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BIPOLAR NEUTROSOPHIC WEAKLY $BG^{\oplus}$-CLOSED SETS

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ABSTRACT. In this paper are presented and explored new sort of bipolar Neutrosophic closed set which is known as bipolar Neutrosophic feebly $Bg^{\oplus}$- closed sets in BNTSs and furthermore talked about properties and portrayal.

Key Words: Bipolar Neutrosophic sets, Bipolar Neutrosophic week closed sets, Bipolar Neutrosophic regular open sets and Bipolar Neutrosophic regular closed sets

1. INTRODUCTION

A. Salama presented NTSs in [2, 3] by utilizing Smarandache's NSs, [7, 8]. Neutrosophic g closed set presented by R. Dhavasheelan et al. in [5, 6], what's more, Neutrosophic $g^{\oplus}$-closed sets introduced by A. Atkinswesley et al. in [1]. Point of this current paper is, to present and research about new sort of Bipolar Neutrosophic closed set is known as bipolar Neutrosophic weakly B$g^{\oplus}$-closed sets in BNTS and furthermore examined about properties and portrayal. In 2016 derived the idea of the neutrosophic topology. The author also have the some more research work on neutrosophic theory see the references [9- 17].

2. PRELIMINARIES

Definition 2.1: Consider Bipolar Neutrosophic set $U_1^{\otimes}$ is in the form

$$U_1^{\otimes}=\left\{<u,\varepsilon_{U_p^{\otimes}}(u),\phi_{U_P^{\otimes}}(u),\varphi_{U_p^{\otimes}}(u)\varepsilon_{U_N^{\otimes}}(u),\phi_{U_N^{\otimes}}(u),\varphi_{U_N^{\otimes}}(u)>:u\in BN_{u_Y^{\otimes}}\right\},$$

where $\varepsilon_{U_P^{\otimes}}(u),\varepsilon_{U_N^{\otimes}}(u)$ denotes membership function, $\phi_{U_P^{\otimes}}(u),\phi_{U_N^{\otimes}}(u)$ denotes indeterminacy and $\varphi_{U_P^{\otimes}}(u),\varphi_{U_P^{\otimes}}(u)$ denotes non-membership function w.r.t. positive and negative ways.

Definition 2.2: Bipolar Neutrosophic set is the set $U_1^{\otimes}=\left\{<u,\varepsilon_{U_p^{\otimes}}(u),\phi_{U_P^{\otimes}}(u),\varphi_{U_p^{\otimes}}(u)\varepsilon_{U_N^{\otimes}}(u),\phi_{U_N^{\otimes}}(u),\varphi_{U_N^{\otimes}}(u)>:u\in BN_{u_Y^{\otimes}}\right\}$ on $BN_{u_Y^{\otimes}}$ and $\forall u\in BN_{u_Y^{\otimes}}$.

Then complement of $U_1^{\otimes}$ is

$$U_1^{\otimes C}=\left\{<u,\varphi_{U_P^{\otimes}}(u),1-\phi_{U_P^{\otimes}}(u),\varepsilon_{U_P^{\otimes}}(u),\varphi_{U_N^{\otimes}}(u),1-\phi_{U_N^{\otimes}}(u),\varepsilon_{U_N^{\otimes}}(u)>:u\in BN_{u_Y^{\otimes}}\right\}$$

Definition 2.3. Let $U_1^{\otimes}$ and $U_2^{\otimes}$ are two BNSs,

$$\forall u \in BN_{u_Y^{\otimes}}, U_1^{\otimes} = \left\{ < u, \varepsilon_{U_{1p}^{\otimes}}(u), \phi_{U_{1P}^{\otimes}}(u), \varphi_{U_{1p}^{\otimes}}(u) \varepsilon_{U_{1N}^{\otimes}}(u), \phi_{U_{1N}^{\otimes}}(u), \varphi_{U_{1N}^{\otimes}}(u) >: u \in BN_{u_Y^{\otimes}} \right\},$$

$$\forall u \in BN_{u_Y^{\otimes}}, U_2^{\otimes} = \left\{ < u, \varepsilon_{U_{2p}^{\otimes}}(u), \phi_{U_{2P}^{\otimes}}(u), \varphi_{U_{2p}^{\otimes}}(u) \varepsilon_{U_{2N}^{\otimes}}(u), \phi_{U_{2N}^{\otimes}}(u), \varphi_{U_{2N}^{\otimes}}(u) >: u \in BN_{u_Y^{\otimes}} \right\}.$$

