 = (s_{4.2708}, s_{1.9873}, s_{2.6949}; 0.4826)$, $\alpha_3 = (s_{3.9177}, s_{2.5380}, s_{2.9814}; 0.5806)$, $\alpha_4 = (s_{3.8576}, s_{2.5382}, s_{2.8849}; 0.6080)$ and $\alpha_5 = (s_{2.9333}, s_{2.2940}, s_{2.9741}; 0.5278)$.
Step 3b: On the other hand, by utilizing the information values given in Table 10 and PLSVNWG operator to compute the aggregated value for each alterative $A_i (i = 1, 2, \ldots, 5)$, we get $\alpha_1 = (s_{3.9210}, s_{1.8807}, s_{3.8801}; 0.4893)$, $\alpha_2 = (s_{3.2653}, s_{2.3319}, s_{3.3192}; 0.4396)$, $\alpha_3 = (s_{3.6117}, s_{2.6842}, s_{3.3163}; 0.5199)$, $\alpha_4 = (s_{3.3501}, s_{2.8224}, s_{4.1309}; 0.5920)$ and $\alpha_5 = (s_{2.2265}, s_{2.4149}, s_{3.7717}; 0.4811)$.

Table 6
Normalized PLSVN decision matrix for $K^{(1)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_6, s_2, s_1;\ 0.3)$	$(s_6, s_2, s_6;\ 0.4)$	$(s_4, s_2, s_6;\ 0.3)$	$(s_2, s_2, s_3;\ 0.4)$
A_2	$(s_5, s_1, s_2;\ 0.4)$	$(s_5, s_1, s_1;\ 0.6)$	$(s_2, s_2, s_4;\ 0.3)$	$(s_2, s_1, s_6;\ 0.5)$
A_3	$(s_4, s_2, s_3;\ 0.5)$	$(s_5, s_2, s_3;\ 0.7)$	$(s_2, s_4, s_4;\ 0.2)$	$(s_4, s_2, s_5;\ 0.6)$
A_4	$(s_6, s_3, s_1;\ 0.6)$	$(s_3, s_2, s_5;\ 0.7)$	$(s_2, s_2, s_3;\ 0.4)$	$(s_3, s_3, s_6;\ 0.5)$
A_5	$(s_4, s_2, s_4;\ 0.4)$	$(s_5, s_3, s_2;\ 0.3)$	$(s_1, s_2, s_5;\ 0.5)$	$(s_1, s_1, s_4;\ 0.4)$

Table 7
Normalized PLSVN decision matrix given for $K^{(2)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_6, s_2, s_2;\ 0.4)$	$(s_6, s_1, s_2;\ 0.8)$	$(s_3, s_2, s_6;\ 0.4)$	$(s_3, s_4, s_6;\ 0.5)$
A_2	$(s_5, s_1, s_2;\ 0.4)$	$(s_6, s_2, s_3;\ 0.2)$	$(s_2, s_3, s_4;\ 0.7)$	$(s_2, s_1, s_5;\ 0.6)$
A_3	$(s_4, s_2, s_2;\ 0.8)$	$(s_3, s_2, s_2;\ 0.9)$	$(s_2, s_2, s_4;\ 0.4)$	$(s_3, s_2, s_5;\ 0.3)$
A_4	$(s_6, s_2, s_1;\ 0.7)$	$(s_3, s_1, s_2;\ 0.6)$	$(s_2, s_4, s_5;\ 0.8)$	$(s_2, s_2, s_6;\ 0.5)$
A_5	$(s_4, s_4, s_1;\ 0.4)$	$(s_3, s_3, s_3;\ 0.3)$	$(s_1, s_2, s_5;\ 0.8)$	$(s_1, s_2, s_4;\ 0.5)$

Table 8
Normalized PLSVN decision matrix for $K^{(3)}$.

	C_1	C_2	C_3	C_4
A_1	$(s_5, s_1, s_2;\ 0.5)$	$(s_5, s_4, s_2;\ 0.7)$	$(s_2, s_1, s_2;\ 0.4)$	$(s_2, s_2, s_5;\ 0.8)$
A_2	$(s_6, s_2, s_2;\ 0.2)$	$(s_6, s_3, s_2;\ 0.5)$	$(s_2, s_4, s_3;\ 0.5)$	$(s_1, s_2, s_4;\ 0.5)$
A_3	$(s_5, s_3, s_2;\ 0.6)$	$(s_4, s_3, s_2;\ 0.4)$	$(s_2, s_3, s_4;\ 0.3)$	$(s_2, s_1, s_3;\ 0.3)$
A_4	$(s_4, s_3, s_2;\ 0.7)$	$(s_3, s_2, s_2;\ 0.7)$	$(s_2, s_4, s_6;\ 0.4)$	$(s_3, s_3, s_6;\ 0.4)$
A_5	$(s_4, s_2, s_1;\ 0.8)$	$(s_4, s_2, s_4;\ 0.6)$	$(s_2, s_2, s_5;\ 0.5)$	$(s_1, s_3, s_6;\ 0.5)$

Table 4
Aggregated decision matrix by PLSVNWA operator used in COPRAS method.

