Bipolar neutrosophic distance measure in multi-attribute decision making

Chunfang Liu
College of Science,
Shenyang Aerospace University, 110000, Shenyang, China
liuchunfang1112@163.com

Abstract

Bipolar neutrosophic set(BNS) is a generalization of bipolar fuzzy set and neutrosophic set that can describe the uncertain information from both positive and negative perspectives. In this contribution, we study the multi-attribute decision making methods based on the distance measure under the uncertain information which the attribute weights are incompletely known or completely unknown. We first propose the distance measures of the bipolar neutrosophic sets and analyze the properties of the distance measures. Then, based on the bipolar neutrosophic information, we establish the programming models to derive the attribute weights of the alternatives. Furthermore, we give the multi-attribute decision making method using the distance measure under the environment of BNS. At last, we give a practical application and the result shows the reasonable and effective of the proposed method in dealing with decision making problems.

Keywords : bipolar neutrosophic set (BNS); multi-attribute decision making (MADM), distance measure.

1 Introduction

Fuzzy set was first proposed by Zadeh to deal with various kinds of uncertain information by a membership degree of an element to the given set[1]. After that, many new higher order fuzzy sets have been developed, such as interval-valued fuzzy set[2], intuitionistic fuzzy set[3], interval-valued intuitionistic fuzzy set[4], rough set[5], soft set[6], Pythagorean fuzzy set[7], neutrosophic set[8], hesitant fuzzy set[9], bipolar fuzzy set[10], and so on. They have been widely used in many fields and caused widespread concern, such as multi-attribute decision making[11, 12, 13], clustering analysis[14], supplier selection[15], imagine processing[16], and so on.

From the perspective of people's habit of thinking, Lee proposed the bipolar fuzzy set which extend the values of membership degrees from the smaller closed interval [0,1] to the larger closed interval [-1,1][10]. Afterwards, bipolar fuzzy set has been extended to many hybrid fuzzy sets, such as bipolar hesitant fuzzy soft set, bipolar fuzzy rough set, bipolar fuzzy soft set, and so on. Guo proposed the bipolar hesitant fuzzy soft set and developed the basic operations, such as intersection, union, and so on[17]. Yang presented the bipolar fuzzy rough set model on the two different universes and discussed the properties and gave two extended models of the bipolar fuzzy rough set model and obtained some related results[18]. Muhammed developed the bipolar fuzzy soft set and studied the fundamental properties and defined the operators. Then they applied them to decision making problems[19]. Nazm studied the basic operations on the bipolar fuzzy soft sets and discussed the algebraic properties and established the equivalence of both structures[20]. Han developed the bipolar-valued rough set and applied to the attribute reduction method and knowledge discovery method[21]. Gao investigated the dual hesitant bipolar fuzzy set and proposed some Hamacher prioritized operators and applied them to solve the dual hesitant bipolar fuzzy multi-attribute decision making problems[22]. Qudah proposed the bipolar fuzzy soft expert set and defined the basic theoretic operations and studied their properties[23]. Pythagorean fuzzy bipolar soft set model and rough Pythagorean fuzzy bipolar soft set model were introduced by Akram. The basic property and operations were studied and applied to multi-attribute decision making[24].

From the perspective of philosophy, the neutrosophic set (NS) was proposed by Smarandache to deal with the uncertain information which contains incomplete, indeterminate, and inconsistant information. Since its appearance,it has been successfully applied to many fields[25, 26, 27, 28], including decision making[29], imagine processing[30]. For simplicity and practical application, Wang proposed the single valued NS (SVNS) and the interval valued NS (IVNS) which are the instances of NS and gave some operations on these sets [31, 32]. Afterwards, many fruitful results have been appeared. Guo proposed a novel algorithm based on neutrosophic similarity score to perform thresholding on image and utilized the neutrosophic set in image processing field and defined a new concept for image thresholding[33]. Ye proposed improved cosine similarity measures of single valued neutrosophic sets and used to solve the medical diagnosis problems[34]. Liang used the information acquisition module to gather the single valued trapezoidal neutrosophic information provided by experts, and applied the single valued trapezoidal neutrosophic-decision making trial[35]. Ye firstly introduced neutrosophic state space models and the neutrosophic controllability and observability in indeterminate linear systems[36]. Sodenkamp developed a novel method to handle independent multisource uncertainty measures affecting the reliability of experts' assessments in group multi-criteria decision-making problems under the environment of single-valued neutrosophic sets[37]. Mohamed presented a new evaluation function to calculate the weights of alternatives under the environment of neutrosophic set and applied to a supplier selection problem[38]. Mohamed also proposed some novel similarity measures for interval-valued bipolar neutrosophic set and examined the propositions of these similarity measures[39]. Liu defined a new distance measure between two linguistic neutrosophic sets, and built a model based on the maximum deviation to obtain fuzzy measure[40]. Liu also extended the Schweizer-Sklar t-norm and t-conorm to single-valued neutrosophic numbers and proposed the Schweizer-Sklar operational laws, and developed the operators[41]. Cui presented dynamic neutrosophic cubic set to express the patient's disease symptoms in a time sequence and put forward the logarithmic similarity measure[42]. Anil established a novel symmetric single-valued neutrosophic cross entropy measure and applied it for identifying defects of bearings installed in a test rig and axial piston pump[43]. Sujit proposed a new idea for assigning relative weights to the experts based on cardinalities of neutrosophic soft sets[44].

