An extended multiple criteria decision making method based on neutrosophic hesitant fuzzy information

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Abstract: Neutrosophic hesitant fuzzy set is the generalization of neutrosophic set and the hesitant fuzzy set, which can easily express the uncertain, incomplete and inconsistent information in cognitive activity, and the VIKOR (from Serbian:VIseKriterijumska Optimizacija I Kompromisno Resenje) method is an effective decision making tool which can select the optimal alternative by the maximum ‘‘group utility’’ and minimum of an ‘‘individual regret’’ with cognitive computation. In this paper, we firstly introduced some operational laws, comparison rules and the Hamming distance measure of neutrosophic hesitant fuzzy set, and described the traditional VIKOR method which only processes the crisp numbers. Then we extended the VIKOR method to process the Neutrosophic hesitant fuzzy information, and proposed an extended VIKOR method for the multiple criteria decision making problems with neutrosophic hesitant fuzzy information, and an illustrative example shows the effectiveness and feasibility of the proposed approach.

Key words: Neutrosophic hesitant fuzzy set; VIKOR method; multiple criteria decision making (MCDM)

1. Introduction

Decision making has been widely used in the politics, economic, military, management and the other fields. But in real decision making, the decision-making information is often inconsistent, incomplete and indeterminate, and how to express the decision-making information is very important. Since the fuzzy set (FS) theory was proposed by Zadeh [1], fuzzy multiple criteria decision-making problems have been widely researched. But FS only has one membership, and it cannot denote some complex fuzzy information. For example, during voting, there are ten persons voting for an issue, three of them give the “agree’’, four of them give the “disagree”, and the others abstain from voting. Obviously, FS cannot fully express the polling information. Atanassov [2, 3] defined the intuitionistic fuzzy set (IFS) by adding a non-membership function based on FS, i.e., IFS consists of truth-membership $T_A(x)$ and falsity-membership $F_A(x)$. The above example can be expressed by membership 0.3 and non-membership 0.4. However, IFSs can only handle incomplete information, and cannot deal with the inconsistent and indeterminate information. And the indeterminacy degree $1\text{-}T_A(x)\text{-}F_A(x)$ in IFSs is by default. In some complicated decision making environment, IFS also has some limitations in some complex decision-making situation. For instance, when an expert is called to make an opinion about a statement, he/she may give the possibility of right is 0.5 and the possibility of false is 0.6 and the uncertain possibility is 0.2 [4]. Obviously, this is a typical cognitive activity. However, on this occasion, IFS doesn’t cope with this type of information. To handle this type of decision-making problems, Smarandache [5] proposed the neutrosophic set (NS) by adding an indeterminacy-membership function based on IFS. In NS, the truth-membership, false-membership and indeterminacy-membership are totally independent. To simplify neutrosophic set and apply it to practical problems, Wang et al. [6] defined a single valued neutrosophic set (SVNS) with some examples. Ye [7, 8] defined the cross-entropy and the correlation coefficient of SVNS which was applied to single valued neutrosophic decision-making problems.

On the other hand, FS only has one membership which will limit some decision making problems. As a generalization of fuzzy set, Torra and Narukawa [9], Torra [10] put forward the hesitant fuzzy sets (HFSs) which use several possible values instead of the single membership degree. Then, Chen et al. [11] defined interval valued hesitant fuzzy sets (IVHFSs) which each membership degree is extended to interval numbers. Zhao et al. [12] developed hesitant triangular fuzzy set and series of aggregation operators for the hesitant triangular fuzzy sets based on the Einstein operations. Meng et al. [13] gave the linguistic hesitant fuzzy sets (LHFSs) and developed a series of linguistic hesitant fuzzy hybrid weighted operators. Farhadinia [14] and Ye [15] proposed the dual hesitant fuzzy sets and dual interval hesitant fuzzy sets. Peng et al. [16] represented the hesitant interval-valued intuitionistic fuzzy sets (HIVIFSs), and developed some hesitant interval intuitionistic fuzzy number weighted averaging operators based on t-conorms and t-norms.

As mentioned above, HFS and NS are extended in two directions based on FS, the HFS assigns the membership function a set of possible values, which is a good method to deal with uncertain information in practical decision making; however, it cannot process indeterminate and inconsistent information, while the NS can easily character uncertainty, incomplete and inconsistent information. Obviously, each of them has its advantages and disadvantages. Further, Ye [17] proposed a neutrosophic hesitant fuzzy set by combining the hesitant fuzzy sets with single-valued neutrosophic sets (SVNHFS), and then some weighted averaging and weighted geometric operators for SVNHFS are developed. Obviously, the neutrosophic hesitant fuzzy sets (NHFSs), which extend truth-membership degree, indeterminacy-membership degree, and falsity-membership degree of NS to a set of possible values in interval [0,1], can easily express the uncertain, incomplete and inconsistent information in .

In addition, the VIKOR (from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje) method is an important tool to process the fuzzy decision-making problems, which is based on the particular measure of “closeness” to the ‘‘ideal” solution and the ‘‘negative ideal” solution, and can achieve the maximum ‘‘group utility’’ and minimum of ‘‘individual regret’’. Obviously, this is a cognitive computation [18-21]. Because the traditional VIKOR method can only deal with the crisp numbers, some new extensions of VIKOR for the different fuzzy information have been studied. Zhang and Wei [22] extended VIKOR to deal with hesitant fuzzy set. Liu and Wu [23] extended VIKOR to process the multi-granularity linguistic variables and apply it to the competency evaluation of human resources managers. Zhang and Liu [24] VIKOR to process the hybrid information, including crisp numbers, interval numbers, triangular fuzzy numbers, trapezoid fuzzy number s, linguistic variables, and so on. Du and Liu [25] extended VIKOR to deal with intuitionistic trapezoidal fuzzy numbers. However, until now, the extended VIKOR cannot process the neutrosophic hesitant fuzzy information, so it is useful and necessary to extend the VIKOR to neutrosophic hesitant fuzzy information.