Then

$$\begin{aligned} U_1^{\otimes} \subseteq U_2^{\otimes} \Leftrightarrow\; & \varepsilon_{U_{1P}^{\otimes}}(u) \le \varepsilon_{U_{2P}^{\otimes}}(u), \varepsilon_{U_{1N}^{\otimes}}(u) \le \varepsilon_{U_{2N}^{\otimes}}(u), \\ & \phi_{U_{1P}^{\otimes}}(u) \le \phi_{U_{2P}^{\otimes}}(u), \phi_{U_{1N}^{\otimes}}(u) \le \phi_{U_{2N}^{\otimes}}(u), \\ & \varphi_{U_{1P}^{\otimes}}(u) \ge \varphi_{U_{2P}^{\otimes}}(u), \varphi_{U_{1N}^{\otimes}}(u) \ge \varphi_{U_{2N}^{\otimes}}(u) \end{aligned}$$

Definition 2.4. Let $U_1^{\otimes}$ and $U_2^{\otimes}$ be two BNSs are

$$\forall u \in BN_{u_Y^{\otimes}}, U_1^{\otimes} = \left\{ < u, \varepsilon_{U_{1p}^{\otimes}}(u), \phi_{U_{1P}^{\otimes}}(u), \varphi_{U_{1p}^{\otimes}}(u) \varepsilon_{U_{1N}^{\otimes}}(u), \phi_{U_{1N}^{\otimes}}(u), \varphi_{U_{1N}^{\otimes}}(u) >: u \in BN_{u_Y^{\otimes}} \right\}$$

$$\forall u \in BN_{u_Y^{\otimes}}, U_2^{\otimes} = \left\{ < u, \varepsilon_{U_{2p}^{\otimes}}(u), \phi_{U_{2P}^{\otimes}}(u), \varphi_{U_{2p}^{\otimes}}(u) \varepsilon_{U_{2N}^{\otimes}}(u), \phi_{U_{2N}^{\otimes}}(u), \varphi_{U_{2N}^{\otimes}}(u) >: u \in BN_{u_Y^{\otimes}} \right\}$$

Then

$$U_1^{\otimes} \cap U_2^{\otimes} = \left\{ \begin{array}{l} < r, \varepsilon_{U_{1P}^{\otimes}}(u) \cap \varepsilon_{U_{2P}^{\otimes}}(u), \varepsilon_{U_{1N}^{\otimes}}(u) \cap \varepsilon_{U_{2N}^{\otimes}}(u) \\ \phi_{U_{1P}^{\otimes}}(u) \cap \phi_{U_{2P}^{\otimes}}(u), \phi_{U_{1N}^{\otimes}}(u) \cap \phi_{U_{2N}^{\otimes}}(u) \\ \varphi_{U_{1P}^{\otimes}}(u) \cup \varphi_{U_{2P}^{\otimes}}(u), \varphi_{U_{1N}^{\otimes}}(u) \cup \varphi_{U_{2N}^{\otimes}}(u) >: u \in N_{u_Y^{\otimes}} \end{array} \right\},$$

$$U_1^{\otimes} \cup U_2^{\otimes} = \left\{ \begin{array}{l} < r, \varepsilon_{U_{1P}^{\otimes}}(u) \cup \varepsilon_{U_{2P}^{\otimes}}(u), \varepsilon_{U_{1N}^{\otimes}}(u) \cup \varepsilon_{U_{2N}^{\otimes}}(u) \\ \phi_{U_{1P}^{\otimes}}(u) \cup \phi_{U_{2P}^{\otimes}}(u), \phi_{U_{1N}^{\otimes}}(u) \cup \phi_{U_{2N}^{\otimes}}(u) \\ \varphi_{U_{1P}^{\otimes}}(u) \cap \varphi_{U_{2P}^{\otimes}}(u), \varphi_{U_{1N}^{\otimes}}(u) \cap \varphi_{U_{2N}^{\otimes}}(u) >: u \in N_{u_Y^{\otimes}} \end{array} \right\}.$$

Definition 2.5 Let $BN_{u_Y^{\otimes}}$ be non-empty set and $BN_{S\zeta}$ be the collection of bipolar Neutrosophic subsets of B $N_{u_Y^{\otimes}}$ satisfying the accompanying properties:

(1) $0_{N_u^*}, 1_{N_u^*} \in BN_{S\zeta}$

(2) $BN_{us_1}^* \cap BN_{us_2}^* \in BN_{S\zeta}$ for any $BN_{us_1}^*, BN_{us_2}^* \in BN_{S\zeta}$

(3) $\cup BN_{us_i}^* \in BN_{S\zeta}$ for every $BN_{us_i}^*$: $i \in j \subseteq BN_{S\zeta}$.