Deli et al combined the bipolar fuzzy set with the neutrosophic set which has the advantages of both sets to evaluate the uncertain information from positive and negative affects with six degrees[45]. Deli also proposed the interval valued bipolar neutrosophic set and its operations. They also gave the interval valued bipolar neutrosophic weighted average operator and interval valued bipolar neutrosophic weighted geometric operator[46]. Ulucay introduced some similarity measures for bipolar neutrosophic sets and developed the multi-attribute decision making method[47]. Sahin proposed the Jaccard vector similarity measure of bipolar neutrosophic set and applied it to multi-attribute decision making[48]. Fan proposed the heronian mean operators and applied them to multi-attribute decision making problems[49]. Akram developed the bipolar neutrosophic graphs and proposed an algorithm for computing domination[50]. Akram introduced the bipolar single valued neutrosophic competition graphs and discussed the propositions related to the graphs[51]. Up to now there is little study about the decision making problem about the incompletely known or completely unknown attribute weights of the decision making information under the environment of bipolar neutrosophic set.

In this paper, we investigate the MADM problems which the information expressed by BNS, and the attribute weights are incompletely known or completely unknown. In order to solve the problem, the rest of the paper is organized as follows. In Section 2, we recall some definitions. In Section 3, we give the distance measures of BNSs and study their properties. In Section 4, we establish the mathematical programming models to solve the unknown weight of the attribute and give the method to multi-attribute decision making based on the distance measure. Finally, a conclusion is given in Section 5.

2 Preliminaries

Definition 2.1 [10] Assume X be an empty set with a generic element in X denoted by x. A bipolar fuzzy set A on X is defined by a positive membership degree $\mu^+(x)$ and a negative membership degree $\mu^-(x)$, where $\mu^+(x) : X \to [0,1]$ represents the satisfaction degree of an element x to the property corresponding to A, and $\mu^-(x) : X \to [-1,0]$ represents the dissatisfaction degree of an element x to some implicit counter property. A is denoted by $A = \{< \mu^+(x), \mu^-(x) >: x \in X\}$.

For simplicity and practical application, Wang proposed the single valued neutrosophic set (SVNS) which is a subclass of NS and preserve all the operations on NS. In the following part, we recall the definition of SVNS.

Definition 2.2 [32] Assume X be a universe of discourse with a generic element in X denoted by x. A single valued neutrosophic set (SVNS) A on X is defined by a truth membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$ and a falsity membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are defined by

$$T_A(x) : X \to [0,1]$$
$$I_A(x) : X \to [0,1]$$
$$F_A(x) : X \to [0,1]$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are subsets of $[0,1]$, and satisfy $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.3 [46] Assume X be a universe of discourse with a generic element in X denoted by x. A bipolar neutrosophic set (BNS) A on X is defined by positive membership functions $T^+(x), I^+(x), F^+(x)$ and negative membership functions $T^-(x), I^-(x), F^-(x)$, and A is denoted by

$$A = \{T^+(x), I^+(x), F^+(x), T^-(x), I^-(x), F^-(x)\}$$

where $T^+(x), I^+(x), F^+(x)$ are positive truth membership function, positive indeterminacy membership function and positive falsity membership function, respectively, and $T^-(x), I^-(x), F^-(x)$ are negative truth membership function, negative indeterminacy membership function and negative falsity membership function, respectively. They are defined by

$$\begin{aligned} T^+(x) &: X \to [0,1] \\ I^+(x) &: X \to [0,1] \\ F^+(x) &: X \to [0,1] \\ T^-(x) &: X \to [-1,0] \\ I^-(x) &: X \to [-1,0] \\ F^-(x) &: X \to [-1,0] \end{aligned}$$

