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An Application of Neutrosophic Hypersoft Mapping to Diagnose Hepatitis and Proposing Appropriate Treatment

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ABSTRACT Hepatitis is regarded as one of the leading causes of death around the globe. This paper aims to characterize the discussions related to the diagnosis of Hepatitis with their related problems. After examining the side effects of Hepatitis, it encases similar indications, and it is hard to distinguish the precise type of Hepatitis with its seriousness. Since in practical assessment, the indeterminacy and falsity parts are frequently dismissed, and because of this issue, it's hard to notice the precision in the patient's progress history and can't foresee the period of treatment. The Neutrosophic Hypersoft (NHS) set and NHS mapping with its inverse mapping to eliminate these limits are presented in this paper. These ideas are capable and essential to analyze the issue properly by interfacing it with scientific modeling. This investigation builds up a connection between symptoms and medicines, which diminishes the difficulty of the narrative. A table depending on a fuzzy interval between $[0, 1]$ for the sorts of Hepatitis is constructed. The calculation depends on NHS mapping to adequately recognize the disease and choose the best medication for each patient's relating sickness. Finally, the generalized NHS mapping is presented, which will encourage a specialist to extricate the patient's progress history and to foresee the time of treatment till the infection is relieved.

INDEX TERMS Hepatitis; Neutrosophic Hypersoft; Mapping; Inverse mapping.

I. INTRODUCTION

Viruses are obligate parasites that can exclusively replicate living organisms like animals, plants, fungi, and bacteria [2]. Hepatitis is a viral disease responsible for inflammation of the liver. There are a total of 7 types of causative agents of Hepatitis reported till this, namely Hepatitis A, B, C, D, E, and G [1]. This disease has been around for a while and has been responsible for thousands of casualties in different periods, and still is an imminent threat to humans to this day. Each virus has a specific mode of transmission depending upon its nature. Hepatitis A and E are the only two viruses whose proliferation is influenced by environmental factors such as temperature, humidity, etc. Their mode of transmission is contaminated and infected water. Environment plays a vital role in the transmission of these viruses.

For solving multifaceted problems in robotics, engineering, economics, and the environment, the use of conventional means is just isn't enough. Despite the variety of incomplete information, there are four theories specific to these problems Probability set theory (PST), Fuzzy set theory (FST) Zadeh [9], Rough set theory (RST) Pawlak [22] and Period mathematics (PM) that is assumed as a scientific apparatus for managing with lacking information. Every one of these apparatuses acquires the pre-determination of few parameters, to begin with, density function (DF) in PST, membership degree in FST, and a congruence relation in RST. Such a prerequisite, observed in the scrim of flawed or deficient information, escalate numerous issues. Simultaneously, fragmented information stays the most glaring attribute of humanitarian, organic, monetary, social, political, and large man-machine frameworks of different kinds. It was Zadeh [9], who introduced the notion of fuzzy sets to handle

the uncertainties. Since then fuzzy set has been pragmatic in many commands such as medical diagnosis [30], decision making [24] and pattern recognition [29]. Keeping in view the prominence of fuzzy sets many generalizations of fuzzy sets have been familiarized like rough sets [22], soft sets [20], intuitionistic fuzzy set [44], bipolar valued fuzzy sets [46] and bipolar soft sets [55], m-polar soft set [49], hypersoft set [12], parameterized hypersoft set [31]. Although all these generalizations have their own benefits but the notion of neutrosophic set [45] has expanded more responsiveness.

Smarandache discussed neutrosophy in 1998 [45], 1999 [50] and 2005 [51]. He started the neutrosophic hypothesis as another numerical mechanism for dealing with imprecise, indeterminacy, and uncertain information. Broumi et al. [52] introduced mapping on neutrosophic soft expert sets through which studied the images and inverse images of neutrosophic soft expert sets. Heilpern [15] presented the idea of fuzzy mapping and demonstrated a fixed point theory for fuzzy contraction mappings, which speculates the fixed point hypothesis for multivalued mappings of Nadler [21]. Estruch and Vidal [35] refer to a fixed point theory for fuzzy contraction mappings on a complete metric space that generalizes the fixed point hypothesis given by Heilpern's fixed point hypothesis. Yan and Xu [37] have examined convexity, quasiconvexity of fuzzy mappings by considering the idea of ordering due to Goetschel-Voxman [13]. Syau [23], [27] demonstrated the concept of convex and concave fuzzy mappings and presented the idea of differentiability, summed up convexity, e.g., pseudoconvexity and invexity for fuzzy mappings of a few factors. His methodology is equal to Goetschel- Voxman approach for fuzzy mapping of a single variable in which the arrangement of fuzzy numbers is embedded in a topological vector space.

Molodtsov [20] successfully implemented soft set (SS) theory in several directions likes the ease of function, Riemann integration (RI), Peran integration (PI), Probability theory (P-T), Measurement theory (MT), and so on. Promising results have been found by Kovkov et al. [16]. The SS theory is used for the process of optimization in Optimization Theory (OPT), Game theory (GT), and Operations research (OR). Maji et al. [19] presented S-sets applications in decision-making problems. In [36], Yang et al. highlighted S-sets' requirements in engineering extended applications. Maji et al. [18] presented the concept of fuzzy SS and its many features. They presented it as an attractive enlargement of S-sets, additional features to vagueness and ambiguity on the highest level of incompleteness. Present researchers have explained [17], [19], [24] how to combine the two ideas into a more flexible, high expression structure for modeling and refined foggy data in the information system. Karaaslan [14] presented the soft class and its relevant operations. He applied its utilization in decision-making successfully. Athar et al. [10], [11] presented the concept of mappings on fuzzy soft classes and mappings on soft classes in 2009 and 2011, respectively. They considered S-images' properties, S-inverse images, fuzzy SS, fuzzy S-images, fuzzy S-inverse images of fuzzy S-psets and illustrated these concepts with examples and counterexamples.

Alkhazaleh [39] et al. defined the idea of a mapping on classes where the neutrosophic soft classes are gatherings of the neutrosophic soft sets. Additionally, they characterized and studied the properties of neutrosophic soft images and neutrosophic soft inverse images of neutrosophic soft sets. Sulaiman et al. [41] presented the idea of mappings on multi-aspect fuzzy soft classes. They explored a few characteristics related to the image and pre-image of multi-aspect fuzzy soft sets and further illustrate with some numerical examples. Bashir and Salleh [40] characterized the notation of mapping on intuitionistic fuzzy soft classes with some properties of intuitionistic fuzzy soft images and inverse images. Borah and Hazarika [42] gave the idea of composite mappings on hesitant fuzzy soft classes in 2016 and discussed some interesting properties of this idea.

Samarandache [12] presented the hypersoft set (HSS) notion as a generalization of soft set in 2018. At that point, he made the differentiation between the sorts of initial universes, crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic, respectively. Thus, he also showed that a HS set could be crisp, fuzzy, intuitionistic fuzzy, neutrosophic, and plithogenic. Saeed et al. [28], [54] explained some basic concepts like HS subset, HS complement, not HS set, absolute set, union, intersection, AND, OR, restricted union, extended intersection, relevant complement, restricted difference, restricted symmetric difference, HS set relation, sub relation, complement relation, HS representation in matrices form, and different operations on matrices. Saeed et al. [26], [29]–[32], [38], [53] presented several applications of SS, NS, NHS in the field of medical diagnosis, pattern recognition of COVID-19, decision making techniques and characterized mapping under a hypersoft set environment. Ye [56] presented the improved cosine similarity measures of simplified neutrosophic sets to solve medical diagnosis problems with simple neutrosophic information.

A. MOTIVATION

The primary motivation of this paper is to simulate a real-life problem of clinical diagnosis of a Hepatitis disease, and it's appropriate and effective treatment since it is hard to distinguish the precise kind of Hepatitis with its seriousness via previous existing theories and methodologies [10], [11], [55] and [49] because these methodologies are limited to comprehensive models. In [10], [11] and [55], the techniques presented are insufficient to study the information in a deep sense for a better understanding to get proper treatment when parameters are divided into sets of sub-parametric values. Also, they can only target the truthness (membership) of objects and do not evaluate the falsity (nonmembership) and indeterminacy parts. Whereas the model presented in [49] evaluates the data in multipolar and studies the data in neutrosophic nature; however it still has deficiencies when a parameter has sub-parametric values. To overcome this issue, we generalized these models to the hybrid of a hypersoft set

which can deal with the parametric values of such disease in more detail considering/concerning the sub-parametric values, since this structure is based on the sub-parametric values of the different parameter and their order and arrangement. A hypersoft set can also arrange the information/data so that it can be easily studied and evaluated. The second hybrid of this model is a structure of neutrosophic which studies the data in all three possible dimensions of positive, indeterminant, and negative parts regarding patient's disease in accordance with parametric values, where these dimensions are independent to each other. A mapping is an association between two or more domains under some specific rules that maps the entangled parameter to the associated fundamental parametric value by considering the similarity in the structure and basis of these parameters. With the help of this mapping, one can deal similar nature of parameters in a single associated fundamental parameter. The study aims to characterize the nearby diagnosis of Hepatitis with their related symptoms. After investigating different side-effects of Hepatitis, we see that these Hepatitis viruses encase related symptoms, and it is hard to distinguish the dissimilarities of Hepatitis. In practical diagnosis, the indeterminacy and falsity parts are frequently dismissed. Because of this issue, it is hard to observe the precision in the patient's progress history and can't foresee the period of medication. To remove these hurdles, we present the NHS set and NHS mapping with its inverse mapping. With the help of our presented model of NHS set and mapping, we can study the data of patient's disease in more depth, and all possible directions under the definition of NHS set and point out the fundamental symptoms of disease using mapping. These ideas are capable and essential to analyze the issue properly by interfacing it with scientific modeling. This investigation builds up a connection between symptoms and medicines, diminishing the proposed study's multifaceted nature. To start, a table depending on a fuzzy interval $[0, 1]$ to range the types of Hepatitis was constructed. A calculation was set up dependent on NHS mapping to recognize the disease properly and choose the best medication for each patient's relating infection. Finally, the generalized NHS-mapping is presented to predict patient's progress reports and if the proposed treatment results in negative effects to the patient, then the inverse mapping is defined to retain the preceding stage of the patient's report under consideration for the suggestion of new treatment; and encourages a specialist to save the patient's progress history until the infection is relieved. The rest of the article is arranged as follows. Section 2 presents some fundamental definitions regarding fuzzy set (FS), SS, fuzzy SS, neutrosophic set, neutrosophic soft class, neutrosophic image, neutrosophic inverse image, HS set, NHS set are re-imagined. In Section 3, mapping on NHS classes, NHS image, NHS inverse image, and its relevant theorems with proofs are characterized. In Section 4, a practical application and comparative analysis are given to exhibit the proposed approach's validity. In the last part, the concluding remarks are described.

II. PRELIMINARIES

In this section, some basic definitions are presented over the universe $\mathbb{S}$.

Definition 1: [9] The FS, $Q = \{(y, I(y))|\ y \in \mathbb{S}\}$ such that

$$I : \mathbb{S} \to [0, 1],$$

where $\mathbb{S}$ is the collection of objects and $I(y)$ specifies the percentage of membership of $y \in \mathbb{S}$.

Definition 2: [20] A pair (I, H) is said to be SS over $\mathbb{S}$, where I is a mapping given as

$$I : H \to P(\mathbb{S}),$$

In other words, a SS over $\mathbb{S}$ is a parameterized family of subsets of the universe $\mathbb{S}$. For $\epsilon \in H$, $I(\epsilon)$ represented the set of ϵ approximate elements of the SS (I, H).

Definition 3: [24] Let $\mathbb{S}$ and E be initial universe and set of parameters respectively. Let $P(\mathbb{S})$ denotes the power set of all fuzzy subsets of $\mathbb{S}$ and $H \subseteq E$. A pair (I, H) is said to be fuzzy SS over $\mathbb{S}$, where $I : H \to P(\mathbb{S})$.

Definition 4: [45] A set ϑ in $\mathbb{S}$ is said to be neutrosophic set if it can be represented by using the membership T, indeterminacy I and non-membership F, where $T(y), I(y)$ and $F(y)$ are elements of $]0^-, 1^+[$ for the alternative y. It can be scripted as $\vartheta = \{(y, \langle T(y), I(y), F(y)\rangle) : y \in \mathbb{S}; T(y), I(y), F(y) \in]0^-, 1^+[\}$ satisfying the constraint $0^- \leq T(y) + I(y) + F(y) \leq 3^+$.

Definition 5: [39] Let $\mathbb{S}$, κ be universal set and set of parameters respectively. Then the gathering of all neutrosophic soft sets over $\mathbb{S}$ having parameters from κ is called a neutrosophic soft class and is expressed as $(\mathbb{S}, \kappa)$.

Definition 6: [39] Let $(\mathcal{R}, \kappa)$ and $(\mathcal{P}, \kappa')$ be two classes of neutrosophic-sets over the universal set $\mathcal{R}$ and $\mathcal{P}$ respectively. Let $\theta : \mathcal{R} \to \mathcal{P}$ and $\chi : \kappa \to \kappa'$ be mappings. Then a mapping $\xi = (\theta, \chi) : (\mathcal{R}, \kappa) \to (\mathcal{P}, \kappa')$ is defined as, for neutrosophic set (ζ, λ) in $(\mathcal{R}, \kappa)$ and $\xi(\zeta, \lambda)$ is neutrosophic set in $(\mathcal{P}, \kappa')$ gained as follows, For $\vartheta \in \chi(\kappa) \subseteq \kappa'$ and $y \in \mathcal{P}$, then

$$\xi(\zeta, \lambda)(\vartheta)(y) = \begin{cases} \bigvee\limits_{x \in \theta^{-1}(y)} \left(\bigvee\limits_{\varrho \in \chi^{-1}(\vartheta) \wedge \lambda} \zeta(\varrho) \right)(x), & \text{if } \theta^{-1}(y) \neq \emptyset, \\ & \chi^{-1}(\vartheta) \wedge \lambda \neq \emptyset \\ 0 \quad \text{if} & otherwise \end{cases} \tag{1}$$

$\xi(\zeta, \lambda)$ is called a neutrosophic image of neutrosophic set (ζ, λ).