	C_1	C_2	C_3	C_4
A_1	$(s_{5.7109}, s_{1.5878}, s_{1.5871};\ 0.4056)$	$(s_{5.7108}, s_{1.9991}, s_{2.8845};\ 0.6699)$	$(s_{5.116}, s_{1.5878}, s_{2.8849};\ 0.6366)$	$(s_{4.8932}, s_{2.5204}, s_{2.2897};\ 0.4572)$
A_2	$(s_{5.3789}, s_{1.2596}, s_{2.0000};\ 0.4233)$	$(s_{5.7106}, s_{1.8169}, s_{1.5874};\ 0.4570)$	$(s_{3.6914}, s_{2.8842}, s_{2.0000};\ 0.5281)$	$(s_{5.1158}, s_{1.2596}, s_{1.5878};\ 0.4686)$
A_3	$(s_{4.3654}, s_{2.2891}, s_{2.2894};\ 0.6581)$	$(s_{4.0848}, s_{2.2891}, s_{2.2894};\ 0.7382)$	$(s_{4.0000}, s_{2.8841}, s_{2.0000};\ 0.7115)$	$(s_{4.4437}, s_{1.5878}, s_{2.8849};\ 0.6220)$
A_4	$(s_{5.4807}, s_{2.6204}, s_{1.2596};\ 0.6698)$	$(s_{3.0000}, s_{1.5870}, s_{2.7144};\ 0.6698)$	$(s_{4.8924}, s_{3.1748}, s_{2.0000};\ 0.4959)$	$(s_{6.0000}, s_{2.6204}, s_{2.6204};\ 0.5358)$
A_5	$(s_{4.0000}, s_{2.5204}, s_{1.5874};\ 0.5838)$	$(s_{4.0848}, s_{2.6211}, s_{2.8842};\ 0.4190)$	$(s_{5.0000}, s_{2.0000}, s_{1.2596};\ 0.4151)$	$(s_{4.8245}, s_{1.8169}, s_{1.0000};\ 0.5358)$

Table 5
Weighted decision matrix.

	C_1	C_2	C_3	C_4
A_1	$(s_{2.6315}, s_{4.7775}, s_{4.7772};\ 0.1316)$	$(s_{2.0346}, s_{5.7788}, s_{6.2977};\ 0.1624)$	$(s_{1.2506}, s_{6.1108}, s_{6.7500};\ 0.1097)$	$(s_{1.8619}, s_{5.0860}, s_{5.6353};\ 0.1308)$
A_2	$(s_{2.3946}, s_{4.4376}, s_{5.1423};\ 0.1347)$	$(s_{2.0344}, s_{5.6507}, s_{5.4746};\ 0.1096)$	$(s_{0.7836}, s_{6.7497}, s_{6.3504};\ 0.0905)$	$(s_{1.9884}, s_{6.0116}, s_{5.0860};\ 0.1341)$
A_3	$(s_{1.7790}, s_{5.3685}, s_{5.3687};\ 0.2160)$	$(s_{1.2344}, s_{5.9654}, s_{5.9656};\ 0.1796)$	$(s_{0.8724}, s_{6.7496}, s_{6.3504};\ 0.1232)$	$(s_{1.6251}, s_{6.0115}, s_{6.0120};\ 0.1793)$
A_4	$(s_{2.4650}, s_{5.6049}, s_{4.4376};\ 0.2199)$	$(s_{0.8350}, s_{5.4743}, s_{6.2086};\ 0.1624)$	$(s_{1.1659}, s_{6.8585}, s_{6.3504};\ 0.0848)$	$(s_{2.5743}, s_{6.1754}, s_{5.8522};\ 0.1538)$
A_5	$(s_{1.5861}, s_{5.5358}, s_{4.7772};\ 0.1909)$	$(s_{1.2344}, s_{6.1579}, s_{6.2976};\ 0.1003)$	$(s_{1.2059}, s_{6.3504}, s_{5.8796};\ 0.0707)$	$(s_{1.8242}, s_{5.4257}, s_{4.4683};\ 0.1538)$

Table 9
Aggregated decision matrix by PLSVNWA operator.