Especially, if X has only one element, for convenience, the BNS is reduced to the bipolar neutrosophic number (BNN), and denoted by

$$A = \{T^+, I^+, F^+, T^-, I^-, F^-\}$$

3 The distance measure of bipolar neutrosophic sets

Distance measure is one of the most important methods to compare the bipolar neutrosophic sets to determine whether they are closely related or not. They have widely used in comprehensive evaluation, decision making, pattern recognition, machine learning, and so on. The common method of the distance measures are Euclidean distance measure, Hamming distance measure, Manhattan distance measure, Minkowski distance measure. For the widely use of Euclidean distance measure and Hamming distance measure, in this section, we give the distance measure of bipolar neutrosophic sets.

Definition 3.1 Let A_1,A_2 be two bipolar neutrosophic sets, $d : BNS \times BNS \to [0,1]$ be a real valued function, which satisfies the following actions:
(1)$0 \leq d(A_1, A_2) \leq 1$
(2)$d(A_1, A_2) = 0$ if and only if $A_1 = A_2$
(3)$d(A_1, A_2) = d(A_2, A_1)$
(4)If $A_1 \subset A_2 \subset A_3$, A_1, A_2, A_3 are bipolar neutrosophic sets in X, then $d(A_1, A_2) \leq d(A_1, A_3)$, $d(A_2, A_3) \leq d(A_1, A_3)$.
Then d is called the distance measure of bipolar neutrosophic sets.

Definition 3.2 Let A_1,A_2be two bipolar neutrosophic sets in a universe of discourse $X = \{x_1, x_2, \cdots, x_n\}$ which are denoted by

$$A_k = \{T_k^+(x_i), I_k^+(x_i), F_k^+(x_i), T_k^-(x_i), I_k^-(x_i), F_k^-(x_i)\}, k = 1, 2$$

(1)the normalized Hamming distance measure between A_1 and A_2 is defined by

$$\begin{aligned} d(A_1, A_2) = \ & \frac{1}{6n}\sum_{i=1}^{n}(|T_1^+(x_i) - T_2^+(x_i)| + |I_1^+(x_i) - I_2^+(x_i)| + |F_1^+(x_i) - F_2^+(x_i)| \\ & +|T_1^-(x_i) - T_2^-(x_i)| + |I_1^-(x_i) - I_2^-(x_i)| + |F_1^-(x_i) - F_2^-(x_i)|) \end{aligned} \quad (1)$$

Especially, if X has only one element, then the Hamming distance measure of bipolar neutrosophic number is denoted as follows:

$$\begin{aligned} d(A_1, A_2) = \ & \frac{1}{6}(|T_1^+ - T_2^+| + |I_1^+ - I_2^+| + |F_1^+ - F_2^+| \\ & +|T_1^- - T_2^-| + |I_1^- - I_2^-| + |F_1^- - F_2^-|) \end{aligned} \quad (2)$$

(2)the normalized Euclidean distance measure between A_1 and A_2 is defined by

$$\begin{aligned} e(A_1, A_2) = \ & \frac{1}{6n}\sum_{i=1}^{n}(|T_1^+(x_i) - T_2^+(x_i)|^2 + |I_1^+(x_i) - I_2^+(x_i)|^2 + |F_1^+(x_i) - F_2^+(x_i)|^2 \\ & +|T_1^-(x_i) - T_2^-(x_i)|^2 + |I_1^-(x_i) - I_2^-(x_i)|^2 + |F_1^-(x_i) - F_2^-(x_i)|^2)^{\frac{1}{2}} \end{aligned} \quad (3)$$

We just prove the Hamming distance measure of bipolar neutrosophic number satisfies the conditions of Definition 3.1.

Proof. In this part, (1), (2), (3) are obvious, we need to prove (4).
(4) If $A_1 \subset A_2 \subset A_3$, then $T_1^+ \le T_2^+ \le T_3^+, T_1^- \ge T_2^- \ge T_3^-$, $I_1^+ \le I_2^+ \le I_3^+, I_1^- \ge I_2^- \ge I_3^-$, $F_1^+ \ge F_2^+ \ge F_3^+, F_1^- \le F_2^- \le F_3^-$.