Now, let $(\varphi, \mathcal{H})$ a neutrosophic set in $(\mathcal{P}, \kappa')$, where $\mathcal{H} \subseteq \kappa'$ then $\xi^{-1}(\varphi, \mathcal{H})$ is a neutrosophic set in $(\mathcal{R}, \kappa)$ given as follows,

$$\xi^{-1}(\varphi, \mathcal{H})(\varrho)(x) = \begin{cases} \varphi(\chi(\varrho)(\theta(x) & \text{if } \chi(\varrho) \in \mathcal{H} \\ 0 & \text{if } otherwise \end{cases} \tag{2}$$

, where $\varrho \in \chi^{-1}(\mathcal{H}) \subset \kappa$, then $\xi^{-1}(\varphi, \mathcal{H})$ said to be the neutrosophic inverse image of $(\varphi, \mathcal{H})$.

Definition 7: [12] Let $o_1, o_2, o_3, \cdots, o_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $\kappa_1, \kappa_2, \kappa_3, \cdots, \kappa_n$ respectively, where $\kappa_i \wedge \kappa_j = \emptyset$ for $i \neq j$. A pair $(\Upsilon, \mathbb{O})$ is called a HS set over the universal set $\mathbb{S}$, where Υ is the mapping given by $\Upsilon : \mathbb{O} \longrightarrow P(\mathbb{S})$, where $\mathbb{O} = \kappa_1 \times \kappa_2 \times \kappa_3 \times ... \times \kappa_n$. For more definition see [25], [28], [33], [34].

Definition 8: [12] Let $\mathbb{S}$, $P(\mathbb{S})$ be the universal set and power set of $\mathbb{S}$ respectively. Let $o_1, o_2, o_3, \cdots, o_n$ be the distinct attributes whose corresponding attribute values belongs to the sets $\kappa_1, \kappa_2, \kappa_3, \cdots, \kappa_n$ respectively, where $\kappa_i \wedge \kappa_j = \emptyset$ for $i \neq j$, and their relation $\mathbb{O} = \kappa_1 \times \kappa_2 \times \kappa_3 \times ... \times \kappa_n$, then the pair $(\Upsilon, \mathbb{O})$ is called a NHS set over the universal set $\mathbb{S}$, where Υ is the mapping given by $\Upsilon : \mathbb{O} \longrightarrow P(\mathbb{S})$ and $\Upsilon(\mathbb{O}) = \{\langle x, T(\Upsilon(\mathbb{O})), T(\Upsilon(\mathbb{O})), T(\Upsilon(\mathbb{O}))\rangle\}$, where T is the membership value of truthiness, I is the membership value of indeterminacy and F is the membership value of falsity such that $T, I, F : \mathbb{S} \to [0, 1]$ also $0 \leq T(\Upsilon(\mathbb{O})) + I(\Upsilon(\mathbb{O})) + F(\Upsilon(\mathbb{O})) \leq 3$.

III. MAPPINGS ON NEUTROSOPHIC HYPERSOFT CLASSES

In this part, we present the idea of mapping on NHS classes. NHS classes are gathering of NHS sets. We moreover characterize the properties of NHS like NHS images, NHS inverse images of NHS sets, and backing them with models and theorems. Throughout this section, consider $\kappa_1 \times \kappa_2 \times \kappa_3 \times ... \times \kappa_n = \mathbb{O}$, $\kappa_1' \times, \kappa_2' \times \kappa_3' \times ... \times \kappa_n' = \mathcal{K}$, $\lambda_1 \times \lambda_2 \times \lambda_3 \times ... \times \lambda_n = \lambda$, $\mathcal{H}_1 \times \mathcal{H}_2 \times \mathcal{H}_3 \times ... \times \mathcal{H}_n = \mathcal{H}$, $(\varrho_1, \varrho_2, \varrho_3, ..., \varrho_n) = \varrho$ and $(\vartheta_1, \vartheta_2, \vartheta_3, ..., \vartheta_n) = \vartheta$.

Definition 9: Suppose $\mathbb{S}$ be an initial universe, let $\epsilon_1, \epsilon_2, \epsilon_3, \cdots, \epsilon_n$ be the distinct attributes whose attribute values belongs to the sets $\kappa_1, \kappa_2, \kappa_3, \cdots, \kappa_n$ respectively, where $\kappa_i \wedge \kappa_j = \emptyset$, for $i \neq j$, let $\Omega = \{\varpi_i : i = 1, 2, ..., n\}$ be a gathering of decision makers. Indexed class of NHS set $\{\xi_{\varpi_i} : \xi_{\varpi_i} : \mathbb{O} \to P(\mathbb{S}), \varpi_i \in \Omega\}$, where $\mathbb{O} = \kappa_1 \times \kappa_2 \times \kappa_3 \times \cdots \times \kappa_n$ is said to be NHS class and it can be symbolized in such a form ξ_Ω. If, for any $\varpi_i \in \Omega$, $\xi_{\varpi_i} = \emptyset$, the NHS set $\xi_{\varpi_i} \notin \xi_\Omega$.

Example 1: Let $\mathcal{R} = \{a =$ Line Interactive, $b =$ Standby-Ferro, $c =$ Delta Conversion On-Line$\}$ be types of UPS (Uninterruptible Power Supply) is considered as universe of discourse. Let $\epsilon_1 =$ efficiency, $\epsilon_2 =$ size, $\epsilon_3 =$ colour, distinct attributes whose attribute values belong to the sets $\kappa_1, \kappa_2, \kappa_3$. Let $\kappa_1 = \{j_1 =$ Good, $j_2 =$ Very Good $\}$, $\kappa_2 = \{j_3 =$ medium, $j_4 = small\}$, $\kappa_3 = \{j_5 =$ brown $\}$ and let $\Omega = \{\varpi_1, \varpi_1, \varpi_1\}$ be a set of decision makers. If we consider NHS sets $\xi_{\varpi_1}, \xi_{\varpi_2}, \xi_{\varpi_3}$ given as

$$\xi_{\varpi_1}(j_1, j_3, j_5) = \{a(0.4, 0.3, 0.5), b(0.1, 0.2, 0.7), c(0.8, 0.3, 0.1)\},$$
$$\xi_{\varpi_2}(j_1, j_4, j_5) = \{a(0.6, 0.1, 0.2), b(0.3, 0.2, 0.5), c(0.2, 0.6, 0.3)\},$$
$$\xi_{\varpi_3}(j_2, j_3, j_5) = \{a(0.2, 0.3, 0.7), b(0.1, 0.2, 0.8), c(0.3, 0.6, 0.9)\},$$
$$\xi_{\varpi_3}(j_2, j_4, j_5) = \{a(0.1, 0.1, 0.2), b(0.8, 0.2, 0.5), c(0.2, 0.9, 0.2)\},$$

then $\xi_\Omega = \{\xi_{\varpi_1}, \xi_{\varpi_2}, \xi_{\varpi_3}\}$ is a NHS class. Now let

$$g_{\varpi_1}(j_1, j_3, j_5) = \{a(0.4, 0.3, 0.5), b(0.1, 0.2, 0.7), c(0.8, 0.3, 0.1)\},$$
$$g_{\varpi_1}(j_1, j_4, j_5) = \{a(0.2, 0.9, 0.1), b(0.2, 0.4, 0.5), c(0.4, 0.2, 0.8)\},$$
$$g_{\varpi_1}(j_2, j_3, j_5) = \{a(0.2, 0.2, 0.4), b(0.6, 0.4, 0.5), c(0.2, 0.7, 0.4)\},$$
$$g_{\varpi_1}(j_2, j_4, j_5) = \{a(0.6, 0.1, 0.8), b(0.4, 0.2, 0.7), c(0.1, 0.4, 0.3)\},$$

is also NHS class. Then NHS classes can be written as $\{\xi_{\varpi_1}, \xi_{\varpi_2}, \xi_{\varpi_3}\}$,$\{g_{\varpi_1}, g_{\varpi_2}, g_{\varpi_3}\}$.

Definition 10: Let $(\mathcal{R}, \mathbb{O})$ and $(\mathcal{P}, \mathcal{K})$ be two classes of NHS sets over the universal set $\mathcal{R}$ and $\mathcal{P}$ respectively. Let $\theta : \mathcal{R} \to \mathcal{P}$ and $\chi : \mathbb{O} \to \mathcal{K}$ be mappings. Then a mapping $\xi = (\theta, \chi) : (\mathcal{R}, \mathbb{O}) \to (\mathcal{P}, \mathcal{K})$ is defined as for NHS set (ζ, λ) in $(\mathcal{R}, \mathbb{O})$ and $\xi(\zeta, \lambda)$ is NHS set in $(\mathcal{P}, \mathcal{K})$ obtained in such a way, For $\vartheta \in \chi(\mathbb{O}) \subseteq \mathcal{K}$ and $y \in \mathcal{P}$, then

$$\xi(\zeta, \lambda)(\vartheta)(y) = \begin{cases} \bigvee\limits_{x \in \theta^{-1}(y)} \left(\bigvee\limits_{\varrho \in \chi^{-1}(\vartheta) \wedge \lambda} \zeta(\varrho) \right)(x), \text{if } \theta^{-1}(y) \neq \emptyset, \\ \qquad\qquad \chi^{-1}(\vartheta) \wedge \lambda \neq \emptyset \\ 0 \quad \text{if} \quad otherwise \end{cases} \tag{3}$$

$\xi(\zeta, \lambda)$ is called a NHS image of NHS set (ζ, λ).

Definition 11: Let $(\mathcal{R}, \mathbb{O})$ and $(\mathcal{P}, \mathcal{K})$ be two classes of NHS sets over the universal set $\mathcal{R}$ and $\mathcal{P}$ respectively. Let $\theta : \mathcal{R} \to \mathcal{P}$ and $\chi : \mathbb{O} \to \mathcal{K}$ be mappings. Now, let $(\varphi, \mathcal{H})$ be a NHS set in $(\mathcal{P}, \mathcal{K})$, where $\mathcal{H} \subseteq \mathcal{K}$ then $\xi^{-1}(\varphi, \mathcal{H})$ is a NHS set in $(\mathcal{R}, \mathbb{O})$ defined as follows,

$$\xi^{-1}(\varphi, \mathcal{H})(\varrho)(x) = \begin{cases} \varphi(\chi(\varrho)(\theta(x) & \text{if } \chi(\varrho) \in \mathcal{H} \\ 0 & \text{if } otherwise \end{cases} \tag{4}$$

where $\varrho \in \chi^{-1}(\mathcal{H}) \subset \mathbb{O}$, then $\xi^{-1}(\varphi, \mathcal{H})$ said to be the NHS inverse image of NHS set $(\varphi, \mathcal{H})$.

Example 2: Let $\mathcal{R} = \{a =$ Line Interactive, $b =$ Standby-Ferro, $c =$ Delta Conversion On-Line$\}$ and $\mathcal{P} = \{x =$ Microwave cum Convection, $y =$ Conventional oven, $z =$ Convection oven $\}$ be types of UPS (Uninterruptible Power

Supply) and OVENS respectively, considered as universes of discourses. A Marketing manger wants to know the relationship between UPS and OVENS characteristics which will be more effective regarding his marketing. Let $\epsilon_1 =$ efficiency, $\epsilon_2 =$ size, $\epsilon_3 =$ colour, $\epsilon_4 =$ price and $b_1 =$ efficiency, $b_2 =$ colour, $b_3 =$ price, $b_4 =$ size be the two types of distinct attributes whose attribute values belong to the sets $\kappa_1, \kappa_2, \kappa_3, \kappa_4$ and $\kappa_1', \kappa_2', \kappa_3', \kappa_4'$ respectively. Let $\kappa_1 = \{j_1 =$ Good, $j_2 =$ Very Good$\}$, $\kappa_2 = \{j_3 =$ medium, $j_4 =$ small $\}$, $\kappa_3 = \{j_5 =$ brown $\}$, $\kappa_4 = \{j_7 =$ low price $\}$. Similarly, $\kappa_1' = \{j_1' =$ Good, $j_2' =$ effective $\}$, $\kappa_2' = \{j_3' = black, j_4' =$ white $\}$, $\kappa_3' = \{j_5' =$ low price $\}$, $\kappa_4' = \{j_7' =$ medium $\}$ and $(\mathcal{R}, \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$ and $(\mathcal{P}, \kappa_1' \times \kappa_2' \times \kappa_3' \times \kappa_4')$ be the classes of HSS. Let $\theta : \mathcal{R} \to \mathcal{P}$, $\chi : \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4 \to \kappa_1' \times \kappa_2' \times \kappa_3' \times \kappa_4'$ be mappings as follows:

$\theta(a) = z, \theta(b) = y, \theta(c) = y$ and

$$\chi(j_1, j_3, j_5, j_7) = (j_1', j_3', j_5', j_7'),$$
$$\chi(j_1, j_4, j_5, j_7) = (j_1', j_3', j_5', j_7'),$$
$$\chi(j_2, j_3, j_5, j_7) = (j_2', j_3', j_5', j_7'),$$
$$\chi(j_2, j_4, j_5, j_7) = (j_2', j_4', j_5', j_7'),$$

choose two NHS set in $(\mathcal{R}, \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$ and $(\mathcal{P}, \kappa_1' \times \kappa_2' \times \kappa_3' \times \kappa_4')$ respectively,

$$(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4) =$$
$$\{\{j_1, j_3, j_5, j_7\} = \{a(0.7, 0.2, 0.5), b(0.4, 0.2, 0.8), c(0.4, 0.1, 0.3)\},$$
$$\{j_1, j_4, j_5, j_7\} = \{a(0.8, 0.3, 0.6), b(0.2, 0.2, 0.9), c(0.5, 0.3, 0.1)\},$$
$$\{j_2, j_4, j_5, j_7\} = \{a(0.8, 0.5, 0.3), b(0.3, 0.1, 0.9), c(0.8, 0.2, 0.6)\}\},$$
$$(\varphi, \mathcal{H}_1 \times \mathcal{H}_2 \times \mathcal{H}_3 \times \mathcal{H}_4) =$$
$$\{\{j_1', j_3', j_5', j_7'\} = \{x(0.7, 0.2, 0.8), y(0.2, 0.4, 0.6), z(0.1, 0.7, 0.1)\},$$
$$\{j_1', j_4', j_5', j_7'\} = \{z(0.5, 0.9, 0.5), y(0.3, 0.5, 0.8), z(0.4, 0.3, 0.7)\},,$$