	C_1	C_2	C_3	C_4
A_1	$(s_{5.7109}, s_{1.5878}, s_{1.5874};\ 0.4056)$	$(s_{5.7109}, s_{1.9990}, s_{2.8844};\ 0.6699)$	$(s_{3.0678}, s_{1.5878}, s_{4.1617};\ 0.3684)$	$(s_{2.3542}, s_{2.5205}, s_{4.4818};\ 0.6084)$
A_2	$(s_{5.3789}, s_{1.2596}, s_{2.0000};\ 0.4232)$	$(s_{5.7106}, s_{1.8169}, s_{1.5874};\ 0.4570)$	$(s_{2.0000}, s_{2.8843}, s_{3.6346};\ 0.5283)$	$(s_{1.6840}, s_{1.2596}, s_{4.9328};\ 0.5359)$
A_3	$(s_{4.3654}, s_{2.2891}, s_{2.2894};\ 0.6581)$	$(s_{4.0848}, s_{2.2891}, s_{2.2894};\ 0.7381)$	$(s_{2.7593}, s_{2.8840}, s_{4.0000};\ 0.3048)$	$(s_{3.8408}, s_{1.5878}, s_{4.2179};\ 0.4191)$
A_4	$(s_{5.4807}, s_{2.6204}, s_{1.2596};\ 0.6698)$	$(s_{3.0000}, s_{1.5870}, s_{2.7143};\ 0.6698)$	$(s_{2.0000}, s_{3.1749}, s_{4.4812};\ 0.5842)$	$(s_{3.0000}, s_{2.6204}, s_{6.0000};\ 0.4687)$
A_5	$(s_{4.0000}, s_{2.5205}, s_{1.5873};\ 0.5838)$	$(s_{4.0848}, s_{2.6211}, s_{2.8843};\ 0.4190)$	$(s_{1.3503}, s_{2.0000}, s_{5.0000};\ 0.6317)$	$(s_{1.0000}, s_{1.8169}, s_{4.5782};\ 0.4687)$

Table 10
Aggregated decision matrix by PLSVNWG operator.

	C_1	C_2	C_3	C_4
A_1	$(s_{5.6466}, s_{1.6840}, s_{1.6837};\ 0.3915)$	$(s_{5.6467}, s_{2.4811}, s_{3.8397};\ 0.6074)$	$(s_{2.8849}, s_{1.6840}, s_{5.1166};\ 0.3634)$	$(s_{2.2898}, s_{2.7593}, s_{4.8933};\ 0.5428)$
A_2	$(s_{5.3130}, s_{1.3503}, s_{2.0000};\ 0.3636)$	$(s_{5.6463}, s_{2.0577}, s_{1.6837};\ 0.3914)$	$(s_{2.0000}, s_{3.0672}, s_{3.6915};\ 0.4718)$	$(s_{1.5878}, s_{1.3503}, s_{5.1157};\ 0.5314)$
A_3	$(s_{4.3085}, s_{2.3534}, s_{2.3537};\ 0.6215)$	$(s_{3.9144}, s_{2.3534}, s_{2.3537};\ 0.6318)$	$(s_{2.5205}, s_{3.0672}, s_{4.0000};\ 0.2885)$	$(s_{3.4210}, s_{1.6840}, s_{4.4437};\ 0.3780)$
A_4	$(s_{5.2422}, s_{2.6864}, s_{1.3503};\ 0.6649)$	$(s_{3.0000}, s_{1.6833}, s_{3.2377};\ 0.6649)$	$(s_{2.0000}, s_{3.4212}, s_{4.8924};\ 0.5041)$	$(s_{3.0000}, s_{2.6864}, s_{6.0000};\ 0.4642)$
A_5	$(s_{4.0000}, s_{2.7593}, s_{2.1911};\ 0.5039)$	$(s_{3.9144}, s_{2.6870}, s_{3.0672};\ 0.3779)$	$(s_{1.2596}, s_{2.0000}, s_{5.0000};\ 0.5849)$	$(s_{1.0000}, s_{2.0557}, s_{4.8245};\ 0.4642)$

Step 4: The score values of the collective values obtained during Step 3a are computed by using Eq. (28) and get $\text{Sc}(\alpha_1) = 0.3588, \text{Sc}(\alpha_2) = 0.3134, \text{Sc}(\alpha_3) = 0.3483, \text{Sc}(\alpha_4) = 0.3657$ and $\text{Sc}(\alpha_5) = 0.3005$. On the other hand, the score values of the aggregated PLSVNN obtained in Step 3b are $\text{Sc}(\alpha_1) = 0.2887, \text{Sc}(\alpha_2) = 0.2494, \text{Sc}(\alpha_3) = 0.2948, \text{Sc}(\alpha_4) = 0.3058$ and $\text{Sc}(\alpha_5) = 0.2413$.