In order to achieve the above purposes, the organization structure is shown as follows. In the next section, we introduce the single valued neutrosophic set, HFSs, NHFSs, and the traditional VIKOR method. In section 3, we extend the traditional VIKOR method to the neutrosophic hesitant fuzzy information, and a multiple criteria decision making approach is proposed. In Section 4, we give a numerical example to elaborate the effectiveness and feasibility of our approach. In Section 5, we give the main concluding remarks of this paper.

2. Preliminaries

2.1 The single valued neutrosophic set

Definition 1 [5, 26]. Let X be a universe of discourse, with a generic element in X denoted by x . A single valued neutrosophic set A in X is characterized by:

$$A = \left\{ x = (T_A(x), I_A(x), F_A(x)) \middle| x \in X \right\}, \tag{1}$$

where the functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ denote the truth-membership, the indeterminacy-membership and the falsity-membership of the element $x \in X$ to the set A respectively. For each point x in X , we have $T_A(x), I_A(x), F_A(x) \in [0,1]$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

For convenience, we can use $x = (T_x, I_x, F_x)$ to denote an element x in SVNS, and the element x is called a single valued neutrosophic number (SVNN).

To compare two SVNNs, Smarandache and Vlădăreanu [27] proposed the partial order relation between two neutrosophic numbers as follows.

Definition 2 [27]. For two SVNNs $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$, iff (if and only if) $T_1 \le T_2$, $I_1 \ge I_2$, $F_1 \ge F_2$, then $x \le y$.

Obviously, in practical applications, many cases can't satisfy the above conditions. With respect to these, Ye [28] presented a comparison method based on the cosine similarity measure of a SVNN $x = (T, I, F)$ to ideal solution (1,0,0), and offered the definition of the cosine similarity:

$$S(x) = \frac{T}{\sqrt{T^2 + I^2 + F^2}}. \tag{2}$$

Definition 3 [28]. Suppose $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ are two SVNNs, if $S(x) \le S(y)$, then $x \le y$.

Definition 4. Let $x = (T_1, I_1, F_1)$ and $y = (T_2, I_2, F_2)$ are two SVNNs, then the normalized Hamming distance between x and y is defined as follows:

$$d(x, y) = \frac{1}{3}\left(|T_1 - T_2| + |I_1 - I_2| + |F_1 - F_2|\right). \tag{3}$$

2.2 The hesitant fuzzy set (HFS)

Definition 5 [29]. Let X be a non-empty fixed set, a HFS A on X is in terms of a function $h_A(x)$ that when applied to X returns a subset of [0,1], which can be denoted by the following mathematical symbol:

$$A = \left\{ \langle x, h_A(x) \rangle \middle| x \in X \right\} \tag{4}$$

where $h_A(x)$ is a set of some values in $[0,1]$, representing the possible membership degrees of the element $x \in X$ to A .For convenience, we call $h_A(x)$ a hesitant fuzzy element (HFE), denoted by h , which reads $h = \{\gamma | \gamma \in h\}$.

For any three HFEs $h = \{\gamma | \gamma \in h\}$, $h_1 = \{\gamma_1 | \gamma_1 \in h_1\}$ and $h_2 = \{\gamma_2 | \gamma_2 \in h_2\}$, Torra [29] defined some operations as follows:

(1) $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$ (5)

(2) $h_1 \cup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \max\{\gamma_1, \gamma_2\}$. (6)

(3) $h_1 \cap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \min\{\gamma_1, \gamma_2\}$. (7)

After that, Xia and Xu [30] gave four operations about the HFEs $h = \{\gamma | \gamma \in h\}$, $h_1 = \{\gamma_1 | \gamma_1 \in h_1\}$ and $h_2 = \{\gamma_2 | \gamma_2 \in h_2\}$ with a positive scale n :

(1) $h^n = \bigcup_{\gamma \in h} \{\gamma^n\}$ (8)

(2) $nh = \bigcup_{\gamma \in h} \{1 - (1-\gamma)^n\}$, (9)

(3) $h_1 \oplus h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$, (10)

(4) $h_1 \otimes h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$. (11)

Definition 6. Let h_1 and h_2 be two HFSs on $X = \{x_1, x_2, \cdots, x_n\}$, then the hesitant normalized Hamming distance measure between h_1 and h_2 is defined as:

$$\|h_1 - h_2\| = \frac{1}{l}\sum_{j=1}^{l} \left|h_{1\sigma(j)} - h_{2\sigma(j)}\right|, \quad (12)$$

where $l(h)$ is the number of the elements in the h, in most cases, $l(h_1) \neq l(h_2)$, and for convenience, let $l = \max\{l(h_1), l(h_2)\}$. $h_{1\sigma(j)}$ and $h_{2\sigma(j)}$ express the jth element in h_1 and h_2, respectively. For operability, we should extend the shorter ones until both of them have the same length when compared. The best way to extend the shorter one is to add the same value in it. In fact, we can extend the shorter one by adding any value in it. The selection of the value mainly depends on the decision makers' risk preferences. Optimists anticipate desirable outcomes and may add the maximum value, while pessimists expect unfavorable outcomes and may add the minimum value.