For a NHS set $(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4)$ in $(\mathcal{R}, \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$, then the NHS image can be written as $(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$ is a NHS set in $(\mathcal{P}, \kappa_1' \times \kappa_2' \times \kappa_3' \times \kappa_4')$, where $(\kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4) = \chi(\lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4) = \{(j_1', j_3', j_5', j_7'), (j_2', j_4', j_5', j_7'))\}$ obtained as follows:

$$= \left(\bigvee_{x \in \theta^{-1}(x)} \left(\bigwedge_{\varrho \in \chi^{-1}(j_1', j_3', j_5', j_7')} (\lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4) \vee \zeta(\varrho) \right) \right)$$
$$= \left(\bigvee_{x \in \theta^{-1}(x)} \left(\bigvee_{\varrho \in \{(j_1, j_3, j_5, j_7), (j_1, j_4, j_5, j_7)\}} \zeta(\varrho) \right) \right)$$
$$= \left(\bigvee_{x \in \theta^{-1}(x)} \left(\zeta(j_1, j_3, j_5, j_7) \vee \zeta(j_2, j_4, j_5, j_7) \right) \right)$$
$$= \left(\bigvee_{x \in \emptyset} \left(\{a(0.7, 0.2, 0.5), b(0.4, 0.2, 0.8), c(0.4, 0.1, 0.3)\} \right.\right.$$
$$\left.\left. \vee \{a(0.8, 0.3, 0.6), b(0.2, 0.2, 0.9), c(0.5, 0.3, 0.1)\} \right) \right),$$
$$\left(\bigvee_{x \in \emptyset} \{a(0.8, 0.25, 0.5), b(0.4, 0.2, 0.9), c(0.5, 0.2, 0.3)\} \right)$$
$$= \{0, 0, 0\}$$

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_1', j_3', j_5', j_7')(x) = \{0, 0, 0\},$$

Similarly,

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_1', j_3', j_5', j_7')(y) = \{0.5, 0.2, 0.3\},$$

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_1', j_3', j_5', j_7')(z) = \{0.8, 0.25, 0.5\},$$

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_2', j_4', j_5', j_7')(x) = \{0, 0, 0\},$$

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_2', j_4', j_5', j_7')(y) = \{0.8, 0.5, 0.6\},$$

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$(j_2', j_4', j_5', j_7')(z) = \{0.8, 0.5, 0.3\},$$

The NHS image form can be written as,

$$(\xi(\zeta, \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4), \kappa_1 \times \kappa_2 \times \kappa_3 \times \kappa_4)$$
$$= \{(j_1', j_3', j_5', j_7')$$
$$= \{x(0, 0, 0), y(0.8, 0.15, 0.3), z(0.8, 0.35, 0.3)\},$$
$$= (j_2', j_4', j_5', j_7')$$
$$= \{x(0, 0, 0), y(0.8, 0.5, 0.6), z(0.8, 0.5, 0.3)\}\},$$

Now for NHS inverse image,

$$\varphi(\xi(j_1, j_3, j_5, j_7)\theta(a)$$
$$= \varphi(j_1', j_3', j_5', j_7')z$$
$$= \{0.1, 0.7, 0.1\}$$

Similarly,

$$\xi^{-1}((\varphi, \mathcal{H}_1 \times \mathcal{H}_2 \times \mathcal{H}_3 \times \mathcal{H}_4), \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4)$$
$$((j_1, j_3, j_5, j_7)(b) = \{0.2, 0.4, 0.6\},$$

$$\xi^{-1}((\varphi, \mathcal{H}_1 \times \mathcal{H}_2 \times \mathcal{H}_3 \times \mathcal{H}_4), \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4)$$
$$((j_1, j_3, j_5, j_7)(c) = \{0.2, 0.4, 0.6\},$$

$$\xi^{-1}((\varphi, \mathcal{H}_1 \times \mathcal{H}_2 \times \mathcal{H}_3 \times \mathcal{H}_4), \Theta_1 \times \Theta_2 \times \Theta_3 \times \Theta_4)$$
$$((j_1, j_4, j_5, j_7)(a) = \{0.1, 0.7, 0.1\},$$

$$\xi^{-1}((\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4),\Theta_1\times\Theta_2\times\Theta_3\times\Theta_4)((j_1,j_4,j_5,j_7)(b)=\{0.2,0.4,0.6\},$$

$$\xi^{-1}((\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4),\Theta_1\times\Theta_2\times\Theta_3\times\Theta_4)((j_1,j_4,j_5,j_7)(c)=\{0.2,0.4,0.6\},$$

and it can be written as,

$$\xi^{-1}((\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4),\Theta_1\times\Theta_2\times\Theta_3\times\Theta_4)=\{(j_1,j_3,j_5,j_7)=\{a(0.1,0.7,0.1),b(0.2,0.4,0.6),c(0.2,0.4,0.6)\},=(j_1,j_4,j_5,j_7)=\{a(0.1,0.7,0.1),b(0.2,0.4,0.6),c(0.2,0.4,0.6)\}\},$$

where $\Theta_1\times\Theta_2\times\Theta_3\times\Theta_4$
$=\chi^{-1}(\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4)$
$=\{(j_1,j_3,j_5,j_7),(j_1,j_4,j_5,j_7)\}$.

Definition 12: Let $\xi:(\mathcal{R},\mathbb{O})\to(\mathcal{P},\mathcal{K})$ be a mapping and (ζ,λ) and $(\varphi,\mathcal{H})$ be the two NHS sets in $(\mathcal{R},\mathbb{O})$. Then for $\vartheta\in\mathcal{K}$, NHS union and intersection of NHS images of (ζ,λ) and $(\varphi,\mathcal{H})$ in $(\mathcal{R},\mathbb{O})$ are defined as;
$(\xi(\zeta,\lambda)\vee\xi(\varphi,\mathcal{H}))(\vartheta)=\xi(\zeta,\lambda)(\vartheta)\vee\xi(\varphi,\mathcal{H})(\vartheta)$,
$(\xi(\zeta,\lambda)\wedge\xi(\varphi,\mathcal{H}))(\vartheta)=\xi(\zeta,\lambda)(\vartheta)\wedge\xi(\varphi,\mathcal{H})(\vartheta)$.

Definition 13: Let $\xi:(\mathcal{R},\mathbb{O})\to(\mathcal{P},\mathcal{K})$ be a mapping and (ζ,λ) and $(\varphi,\mathcal{H})$ be the two NHS sets in $(\mathcal{R},\mathbb{O})$. Then for $\varrho\in\mathbb{O}$, NHS union and intersection of NHS inverse images of (ζ,λ) and $(\varphi,\mathcal{H})$ in $(\mathcal{R},\mathbb{O})$ are defined as;
$(\xi^{-1}(\zeta,\lambda)\vee\xi^{-1}(\varphi,\mathcal{H}))(\varrho)=\xi^{-1}(\zeta,\lambda)(\varrho)\vee\xi^{-1}(\varphi,\mathcal{H})(\varrho)$,
$(\xi^{-1}(\zeta,\lambda)\wedge\xi^{-1}(\varphi,\mathcal{H}))(\varrho)=\xi^{-1}(\zeta,\lambda)(\varrho)\wedge\xi^{-1}(\varphi,\mathcal{H})(\varrho)$.

Definition 14: Let $M_1\in NHS(\mathcal{R}\times\mathcal{P})$ and $M_2\in NHS(\mathcal{P}\times\mathcal{S})$, then max-min composition of M_1 and M_2 can be denoted as M_1oM_2 and defined as
$M_1oM_2=\{(\mathcal{W},\mathcal{W}'),T_{M_1oM_2}(\mathcal{W},\mathcal{W}'),I_{M_1oM_2}(\mathcal{W},\mathcal{W}'),F_{M_1oM_2}(\mathcal{W},\mathcal{W}'):\mathcal{W}\in\mathcal{R},\mathcal{W}'\in\mathcal{S}\}$, where
$T_{M_1oM_2}(\mathcal{W},\mathcal{W}')=\underset{\mathcal{W}'\in\mathcal{P}}{max}(T_{M_1}(\mathcal{W},\mathcal{W}'),T_{M_1}(\mathcal{W}',\mathcal{W}''))$,
$I_{M_1oM_2}(\mathcal{W},\mathcal{W}')=\underset{\mathcal{W}'\in\mathcal{P}}{min}(I_{M_1}(\mathcal{W},\mathcal{W}'),I_{M_1}(\mathcal{W}',\mathcal{W}''))$,
$F_{M_1oM_2}(\mathcal{W},\mathcal{W}')=\underset{\mathcal{W}'\in\mathcal{P}}{min}(F_{M_1}(\mathcal{W},\mathcal{W}'),F_{M_1}(\mathcal{W}',\mathcal{W}''))$.

Theorem 1: Let $\xi:(\mathcal{R},\mathbb{O})\to(\mathcal{P},\mathcal{K})$ be a NHS-mapping, let (ζ,λ), $(\varphi,\mathcal{H})$ be the two NHS sets in $(\mathcal{R},\mathbb{O})$ and with (ζ_i,λ_i) as the family of a NHS sets in $(\mathcal{R},\mathbb{O})$ we have,

1) $\xi(\emptyset)=\emptyset$
2) $\xi(\mathcal{R})\subset\mathcal{P}$
3) $\xi((\zeta,\lambda)\vee(\varphi,\mathcal{H}))=\xi(\zeta,\lambda)\vee\xi(\varphi,\mathcal{H})$. Generally, $\xi(\vee_i(\zeta_i,\lambda_i)=\vee_i\xi(\zeta_i,\lambda_i)$.
4) $\xi((\zeta,\lambda)\wedge(\varphi,\mathcal{H}))\supseteq\xi(\zeta,\lambda)\wedge\xi(\varphi,\mathcal{H})$. Generally, $\xi(\wedge_i(\zeta_i,\lambda_i))\subseteq\wedge_i\xi(\zeta_i,\lambda_i)$.
5) If $(\zeta,\lambda)\subseteq(\varphi,\mathcal{H})$ then $\xi(\zeta,\lambda)\subseteq\xi(\varphi,\mathcal{H})$.

Proof: Here, points 1 and 2 are trivial case,
3: For $\vartheta\in\mathcal{K}$, we have to prove $\xi((\zeta,\lambda)\vee(\varphi,\mathcal{H}))(\vartheta)=(\xi(\zeta,\lambda)\vee\xi(\varphi,\mathcal{H}))(\vartheta)$.
Take, $\xi((\zeta,\lambda)\vee(\varphi,\mathcal{H}))(\vartheta)=\xi(s,\lambda\vee\mathcal{H})(\vartheta)$

$$=\begin{cases}\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\vee\mathcal{H})}s(\varrho)\right)\text{ if}\\ \quad\chi^{-1}(\vartheta)\wedge(\lambda\vee\mathcal{H})\neq\emptyset,\\ \emptyset,\ \text{Otherwise}\end{cases}\tag{5}$$

where,

$$s(\varrho)=\begin{cases}\zeta(\varrho) & \text{if } \varrho\in(\lambda-\mathcal{H})\\ \varphi(\varrho) & \text{if } \varrho\in(\mathcal{H}-\lambda)\\ \zeta(\varrho)\vee\varphi(\varrho) & \text{if } \varrho\in(\mathcal{H}\wedge\lambda)\end{cases}\tag{6}$$

L.H.S: For a non trivial case when $\chi^{-1}(\vartheta)\wedge(\lambda\vee\mathcal{H})\neq\emptyset$, we have $\xi((\zeta,\lambda)\vee(\varphi,\mathcal{H}))(\vartheta)$

$$=\theta\left(\vee\begin{cases}\zeta(\varrho) & \text{if } \varrho\in(\lambda-\mathcal{H})\wedge\chi^{-1}(\vartheta)\\ \varphi(\varrho) & \text{if } \varrho\in(\mathcal{H}-\lambda)\wedge\chi^{-1}(\vartheta)\\ \zeta(\varrho)\vee\varphi(\varrho) & \text{if } \varrho\in(\lambda\wedge\mathcal{H})\wedge\chi^{-1}(\vartheta)\end{cases}\right)\tag{7}$$

R.H.S: for non trivial case, we have
$(\xi(\zeta,\lambda)\vee\xi(\varphi,\mathcal{H}))(\vartheta)=$

$$u\left(\vee_{\varrho\in p^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\right)\vee u\left(\vee_{\varrho\in p^{-1}(\vartheta)\wedge\breve{H}}\varphi(\varrho)\right)$$
$$u\left(\vee_{\varrho\in p^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\vee_{\varrho\in p^{-1}(\vartheta)\wedge\breve{H}}\varphi(\varrho)\right)$$

$$=\theta\left(\vee\begin{cases}\zeta(\varrho) & \text{if } \varrho\in(\lambda-\mathcal{H})\wedge\chi^{-1}(\vartheta)\\ \varphi(\varrho) & \text{if } \varrho\in(\mathcal{H}-\lambda)\wedge\chi^{-1}(\vartheta)\\ \zeta(\varrho)\vee\varphi(\varrho) & \text{if } \varrho\in(\lambda\wedge\mathcal{H})\wedge\chi^{-1}(\vartheta)\end{cases}\right)\tag{8}$$

From (7) and (8), we have point 3.
4: For $\vartheta\in\mathcal{K}$ and $y\in Y$ we have to show that

$$\xi((\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta)\supseteq(\xi(\zeta,\lambda)\wedge\xi(\varphi,\mathcal{H}))(\vartheta)$$

$$(\xi(\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta)=\xi(s,\lambda\wedge\mathcal{H})(\vartheta)$$

$$=\begin{cases}\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})}s(\varrho)\right)\text{ if }\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})\neq\emptyset,\\ \emptyset,\text{Otherwise},\end{cases}\tag{9}$$

Where, $s(\varrho)=\zeta(\varrho)\wedge\varphi(\varrho)$.
For the non trivial case, $\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})\neq\emptyset$

$$\xi(s,\lambda\wedge\mathcal{H})(\vartheta)=\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})}s(\varrho)\right)=\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})}(\zeta(\varrho)\wedge\varphi(\varrho))\right)$$

$$(\xi(\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta)=\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})}(\zeta(\varrho)\wedge\varphi(\varrho))\right)$$

on the other side, by using definition III we have,

$$(\xi(\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta)=(\xi(\zeta,\lambda)(\vartheta)\wedge\xi(\varphi,\mathcal{H})(\vartheta))$$

$$= \left(\begin{cases} \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\right) \text{ if } \chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})\neq\emptyset \\ \emptyset, \text{ Otherwise,} \end{cases} \right) \tag{10}$$

$$\wedge\left(\begin{cases} \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\mathcal{H}}\varphi(\varrho)\right) \text{ if } \chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})\neq\emptyset \\ \emptyset, \text{ Otherwise,} \end{cases} \right) \tag{11}$$

Neglecting the trivial case, we obtain

$$\begin{aligned}(\xi(\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta) = \\ \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\right)\wedge\theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\mathcal{H}}\varphi(\varrho)\right) \\ \supseteq \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge(\lambda\wedge\mathcal{H})}(\zeta(\varrho)\wedge\varphi(\varrho))\right) \\ = \xi((\zeta,\lambda)\wedge(\varphi,\mathcal{H}))(\vartheta)\end{aligned}$$

5: For a non-trivial case for $\vartheta\in\mathcal{K}$

$$\xi((\zeta,\lambda)(\vartheta) = \begin{cases} \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\right) \text{ if } \chi^{-1}(\vartheta)\wedge\lambda\neq\emptyset \\ \emptyset, \text{ Otherwise.} \end{cases} \tag{12}$$

Then

$$\begin{aligned}\xi((\zeta,\lambda)(\vartheta) &= \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\lambda}\zeta(\varrho)\right) \\ &\subseteq \theta\left(\vee_{\varrho\in\chi^{-1}(\vartheta)\wedge\mathcal{H}}\varphi(\varrho)\right) \\ &= \xi(\varphi,\mathcal{H})(\vartheta).\end{aligned}$$

This yields point 5. Reversible of points 2 and 4 do not hold. It can be explain in the following example.