Step 5: The final ranking order of the given alternatives are obtained as $A_4 \succ A_1 \succ A_3 \succ A_2 \succ A_5$ corresponding to PLSVNWA operator while $A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5$ to the PLSVNWG operator. Here $\succ$ refers "preferred to". Thus, we conclude that A_4 is the best alternative among the given ones.

6.3. Comparative analysis

In this section, the proposed approaches are compared with the various existing studies in order to defend its dominance in DM problems. It is noticeable that the PLSVN environment reduces to LSVN environment if we take the possibility of each element in the PLSVNS as one. So, to analyze the behavior of our proposed approaches, we compare them to the existing theories [36–38] by taking the same numerical example. But for this comparative study, we firstly convert the PLSVNNs into LSVNNs by fixing values of $p_{ij}^{(q)} = 1; (i = 1, 2, 3, 4, 5;\ j = 1, 2, 3, 4;\ q = 1, 2, 3)$.

(i) Based on the reduction of PLSVNNs to LSVNNs, we apply the approach given by Fang and Ye [36], on the considered example. Under it, we aggregate the given preference values by using the existing linguistic neutrosophic number weighted averaging operator and hence obtain the score values of each alternative $A_i (i = 1, 2, \ldots, 5)$ as

$$\text{Sc}(A_1) = 0.7194; \quad \text{Sc}(A_2) = 0.7423; \quad \text{Sc}(A_3) = 0.6554;$$
$$\text{Sc}(A_4) = 0.6968; \quad \text{Sc}(A_5) = 0.6945.$$

Thus, the ranking order of the given alternatives is obtained as $A_2 \succ A_1 \succ A_4 \succ A_5 \succ A_3$ and hence conclude that A_2 is the best alternative.

On the other hand, if we utilize linguistic neutrosophic number weighted geometric aggregation operator to aggregate the different rating values then the score values corresponding to each alternative are obtained as

$$\text{Sc}(A_1) = 0.7111; \quad \text{Sc}(A_2) = 0.7321; \quad \text{Sc}(A_3) = 0.6513;$$
$$Sc(A_4) = 0.6732; \quad \text{Sc}(A_5) = 0.6838.$$

Based on these score values, we obtain the final ranking order of the alternatives as $A_2 \succ A_1 \succ A_5 \succ A_4 \succ A_3$.

It can be easily seen that ranking results given by these operators and the proposed approach based on operators are totally vary from each other. This fluctuation in results is due to various reasons. The approach used by the existing ones didn't normalize the given data but the our proposed approach do so. Also, the aggregation operators for PLSVNSs takes the possibility of each LSVNS but in the aggregate process of the LSVNSs the possibility didn't come into play for numerical evaluations. Thus, we can conclude that the ignorance of the one factor, that is, possibility degree in the data shows the great divergence in final results. But it is clear that involvement of possibility of each member of the PLSVNS gives more reliable information in case of complex uncertainties. Moreover, LSVNS is the particular case of the PLSVNS. Thus, we can say proposed approach based on operators give more genuine results than existing one.

(ii) If we utilize the prioritized weighted averaging aggregation operator, as proposed by Garg and Nancy [38] to the considered data then the final score values of each alternative $A_i (i = 1, 2, \ldots, 5)$ are obtained as

$$\text{Sc}(A_1) = 0.7006; \quad \text{Sc}(A_2) = 0.7002; \quad \text{Sc}(A_3) = 0.6143;$$
$$\text{Sc}(A_4) = 0.6529; \quad \text{Sc}(A_5) = 0.6226.$$

Thus, the ranking order of the alternatives is obtained as $A_1 \succ A_2 \succ A_4 \succ A_5 \succ A_3$ and get A_1 is the most desirable alternative. On the other hand, if we utilize the prioritized weighted geometric aggregation operator to the given information then the final scores are obtained corresponding to the given alternatives are

$$\text{Sc}(A_1) = 0.6341; \quad \text{Sc}(A_2) = 0.6356; \quad \text{Sc}(A_3) = 0.5917;$$
$$\text{Sc}(A_4) = 0.5771; \quad \text{Sc}(A_5) = 0.5503.$$

Based on these score values, the final ranking order of the given alternatives is obtained as $A_2 \succ A_1 \succ A_3 \succ A_4 \succ A_5$. From this study, it is observed that the final ranking order of the given alternatives obtained through the proposed approach and the existing approaches is entirely different. This is due to the fact that the computational procedure to find the weights of the factors of the proposed approach is different from the existing studies. In the existing studies, weights of each decision maker and criteria are obtained by taking account the priority of each criteria while in the proposed approach, these weights vector are computed by using the entropy measures.