For example, let h_1 ={0.1, 0.2, 0.3}, h_2 ={0.4, 0.5}, and $l(h_1) > l(h_2)$. For operability, we extend h_2 to h_2 ={0.4, 0.4, 0.5} until it has the same length of h_1, the optimist may extend h_2 as h_2 ={0.4, 0.5, 0.5} and the pessimist may extend it as h_2 ={0.4, 0.4, 0.5}. Although the results may be different when we extend the shorter one by adding different values, this is reasonable because the decision makers' risk preferences can directly influence the final decision. The same situation can also be found in many existing Refs. [30]. In this study, we assume that the decision makers are all pessimistic (other situations can be studied similarly).

2.3 The neutrosophic hesitant fuzzy set

In this section, we will introduce the neutrosophic hesitant fuzzy set by combining neutrosophic set with hesitant fuzzy set.

Definition 7 [17]. Let X be a non-empty fixed set, a neutrosophic hesitant fuzzy set (NHFS) on X is expressed by:

$$N = \left\{ \left\langle x, \tilde{t}(x), \tilde{i}(x), \tilde{f}(x) \right\rangle \middle| x \in X \right\}, \quad (13)$$

where $\tilde{t}(x) = \{\tilde{\gamma} | \tilde{\gamma} \in \tilde{t}(x)\}$, $\tilde{i}(x) = \{\tilde{\delta} | \tilde{\delta} \in \tilde{i}(x)\}$, and $\tilde{f}(x) = \{\tilde{\eta} | \tilde{\eta} \in \tilde{f}(x)\}$ are three sets with some values in interval $[0,1]$, which represents the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees of the element $x \in X$ to the set N, and satisfies these limits :

$\tilde{\gamma} \in [0,1]$, $\tilde{\delta} \in [0,1]$, $\tilde{\eta} \in [0,1]$ and $0 \le \sup \tilde{\gamma}^{+} + \sup \tilde{\delta}^{+} + \sup \tilde{\eta}^{+} \le 3$,where

$\tilde{\gamma}^{+} = \bigcup_{\tilde{\gamma} \in \tilde{t}(x)} \max\{\tilde{\gamma}\}$, $\tilde{\delta}^{+} = \bigcup_{\tilde{\delta} \in \tilde{i}(x)} \max\{\tilde{\delta}\}$, and $\tilde{\eta}^{+} = \bigcup_{\tilde{\eta} \in \tilde{f}(x)} \max\{\tilde{\eta}\}$ for $x \in X$.

The $\tilde{n} = \{\tilde{t}(x), \tilde{i}(x), \tilde{f}(x)\}$ is called a neutrosophic hesitant fuzzy element (NHFE) which is the basic unit of the NHFS and is denoted by the symbol $\tilde{n} = \{\tilde{t}, \tilde{i}, \tilde{f}\}$.

Then, some basic operations of NHFEs are defined as follows:

Definition 8. Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ be two NHFEs in a non-empty fixed set X , then

(1) $\tilde{n}_1 \cup \tilde{n}_2 = \{\tilde{t}_1 \cup \tilde{t}_2, \tilde{i}_1 \cap \tilde{i}_2, \tilde{f}_1 \cap \tilde{f}_2\}$; (14)

(2) $\tilde{n}_1 \cap \tilde{n}_2 = \{\tilde{t}_1 \cap \tilde{t}_2, \tilde{i}_1 \cup \tilde{i}_2, \tilde{f}_1 \cap \tilde{f}_2\}$. (15)

Therefore, for two NHFEs $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$, $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ and a positive scale $k > 0$, the operations can be defined as follows:

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \{\tilde{t}_1 \oplus \tilde{t}_2, \tilde{i}_1 \otimes \tilde{i}_2, \tilde{f}_1 \otimes \tilde{f}_2\} = \bigcup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{i}_2, \tilde{\eta}_2 \in \tilde{f}_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2, \delta_1\delta_2, \eta_1\eta_2\}$ (16)

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \{\tilde{t}_1 \otimes \tilde{t}_2, \tilde{i}_1 \oplus \tilde{i}_2, \tilde{f}_1 \oplus \tilde{f}_2\} = \bigcup_{\tilde{\gamma}_1 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1, \tilde{\gamma}_2 \in \tilde{t}_2, \tilde{\delta}_2 \in \tilde{i}_2, \tilde{\eta}_2 \in \tilde{f}_2} \{\gamma_1\gamma_2, \delta_1 + \delta_2 - \delta_1\delta_2, \eta_1 + \eta_2 - \eta_1\eta_2\}$ (17)

(3) $k\tilde{n}_1 = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1} \{1 - (1 - \gamma_1)^k, \delta_1^{\,k}, \eta_1^{\,k}\}$ (18)

(4) $\tilde{n}_1^{\,k} = \bigcup_{\tilde{\gamma}_2 \in \tilde{t}_1, \tilde{\delta}_1 \in \tilde{i}_1, \tilde{\eta}_1 \in \tilde{f}_1} \{\gamma_1^{\,k}, 1 - (1 - \delta_1)^k, 1 - (1 - \eta_1)^k\}$ (19)

Example 1. Let $\tilde{n}_1 = \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle$ and $\tilde{n}_2 = \langle\{0.5\},\{0.3\},\{0.2,0.3\}\rangle$ be two NHFEs, and $k = 2$, then

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \langle\{0.80\}, \{0.03, 0.06\}, \{0.04, 0.06\rangle$

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \langle\{0.30\}, \{0.37, 0.44\}, \{0..36, 0.44\}\rangle$

(3) $2 \cdot \tilde{n}_1 = \langle\{0.84\}, \{0.01, 0.04\}, \{0.04\}\rangle$

(4) $\tilde{n}_1^{\,2} = \langle\{0.36\}, \{0.19, 0.36\}, \{0.36\}\rangle$

Theorem 1. Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ be two NHFEs in a non-empty fixed set X , and $\eta, \eta_1, \eta_2 > 0$, then we have