The above inequality relations are brought into Eq. (2), the relationships of the distance measures are obtained as follows:

$$d(A_1, A_2) \le d(A_1, A_3), d(A_2, A_3) \le d(A_1, A_3)$$

4 Multi-attribute decision making method based on the distance measure

Let $X = \{X_1, X_2, \ldots, X_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of attributes and $w = \{w_1, w_2, \ldots, w_n\}$ be the weight vector of the attribute with $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$. Let $A = (r_{ij})_{m\times n}$ be a single valued neutrosophic decision matrix, where $r_{ij} = \{T_{ij}^+, I_{ij}^+, F_{ij}^+, T_{ij}^-, I_{ij}^-, F_{ij}^-\}$ is the value of the attribute, expressed by BNs.

In multi-attribute decision making environments, the ideal point is used to help the identification of the best alternative in the decision set which can keep the advantages of the most properties of the alternatives and eliminate the disadvantages of the individual ones. Although the ideal point does not exist in the real world, it does provide an effective way to evaluate the best alternative. Now we suppose the ideal BNN as $\alpha_j^* = \{t^{+*}, i^{+*}, f^{+*}, t^{-*}, i^{-*}, f^{-*}\} = \{1, 0, 0, 0, 0, 1\}$. Based on the ideal BNN, we define the bipolar neutrosophic positive ideal-solution (BNPIS).

Definition 4.1 Let $\alpha_j^* = \{1,0,0,0,0,1\}$ $(j = 1,2,\ldots,n)$ be n ideal BNNs, then a BNPIS is defined by $A^* = \{\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\}$.

Definition 4.2 Let $A_i = \{r_{i1}, r_{i2}, \ldots, r_{in}\}$ $(i = 1,2,\ldots,m)$ be the i th alternative, $A^* = \{\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\}$ be the BNPIS, then the Hamming distance measure (HDM) between A_i and A^* is defined by

$$d(A_i, A^*) = \sum_{j=1}^{n} w_j d(r_{ij}, \alpha_j^*). \tag{4}$$

4.1 The programming models for solve the attribute weight

In the decision making process, the unknown information of the attribute weight provided by the decision makers can usually be constructed using several basic ranking forms [52]. The information of the attribute weight is divided into three cases, the value attribute weight is known, incompletely known, completely unknown. For the latter two cases, we establish mathematical models to solve them.

Case 1 incompletely known attribute weights of the bipolar neutrosophic information

Let H be the set of information about the incompletely known attribute weights, which may be constructed in the following forms [53], for $i \neq j$:
(a) A weak ranking:$\{w_i \geq w_j\}$;
(b) A strict ranking: $\{w_i - w_j \geq \delta_i (> 0)\}$;
(c) A ranking with multiples: $\{w_i \geq \delta_i w_j\}, 0 \leq \delta_i \leq 1$;

In general, the single-objective programming model based on the distance measure can be expressed as follows:

$$(\text{M1}) \begin{cases} \text{Min} f(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j d(r_{ij}, \alpha_j^*) \\ s.t. w_j \in H, \sum_{j=1}^{n} w_j = 1, w_j \geq 0, j = 1,2,\ldots,n. \end{cases}$$

and

$$d(r_{ij}, \alpha_j^*) = \frac{1}{6}(|T_{ij}^+ - 1| + I_{ij}^+ + F_{ij}^+ + |T_{ij}^-| + |I_{ij}^-| + |F_{ij}^- - 1|) \tag{5}$$

$d(r_{ij}, \alpha_j^*)$ represents the distance measure between the attribute value r_{ij} and the BNPIS α_j^*. The desirable weight vector $w = (w_1, w_2, \ldots, w_n)$ should make the sum of all the weighted distance measure (M1) small. So we construct this model to make the overall distance small.
By solving the model (M1) with Matlab software, we get the optimal solution $w^* = (w_1^*, w_2^*, \ldots, w_n^*)$, which is considered as the weight of the attributes $C_1, C_2, \ldots, C_n$.

Case 2 Completely unknown attribute weights of the bipolar neutrosophic information

When the attribute weight of the decision making information is completely unknown, we establish the following programming model:

$$(\text{M2}) \begin{cases} \text{Min} f(w) = \sum_{i=1}^{m} \sum_{j=1}^{n} w_j d(r_{ij}, \alpha_j^*) \\ s.t. \sum_{j=1}^{n} w_j^2 = 1, w_j \geq 0, j = 1,2,\ldots,n. \end{cases}$$