Then the space $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$, is known a BNTS (BNS –T-S). The component of $BN_{S\zeta}$ are called BNS-OS (Bipolar Neutrosophic open set) and its complement is BNS-CS (Bipolar Neutrosophic closed set)

Example 1. Let B $N_{u_Y^{\otimes}} = \{u\}$ and $\forall u \in BN_{u_Y^{\otimes}}$

$$U_1^{\otimes} = \left\langle u, -6\times10^{-1}, -6\times10^{-1}, -6\times10^{-1}, 6\times10^{-1}, 6\times10^{-1}, 6\times10^{-1} \right\rangle,$$
$$U_2^{\otimes} = \left\langle u, -5\times10^{-1}, -7\times10^{-1}, -9\times10^{-1}, 5\times10^{-1}, 7\times10^{-1}, 9\times10^{-1} \right\rangle$$

$$U_3^{\otimes} = \left\langle u, -3\times10^{-1}, -4\times10^{-1}, -7\times10^{-1}, 6\times10^{-1}, 7\times10^{-1}, 5\times10^{-1} \right\rangle,$$
$$U_4^{\otimes} = \left\langle u, -2\times10^{-1}, -6\times10^{-1}, -4\times10^{-1}, 5\times10^{-1}, 6\times10^{-1}, 9\times10^{-1} \right\rangle$$

Then collection $BN_{S\zeta} = \left\{0_{N_u^*}, U_1^{\otimes}, U_2^{\otimes}, U_3^{\otimes}, U_4^{\otimes}, 1_{N_u^*}\right\}$ is known as BNS-T-S on $N_{u_Y^{\otimes}}$.

Definition 2.6. Let $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ be BNTS. Then Bipolar Neutrosophic closure of $U_1^{\otimes}$ is
$$BN_u \approx BCL\left(U_1^{\otimes}\right) = \cap\left\{L : L \text{ is a Bipolar Neutrosophic Closed set in } BN_{u_Y^{\otimes}} \text{ and } U_1^{\otimes} \subseteq L\right\}.$$

Bipolar Neutrosophic interior of $U_1^{\otimes}$ is:
$$BN_u \approx BINT\left(U_1^{\otimes}\right) = \cup\left\{L_1 : L_1 \text{ is a Bipolar Neutrosophic Open set in } BN_{u_Y^{\otimes}} \text{ and } L_1 \subseteq U_1^{\otimes}\right\}.$$

Definition 2.7. Let $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ be a BNTS. Then $U_1^{\otimes}$ is known as

(1) Bipolar Neutrosophic regular Closed set (BNeu-RCS) if
$U_1^{\otimes} = BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$[1];

(2) Bipolar Neutrosophic α -Closed set (Neu-α CS) if
$BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(BN_{eu} \approx BCL\left(U_1^{\otimes}\right)\right)\right) \subseteq U_1^{\otimes}$ [1];

(3) Bipolar Neutrosophic semi Closed set (BNeu-SCS) if
$BN_{eu} \approx BINT\left(BN_{eu} \approx BCL\left(U_1^{\otimes}\right)\right) \subseteq U_1^{\otimes}$ [7];

(4) Bipolar Neutrosophic pre Closed set (BNeu-PCS) if
$BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq U_1^{\otimes}$[15];

Definition 2.8. Let $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ be a BNTS. Then $U_1^{\otimes}$ is called:

(1) Bipolar Neutrosophic (regular open) set BNeu-ROS) if
$U_1^{\otimes} = BN_{eu} \approx BINT\left(BN_{eu} \approx BCL\left(U_1^{\otimes}\right)\right)$,[1];

(2) Bipolar Neutrosophic (_-open)set (BNeu-_ OS) if
$U_1^{\otimes} \subseteq BN_{eu} \approx BINT\left(BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)\right)$, [1];

(3) Bipolar Neutrosophic (semi open) set (BNeu-SOS) if $U_1^{\otimes} \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$,[7];

(4) Bipolar Neutrosophic (pre open) set (BNeu-POS) if $U_1^{\otimes} \subseteq BN_{eu} \approx BINT\left(BN_{eu} \approx BCL\left(U_1^{\otimes}\right)\right)$,[15].

Definition 2.9. A bipolar Neutrosophic set $U_1^{\otimes}$ of a BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ is called

(1) Bipolar Neutrosophic (Bg-closed) if $BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is Bipolar Neutrosophic open, [3];

(2) Bipolar Neutrosophic (Bsg-closed) if $BN_{eu} \approx \left(BS_g\right) BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is Bipolar Neutrosophic semi open, [14];

(3) Bipolar Neutrosophic (Bg_-closed) if $BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is Bipolar Neutrosophic g-open, [2];

(4) Bipolar Neutrosophic (Bg-closed) if $BN_{eu} \approx (\alpha) BCL\left(U_1^{\otimes}\right) \subseteq BG$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is bipolar Neutrosophic - open, [8];

(5) Bipolar Neutrosophic (Bg_-closed) if $BN_{eu} \approx (\alpha) BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is bipolar Neutrosophic _- open, [4];

(6) Bipolar Neutrosophic (Bw-closed) if $BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq BG$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is Bipolar Neutrosophic semi open, [13];

(7) Bipolar Neutrosophic (BgP-closed) if $BN_{eu} \approx (P) BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is Bipolar Neutrosophic open, [9];

(8) Bipolar Neutrosophic (Bgs-closed) if $BN_{eu} \approx (S) BCL\left(U_1^{\otimes}\right) \subseteq BG_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq BG_1^{\oplus}$ and $BG_1^{\oplus}$ is bipolar Neutrosophic open, [14].