(1) $\tilde{n}_1 \oplus \tilde{n}_2 = \tilde{n}_2 \oplus \tilde{n}_1$; (20)

(2) $\tilde{n}_1 \otimes \tilde{n}_2 = \tilde{n}_2 \otimes \tilde{n}_1$; (21)

(3) $\eta(\tilde{n}_1 \oplus \tilde{n}_2) = \eta\tilde{n}_1 \oplus \eta\tilde{n}_2$; (22)

(4) $\eta_1\tilde{n}_1 \oplus \eta_2\tilde{n}_1 = (\eta_1 + \eta_2)\tilde{n}_1$; (23)

(5) $\tilde{n}_1^{\eta} \otimes \tilde{n}_2^{\eta} = (\tilde{n}_2 \otimes \tilde{n}_1)^{\eta}$; (24)

(6) $\tilde{n}_1^{\eta_1} \otimes \tilde{n}_1^{\eta_2} = \tilde{n}_1^{\eta_1+\eta_2}$; (25)

Definition 9. For an NHFE $\tilde{n}$,

$$S(\tilde{n}) = \left[\frac{1}{l}\sum_{i=1}^{l}\tilde{\gamma}_i + \frac{1}{p}\sum_{i=1}^{p}(1-\tilde{\delta}_i) + \frac{1}{q}\sum_{i=1}^{q}(1-\tilde{\eta}_i)\right] \Big/ 3 \tag{26}$$

is called the score function of $\tilde{n}$, where l, p, q are the numbers of the values $\tilde{\gamma}$, $\tilde{\delta}$, $\tilde{\eta}$, respectively. Obviously, $S(\tilde{n})$ is a value belonging $[0,1]$.

Suppose $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ are any two NHFEs, the comparison method of NHFEs is expressed as follows [17]:

(1) If $S(\tilde{n}_1) > S(\tilde{n}_2)$, then $\tilde{n}_1 > \tilde{n}_2$;

(2) If $S(\tilde{n}_1) < S(\tilde{n}_2)$, then $\tilde{n}_1 < \tilde{n}_2$;

(3) If $S(\tilde{n}_1) = S(\tilde{n}_2)$, then $\tilde{n}_1 = \tilde{n}_2$.

Example 2. Let $\tilde{n}_1 = \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle$ and $\tilde{n}_2 = \langle\{0.5\},\{0.3\},\{0.2,0.3\}\rangle$ be two NHFEs, then

$$S(\tilde{n}_1) = \left[0.6 + \frac{1}{2}[(1-0.1)+(1-0.2)] + (1-0.2)\right] \Big/ 3 = 0.75$$

$$S(\tilde{n}_2) = \left[0.5 + (1-0.3) + \frac{1}{2}[(1-0.2)+(1-0.3)]\right] \Big/ 3 = 0.65$$

Because $S(\tilde{n}_1) > S(\tilde{n}_2)$, we can get $\tilde{n}_1 > \tilde{n}_2$.

Definition 10. Let $\tilde{n}_1 = \{\tilde{t}_1, \tilde{i}_1, \tilde{f}_1\}$ and $\tilde{n}_2 = \{\tilde{t}_2, \tilde{i}_2, \tilde{f}_2\}$ are any two NHFEs, then the normalized Hamming distance between $\tilde{n}_1$ and $\tilde{n}_2$ is defined as follows:

$$\begin{aligned} d(\tilde{n}_1, \tilde{n}_2) = \left\|\tilde{n}_1 - \tilde{n}_2\right\| &= \frac{1}{3}\left(\left|\tilde{\gamma}_1 - \tilde{\gamma}_2\right| + \left|\tilde{\delta}_1 - \tilde{\delta}_2\right| + \left|\tilde{\eta}_1 - \tilde{\eta}_2\right|\right) \\ &= \frac{1}{3}\left(\left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\gamma}_{1\sigma(j)} - \tilde{\gamma}_{2\sigma(j)}\right|\right| + \left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\delta}_{1\sigma(j)} - \tilde{\delta}_{2\sigma(j)}\right|\right| + \left|\frac{1}{l}\sum_{j=1}^{l}\left|\tilde{\eta}_{1\sigma(j)} - \tilde{\eta}_{2\sigma(j)}\right|\right|\right). \end{aligned} \tag{27}$$

Example 3. Let $\tilde{n}_1 = \langle\{0.6\},\{0.1,0.2\},\{0.2\}\rangle$ and $\tilde{n}_2 = \langle\{0.5\},\{0.3\},\{0.2,0.3\}\rangle$ be two NHFEs, then

$$d(\tilde{n}_1, \tilde{n}_2) = \left\|\tilde{n}_1 - \tilde{n}_2\right\| = \frac{1}{3}\left(\left\|0.6-0.5\right\| + \left|\frac{1}{2}(\left|0.1-0.3\right| + \left|0.2-0.3\right|)\right| + \left|\frac{1}{2}(\left|0.2-0.2\right| + \left|0.2-0.3\right|)\right|\right) = 0.1$$

2.4 VIKOR method

The VIKOR method was introduced for multi-criteria optimization problem. This method focuses on ranking a set of alternatives and selecting a compromise solution. Here, the compromise means that an agreement was established by mutual concessions [31-33]. The decision making problem, which can be solved by VIKOR, is described as follows.