It is a conditional extremum problem. To solve this model to get the weight vector w_j, we construct the Lagrange function as follows:

$$L(w,\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n} w_j d(r_{ij},\alpha_j^*) + \frac{\lambda}{2}(\sum_{j=1}^{n} w_j^2 - 1) \tag{6}$$

where λ is the Lagrange multiplier.
Differentiating (6) with respect to $w_j (j = 1, 2, \ldots, n)$ and λ, setting these partial derivatives equal to zero, the following set of the equations are obtained:

$$\begin{cases} \frac{\partial L}{\partial w_j} = \sum_{i=1}^{m} d(r_{ij},\alpha_j^*) + w_j\lambda = 0 \\ \frac{\partial L}{\partial \lambda} = \sum_{j=1}^{n} w_j^2 = 1 \end{cases} \tag{7}$$

By solving Eq.(7), we obtain the weight w_j and normalize it with $w_j^* = \frac{w_j}{\sum_{j=1}^{n} w_j}$, then we get

$$w_j^* = \frac{\sum_{i=1}^{m} d(r_{ij},\alpha_j^*)}{\sum_{j=1}^{n}\sum_{i=1}^{m} d(r_{ij},\alpha_j^*)} \tag{8}$$

we get the optimal solution $w^* = (w_1^*, w_2^*, \ldots, w_n^*)$, $\sum_{j=1}^{n} w_j^* = 1$, which is considered as the weight of the attributes $C_1, C_2, \ldots, C_n$.

4.2 The decision making method based on distance measure

In the decision making problem, we have established the mathematical programming model to obtain the information of the attribute weight. Afterwards, we give the process as follows:

Step 1. Establish the programming model according to the given information;

Step 2. Solve the model with Matlab to obtain the optimal solution;

Step 3. Calculate the distance measure of the alternative and the ideal solution;

Step 4. Rank the alternatives according to the distance measure. The smaller the distance measure is, the better the alternative is.

5 Ilustrative example

5.1 Example

Here we choose the decision making problem adapted from [46]. With the development of economy and urbanization, Cars have become a convenient way of transportation for people's life. A customer who desired to buy the most appropriate car. After pre-evaluation, four types of cars have remained as alternatives for further evaluation. In order to evaluate alternative cars, the customer takes into account four attributes to evaluate the alternatives: (1)C_1: fuel economy, (2)C_2: aerod, (3)C_3: comfortable, (4)C_4: safety. The four possible alternatives are to be evaluated under these four attributes and are in the form of BNs, as shown in the following bipolar neutrosophic decision matrix:

$$D = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} \end{bmatrix}$$

$$\gamma_{11} = \{0.5, 0.7, 0.2, -0.7, -0.3, -0.6\}, \gamma_{12} = \{0.4, 0.4, 0.5, -0.7, -0.8, -0.4\}$$

$$\gamma_{13} = \{0.7, 0.7, 0.5, -0.8, -0.7, -0.6\}, \gamma_{14} = \{0.1, 0.5, 0.7, -0.5, -0.2, -0.8\}$$

$$\gamma_{21} = \{0.9, 0.7, 0.5, -0.7, -0.7, -0.1\}, \gamma_{22} = \{0.7, 0.6, 0.8, -0.7, -0.5, -0.1\}$$

$$\gamma_{23} = \{0.9, 0.4, 0.6, -0.1, -0.7, -0.5\}, \gamma_{24} = \{0.5, 0.2, 0.7, -0.5, -0.1, -0.9\}$$

$$\gamma_{31} = \{0.3, 0.4, 0.2, -0.6, -0.3, -0.7\}, \gamma_{32} = \{0.2, 0.2, 0.2 - 0.4, -0.7, -0.4\}$$

$$\gamma_{33} = \{0.9, 0.5, 0.5, -0.6, -0.5, -0.2\}, \gamma_{34} = \{0.7, 0.5, 0.3, -0.4, -0.2, -0.2\}$$

$$\gamma_{41} = \{0.9, 0.7, 0.2, -0.8, -0.6, -0.1\}, \gamma_{42} = \{0.3, 0.5, 0.2, -0.5, -0.5, -0.2\}$$

$$\gamma_{43} = \{0.5, 0.4, 0.5, -0.1, -0.7, -0.2\}, \gamma_{44} = \{0.4, 0.2, 0.8, -0.5, -0.5, -0.6\}$$

In order to show the feasibility of our method, we divide the attribute weight into two cases.

Case 1. Incompletely known attribute weights

Suppose the incompletely known information of the attribute weight is given by $H = \{0.18 \le w_1 \le 0.2, 0.15 \le w_2 \le 0.25, 0.30 \le w_3 \le 0.35, 0.3 \le w_4 \le 0.4, \sum_{j=1}^{4} w_j = 1\}$.