Example 3: From Example 2,

$$\begin{aligned}\mathcal{P}\not\subset\xi(\mathcal{R}) &= \{\{j_1{}', j_3{}', j_5{}', j_7{}'\} = \{x(0.7,0.2,0.8), \\ &\quad y(0.2,0.4,0.6), z(0.1,0.7,0.1)\}, \{j_1{}', j_4{}', j_5{}', j_7{}'\} \\ &= \{z(0.5,0.9,0.5), y(0.3,0.5,0.8), z(0.4,0.3,0.7)\}\}.\end{aligned}$$

This indicate that reverse of point 2 does not hold. Now we are going to show reverse of point 4 also does not hold. For this purpose, we choose two NHS sets in $(\mathcal{R},\kappa_1\times\kappa_2\times\kappa_3\times\kappa_4)$ as

$$\begin{aligned}(\zeta,\lambda_1\times\lambda_2\times\lambda_3\times\lambda_4) &= \{\{j_1,j_3,j_5,j_7\} \\ &= \{a(0.8,0.1,0.5), b(0.8,0.2,0.8), \\ &\quad c(0.4,0.1,0.3)\}, \{j_1,j_4,j_5,j_7\} \\ &= \{a(0.6,0.3,0.6), b(0.2,0.2,0.9), \\ &\quad c(0.5,0.3,0.1)\}, \{j_2,j_4,j_5,j_7\} \\ &= \{a(0.8,0.5,0.3), b(0.7,0.5,0.9), \\ &\quad c(0.1,0.2,0.6)\},\end{aligned}$$

$$\begin{aligned}(\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4) &= \{\{j_1,j_3,j_5,j_7\} \\ &= \{a(0.3,0.1,0.4), b(0.9,0.3,0.6), \\ &\quad c(0.2,0.9,0.3)\}, \{j_1,j_4,j_5,j_7\} \\ &= \{a(0.5,0.3,0.7), b(0.1,0.6,0.9), c(0.2,0.3,0.8)\},\end{aligned}$$

Then calculations indicate that

$$\begin{aligned}&\xi(\zeta,\lambda_1\times\lambda_2\times\lambda_3\times\lambda_4)\wedge\xi(\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4) \\ &\not\subset \xi((\zeta,\lambda_1\times\lambda_2\times\lambda_3\times\lambda_4)\wedge(\varphi,\mathcal{H}_1\times\mathcal{H}_2\times\mathcal{H}_3\times\mathcal{H}_4))\end{aligned}$$

Theorem 2: Let $\xi:(\mathcal{R},\mathbb{O})\to(\mathcal{P},\mathcal{K})$ be a mapping, (ζ,λ) and $(\varphi,\mathcal{H})$ be the two NHS sets in $(\mathcal{R},\mathbb{O})$ and the family of a NHS sets (ζ_i,λ_i) in $(\mathcal{R},\mathbb{O})$ we have,

1) $\xi^{-1}(\emptyset)=\emptyset$
2) $\xi^{-1}(\mathcal{P})=\mathcal{R}$
3) $\xi^{-1}((\zeta,\lambda)\vee(\varphi,\mathcal{H})) = \xi^{-1}(\zeta,\lambda)\vee\xi^{-1}(\varphi,\mathcal{H})$. Generally, $\xi^{-1}(\zeta_i,\lambda_i)=\vee_i\xi^{-1}(\zeta_i,\lambda_i)$.
4) $\xi^{-1}((\zeta,\lambda)\wedge(\varphi,\mathcal{H})) = \xi^{-1}(\zeta,\lambda)\wedge\xi^{-1}(\varphi,\mathcal{H})$. Generally, $\xi^{-1}(\zeta_i,\lambda_i)) = \wedge_i\xi^{-1}(\zeta_i,\lambda_i)$.
5) If $(\zeta,\lambda)\subseteq(\varphi,\mathcal{H})$ then $\xi^{-1}((\zeta,\lambda)\subseteq\xi^{-1}(\varphi,\mathcal{H})$.

Proof: Here, points 1 and 2 are trivial case,
3: For $\varrho\in\mathbb{O}$, we have to prove $\xi^{-1}((\zeta,\lambda)\vee(\varphi,\mathcal{H}))(\varrho) = (\xi^{-1}(\zeta,\lambda)\vee\xi^{-1}(\varphi,\mathcal{H}))(\varrho)$.
Take

$$\begin{aligned}&\xi^{-1}((\zeta,\lambda)\vee(\varphi,\mathcal{H}))(\varrho) \\ =\ &\xi^{-1}(s,\lambda\vee\mathcal{H})(\varrho) \\ =\ &u^{-1}(s(\chi(\varrho)),\ \chi(\varrho)\in\lambda\vee\mathcal{H} \\ =\ &u^{-1}(s(\vartheta)),\ \chi(\varrho)=\vartheta\end{aligned}$$

where,

$$= \theta^{-1}\left(\begin{cases} \zeta(\vartheta) & \text{if } \vartheta\in(\lambda-\mathcal{H}) \\ \varphi(\vartheta) & \text{if } \vartheta\in(\mathcal{H}-\lambda) \\ \zeta(\vartheta)\vee\varphi(\vartheta) & \text{if } \vartheta\in(\mathcal{H}\wedge\lambda) \end{cases} \right) \tag{13}$$

Now by using definition 13, we have

$$\begin{aligned}&(\xi^{-1}(\zeta,\lambda)\vee\xi^{-1}(\varphi,\mathcal{H}))\varrho \\ =\ &\xi^{-1}(\zeta,\lambda)(\varrho)\vee\xi^{-1}(\varphi,\mathcal{H})(\varrho) \\ =\ &u^{-1}(\zeta(\chi(\varrho)))\vee u^{-1}(\varphi(\chi(\varrho))),\ \chi(\varrho)\in\lambda\wedge\mathcal{H}\end{aligned}$$

$$= \theta^{-1}\left(\begin{cases} \zeta(\vartheta) & \text{if } \vartheta\in(\lambda-\mathcal{H}) \\ \varphi(\vartheta) & \text{if } \vartheta\in(\mathcal{H}-\lambda) \\ \zeta(\vartheta)\vee\varphi(\vartheta) & \text{if } \vartheta\in(\mathcal{H}\wedge\lambda) \end{cases} \right) \tag{14}$$

Where $\vartheta=\chi(\varrho)$. From (13) and (14), we get point 3.

Now, for point 4 we take $\varrho\in\mathbb{O}$

$$\begin{aligned}&\xi^{-1}(\zeta,\lambda)\wedge(\varphi,\mathcal{H})(\varrho) \\ =\ &(\xi^{-1}(s,\lambda\wedge\mathcal{H})(\varrho) \\ =\ &\theta^{-1}(s(p(\varrho))), p(\varrho)\in\lambda\wedge\mathcal{H}, \\ =\ &\theta^{-1}(\zeta(\vartheta)\wedge\varphi(\vartheta)), p(\varrho)=\vartheta, \\ =\ &\theta^{-1}(\zeta(\vartheta))\wedge\theta^{-1}(\varphi(\vartheta)) \\ =\ &(\xi^{-1}(\zeta,\lambda)\wedge\xi^{-1}(\varphi,\mathcal{H}))(\varrho)\end{aligned}$$

This gives point 4.

5: For $\varrho \in \mathbb{O}$, consider $\xi^{-1}(\zeta, \lambda)(\varrho)$

$$\begin{aligned} &= \theta^{-1}(\zeta(\chi(\varrho))) \\ &= \theta^{-1}(\zeta(\vartheta)), \chi(\varrho) = \vartheta \\ &\subseteq \theta^{-1}(\varphi(\vartheta)) \\ &= \theta^{-1}(\varphi(\chi(\varrho))) \\ &= \xi^{-1}(\varphi, \mathcal{H})(\varrho) \end{aligned}$$

This yields point 5.

IV. APPLICATION OF NHS MAPPING TO HEPATITIS AND ITS RELEVANT PROPERTIES

In this part of the paper, the diseases that Hepatitis and its relevant problems cause are evaluated. The reasons, symptoms, diagnosis, and treatment of Hepatitis patients are analyzed and discussed with the comprehensive concept of the NHS set and its relative mapping, inverse mapping. This section shows how the proposed mathematical model is suitable to set a plan for Hepatitis patients.

A. INVESTIGATION OF HEPATITIS AND ITS RELATED PROPERTIES

The analytical investigation of Hepatitis and mathematical modelling have eternal significance in the medical field. There are different sorts of Hepatitis in medicine, but here, only four are considered for evaluation.

- Hepatitis A
- Hepatitis B
- Hepatitis C
- Hepatitis D

1) Hepatitis A

Hepatitis A belongs to the Family Picornavirus of viruses. The natural host of this virus is only Primates [3]. The virus is non-enveloped and has a 27-28 nm diameter and belongs to a single serotype [4]. The immunity from the only serotype of the virus after the infection is lifelong. After ingestion, the virus targets the liver and replicates itself, causing excretion in bile found in stool specimens. The virus transmission occurs via the oral-fecal route, either by direct contact with an HAV- an infected person or the ingestion of HAV-contaminated water or food [5]. For more detail see Figure 1, 2.

2) Hepatitis B

Hepatitis B also belongs to the hepatitis virus family. It is regarded as the causative agent of chronic liver sickness for over 350 million people. Its detection is done by the persistence of HBV surface antigen in serum and the production of HBV, DNA and viral antigens in the liver. It was recorded that HBV-related chronic diseases cause over a million deaths. It was also recorded that 15-40 percent of individuals with HBV virus infection led to severe issues like cirrhosis, hepatocellular carcinoma, and liver failure. [6] For more detail, see Figure 3, 4.

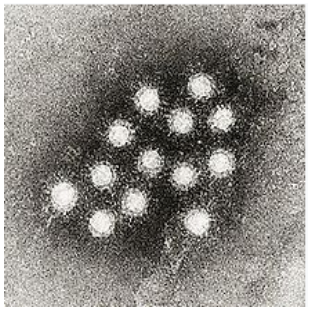

FIGURE 1. Electron micrograph of Hepatovirus A virions.*Source*:"https://www.gesundheitsindustrie-bw.de/en/article/news/systems-biology-and-hepatitis-c-research"

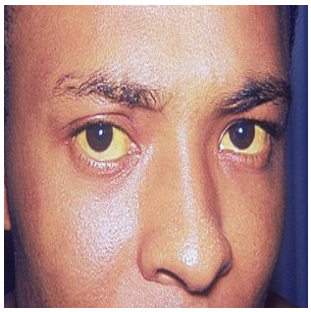

FIGURE 2. A case of jaundice caused by hepatitis A *Source*:"https://www.britannica.com/science/jaundice"

3) Hepatitis C

Hepatitis C virus is the most common indicator of liver transplantation in Europe, Australia and the United States. Its discovery was done when its strain was isolated from a serum of a person with a non-A, non-B virus in 1989. It belongs to the viral family of Flaviviridae and is an RNA virus. After further studies, it was found out that it was the cause of 90 percent of the chronic liver diseases that were non-A, non-B hepatitis in the US. [7] For more detail, see Figure 5, 6.

4) Hepatitis D

The first-time identification of hepatitis D virus was unexpected. Most likely, hepatitis D was confused with a sub type of hepatitis B but with technological advancement, it was said to be a unique RNA virus different from HBV. Structurally, it is made up of 1700 RNA genome base units and due to its size, its regarded as the smallest infectious agent known to man. A total of 8 genotypes have been identified of this virus till date. [8] For more detail, see Figure 7, 8.

The patient is showing some common causes and symptoms of these Hepatitis. These are some indications associated with these problems.