Suppose there are m alternatives which are denoted as $A_1, A_2, \ldots, A_m$, and there are n criteria which are denoted as $C_1, C_2, \ldots, C_n$, the evaluation value of alternative A_i with respect to criterion C_j is expressed by f_{ij}. Suppose the f_j^* and f_j^- express the virtual positive ideal value and virtual negative ideal value under the criterion C_j. $w = (w_1, w_2, \cdots w_n)^T$ is the criterion weight vector satisfying $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. The compromise ranking could be performed by comparing the measure of closeness to the ideal alternative. The multi-criteria measure for compromise ranking is developed from the L_p-metric used as an aggregating function in a compromise programming method [33]. The VIKOR method is started with the following form of L_p-metric:

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[w_j (f_j^* - f_{ij}) \big/ (f_j^* - f_j^-) \right]^p \right\}^{1/p} \quad 1 \le p \le \infty; i = 1, 2, 3, \ldots, m. \tag{28}$$

In the VIKOR method, the $L_{1,i}$ (as S_i) and $L_{\infty,i}$ (as R_i) are used to formulate ranking measure. The solution obtained by $\min S_i$ is with a maximum group utility ("majority" rule), and the solution obtained by $\min R_i$ is with a minimum individual regret of the "opponent".

The compromise ranking algorithm of the VIKOR method has the following steps:

Step 1. Determine the virtual positive ideal f_j^* and the virtual negative ideal f_j^- values under the criterion C_j, we have

$$f_j^* = \max_i f_{ij}, \quad f_j^- = \min_i f_{ij} \tag{29}$$

Step 2. Compute the values S_i and R_i; $i = 1, 2, \ldots, m$, by these relations:

$$S_i = \sum_{j=1}^{n} w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-), \tag{30}$$

$$R_i = \max_j w_j (f_j^* - f_{ij}) / (f_j^* - f_j^-) \tag{31}$$

Step 3. Compute the values Q_i : $i = 1, 2, \ldots, m$, by the following relation:

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1 - v)(R_i - R^*)/(R^- - R^*) \tag{32}$$

where

$$S^* = \min_i S_i, S^- = \max_i S_i, \quad R^* = \min_i R_i, R^- = \max_i R_i$$

v is the weight of the strategy of "the majority of criteria" (or "the maximum group utility").

When $v > 0.5$, considering "the maximum group utility" is more than "minimum individual regret", and when $v < 0.5$, considering "minimum individual regret" is more than "the maximum group utility". In this study, we suppose that $v = 0.5$ which "minimum individual regret" and "the maximum group utility" are the same important.

Step 4. Rank the alternatives. Sorting by the values S, R and Q in decreasing order. The results are three ranking lists.

Step 5. Propose as a compromise solution $A^{(1)}$, which is ranked in the first position by the measure Q (Minimum) if the following two conditions are satisfied:

Condition 1: Acceptable advantage: $Q(A^{(2)}) - Q(A^{(1)}) \geq \frac{1}{m-1}$, where $Q(A^{(2)})$ is the value of alternative with second position in the ranking list. m is the number of alternatives.

Condition 2: Acceptable stability in decision making: Alternative $A^{(1)}$ must also be the best ranked by S or/and R.

If one of above conditions does not meet, then we will get a set of compromise solutions:

(1) If condition 2 is not met, then alternatives $A^{(1)}$ and $A^{(2)}$ are compromise solutions.

(2) If condition 1 is not be met, then the maximum M can be got by the relation $Q(A^{(M)}) - Q(A^{(1)}) < DQ$, and alternatives $A^{(1)}$, $A^{(2)}$,…, $A^{(M)}$ are compromise solutions.

Based on the above analysis, we know that the best alternative is the one with the minimum value of Q when the conditions1 and 2 are met, and if one of two conditions is not met, the compromise solutions may be have more than one. VIKOR is an effective tool in multi-criteria decision making. The obtained compromise solution could be accepted by the decision makers because it provides a maximum "group utility" (represented by min S) of the "majority", and a minimum of the "individual regret" (represented by min R) of the "opponent". The compromise solutions could be the basis for negotiations, involving the decision maker's preference by criteria weights.

3. VIKOR method for decision making problem with neutrosophic hesitant fuzzy numbers

In real decision making, it is difficult or impossible to obtain the criteria values by the exact numbers, however, the neutrosophic hesitant fuzzy set is a very useful tool to deal with uncertain decision making problems in which each criteria can be described as a neutrosophic hesitant fuzzy numbers [17]. The VIKOR is very effective method to solve the decision making problems, however, the traditional VIKOR is only suitable for crisp numbers, and then it has been extended to process the different fuzzy information [22-25]. Until now, it has not been used to process the neutrosophic hesitant fuzzy information. So, in this paper, we will extend the VIKOR method to solve MADM problem with the neutrosophic hesitant fuzzy information.

To do this, we firstly describe the decision making problem.

For a multiple criteria decision making problem, let $A = \{A_1, A_2, \cdots, A_m\}$ be a collection of m alternatives, $C = \{C_1, C_2, \cdots, C_m\}$ be a collection of n criteria, which weight vector is $w = (w_1, w_2, \cdots w_n)^T$ satisfying $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$. Suppose that $\tilde{n}_{ij} = (\tilde{t}_{ij}, \tilde{i}_{ij}, \tilde{f}_{ij})$ is the evaluation value of the alternative A_i with respect to the criteria C_j which is expressed by the neutrosophic hesitant fuzzy information, where $\tilde{t}_{ij} = \{\tilde{\gamma}_{ij} | \tilde{\gamma}_{ij} \in \tilde{t}_{ij}\}$, $\tilde{i}_{ij} = \{\tilde{\delta}_{ij} | \tilde{\delta}_{ij} \in \tilde{i}_{ij}\}$ and $\tilde{f}_{ij} = \{\tilde{\eta}_{ij} | \tilde{\eta}_{ij} \in \tilde{f}_{ij}\}$ are three collections of some values in interval $[0,1]$, which represent the possible truth-membership hesitant degrees, indeterminacy-membership hesitant degrees, and falsity-membership hesitant degrees, and satisfy following limits:

$\tilde{\gamma} \in [0,1]$, $\tilde{\delta} \in [0,1]$, $\tilde{\eta} \in [0,1]$, and $0 \le \sup \tilde{\gamma}^{+} + \sup \tilde{\delta}^{+} + \sup \tilde{\eta}^{+} \le 3$, where $\tilde{\gamma}^{+} = \bigcup_{\tilde{\gamma}_{ij} \in \tilde{t}_{ij}} \max\{\gamma_{ij}\}$, $\tilde{\delta}^{+} = \bigcup_{\tilde{\delta}_{ij} \in \tilde{i}_{ij}} \max\{\delta_{ij}\}$ and $\tilde{\eta}^{+} = \bigcup_{\tilde{\eta}_{ij} \in \tilde{f}_{ij}} \max\{\eta_{ij}\}$. The decision matrix denoted by the neutrosophic hesitant fuzzy numbers are shown in Table 1, and then we can rank the order of the alternatives.

The procedures of the proposed method as follows:

Table 1 Decision making matrix with the neutrosophic hesitant fuzzy information

	C_1	C_2	$\cdots$	C_n
A_1	$\tilde{n}_{11}$	$\tilde{n}_{12}$	$\cdots$	$\tilde{n}_{1n}$
A_2	$\tilde{n}_{21}$	$\tilde{n}_{22}$	$\cdots$	$\tilde{n}_{2n}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
A_m	$\tilde{n}_{m1}$	$\tilde{n}_{m2}$	$\cdots$	$\tilde{n}_{mn}$

Step 1. Normalize the decision matrix.

In MAGDM problems, there are two types in criteria, that is, benefit criteria and cost criteria. To maintain consistency of the criteria, we usually transform the cost criteria into benefit criteria.

For the cost criteria, the normalization formula is

$$(\tilde{n}_{ij}^{k}) = \left\{\bigcup_{\tilde{\gamma} \in \tilde{t}_{ij}} \{1-\tilde{\gamma}\}, \bigcup_{\tilde{\delta} \in \tilde{i}_{ij}} \{1-\tilde{\delta}\}, \bigcup_{\tilde{\eta} \in \tilde{f}_{ij}} \{1-\tilde{\eta}\}\right\} \tag{33}$$

Step 2. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS). There are two methods to determine them.

(1) According to the partial order relation, we have

the positive ideal solution (PIS): $A^{*} = \{\tilde{n}_1^{*}, \cdots, \tilde{n}_n^{*}\}$ (34)

Where

$$\tilde{n}_j^{*} = \left\{\max_i \bigcup_{\tilde{\gamma} \in \tilde{t}_{ij}} \{\tilde{\gamma}\}, \min_i \bigcup_{\tilde{\delta} \in \tilde{i}_{ij}} \{\tilde{\delta}\}, \bigcup_{\tilde{\eta} \in \tilde{f}_{ij}} \min_i \{\tilde{\eta}\}\right\},\ j = 1, 2, \cdots, n \tag{35}$$

the negative ideal solution (NIS) $A^{-} = \{\tilde{n}_1^{-}, \cdots, \tilde{n}_n^{-}\}$ (36)

where $\tilde{n}_j^{-} = \left\{\min_i \bigcup_{\tilde{\gamma} \in \tilde{t}_{ij}} \{\tilde{\gamma}\}, \max_i \bigcup_{\tilde{\delta} \in \tilde{i}_{ij}} \{\tilde{\delta}\}, \bigcup_{\tilde{\eta} \in \tilde{f}_{ij}} \max_i \{\tilde{\eta}\}\right\},\ j = 1, 2, \cdots, n$ (37)

(2) According to score function, we have

$A^{*} = \{\tilde{n}_1^{*}, \cdots, \tilde{n}_n^{*}\}$, where $\tilde{n}_j^{*} = \max\{S(\tilde{n}_{1j}), \cdots, S(\tilde{n}_{mj})\},\ j = 1, 2, \cdots, n$ (38)

$A^{-} = \{\tilde{n}_1^{-}, \cdots, \tilde{n}_n^{-}\}$, where $\tilde{n}_j^{-} = \min\{S(\tilde{n}_{1j}), \cdots, S(\tilde{n}_{mj})\},\ j = 1, 2, \cdots, n$ (39)

Where $S(.)$ is the score function of neutrosophic hesitant fuzzy number which is defined by (26).

Step 3. Compute S_i and R_i, and we have

$$S_i = \sum_{j=1}^{n} w_j \left\| \tilde{n}_j^* - \tilde{n}_{ij} \right\| \Big/ \left\| \tilde{n}_j^* - \tilde{n}_j^- \right\|, i = 1,2,..., m \ , \tag{40}$$

$$R_i = \max_j \ w_j \left\| \tilde{n}_j^* - \tilde{n}_{ij} \right\| \Big/ \left\| \tilde{n}_j^* - \tilde{n}_j^- \right\|, i = 1,2,..., m \tag{41}$$

Where $\left\| \tilde{n}_1 - \tilde{n}_2 \right\|$ is the distance between two neutrosophic hesitant fuzzy numbers $\tilde{n}_1$ and $\tilde{n}_2$, which is defined by (27).

Step 4. Compute the values Q_i, and we have

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(R_i - R^*)/(R^- - R^*) \tag{42}$$

Where $S^* = \min_i \ S_i, S^- = \max_i \ S_i$, $R^* = \min_i \ R_i, R^- = \max_i \ R_i$

where v is introduced as weight of the strategy of "the majority of criteria" (or "the maximum group utility"), here suppose that $v = 0.5$

Step 5. Same as the step 4 of section 2.

Step 6. Same as the step5 of section 2.