The decision making progress is as follows:

Step 1. By model (M1), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 0.7333w_1 + 0.8499w_2 + 0.6667w_3 + 0.8833w_4 \\ s.t. w \in H \end{cases}$$

Step 2. By solving this model with Matlab software, we get the weight vector:

$$w_1 = 0.2, w_2 = 0.15, w_3 = 0.35, w_4 = 0.30.$$

Step 3. Use the distance measure (4), we have

$$d(A_1, A^*) = 0.2320, d(A_2, A^*) = 0.1775, d(A_3, A^*) = 0.1466, d(A_4, A^*) = 0.2166.$$

Step 4. Rank the alternatives. It's easy to see $d(A_1, A^*) \ge d(A_4, A^*) \ge d(A_2, A^*) \ge d(A_3, A^*)$, since $d(A_1, A^*)$ is the biggest, and $d(A_3, A^*)$ is the smallest, we rank the alternatives as $A_3 \succ A_2 \succ A_4 \succ A_1$,, where $\succ$ indicates the relationship superior or preferred to, and A_3 is the best alternative.

Case 2.Completely unknown attribute weights

If the information of the attribute weight is completely unknown, the decision making progress is as follows:

Step 1. By model (M2), we establish the following model:

$$\begin{cases} \text{Min} f(w) = 0.7333w_1 + 0.8499w_2 + 0.6667w_3 + 0.8833w_4 \\ s.t. \sum_{j=1}^{4} w_j^2 = 1, w_j \ge 0, j = 1, 2, 3, 4. \end{cases}$$

Step 2. Use Eq. (8) to obtain the weight vector of attributes:

$$w_1 = 0.4652, w_2 = 0.5392, w_3 = 0.4229, w_4 = 0.5604,$$

then we normalize it as

$$w_1^* = 0.2340, w_2^* = 0.2712, w_3^* = 0.2128, w_4^* = 0.2819.$$

Step 3. Use the distance measure (4), we have

$$d(A_1, A^*) = 0.4746, d(A_2, A^*) = 0.2984, d(A_3, A^*) = 0.3830, d(A_4, A^*) = 0.4204.$$

Step 4. Rank the alternatives. It's easy to see $d(A_1, A^*) \geq d(A_4, A^*) \geq d(A_3, A^*) \geq d(A_2, A^*)$, since $d(A_1, A^*)$ is the biggest, and $d(A_2, A^*)$ is the smallest, we rank the alternatives as $A_2 \succ A_3 \succ A_4 \succ A_1$, where $\succ$ indicates the relationship superior or preferred to, and A_2 is the best alternative.

5.2 Comparative analysis

Considering the proposed method and the aggregation operator method proposed by Deli[45], there exsits some differences. In Delis method, they used the weighted aggregation operators and the score functions to rank the alternatives, the weighted aggregation operators need the full decision making information and consider the decision maker's attitude; while, the proposed method calculates the distance measure between the attributes and the ideal solution, and obtain the weight that make the weighted distance measure small, we use the distance measures to rank the alternatives. Our method is effective to deal with the incompletely known or completely unknown attribute weight by solving the program models. The advantage of the proposed method is that the calculation is to use the distance measure to rank the alternatives, which can deal with the MADM problem effectively.

6 Conclusion

In this paper, we investigate the multi-attribute decision making problems expressed with bipolar neutrosophic set and the attribute weights are incompletely known or completely unknown. We first propose the distance measure of two BNSs and analyze the properties they satisfied; then, we define the bipolar neutrosophic ideal solution (BNIS), and then establish the optimal models to derive the attribute weight. Moreover, an approach to MADM within the framework of BNS is developed, and the example shows that our approach is reasonable and effective in dealing with decision making problems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

This work was financially supported by the Talent Research Startup fund of Shenyang Aerospace University (120419054) and China Natural Science Fund under grant (11401084).

References

[1] Zadeh, L.A.(1965). Fuzzy sets. Information and Control, 8, 338-353.

[2] Zadeh, L.A.(2008). In there a need for fuzzy logic?. Information Sciences, 178, 2751-2779.

[3] Atanassov, K.(1986). Intuitionistic fuzzy sets. Fuzzy sets and systems, 20, 87-96.

[4] Atanassov, K., Gargov, G. (1989). Interval valued intuitionistic fuzzy sets. Fuzzy sets and systems, 31, 343-349.

[5] Pawlak, Z.(1982). Rough sets. International Journal of Computer Information Science. 11(5), 341-356.