(iii) If we apply the TOPSIS method, as proposed by Liang et al. [37], under the LSVN environment to the normalized decision matrices obtained by taking all the possibility degree to be one, then the final ranking order of the given alternatives is obtained as $A_2 \succ A_1 \succ A_3 \succ A_4 \succ A_5$. Compared this ranking order with the proposed ranking order by COPRAS method, we found that ranking results by the TOPSIS method is far apart from ranking results of COPRAS method. This difference is due to the fact that both ranking methods adopt different strategies. In TOPSIS, calculation and final ranking is obtained by involving the concept of worst and the best ideal solutions of the decision-making problems under consideration. In contrast to this, COPRAS method gives the ranking results without finalizing the ideal solutions and use the procedure of aggregating the benefit and cost criteria individually which is not done in TOPSIS method.

6.4. Advantages of proposed approaches

This section explores the advantages of the proposed work as follows.

(i) Real life DM problems are often possibilistic and as well as qualitative in nature. This paper caught the significance of taking the idea of possibility along with the LSVNS in demonstrating the present practical circumstances. As the assigned possibility degree to the element of LSVNS represents the possibility of occurrence of linguistic membership, indeterminacy and non-membership, therefore, this combination has high potential in true representation in the field of computational intelligence.

(ii) From the presented study, it is observed that the proposed operator is a generalization of the existing ones in the LSVNS environment if we put the value of the possibility of each element in LSVNS equals to 1. As we take possibility as 1, so the proposed PLSVNWA and PLSVNWG aggregation operators reduce to existing Linguistic neutrosophic number weighted averaging and geometric operators [36], respectively. This shows the proposed concept is much more generalized than existing ones.

(iii) In this manuscript, we use the COPRAS method as the ranking method which is a suitable strategy to prepare the information in a sensible and effective way. The strategy used by the COPRAS can process the criterion information from distinctive points based on the complex proportional calculation, which contains more accurate information compared with other strategies basically dealing with the benefit criteria or the cost criteria.

(iv) This paper introduces a new entropy measure that not only provides the overall information about the amount of uncertainty imbued in the specific structure but also used as an effective tool in the DM process. In the DM process, the allocation of weights to the decision maker as well as criteria in order to signify the preference of the both, the proposed entropy measure has been utilized. Thus, this paper gives us a way to find the completely unknown criterion weights to decision maker and criteria using entropy measure.

7. Conclusion

In this manuscript, we have introduced the new concept of PLSVNSs along with information measures and aggregation operators under the same scenario. The current speculations just manage the qualitative aspects without incorporation of possibility and hence the final outcomes are sometimes inadmissible in grabbing the best choice. To resolve this, this paper presents the LSVNSs which are imbued with the possibility in the decision process and are named as PLSVNSs. Also, in this paper, we give the operational laws for this proposed set-theoretic structure with uncertainties and then explore the various relationships among these operations. Further, some weighted averaging and geometric AOs for the PLSVNSs are proposed based on the averaging and geometric conception for assembled the PLSVNSs in the single value. Moreover, this paper presents a new way to rank the alternatives evaluated under PLSVN domain by introducing the COPRAS method. Also, the two approaches are given for evaluating the information with completely unknown weights and these weights are calculated by utilizing the entropy measure. The illustrative example demonstrates all the concepts of the proposed work in this paper. Thus, we can conclude that the proposed work is widely used in the different scenarios like: when decision maker provides the information about the fact that 'how much he/she sure about the uncertain information evaluated by him/her?'; in the situations, when we need to know the gap between the most beneficial choice and the others choices; when the evaluators have no knowledge of the importance of their decision as well the considered criteria. Thus, the proposed concepts are efficaciously applicable to the situation under uncertainties and expected to have wide applications in complex DM problems. In the future, there is a scope of extending the proposed method to some different environment and its application in the various fields related to decision-theory [46–49].

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