4. An numerical example

We consider an example [34] where one investment company intends to select an enterprise from the following four alternatives to invest. The four enterprises are marked by $A_i \ (i = 1,2,3.4)$, and they are measured by three criteria: (1) C_1 (the risk index); (2) C_2 (the growth index); (3) C_3 (environmental impact index) (suppose it is cost type), and the evaluation values are denoted by NHFNs and their weight is $w = (0.35, 0.25, 0.4)^T$. The decision matrix R is shown in the Table 2. Then give the ranking the alternatives.

Table 2 The neutrosophic hesitant fuzzy decision matrix

	C_1	C_2	C_3
A_1	{{0.4, 0.5},{0.2}, {0.3}}	{{0.4 },{0.2,0.3},{0.3}}	{{0.2},{0.2},{0.5}}
A_2	{{0.6},{0.1,0.2},{0.2}}	{{0.6},{0.1},{0.2}}	{{0.5},{0.2},{0.1,0.2}}
A_3	{{0.3,0.4},{0.2},{0.3}}	{{0.5},{0.2},{0.3}}	{{0.5},{0.2,0.3},{0.2}}
A_4	{{0.7},{0.1,0.2},{0.1}}	{{0.6},{0.1},{0.2}}	{{0.6},{0.3},{0.2}}

4.1 The evaluation steps by the proposed method

Step 1. Normalize the decision matrix.

Considering all the criteria should be uniform types, the cost type C_3 should be transformed into benefit type, and then we obtain the normalized NHFNS decision matrix $\tilde{R}^k = (\beta_{ij}^k)_{4\times 3}$ by (33) as follows:

Table 3 The normalized neutrosophic hesitant fuzzy decision matrix

	C_1	C_2	C_3
A_1	{{0.4, 0.5},{0.2}, {0.3}}	{{0.4 },{0.2,0.3},{0.3}}	{{0.8},{0.8},{0.5}}
A_2	{{0.6},{0.1,0.2},{0.2}}	{{0.6},{0.1},{0.2}}	{{0.5},{0.8},{0.9,0.8}}
A_3	{{0.3,0.4},{0.2},{0.3}}	{{0.5},{0.2},{0.3}}	{{0.5},{0.8,0.7},{0.8}}
A_4	{{0.7},{0.1,0.2},{0.1}}	{{0.6},{0.1},{0.2}}	{{0.6},{0.7},{0.8}}

Step 2. Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) by (34) - (37), we can get

$$A^* = \{\tilde{n}_1^*, \tilde{n}_2^*, \tilde{n}_3^*\} = \{\{\{0.7\},\{0.1\},\{0.1\}\}, \{\{0.6\},\{0.1\},\{0.2\}\}, \{\{0.8\},\{0.7\},\{0.5\}\}\}$$

$$A^- = \{\tilde{n}_1^-, \tilde{n}_2^-, \tilde{n}_3^-\} = \{\{\{0.3\},\{0.2\},\{0.3\}\}, \{\{0.4\},\{0.3\},\{0.3\}\}, \{\{0.5\},\{0.8\},\{0.9\}\}\}$$

Step 3. Compute S_i and R_i by (40) and (41), and we have

$$S_1 = \frac{w_1\left\|n_1^* - n_{11}\right\|}{\left\|n_1^* - n_1^-\right\|} + \frac{w_2\left\|n_2^* - n_{12}\right\|}{\left\|n_2^* - n_2^-\right\|} + \frac{w_3\left\|n_3^* - n_{13}\right\|}{\left\|n_3^* - n_3^-\right\|} = 0.450$$

$$S_2 = \frac{w_1\left\|n_1^* - n_{21}\right\|}{\left\|n_1^* - n_1^-\right\|} + \frac{w_2\left\|n_2^* - n_{22}\right\|}{\left\|n_2^* - n_2^-\right\|} + \frac{w_3\left\|n_3^* - n_{23}\right\|}{\left\|n_3^* - n_3^-\right\|} = 0.587$$

$$S_3 = \frac{w_1\left\|n_1^* - n_{31}\right\|}{\left\|n_1^* - n_1^-\right\|} + \frac{w_2\left\|n_2^* - n_{32}\right\|}{\left\|n_2^* - n_2^-\right\|} + \frac{w_3\left\|n_3^* - n_{33}\right\|}{\left\|n_3^* - n_3^-\right\|} = 0.783$$

$$S_4 = \frac{w_1\left\|n_1^* - n_{41}\right\|}{\left\|n_1^* - n_1^-\right\|} + \frac{w_2\left\|n_2^* - n_{42}\right\|}{\left\|n_2^* - n_2^-\right\|} + \frac{w_3\left\|n_3^* - n_{43}\right\|}{\left\|n_3^* - n_3^-\right\|} = 0.184$$

$$R_1 = \max_3 \left\{ \frac{w_1\left\|n_1^* - n_{11}\right\|}{\left\|n_1^* - n_1^-\right\|}, \frac{w_2\left\|n_2^* - n_{12}\right\|}{\left\|n_2^* - n_2^-\right\|}, \frac{w_3\left\|n_3^* - n_{13}\right\|}{\left\|n_3^* - n_3^-\right\|} \right\} = 0.225$$

$$R_2 = \max_3 \left\{ \frac{w_1\left\|n_1^* - n_{21}\right\|}{\left\|n_1^* - n_1^-\right\|}, \frac{w_2\left\|n_2^* - n_{22}\right\|}{\left\|n_2^* - n_2^-\right\|}, \frac{w_3\left\|n_3^* - n_{23}\right\|}{\left\|n_3^* - n_3^-\right\|} \right\} = 0.516$$

$$R_3 = \max_3 \left\{ \frac{w_1\left\|n_1^* - n_{31}\right\|}{\left\|n_1^* - n_1^-\right\|}, \frac{w_2\left\|n_2^* - n_{32}\right\|}{\left\|n_2^* - n_2^-\right\|}, \frac{w_3\left\|n_3^* - n_{33}\right\|}{\left\|n_3^* - n_3^-\right\|} \right\} = 0.447$$

$$R_4 = \max_3 \left\{ \frac{w_1\left\|n_1^* - n_{41}\right\|}{\left\|n_1^* - n_1^-\right\|}, \frac{w_2\left\|n_2^* - n_{42}\right\|}{\left\|n_2^* - n_2^-\right\|}, \frac{w_3\left\|n_3^* - n_{43}\right\|}{\left\|n_3^* - n_3^-\right\|} \right\} = 0.167$$