- Fever
- Fatigue
- Loss of appetite
- Nausea
- Vomiting
- Abdominal pain

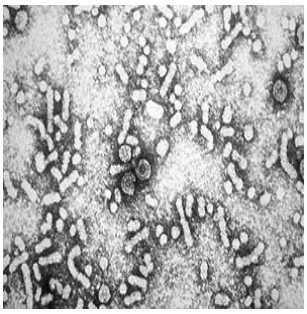

FIGURE 3. Electron micrograph of hepatitis B virus. *Source*:"https://www.nature.com/articles/nrdp201835?WT.feed_name=subjects_hepatitis-b"

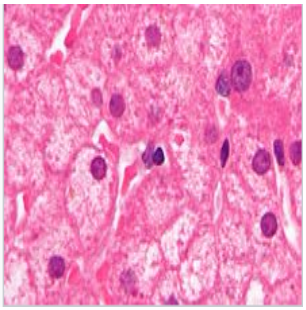

FIGURE 4. Micrograph showing ground glass hepatocytes. H E stain.*Source*:"https://www.webpathology.com/image.asp?case=232&n=5"

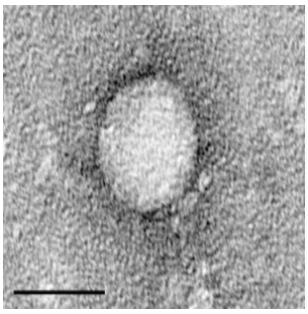

FIGURE 5. Electron micrograph of hepatitis C virus from cell culture (scale = 50 nanometers).*Source*:"https://medicalxpress.com/news/2020-10-hepatitis-silent-killer.html"

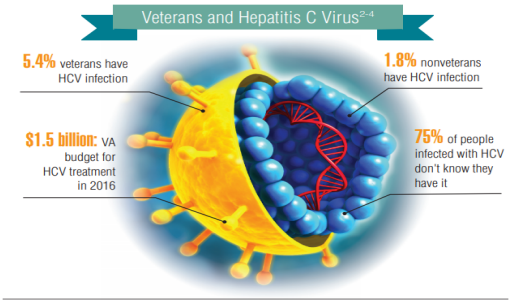

FIGURE 6. Structure of Hepatitis C Virus.*Source*:"https://cdn.mdedge.com/files/s3fs-public/Document/March-2018/fdp_s16-s17.pdf"

- Dark urine
- Light-colored stools
- Joint pain
- Jaundice

In the following section, the technique is discussed that is used to address the problem. A unique algorithm dependent on NHS-mapping is used to analyze the sickness, to provide good treatment, and progress of treatment stages.

B. METHODOLOGY

1) Pre Step

A specialist faces a few complexities when analyzing a Hepatitis patient because of the similar symptoms of Hepatitis. It is very difficult to get the distinction between these classifications. It implies that these sorts of difficulties consist of uncertainties and unclearness, so the NHS is appropriate to deal with such information. Initially, a fuzzy interval [0, 1] is established for different types of Hepatitis to interface verbal data into numerical language. For various kinds of Hepatitis, a graph is established to evaluate the unique sort of Hepatitis. This graph is given in Table 1.

TABLE 1. Diagnosis chart of Hepatitis with ranges

Types of Hepatitis	Different ranges of $[0, 1]$
Hepatitis A	$[0.6, 1]$
Hepatitis B	$[0.5, 0.6)$
Hepatitis C	$(0.2, 0.5)$
Hepatitis D	$[0.1, 0.2]$
No Hepatitis	$[0, 0.1)$

Since each issue has its concentration with the progression of time. To gather the more beneficial information history of a patient, each doctor wants to observe at least 2-3 days of information compared to the showing up side effects for well finding. Different graphs are set of conditions and their day after day fixation to analyze the Hepatitis. This graph is given in Table 2 or Figure 9. There are three stages for each kind of turmoil; one is serious confusion, the second is moderate, and the third is a common issue. The flow chart of various ranges assigned to these restrictions is given in Figure 9.

2) Algorithm

Step 1.

Different hepatitis diseases are distinguished. Let $R = \{r_1, r_2, r_3, ..., r_n\}$ be collection of patients suffering from Hepatitis and $A = \{s_1, s_2, s_3, ..., s_v\}$ be the gathering of symptoms of Hepatitis whose corresponding attribute values belong to sets S_i's, where $S = \prod_{i=1}^{v} S_i$. The administration assemble a "t" number of daily diagnostic chart (which can be fit out as NHS set) by the assistance of a mathematician utilizing etymological terms. This chart will assist us to find the proper infection of the patient. The NHS set chart gave by the specialist after essential evaluation at εth times can be fit out as

$z_S^{\varepsilon} = \{z_p^{\varepsilon} = \{r, \langle T_p^{\varepsilon}(r), I_p^{\varepsilon}(r), F_p^{\varepsilon}(r)\rangle\} : r \in R, p \in S\}$, where $T_p^{\varepsilon}(r)$, $I_p^{\varepsilon}(r)$ and $F_p^{\varepsilon}(r)$ are membership, indeterminacy and non membership grades of Hepatitis A, Hepatitis

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TABLE 2. Related conditions and their day after day fixation to analyze Hepatitis

conditions	During 1st day	During 2nd and third day	After 3rd day
serious Hepatitis A (SHA)	$0.72 \leq \gamma < 0.8$	$0.8 \leq \gamma < 1$	$= 1$
moderate Hepatitis A (MHA)	$0.75 \leq \gamma < 0.82$	$0.82 \leq \gamma < 0.87$	$0.87 \leq \gamma < 0.92$
low Hepatitis A (LHA)	$0.6 \leq \gamma < 0.65$	$0.65 \leq \gamma < 0.69$	$0.69 \leq \gamma < 0.74$
serious Hepatitis B (SHB)	$0.55 \leq \gamma < 0.57$	$0.57 \leq \gamma < 0.58$	$0.58 \leq \gamma < 0.59$
moderate Hepatitis B (MHB)	$0.551 \leq \gamma < 0.558$	$0.558 \leq \gamma < 0.559$	$0.559 \leq \gamma < 0.5596$
low Hepatitis B (LHB)	$0.557 \leq \gamma < 0.559$	$0.559 \leq \gamma < 0.5597$	$0.5597 \leq \gamma < 0.5593$
serious Hepatitis C (SHC)	$0.2 \leq \gamma < 0.3$	$0.3 \leq \gamma < 0.4$	$0.4 \leq \gamma < 0.49$
moderate Hepatitis C (MHC)	$0.23 \leq \gamma < 0.25$	$0.25 \leq \gamma < 0.27$	$0.27 \leq \gamma < 0.4$
low Hepatitis C (LHO)	$0.22 \leq \gamma < 0.23$	$0.23 \leq \gamma < 0.235$	$0.235 \leq \gamma < 0.37$
serious Hepatitis D (SHD)	$0.1 \leq \gamma < 0.15$	$0.15 \leq \gamma < 0.17$	$0.17 \leq \gamma < 0.176$
moderate Hepatitis D (MHD)	$0.12 \leq \gamma < 0.13$	$0.13 \leq \gamma < 0.15$	$0.15 \leq \gamma < 0.157$
low Hepatitis D (LHD)	$0.123 \leq \gamma < 0.125$	$0.125 \leq \gamma < 0.129$	$0.129 \leq \gamma < 0.189$
No Hepatitis (NH)	$0.00 \leq \gamma < 0.01$	$0.01 \leq \gamma < 0.06$	$0.06 \leq \gamma < 0.08$

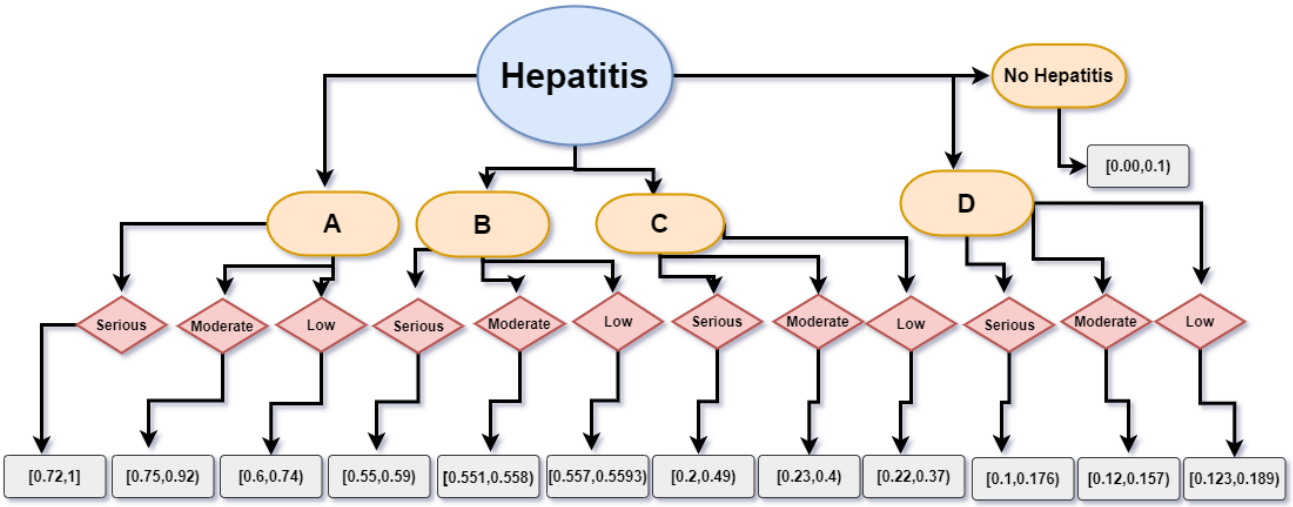

FIGURE 9. Flow chart of diferent ranges corresponding to the listed conditions of Hepatitis.

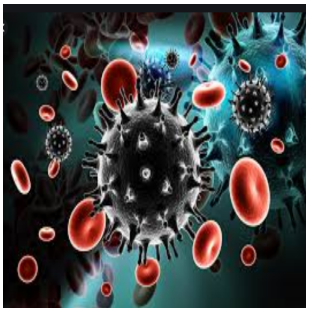

FIGURE 7. First Patient in Phase 2 Trial Receives Lambda as Therapy for Hepatitis D Virus Infection.*Source*:"https://hepatitisnewstoday.com/2016/10/20/eiger-announces-first-patient-dosed-in-phase-2-study-of-pegylated-interferon-lambda-in-hepatitis-d-virus-hdv-infection/"

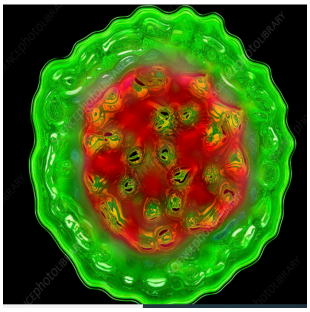

FIGURE 8. HDV- Hepatitis D virus. Image HDRI made according to a view under transmission electron microscope, viral diameter 35 nm. Single-stranded RNA virion. Hepatitis D virus is unable to multiply by itself ; for this it is dependent on hepatitis B virus (satellite virus). As a result, the infection by HDV is always concomitant to the one of hepatitis B virus (co-infection or overinfection) of which it worsen the effects.*Source*:"https://www.sciencephoto.com/media/126633/view/hepatitis-d-virus"

B, Hepatitis C, and Hepatitis D for kth symptoms and lth patients respectively, $(l = 1, 2, 3, ..., n, k = 1, 2, 3, ..., |S|, \varepsilon = 1, 2, 3, ..., t)$. We take NHS union of all "t" number of daily diagnostic charts to gain maximum information of all patients.

Step 2.

We expect that $B = \{s'_1, s'_2, s'_3, ..., s'_w\}$ be the gathering of associated symptoms to A whose corresponding attribute

values belong to sets S_i''s, where $S' = \prod_{i=1}^{w} S_i'$. We consider an NHS set (specialist designating weights by remembering that patients ε number day assessment of fundamental symptoms) in view of major or introductory signs of the patients.

Step 3.

We build a mapping defining as $\zeta : R \to R$ and $\rho : S \to S'$ characterized as follows; $\zeta(r_l) = r_l$, $\rho(p_k) = (p'_{k'})$, $(l = 1, 2, 3, ..., n, k = 1, 2, 3, ..., |S|, k' = 1, 2, 3, ..., |S'|)$ depends upon the relationship among fundamental and primary symptoms.

Let NHS-mapping $\Psi = (\zeta, \rho) : NHS(R) \to NHS(R)$ defined as

$$T_{\Psi(z_S)}(p')(r) = |T_{p'_{k'}}| \begin{cases} \max_{r\in\zeta^{-1}(r)}\left(\max_{p\in\rho^{-1}(p')\cap S} T_{z_S}\right)(r) \text{ if} \\ \qquad \zeta^{-1}(r) \neq \emptyset, \rho^{-1}(p') \cap S \neq \emptyset, \\ 0 \text{ if } otherwise \end{cases} \tag{15}$$

$$I_{\Psi(z_S)}(p')(r) = |I_{p'_{k'}}| \begin{cases} \min_{r\in\zeta^{-1}(r)}\left(\min_{p\in\rho^{-1}(p')\cap S} I_{z_S}\right)(r) \text{ if} \\ \qquad \zeta^{-1}(r) \neq \emptyset, \rho^{-1}(p') \cap S \neq \emptyset, \\ 1 \text{ if } otherwise \end{cases} \tag{16}$$

$$F_{\Psi(z_S)}(p')(r) = |F_{p'_{k'}}| \begin{cases} \min_{r\in\zeta^{-1}(r)}\left(\min_{p\in\rho^{-1}(p')\cap S} F_{z_S}\right)(r) \text{ if} \\ \qquad \zeta^{-1}(r) \neq \emptyset, \rho^{-1}(p') \cap S \neq \emptyset, \\ 1 \text{ if } otherwise \end{cases} \tag{17}$$

where $T_{p'_{k'}}$, $I_{p'_{k'}}$ and $F_{p'_{k'}}$ are related weights from $z_{S'}$. Get the image of $\sqcup z_S^{\varepsilon}$ under the characterized mapping Ψ, which can be composed as $z'_{S'}$.

Step 4.

Then define the after effects set with the values given in Table 2 and gather the pre-diagnosis table from which we can see the precision of final results.

Step 5.

Calculate the score function values of the acquiring NHS set $z'_{S'}$ and gain average of every score value corresponding to related symptoms by using

$$|T_s^{\varepsilon}(r) - 2I_s^{\varepsilon}(r) - F_s^{\varepsilon}(r)|.$$

Then, we carry out our final outcome from Table 1.

Step 6.