[6] Maji, P.K., Biswas, R., Roy, A.R.(2003). Soft set theory. Computers and Mathematics with Applications, 45, 555-562.

[7] Yager, R.R.(2014). Pythagorean membership grades in multi-criteria decision making. IEEE Transactions on Fuzzy Systems, 22(4), 958-965.

[8] Smarandache, F.(1998). Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information and Learning. Ann Arbor, Michigan, USA.

[9] Torra, V.(2010). Hesitant fuzzy sets. International Journal of Intelligent Systems, 25, 529-539.

[10] Lee, K.J.(2000). Bipolar valued fuzzy sets and their operations. Proceedings of International Conference on Intelligent Technologies, Bangkok, Thailand, 307-312.

[11] Broumi, S., Smarandache, F.(2014). Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making. Bull Pure Appl. Sci, 33(2), 135-155.

[12] Liu, C.F., Luo, Y.S.(2016). Correlated aggregation operators for simplified neutrosophic set and their application in multi-attribute group decision making, Journal of Intelligent and Fuzzy Systems, 30, 1755-1761.

[13] Yang, M.S., Lin, D.C.(2009). On Similarity and inclusion measures between type-2 fuzzy sets with an application to clustering. Computers and Mathematics with Applications, 57, 896-967.

[14] Zhang, Y., Li, P., Wang, Y., etc.(2013). Multiattribute decision making based on entropy under interval-valued intuitionistic fuzzy environment. Mathematical Problems in Engineering, 69, 831-842.

[15] Guo, Y., Cheng, H.D. (2009). New neutrosophic approach to imagine segmentation. Pattern Recognition, 42, 587-595.

[16] Xu, Z.S., Cai, X.Q. (2010). Nonlinear optimization models for multiple attribute group decision making with intuitionistic fuzzy information. International Journal of Intelligent Systems, 25, 489-513.

[17] GUO, J.S. (2008). A new bipolar hesitant fuzzy soft set and its applications in decision making problem. Journal of Jilin University(Science Edition), 5.

[18] Yang, H.L., Li, S.G., Wang, S.Y., Wang, J.(2012). Bipolar fuzzy rough set model on two different universes and its application. Knowledge-Based Systems, 94-101.

[19] Muhammad, A., Saleem, A., Kifayat, U.(2014). Bipolar fuzzy soft sets and its applications in decision making problem. Journal of Intelligent and Fuzzy Systems, DOI:10.3233/IFS-131031.

[20] Naz, M., Shabir, M.(2014). On fuzzy bipolar soft sets, their algebraic structures and applications. Journal of Intelligent Fuzzy Systems, DOI: 10.3233/IFS-130844.

[21] Han, Y.P., Shi, S. (2015). Bipolar-valued rough fuzzy set and its applications to decision information system. IEEE Transactions on Fuzzy Systems, 23(6), 1-11.

[22] Gao, H., Wei, G.W., Huang, Y. H. (2017). Dual hesitant bipolar fuzzy Hamacher Prioritized aggregation operators in multiple attribute decision making. IEEE Access, 99, 1-15.

[23] Qudah, Y., Hassan, A.N. (2017). Bipolar fuzzy soft expert set and its application in decision making. International Journal of Applied Decision Sciences, 10(2), 175-191.

[24] Akram, M., Ali, G. (2020). Hybrid models for decision-making based on rough Pythagorean fuzzy bipolar soft information. Granul. Computer, 5, 1-15.

[25] Liu, P.D., Wang, Y.M.(2015). Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making, Journal of Systems Science and Complextity, DOI: 10.1007/s11424-015-4010-7.

[26] Deli, I., Broumi, S.(2015). Neutrosophic soft matrices and NSM-decision making. Journal of Intelligent and Fuzzy Systems, 28, 2231-2241.

[27] Broumi, S., Talea, M., Bakali, A., Smarandache, F.(2016). Single valued neutrosophic graphs. Journal of New Theory, 10, 86-101.

[28] Broumi, S., Talea, M., Bakali, A., Smarandache, F.(2016). On bipolar single valued neutrosophic graphs, Journal of New Theory, 11, 84-102.

[29] Liu, C.F., Luo, Y.S. (2017). Power aggregation operators of simplified neutrosophic sets and their use in multi-attribute group decision making. IEEE/CAA J. Automatica Sinica, 1-10.

[30] Peng, J., Wang, J., Zhang, H., Chen, X.(2014). An outranking approach for multicriteria decision-making problems with simplified neutrosophic sets. Applied Soft Computing, 25, 336-346.