Step 4. Compute the values $Q_i\,(i = 1,2,3,4)$ by (42) (suppose $v = 0.5$), we have

$$Q_1 = 0.305,\quad Q_2 = 0.836,\quad Q_3 = 0.901,\quad Q_4 = 0$$

Step 5. Rank the alternatives. Sorting by the values S , R and Q in decreasing order. The results are three ranking lists, which is depicted in Table 4.

Step 6. The ranking of alternatives by Q in decreasing order, the alternative with first position is A_4 with $Q(A_4)=0$, and A_1 is the alternative with second position with $Q(A_1)$ =0.305. As DQ=1/(m-1)=1/(4-1)=0.333, so

$$Q(A_1)-Q(A_4)=0.305 < 0.333$$

Which is not satisfied $Q(A_1)-Q(A_4)\geq \frac{1}{4-1}$, but alternative A_4 is the best ranked by S and R , which satisfies the condition 2. By computing, we get:

$$Q(A_3)-Q(A_4)=0.901 > 0.333$$

$$Q(A_2)-Q(A_4)=0.836 > 0.333$$

$$Q(A_1)-Q(A_4)=0.305 < 0.333$$

so A_4 , A_1 are both compromise solutions.

Table 4 The ranking and the compromise solutions.

	A_1	A_2	A_3	A_4	Ranking	Compromise solutions
S	0.450	0.587	0.783	0.184	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4
R	0.225	0.516	0.447	0.167	$A_4 \succ A_1 \succ A_3 \succ A_2$	A_4
$Q(v=0.5)$	0.305	0.836	0.901	0	$A_4 \succ A_1 \succ A_2 \succ A_3$	A_4
Compromise solutions						A_4 , A_1

4.2 Comparison analysis

In order to verify the feasibility and effectiveness of the proposed decision-making approach, a comparison analysis with multi-valued neutrosophic TODIM (an acronym in Portuguese of Interactive and Multicriteria Decision Making) method introduced by Wang and Li [34] is given based on the same illustrative example.

With regard to the method in Wang and Li [34], the multi-valued neutrosophic number is defined, and the traditional TODIM method is extended to the neutrosophic environment. In this new method, a reference criterion is selected first and then built the value function based on the Hamming distance between multi-valued neutrosophic numbers. Its decision-making steps are shown as below:

Step 1: Select the highest weight criterion as the reference criterion, so select C_3 as the reference criterion.

Step 2: Calculate the degree of alternative A_i superior to alternative A_j which are shown in the Table 5.

Step 3: Calculate the comprehensive ranking value of A_i , and get $\xi_1=0.492$, $\xi_2=0.702$, $\xi_3=0$, $\xi_4=1$

Step 4: Rank all the alternatives $A_i(i=1,2,3,4)$ based on the values of ξ_i . The bigger the ξ_i is, the better the alternative is $A_i(i=1,2,3,4)$, we can get $A_4 \succ A_2 \succ A_1 \succ A_3$.

Clearly, the ranking has a little difference; however, the best alternative is the same as A_4 . The advantage of the proposed method is that it can select the optimal alternative by the maximum ‘‘group utility’’ and minimum of an ‘‘individual regret’’ and the advantage of the extended TODIM method is that it can consider the bounded rationality of decision makers. Because the ranking principle is different, it is reasonable for not completely same ranking results. In this example, these two methods produced the same best and worst alternatives, and this can show the validity of the proposed in this paper.

Table 5 The degree of priority among alternatives [34]

	A_1	A_2	A_3	A_4
A_1	0	-0.893	0.236	-1.358
A_2	0.539	0	0.272	-0.765
A_3	0.927	-1.406	0	-1.955
A_4	-0.415	0.248	0.498	0

5. Conclusion

Neutrosophic hesitant fuzzy set is the generalization of neutrosophic set and the hesitant fuzzy set. Some operational laws, comparison rules of neutrosophic hesitant fuzzy set and the Hamming distance between two neutrosophic hesitant fuzzy numbers are defined. For multiple criteria decision making with neutrosophic hesitant fuzzy sets, the traditional VIKOR method is extended, and an approach is given. In this method, which is based on the particular measure of “closeness” to the ‘‘ideal” solution, using linear programing method during the process of decision-making, and order the hesitant fuzzy numbers by index of attitude and choose the alternatives under the acceptable advantage and the stability of the decision-making process to get a compromise solution, which achieving the maximum ‘‘group utility’’ and minimum of an ‘‘individual regret’’. This method has its own advantages compared with other multiple criteria decision making method based on distance, but it can only solve the decision making problems in which the criteria is neutrosophic hesitant fuzzy numbers and fixed weights, in the case of uncertain weights is universal in real life, which needs further study.

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Compliance with Ethical Standards

(1) Disclosure of potential conflicts of interest

We declare that we do have no commercial or associative interests that represent a conflict of interests in connection with this manuscript. There are no professional or other personal interests that can inappropriately influence our submitted work.

(2) Research involving human participants and/or animals

This article does not contain any studies with human participants or animals performed by any of the authors.

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