We suppose that $B = \{s'_1, s'_2, s'_3, ..., s'_w\}$ be the collection of associated symptoms, where $k = \prod_{i=1}^{w} |S'_i|$ and $C = \{c_1, c_2, c_3, ..., c_x\}$ be a collection of possible medications (treatments) then we can build $\chi_{S'}$, where χ is NHS function from S' to $P(C)$ which is the set of doctor's recommendations with the appropriate treatment corresponding to the symptoms of Hepatitis.

Step 7.

We utilize min-max composition over $z'_{S'}$ and $\chi_{S'}$ and get R_C^1 by using the definition 14.

Step 8.

We choose the medications (treatments) having additional advantages and fewer side impacts. The following steps are necessary for the progress history of the patient.

Step 9.

We characterize a unique generalized mappings; $\zeta' : R^{q-1} \to R^q$ and $\rho' : C^{q-1} \to C^q$ such that $\zeta'(r_l) = r_l$ and $\rho'(c_x) = c_x$. Then NHS-mapping can be written in such a way $\Psi' = (\zeta', \rho') : R_C^{q-1} \to R_C^q$ and can be evaluated as: $R_C^q = \Psi'(R_C^{q-1})(c)(r)$

$$= \frac{1}{q} \begin{cases} \sqcup_{\pi\in\zeta'^{-1}(r)} \left(\sqcup_{\vartheta\in\rho'^{-1}(c)\cap C} R_C^{q-1}\right)(\pi) \text{ if} \\ \qquad \zeta'^{-1}(r) \neq \emptyset, \rho'^{-1}(c) \cap C \neq \emptyset, \\ 0 \text{ if } otherwise \end{cases} \tag{18}$$

where $q = 2, 3, 4...$ is the number of medications(treatments) episodes and $c \in \rho'(C) \subseteq C$, $r \in R^q$, $\pi \in R^{q-1}$, $\vartheta \in (C)^{q-1}$.

Step 10.

Redo step 9 again and again till we get our required outcomes.

3) Limitation of the Method

There are several limitations of the method that must be assured before implementing the algorithm as follows;

1) There must be a mapping that maps the entangled parameter to the associated fundamental parametric value by considering the similarity in the structure and basis of these parameters.
2) The two sets on which mapping or compositions are defined must be independent of each other and must be from the same class of structure (NHS).
3) A doctor's recommendations are needed with the appropriate treatment concerning the symptoms of the disease should be known based on the historical record of the disease.
4) The different ranges of concern types of disease must be known with the help of a doctor.
5) If the proposed treatment results in negative effects to the patient, then the inverse mapping is needed to retain the preceding stage of the patient's report under consideration for the suggestion of new treatment to save the patient's progress history until the infection is relieved.

C. PROPOSED STUDY AND NUMERICAL EXAMPLE

This section of the article is devoted to the application of the suggested algorithm to a medical scenario. The aid of medical personnel is used to translate and gather the input samples in mathematical language. The next step involves selecting the group of patients that has Hepatitis symptoms pointed out by the doctor. From here, a descriptive map was built under the supervision of the doctor for the conditions of different Hepatitis diseases (Table 1) and their daily conditions (Table 2) concerning the diagnosis. With these tables, the symptoms related to Hepatitis can be analyzed in terms of the severity of the disorder. The best feature

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of the technique is that one can add the initial data in our proposed model to determine the particular form of the diseases. The algorithm will also recommend the best course of medication for the specific form of the diagnosed conditions. The algorithm will allow complete generalized mapping of the patient's recovery with efficient recovery graphs for individual patients and comparative case analysis and suitable criteria that will help optimize the technique shortly. For the algorithm, information was collected from several immune-challenged individuals, and with the help of the proposed model, the data can be applied for the statistical formulation and modelling of descriptive concepts. Four patients are considered that have a particular type of Hepatitis disease and needed to be diagnosed by a doctor. As there are multiple overlapping symptoms for a similar disease, it is challenging to pinpoint the right one. After a few patients' examinations, the doctor rules out some dynamics based on the patient's health, recent and past changes in skin colour, the patient's history, genetic factors, etc. From there, the doctor devises a treatment and rehabilitation plan after the initial evaluation of the patient.

Step 1.
Let $R = \{r_1, r_2, r_3, r_4\}$ be collection of four patients. Let s_1 = Weight, s_2 = Strength and s_3 = Color of eye, be distinct attributes of symptoms whose corresponding attribute values belong to the sets S_1, S_2 and S_3 respectively. Let $S_1 = \{s_{11} =$ Sustained weight, $s_{12} =$ Low weight$\}$, $S_2 = \{s_{21} =$ Weakness$\}$, $S_3 = \{s_{31} =$ Dark yellow, $s_{32} =$ Light yellow$\}$, which can be assessed by the doctor after complete checkup. As indicated by the underlying initial information of patients with the above defined symptoms, we can build a chart of two $(\varepsilon = 2)$ days with the information gathered by the doctor given as $z_S^{\varepsilon} \in NHS(R)$ for the 1st and 2nd-day given as (Table 3) and (Table 4) respectively, which are in the form of NHS. Next, we take NHS-union over the z_S^1 and z_S^2. The resultant NHS $\sqcup z_S^{\varepsilon}$ is given as Table 5.
Step 2.
Supposee s_1' = Loss of appetite, s_2' = Fatigue, s_3' = Jaundice, be distinct attributes of connected symptoms of Hepatitis whose corresponding attribute values belong to the sets S_1', S_2', S_3'. Let $S_1' = \{s_{11}' =$ Digestive issues, $s_{12}' =$ Acid reflux$\}$, $S_2' = \{s_{21}' =$ Tiredness$\}$, $S_3' = \{s_{31}' =$ Dark-coloured urine, $s_{32}' =$ Inflamed liver $\}$. Doctor allot the weight to the associated symptoms regarding to the gathered information of patients and we convert verbal data into numerical terminology into the type of NHS $z_{S'}$ given as Table 6.
Step 3.
Now, we characterize two mappings; $\zeta : R \to R$ and $\rho : S \to S'$ such that
$\zeta(r_1) = r_1, \zeta(r_2) = r_2, \zeta(r_3) = r_3, \zeta(r_4) = r_4$, and
$\rho(s_{11}, s_{21}, s_{31}) = (s_{11}', s_{21}', s_{31}')$,
$\rho(s_{11}, s_{21}, s_{32}) = (s_{12}', s_{21}', s_{31}')$,
$\rho(s_{12}, s_{21}, s_{31}) = (s_{11}', s_{21}', s_{32}')$,
$\rho(s_{12}, s_{21}, s_{32}) = (s_{12}', s_{21}', s_{32}')$.
Then NHS-mapping can be written in such a way $\Psi = (\zeta, \rho) : NHS(R) \to NHS(R)$. Now, we evaluate the image of $\sqcup z_S^{\varepsilon}$ given as $z_{S'}'$ in table 7 by utilizing the above mapping in Step 3 in algorithm.
Step 4.
Comparing Table 7 with the Table 2 to get the table of initial diagnosis (Table 8). We will utilize this table later to analyze the precision of our outcomes.
Step 5.
Computing the score estimations of NHS from table 7 using

$$|T_s^{\varepsilon}(r) - 2I_s^{\varepsilon}(r) - F_s^{\varepsilon}(r)|.$$

for each patient concerning to their associated symptoms. After the score estimation values, taking the average score for each patient. These values are drawn in Table 9. Now, the diagnosis chart (Table 1) of Hepatitis is used to compare with the outcomes obtained in Table 9. Correlation shows that patients r_1, r_3 and r_4 are determined to have Hepatitis C and patient r_2 is determined to have Hepatitis D.
Step 6.
After a diagnosis of the genuine kind of illness of each patient, the specialist will propose some medicine to the patients. We developed the NHS set according to the specialist's recommendations with the proper treatment relating to the types of Hepatitis. Consider $C = \{c_1 =$ entecavir (Baraclude), $c_2 =$ tenofovir (Viread), $c_3 =$ lamivudine (Epivir)$\}$ be distinct possible medications (treatments) then we build $\chi_{S'}$, which is the set of doctor's recommendations with the appropriate treatment corresponding to the symptoms of Hepatitis. The set $\chi_{S'} \in NHS(R)$ given as Table 10. In Table 10 the evaluations are given as indicated by the historical backdrop of each patient. The positive effects of medication (treatment) for each kind can be seen from membership grades, the indeterminacy grades represent the impartial impacts of each type, and falsity grades speak to the side effects of medications (treatments) kind of Hepatitis along with its symptoms.
Step 7.
We calculate NHS set min-max composition among $\chi_{S'}$ and $z_{S'}'$ to acquire the connection among suggested medicines and patients as Neutrosophic soft set $\chi_{S'} \circ z_{S'}' = R_C^1$, see Table 11.
Step 8.
The medication (treatment) is suitable for the patients having greater benefits and less side effects. In this way, we find the score esteems by using score function that is given in algorithm step 4 relating to the medicines for each patient (Table 12).

From Table 12, it can be determined that the treatments c_1 is best fit for patient r_1; while c_1 or c_2 can be suggested for patient r_2 for the best medication; r_3 patient can be treated with c_1 or c_3; for r_4 any one medication (treatment) can be chosen. The final choice is relying on the state of the patient as per his past clinical past history and kind of sickness.

TABLE 3. Tabular representation of z_S^1: Chart of first day of patients regarding symptoms from S

symptoms / patients	r_1	r_2	r_3	r_4
(s_{11}, s_{21}, s_{31})	$(0.4, 0.1, 0.5)$	$(0.7, 0.1, 0.9)$	$(0.5, 0.3, 0.5)$	$(0.2, 0.3, 0.8)$
(s_{11}, s_{21}, s_{32})	$(0.7, 0.2, 0.8)$	$(0.1, 0.2, 0.4)$	$(0.4, 0.1, 0.6)$	$(0.8, 0.2, 0.9)$
(s_{12}, s_{21}, s_{31})	$(0.4, 0.1, 0.5)$	$(0.4, 0.1, 0.5)$	$(0.4, 0.1, 0.5)$	$(0.6, 0.1, 0.5)$
(s_{12}, s_{21}, s_{32})	$(0.5, 0.7, 0.3)$	$(0.3, 0.8, 0.6)$	$(0.6, 0.1, 0.3)$	$(0.9, 0.4, 0.3)$

TABLE 4. Tabular representation of z_S^2: Chart of second day of patients regarding symptoms from S

symptoms / patients	r_1	r_2	r_3	r_4
(s_{11}, s_{21}, s_{31})	$(0.5, 0.3, 0.9)$	$(0.5, 0.2, 0.9)$	$(0.5, 0.7, 0.8)$	$(0.6, 0.8, 0.3)$
(s_{11}, s_{21}, s_{32})	$(0.4, 0.6, 0.5)$	$(0.4, 0.1, 0.7)$	$(0.4, 0.3, 0.8)$	$(0.6, 0.7, 0.6)$
(s_{12}, s_{21}, s_{31})	$(0.1, 0.7, 0.6)$	$(0.2, 0.7, 0.5)$	$(0.6, 0.1, 0.5)$	$(0.3, 0.3, 0.8)$
(s_{12}, s_{21}, s_{32})	$(0.6, 0.1, 0.5)$	$(0.3, 0.1, 0.5)$	$(0.4, 0.1, 0.5)$	$(0.4, 0.1, 0.5)$

TABLE 5. Tabular representation of $\sqcup z_S^{\varepsilon}$: NHS union of z_S^1 and z_S^2

symptoms / patients	r_1	r_2	r_3	r_4
(s_{11}, s_{21}, s_{31})	$(0.5, 0.2, 0.5)$	$(0.7, 0.15, 0.9)$	$(0.5, 0.5, 0.5)$	$(0.6, 0.55, 0.3)$
(s_{11}, s_{21}, s_{32})	$(0.7, 0.4, 0.5)$	$(0.4, 0.15, 0.4)$	$(0.4, 0.2, 0.6)$	$(0.8, 0.45, 0.6)$
(s_{12}, s_{21}, s_{31})	$(0.4, 0.4, 0.5)$	$(0.4, 0.4, 0.5)$	$(0.6, 0.1, 0.5)$	$(0.6, 0.2, 0.5)$
(s_{12}, s_{21}, s_{32})	$(0.6, 0.4, 0.3)$	$(0.3, 0.45, 0.5)$	$(0.6, 0.1, 0.3)$	$(0.9, 0.25, 0.3)$

TABLE 6. Tabular representation of $z_{S'}$: Weights of the associated symptoms S' regarding to each patient in NHS

symptoms / patients	r_1	r_2	r_3	r_4
$(s'_{11}, s'_{21}, s'_{31})$	$(0.4, 0.1, 0.7)$	$(0.6, 0.1, 0.3)$	$(0.5, 0.3, 0.2)$	$(0.6, 0.2, 0.1)$
$(s'_{11}, s'_{21}, s'_{32})$	$(0.8, 0.3, 0.2)$	$(0.9, 0.2, 0.4)$	$(0.7, 0.1, 0.3)$	$(0.8, 0.2, 0.2)$
$(s'_{12}, s'_{21}, s'_{31})$	$(0.9, 0.2, 0.3)$	$(0.7, 0.3, 0.4)$	$(0.9, 0.2, 0.3)$	$(0.6, 0.5, 0.4)$
$(s'_{12}, s'_{21}, s'_{32})$	$(0.9, 0.1, 0.1)$	$(0.8, 0.2, 0.4)$	$(0.7, 0.3, 0.1)$	$(0.8, 0.2, 0.1)$

Step 9.
Every patient's scenario depends on the type of illness and its history of the patient. One can repeat the episodes till the illness is healed totally. We can observe the improvement of each patient by utilizing the NHS-mapping by defining two mappings $\zeta' : R^{q-1} \to R^q$ and $\rho' : C^{q-1} \to C^q$ such that

$$\zeta'(r_1) = r_1, \zeta'(r_2) = r_2, \zeta'(r_3) = r_3, \zeta'(r_4) = r_4;$$

and

$$\rho'(c_1) = c_1, \rho'(c_2) = c_2, \rho'(c_3) = c_3.$$

Then NHS-mapping can be written in such a way

$$\Psi' = (\zeta', \rho') : R_C^{q-1} \to R_C^q$$

The NHS-mapping is given as

$$R_C^q = \Psi'(R_C^{q-1})(c)(r) = \frac{1}{q} \begin{cases} \vee_{\pi \in \zeta'^{-1}(r)} (\vee_{\vartheta \in \rho'^{-1}(c) \cap C} R_C^{q-1}(\pi) \text{ if} \\ \qquad \zeta'^{-1}(r) \neq \emptyset, \rho'^{-1}(c) \cap C \neq \emptyset \\ 0 \quad \text{if } otherwise \end{cases} \tag{19}$$

where $q = 2, 3, 4, ...$ represents the number of episodes of treatments and $c \in \rho'(C) \subseteq C$, $r \in R^q$, $\pi \in R^{q-1}$, $\vartheta \in C^{q-1}$ and episodes of treatment can be written in Tables 13, 14, 15 and 16 are given for $q = 2, 3, 4$ and 5 respectively.
Step 10.
We redo step 9 again and again till we get outcomes for the patients. The progressive report for every patient are depicted in Figure 10, 11, 12 and 13.