[31] Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R.(2005). Interval neutrosophic sets and logic: theory and applications in computing. Hexis, Phoenix, AZ.

[32] Wang, H., Smarandache, F., Zhang, Y.Q., Sunderraman, R.(2010). Single valued neutrosophic sets. Multispace Multistruct, 4, 410-413.

[33] Guo, Y., Abdulkadir, S, Ye, J.(2014). A novel image thresholding algorithm based on neutrosophic similarity score. Measurement, 58, 175-186.

[34] Ye, J.(2015). Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. Artificial Intelligence in Medicine, 63, 171-179

[35] Liang, R., Wang, J., Zhang H.(2017). Evaluation of e-commerce websites: An integrated approach under a single-valued trapezoidal neutrosophic environment. Knowledge-Based Systems, 135, 44-59.

[36] Ye, J., Cui, W. (2018). Neutrosophic state feedback design method for SISO neutrosophic linear systems. Cognitive Systems Research, 52, 1056-1065.

[37] Sodenkamp, M. A., Tavana, M., Caprio, D.(2018). An aggregation method for solving group multi-criteria decision-making problems with single-valued neutrosophic sets. Applied Soft Computing, 71, 715-727.

[38] Mohamed, A.B., Mohamed, M. (2018).Three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem. Future Generation Computer Systems, 89, 19-30.

[39] Mohamed A.B., Mohamed, M., Mohamed E.,etc (2019). Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases. Artificial Intelligence In Medicine, 101, 101735.

[40] Liu, P., You X.(2019). Bidirectional projection measure of linguistic neutrosophic numbers and their application to multi-criteria group decision making. Computers Industrial Engineering, 128, 447-457.

[41] Liu, P., Khan, Q., Mahmood, T.(2019). Multiple-attribute decision making based on single-valued neutrosophic Schweizer-Sklar prioritized aggregation operator. Cognitive Systems Research, 57, 175-196.

[42] Cui, W.H., Ye, J.(2019). Logarithmic similarity measure of dynamic neutrosophic cubic sets and its application in medical diagnosis. Computers in Industry 111, 198-206.

[43] Anil, K., Gandhi, C.P., Zhou, Y.Q.,etc. (2020). Fault diagnosis of rolling element bearing based on symmetric cross entropy of neutrosophic sets. Measurement, 152, 107318.

[44] Sujit, D., Saurabh, K., Samarjit, K., Tandra, P.(2019). Group decision making using neutrosophic soft matrix: An algorithmic approach Journal of King Saud University - Computer and Information Sciences, 31, 459-468.

[45] Deli, I., Ali, M., Smarandache, F. (2015). Bipolar neutrosophic sets and their application based on multi-criteria decision making problems. Proceedings of the 2015 International Conference on Advanced Mechanic Systems. Beijing, China,August, 22-24.

[46] Deli, I., Subas, Y., Smarandache, F. (2016) . Interval valued bipolar neutrosophic sets and their application in pattern recognition. Conference Paper. arXiv 2016, arXiv: 289587637.

[47] Ulucay, V., Deli, I., Sahin, M.(2016). Similarity measures of bipolar neutrosophic sets and their application to multiple criteria decision making. Neural Computing and Applications. DOI: 10.1007/s00521-016-2479-1.

[48] Sahin, M., Deli, I., Ulucay, V. (2016). Jaccard vector similarity measure of bipolar neutrosophic set based on multi-criteria decision making. In: International Conference on natural Sciences and Engineering (ICNASE16), March 19-20, Kilis.

[49] Fan, C., Ye, J., Fen, S., etc. (2019). Multi-criteria decision making method using heronian mean operators under a bipolar neutrosophic environment. Mathematics, 7, 97-109.

[50] Akram, M., Sarwar, M. (2019). Novel multiple criteria decision making methods based on bipolar neutrosophic sets and bipolar neutrosophic graphs. Symmetry, 11, 1-21.

[51] Akram, M., Nasir, M., Shum, K. P.(2018). Novel application of bipolar single valued neutrosophic competition graphs. Applied Mathematics: A Journal of Chinese Universities, 33(4), 436-467.

[52] Kim, S.H., Choi, S.H., Kim, J.K. (1999). Interactive procedure for multiple attribute group decision making with incomplete information: range-based approach. European Journal of Operational Research, 118(1), 139-152.

[53] Park, K.S.(2004). Mathematical programming models for characterizing dominance and potential optimality when multicriteria alternative values and weights are simultaneously incomplete. IEEE Transactions on Systems, Man, and Cybernetics Part A:Systems and Humans, 34(5), 601-614.