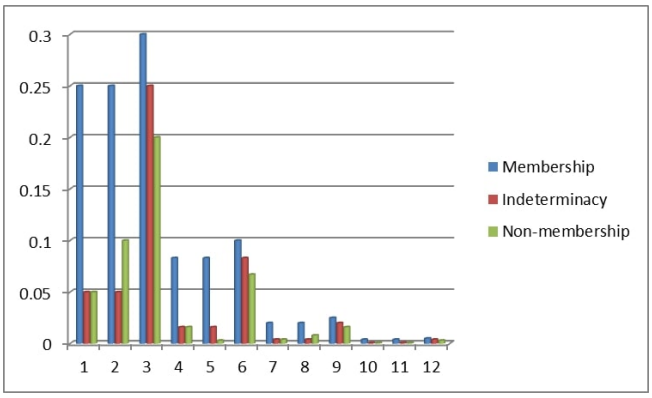

FIGURE 10. Progress chart of patient r_1

D. DISCUSSION AND COMPARATIVE ANALYSIS

The proposed idea of NHS mapping is generally broad and proper for these kinds of sicknesses. These problems can't be dealt with by utilizing existing theories because of their limitations (see Table 17). Because of these shortcomings, the collection of initial data of the patient isn't possible. Yet, our proposed structure can change the patient history into a mathematical format with no deficiency of data, and we

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TABLE 7. Tabular representation of $z'_{S'}$: The image of $\sqcup z^{\varepsilon}_{S}$ under NHS mapping

symptoms / patients	r_1	r_2	r_3	r_4
$(s'_{11}, s'_{21}, s'_{31})$	$(0.2, 0.002, 0.245)$	$(0.42, 0.0015, 0.081)$	$(0.25, 0.045, 0.02)$	$(0.36, 0.022, 0.003)$
$(s'_{11}, s'_{21}, s'_{32})$	$(0.32, 0.036, 0.02)$	$(0.36, 0.06, 0.08)$	$(0.42, 0.003, 0.45)$	$(0.48, 0.008, 0.08)$
$(s'_{12}, s'_{21}, s'_{31})$	$(0.63, 0.016, 0.045)$	$(0.28, 0.013, 0.064)$	$(0.36, 0.008, 0.054)$	$(0.48, 0.112, 0.096)$
$(s'_{12}, s'_{21}, s'_{32})$	$(0.54, 0.004, 0.003)$	$(0.24, 0.018, 0.08)$	$(0.42, 0.009, 0.003)$	$(0.72, 0.01, 0.003)$

TABLE 8. Tabular representation of initial diagnosis chart to analyze the precision of outcomes.

symptoms / patients	r_1	r_2	r_3	r_4
$(s'_{11}, s'_{21}, s'_{31})$	(SO, NT, MO)	(SO, NT, LE)	(LO, NT, NT)	(SO, NT, NT)
$(s'_{11}, s'_{21}, s'_{32})$	(SO, NT, NT)	(SO, NT, LE)	(SO, NT, SO)	(SO, NT, LE)
$(s'_{12}, s'_{21}, s'_{31})$	(LA, NT, NT)	(LO, NT, NT)	(SO, NT, NT)	(SO, LE, LE)
$(s'_{12}, s'_{21}, s'_{32})$	(MM, NT, NT)	(SO, NT, NT)	(SO, NT, NT)	(SA, NT, NT)

TABLE 9. Tabular representation of patient score values data related to connected symptoms

patients / symptoms	$(s'_{11}, s'_{21}, s'_{31})$	$(s'_{11}, s'_{21}, s'_{32})$	$(s'_{12}, s'_{21}, s'_{31})$	$(s'_{12}, s'_{21}, s'_{32})$	Average score
r_1	0.049	0.336	0.14	0.313	0.209
r_2	0.22	0.16	0.036	0.384	0.2
r_3	0.553	0.19	0.29	0.16	0.29
r_4	0.529	0.124	0.399	0.715	0.44

TABLE 10. Tabular representation of $\chi_{S'}$: Doctor's recommendations with the appropriate treatment corresponding to the symptoms of Hepatitis.

treatments / symptoms	$(s'_{11}, s'_{21}, s'_{31})$	$(s'_{11}, s'_{21}, s'_{32})$	$(s'_{12}, s'_{21}, s'_{31})$	$(s'_{12}, s'_{21}, s'_{32})$
c_1	$(0.5, 0.3, 0.1)$	$(0.9, 0.5, 0.8)$	$(0.2, 0.5, 0.2)$	$(0.6, 0.3, 0.4)$
c_2	$(0.5, 0.3, 0.2)$	$(0.4, 0.6, 0.1)$	$(0.7, 0.3, 0.4)$	$(0.6, 0.3, 0.2)$
c_3	$(0.6, 0.1, 0.4)$	$(0.8, 0.1, 0.6)$	$(0.6, 0.3, 0.2)$	$(0.8, 0.2, 0.3)$

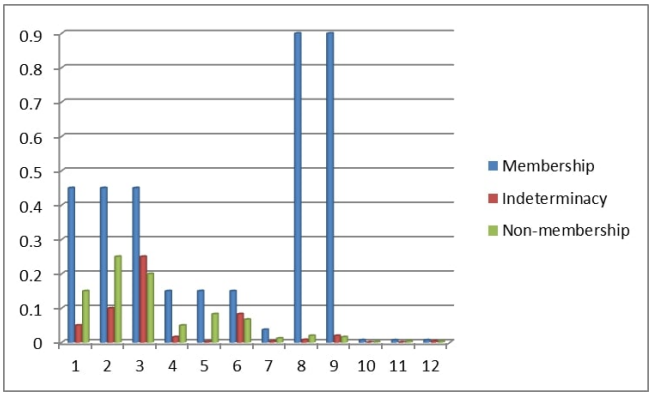

FIGURE 11. Progress chart of patient r_2

obtain predominant outcomes for diagnosis and medication of the patient. In Table 17, we compare our proposed model with the existing theories. However, when the attributes are further sub-divided into attribute values, all current theories fail to manage. This need is fulfilled in the proposed NHS mapping. It shows that our structure is solid compared with existing procedures and can positively deal with these sorts of issues. Now, we discuss our recommended approach and its precision.

- In this calculation, we add numerous days because the Hepatitis patient can't analyze totally after the first checkup. The NHS set and its union tell all the patient's statistics, and we can relate seriousness with its symptoms.
- We see that the relationship among the related and essential signs with its allocated appropriate weights is important in each patient trial. Suppose if we choose just initial symptoms at that point, outcomes will be unspecific.
- In the second stage, we select the treatment for the patients as indicated by their kind of Hepatitis. The score function can be used to rank the selected treatments.
- Thirdly, we utilize a more generalized form of NHS mapping to observe the patients' improvement history. With each episode, all the evaluations are diminishing up to zero, which implies that the symptoms of Hepatitis, neutral impacts of medicine with treatments, and side impacts are diminishing. This model represents the progress of patients with the progression of time.
- If a patient does not progress in the first episode, then inverse NHS-mapping can be utilized to get him back on the preceding episode to start medication from here once more.
- The proposed procedure helps an enormous number of patients with different illnesses and multiform criteria under the impact of parameterizations. This study is steady and consistent to deal with such problems in the medical field and MCDM.

TABLE 11. Tabular representation of R_C^1: Composition among $\chi_{S'}$ and $z'_{S'}$ to acquire the connection among suggested medicines and patients

patients / treatments	c_1	c_2	c_3
r_1	$(0.5, 0.1, 0.1)$	$(0.5, 0.1, 0.2)$	$(0.6, 0.1, 0.4)$
r_2	$(0.9, 0.1, 0.3)$	$(0.9, 0.2, 0.1)$	$(0.9, 0.1, 0.4)$
r_3	$(0.9, 0.2, 0.2)$	$(0.9, 0.2, 0.3)$	$(0.9, 0.2, 0.2)$
r_4	$(0.8, 0.2, 0.1)$	$(0.8, 0.2, 0.1)$	$(0.8, 0.2, 0.1)$

TABLE 12. Tabular representation of patient score values data related to recommended treatment

patients / treatments	c_1	c_2	c_3	Maximum esteems	Selected treatment
r_1	0.2	0.1	0	0.2	c_1
r_2	0.4	0.4	0.3	0.4	c_1 or c_2
r_3	0.3	0.2	0.3	0.3	c_1 or c_3
r_4	0.3	0.3	0.3	0.3	any one

TABLE 13. Tabular representation of R_C^2: Progress report of patients after second episode of treatments

patients / treatments	c_1	c_2	c_3
r_1	$(0.25, 0.05, 0.05)$	$(0.25, 0.05, 0.1)$	$(0.3, 0.25, 0.2)$
r_2	$(0.45, 0.05, 0.15)$	$(0.45, 0.1, 0.25)$	$(0.45, 0.25, 0.2)$
r_3	$(0.45, 0.1, 0.2)$	$(0.45, 0.2, 0.3)$	$(0.45, 0.1, 0.1)$
r_4	$(0.4, 0.1, 0.05)$	$(0.8, 0.1, 0.05)$	$(0.8, 0.1, 0.05)$

TABLE 14. Tabular representation of R_C^3: Progress report of patients after third episode of treatments

patients / treatments	c_1	c_2	c_3
r_1	$(0.083, 0.016, 0.016)$	$(0.083, 0.016, 0.003)$	$(0.1, 0.083, 0.067)$
r_2	$(0.15, 0.016, 0.05)$	$(0.15, 0.003, 0.083)$	$(0.15, 0.083, 0.067)$
r_3	$(0.15, 0.003, 0.003)$	$(0.15, 0.003, 0.3)$	$(0.15, 0.003, 0.003)$
r_4	$(0.13, 0.003, 0.016)$	$(0.8, 0.003, 0.016)$	$(0.8, 0.003, 0.016)$

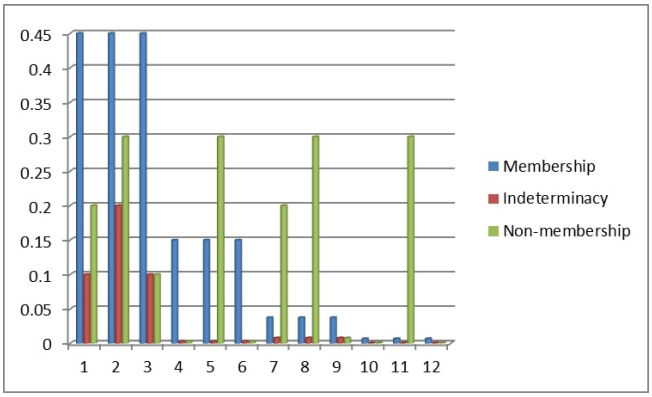

FIGURE 12. Progress chart of patient r_3

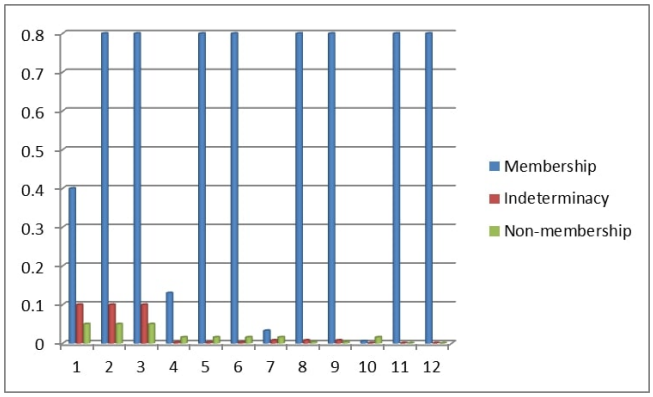

FIGURE 13. Progress chart of patient r_4

V. CONCLUSIONS

In this article, we look at Hepatitis and its related issues. We have proposed a comprehensive way to diagnose the patient's underlying symptoms and analyze their Hepatitis. That is why we have presented NHS mapping with its inverse mapping and few practical tasks with their characteristics. We have developed a calculation having three stages. In the first stage, the model was used to analyze the patients' real sort of Hepatitis. In the second stage, appropriate medicines for the patients were accessed as indicated by the seriousness of the disease by utilizing NHS mapping. Thirdly, generalized NHS mapping was developed to observe the patient's progress history and anticipate the patient's medication time until he reported its normal range in the nervous syndrome region. This procedure is valuable and successful to analyze the infections. Correlation shows that the proposed calculation is predominant, simple to deal with, substantial, solid, and adaptable to solve MCDM problems. In the future, one can expand exploration in the domain of Pythagorean fuzzy uncertain environment, Neutrosophic Hypersoft Set, Plithogenic Crisp Hypersoft Set, Plithogenic Fuzzy Hypersoft Set, Plithogenic Intuitionistic Fuzzy Hyper-

TABLE 15. Tabular representation of R_C^4: Progress report of patients after fourth episode of treatments

patients / treatments	c_1	c_2	c_3
r_1	$(0.020, 0.004, 0.004)$	$(0.020, 0.004, 0.008)$	$(0.025, 0.020, 0.016)$
r_2	$(0.0375, 0.004, 0.0125)$	$(0.9, 0.008, 0.020)$	$(0.9, 0.020, 0.016)$
r_3	$(0.0375, 0.008, 0.2)$	$(0.0375, 0.008, 0.3)$	$(0.0375, 0.008, 0.008)$
r_4	$(0.033, 0.008, 0.016)$	$(0.8, 0.008, 0.004)$	$(0.8, 0.008, 0.004)$

TABLE 16. Tabular representation of R_C^5: Progress report of patients after fifth episode of treatments

patients / treatments	c_1	c_2	c_3
r_1	$(0.004, 0.0008, 0.0008)$	$(0.004, 0.0008, 0.001)$	$(0.005, 0.004, 0.003)$
r_2	$(0.007, 0.0008, 0.0025)$	$(0.007, 0.001, 0.004)$	$(0.007, 0.004, 0.003)$
r_3	$(0.007, 0.001, 0.001)$	$(0.007, 0.001, 0.3)$	$(0.007, 0.001, 0.001)$
r_4	$(0.006, 0.001, 0.016)$	$(0.8, 0.001, 0.0008)$	$(0.8, 0.001, 0.0008)$

TABLE 17. Comparison of the proposed NHS with existing theories

SN	References	Disadvantage	Ranking
1	[9]	lack of indeterminacy and non-membership	Not Possible
2	[43]	lack of indeterminacy and non-membership	Not Possible
3	[44]	lack of indeterminacy	Not Possible
4	[45]	lack of multiple properties	Not Possible
5	[46]	lack of multiplicity and indeterminacy	Not Possible
6	[47]	Does not develop strong relation between falsity and indeterminacy	Not Possible
7	[48]	lack of multiplicity	Not Possible
8	[49]	fail to deal when attributes can be further sub-partitioned into attribute values	Not Possible
9	[10]	lack of indeterminacy and non-membership	Not Possible
10	[11]	lack of indeterminacy and non-membership	Not Possible
11	[14]	lack of indeterminacy and non-membership	Not Possible
12	[15]	lack of indeterminacy and non-membership	Not Possible
22	Proposed Method in this paper	Lengthy and heavy calculations in decision-making	This challenge can be sorted out with the help of computer program

soft Set, Plithogenic Neutrosophic Hypersoft Set, complex fuzzy hypersoft set, complex Intuitionistic Fuzzy Hypersoft Set, complex Neutrosophic Hypersoft Set, and their hybrid structures. We will apply them in medical imaging issues, pattern recognition, recommender frameworks, social; the monetary framework estimated thinking, image processing, and game theory.

REFERENCES

[1] G.F. Hall, "Hepatitis A, B, C, D, E, G: An Update", Ethn. Dis., vol. 17, no. 2, pp. 41-45, 2007.

[2] K. Im, J. Kim and H. Min, "Ginseng, The Natural Effectual Antiviral: Protective Effects Of Korean Red Ginseng Against Viral Infection", J. Ginseng Res., vol. 40, no. 4, pp. 309–314, 2016.

[3] M.S. Balayan, "Natural Hosts Of Hepatitis A Virus", Vaccine, vol. 10, no. 1, pp. 27–31, 1992.

[4] J.G. Melgaco, L.N. Morgado, M.A. Santiago, J.M.D. Oliveira, L.L. Lewis-Ximenez, B. Hasselmann, O.G. Cruz and C.L. Vitral, "A Single Dose Of Inactivated Hepatitis A Vaccine Promotes HAV-Specific Memory Cellular Response Similar To That Induced By A Natural Infection", Vaccine, vol. 33, no. 32, pp. 3813-3820, 2015.

[5] A.E. Fiore, "Hepatitis A Transmitted By Food. Clin Infect Dis", Vaccine, vol. 38, no. 5, pp. 705-715, 2004.

[6] T.L. Wright, "Introduction To Chronic Hepatitis B Infection", Am. J. Gastroenterol., vol. 101, no. 1, pp. 1-S, 2006.

[7] S.L. Chen and T.R. Morgan "The Natural History Of Hepatitis C Virus (HCV) Infection", Int. J. Med. Sci., vol. 3, no. 2, pp. 47-52, 2006.

[8] M. Rizzetto and T.R. Morgan "Hepatitis D virus: Introduction And Epidemiology", Csh. Perspect. Med., vol. 5, no. 7, pp. a021576, 2015.

[9] L.A. Zadeh, "Fuzzy Sets", Inf. Control, vol. 8, no. 3, pp. 338–353, 1965.

[10] A. Kharal and B. Ahmad, "Mappings On Soft Classes", NMNC, vol. 7, no. 3, pp. 471–481, 2011.

[11] A. Kharal and B. Ahmad, "Mappings On Fuzzy Soft Classes", Adv. Fuzzy Syst., vol. 2009, no. 1, pp. 407890, 2009.

[12] F. Smarandache, "Extension Of Soft Set To Hypersoft Set And Then To Plithogenic Hypersoft Set", Neutrosophic Set Syst., vol. 22, no. 1, pp. 168-170, 2018.

[13] J.R. Goetschel and W. Voxman, "Elementary Fuzzy Calculus", Fuzzy Sets Syst., vol. 18, no. 1, pp. 31–43, 1986.

[14] F. Karaaslan, "Soft Classes And Soft Rough Classes With Applications In Decision Making", Math. Probl. Eng., vol. 2016, no. 1, pp. 1584528, 2016.

[15] S. Heilpern, "Fuzzy Mappings And Fixed Point Theorem", J. Math. Anal. Appl., vol. 83, no. 2, pp. 566–569, 1981.

[16] D.V. Kovkov, V.M. Kolbanov and D.A. Molodtsov, "Soft Sets Theory-Based Optimization", J. Comput. Syst. Sci., vol. 46, no. 6, pp. 872–880, 2007.

[17] Z. Kong, L. Gao and L. Wong, "Comment On A Fuzzy Soft Set Theoretic Approach To Decision Making Problems", J. Comput. Appl. Math., vol. 223, no. 2, pp. 540–542, 2009.

[18] P.K. Maji, R. Biswas and A.R. Roy, "Fuzzy Soft Sets", J. Fuzzy Math, vol. 9, no. 3, pp. 589–602, 2001.

[19] P.K. Maji, A.R. Roy and R. Biswas, "An Application Of Soft Sets In A Decisionmaking Problem", Comput. Math. with Appl., vol. 44, no. 8-9, pp. 1077–1083, 2002.

[20] D. Molodtsov, "Soft Set Theory First Results", Comput. Math. with Appl., vol. 37, no. 4-5, pp. 19–31, 1999.

[21] S.B. Nadler, "Multivalued Contraction Mapping", Pac. J. Math., vol. 30, no. 2, pp. 475-488, 1969.

[22] Z. Pawlak, "Rough Set", Int. J. Comput. Inf. Sci., vol. 11, no. 5, pp. 341-356, 1982.

[23] Y.R. Syau, "On Convex And Concave Fuzzy Mappings", Fuzzy Sets Syst., vol. 103, no. 1, pp. 163–168, 1999.

[24] A.R. Roy and P.K. Maji, "A Fuzzy Soft Set Theoretic Approach To

Decision Making Problems", J. Comput. Appl. Math., vol. 203, no. 2, pp. 412–418, 2007.

[25] S. Rana, M. Qayyum, M. Saeed, F. Smarandache, and B.A. Khan, "Plithogenic Fuzzy Whole Hypersoft Set, Construction Of Operators And Their Application In Frequency Matrix Multi Attribute Decision Making Technique", Neutrosophic Set Syst., vol. 28, no. 1, pp. 34-50, 2019.

[26] M. Riaz, M. Saeed, M. Saqlain and N. Jafar, "Impact Of Water Hardness In Instinctive Laundry System Based On Fuzzy Logic Controller", Punjab Univ. j. math., vol. 51, no. 4, pp. 73–84, 2019.

[27] Y.R. Syau, "Differentiability And Convexity Of Fuzzy Mappings", Comput. Math. with Appl., vol. 41, no. 1-2, pp. 73–81, 2001.

[28] M. Saeed, M. Ahsan, M.S. Khubab and M.R. Ahmad, "A Study Of The Fundamentals Of Hypersoft Set Theory", Int. Sci. Eng, vol. 11, no. 1, pp. 320-329, 2020.

[29] M. Saeed, A. Mehmood, T. Abdeljewad, M.H. Saeed and M. Asim, "Application Of Similarity Measure In Pattern Recognition Of COVID-19 Spread And Its Effects In Pakistan", Appl. Comput. Math., vol. 20, no. 1, pp. 108-123, 2021.

[30] M. Saeed, M. Saqlain, A. Mehmood, K. Naseer and S. Yaqoob, "Multi-Polar Neutrosophic Soft Sets With Application In Medical Diagnosis And Decision-Making", Neutrosophic Set Syst., vol. 33, no. 1, pp. 183-207, 2020.

[31] A.U. Rahman, M. Saeed and A. Dhital, "Decision Making Application Based On Neutrosophic Parameterized Hypersoft Set Theory", Neutrosophic Set Syst., vol. 41, no. 1, pp. 1-14, 2021.

[32] M. Saeed, M. Ahsan and T. Abdeljewad, "A Development Of Complex Multi-Fuzzy Hypersoft Set With Application In MCDM Based On Entropy And Similarity Measure", IEEE Access, vol. 9, no. 1, pp. 60026 - 60042, 2021.

[33] M. Saqlain, N. Jafar, S. Moin, M. Saeed, and S. Broumi, "Single And Multi-Valued Neutrosophic Hypersoft Set And Tangent Similarity Measure Of Single Valued Neutrosophic Hypersoft Sets", Neutrosophic Set Syst., vol. 32, no. 1, pp. 317-329, 2020.

[34] M. Saqlain, S. Moin, M.N. Jafar, M. Saeed, and F. Smarandache, "Aggregate Operators Of Neutrosophic Hypersoft Set", Neutrosophic Set Syst., vol. 32, no. 1, pp. 295-303, 2020.

[35] V.D. Estruch, and A. Vidal, "A Note On Fixed Fuzzy Points For Fuzzy Mappings", Rend Istit. Univ. Trieste, vol. 32, no. 2, pp. 39–45, 2001.

[36] X. Yang, D. Yu, J. Yang and C. Wu, "Generalization Of Soft Set Theory From Crisp To Fuzzy Case", Fuzzy Inf. Eng., vol. 40, no. 1, pp. 345–354, 2007.

[37] H. Yan and J. Xu, "A Class Of Convex Fuzzy Mappings", Fuzzy Set Syst., vol. 129, no. 1, pp. 47-56, 2002.

[38] M. Zulqarnain, F. Dayan and M. Saeed, "Topsis Analysis For The Prediction Of Diabetes Based On General Characteristics Of Humans", Int. J. Pharm. Sci. Res., vol. 9, no. 7, pp. 2932–2939, 2018.

[39] S. Alkhazaleh and E. Marei, "Mappings On Neutrosophic Soft Classes", Neutrosophic Set Syst., vol. 2, no. 1, pp. 3-8, 2014.

[40] M. Bashir and A.R. Salleh, "Mappings On Intuitionistic Fuzzy Soft Classes", AIP. Conf. Proc., vol. 1522, no. 1, pp. 1022–1032, 2013.

[41] N.H. Sulaiman and D. Mohamad, "Mappings On Multiaspect Fuzzy Soft Classes", AIP. Conf. Proc., vol. 1602, no. 1, pp. 716–722, 2014.

[42] M.J. Borah and B. Hazarika, "Composite Mapping On Hesitant Fuzzy Soft Classes", arXiv, 2016.

[43] L.A. Zadeh, "The Concept Of A Linguistic Variable And Its Application To Approximate Reasoning", Inf. Sci., vol. 8, no. 3, pp. 199-249, 1975.

[44] K.T. Atanassov, "Intuitionistic Fuzzy Sets", Fuzzy Sets Syst., vol. 20, no. 1, pp. 87-96, 1986.

[45] F. Smarandache, "Neutrosophy Neutrosophic Probability, Set And Logic: Analytic Synthesis Synthetic Analysis", American Research Press, 1998, 105 p.

[46] W.R. Zhang and L. Zhang, "YinYang Bipolar Logic And Bipolar Fuzzy Logic", Inf. Sci., vol. 165, no. 3-4, pp. 265–287, 2004.

[47] J. Chen, S. Li, S. Ma and X. Wang, "M-Polar Fuzzy Sets An Extension Of Bipolar Fuzzy Sets", Sci. World J., vol. 2014, no. 1, pp. 416530, 2014.

[48] I. Deli, M. Ali and F. Smarandache, "Bipolar Neutrosophic Sets And Their Application Based On Multi-Criteria Decision-Making Problems", In Proceedings of 2015 international conference on advanced mechatronic systems Beijing China, 2015, pp. 249-254.

[49] M. Riaz and M.R. Hashmi, "M-Polar Neutrosophic Soft Mapping With Application To Multiple Personality Disorder And Its Associated Mental Disorders", Artif. Intell. Rev., vol. 54, no. 4, pp. 2717-2763, 2021.

[50] F. Smarandache, "A Unifying Field In Logics: Neutrosophic Logic", American Research Press, vol. 54, no. 4, pp. 1-141, 1999.

[51] F. Smarandache, "Neutrosophic Set, A Generalisation Of The Intuitionistic Fuzzy Sets", Int. J. Pure Appl. Math., vol. 24, no. 3, pp. 287-297, 2005.

[52] S. Broumi, A. Mumtaz and F. Smarandache, "Mappings On Neutrosophic Soft Expert Sets", J. New Theory, vol. 5, no. 5, pp. 27-42, 2015.

[53] M. Saeed, M. Ahsan and A.U. Rahman, "A Novel Approach To Mappings On Hypersoft Classes With Application", In Theory and Application of Hypersoft Set, Brussels, Belgium, Pons Publication House, 2021, 175-191.

[54] M. Saeed, M. Ahsan, A.U. Rahman and F. Smarandache, "An Inclusive Study On Fundamentals Of Hypersoft Set", In Theory and Application of Hypersoft Set, Brussels, Belgium, Pons Publication House, 2021, 1-23.

[55] M. Riaz and S.T. Tehrim, "Bipolar Fuzzy Soft Mappings With Application To Bipolar Disorders", Int. J. Biomath., vol. 12, no. 7, pp. 1950080, 2019.

[56] J. Ye, "Improved Cosine Similarity Measures Of Simplified Neutrosophic Sets For Medical Diagnoses", Artif. Intell. Med., vol. 63, no. 3, pp. 171-179, 2015.

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