The complements of the above mentioned closed set are their respective open sets.

Definition 2.10. If $U_1^{\otimes}$ is a Bipolar Neutrosophic set in BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ then

(1) $BN_{eu} \approx (S) BC_L\left(U_1^{\otimes}\right) = \cap\left\{K_1^{\otimes} : U_1^{\otimes} \subseteq K_1^{\otimes},\ K_1^{\otimes} \text{ is } BN_{eu}(S) C_S\right\}$

(2) $BN_{eu} \approx (P) BC_L (U_1^{\otimes}) = \cap \{ K_1^{\otimes} : U_1^{\otimes} \subseteq K_1^{\otimes}, K_1^{\otimes} \text{ is } BN_{eu}(P)C_S \}$

(3) $BN_{eu} \approx (\alpha) BC_L (U_1^{\otimes}) = \cap \{ K_1^{\otimes} : U_1^{\otimes} \subseteq K_1^{\otimes}, K_1^{\otimes} \text{ is } BN_{eu}(\alpha)C_S \}$

Remark 2.1. (1) Every $BN_{eu}C_S$ is $BN_{eu}(g)C_S$.

(2) Every $BN_{eu}(\alpha)C_S$ is $BN_{eu}(\alpha g)C_S$.

(3) Every $BN_{eu}(g)C_S$ is $BN_{eu}(g\alpha)C_S$.

(4) Every $BN_{eu}(\alpha g)C_S$ is $BN_{eu}(g\alpha)C_S$.

(5) Every $BN_{eu}(w)C_S$ is Nu(g)CS.

(6) Every $BN_{eu}(g)C_S$ is $BN_{eu}(w)C_S$.

(7) Every $BN_{eu}(sg)C_S$ is $BN_{eu}(sg)C_S$.

Lemma 2.1. Let $U_1^{\otimes}$ and $U_2^{\otimes}$ be any two BNSs of a BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$. Then:

(a) $U_1^{\otimes}$ is a $BN_{eu}C_S$ in $BN_{u_Y^{\otimes}} \Leftrightarrow BN_{eu} \approx BC_L(U_1^{\otimes}) = (U_1^{\otimes})$

(b) $U_1^{\otimes}$ is a $BN_{eu}O_S$ in $BN_{u_Y^{\otimes}} \Leftrightarrow BN_{eu} \approx BINT(U_1^{\otimes}) = (U_1^{\otimes})$

(c) $BN_{eu} \approx BCL(U_1^{\otimes}) = (BN_{eu} \approx BINT(U_1^{\otimes}))C$.

(d) $BN_{eu} \approx BINT(U_1^{\otimes}) = (BN_{eu} \approx BCL(U_1^{\otimes}))C$

(e) $U_1^{\otimes} \subseteq U_2^{\otimes} \Rightarrow BN_{eu} \approx BINT(U_1^{\otimes}) \subseteq BN_{eu} \approx BINT(U_2^{\otimes})$.

(f) $U_1^{\otimes} \subseteq U_2^{\otimes} \Rightarrow BN_{eu} \approx BCL(U_1^{\otimes}) \subseteq BN_{eu} \approx BCL(U_2^{\otimes})$.

(g) $BN_{eu} \approx BCL(U_1^{\otimes} \cup U_2^{\otimes})_2^{\otimes} \Rightarrow BN_{eu} \approx BCL(U_1^{\otimes}) \cup BN_{eu} \approx BCL(U_2^{\otimes})$.

(h) $BN_{eu} \approx INT(U_1^{\otimes} \cap U_2^{\otimes})_2^{\otimes} \Rightarrow BN_{eu} \approx BINT(U_1^{\otimes}) \cap BN_{eu} \approx BINT(U_2^{\otimes})$.

3. BIPOLAR NEUTROSOPHIC WEAKLY B $g^{\otimes}$-CLOSED

Definition 3.1. A bipolar Neutrosophic set $U_1^{\otimes}$ of a BNTS$\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ is called bipolar Neutrosophic weakly $Bg^{\otimes}$ -closed if $BN_{eu} \approx BC_L\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq P_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq P_1^{\oplus}$ and $P_1^{\oplus}$ is Bipolar Neutrosophic g-open in B $N_{u_Y^{\otimes}}$.

Theorem 3.1. Every $BN_{eu}(W)C_S$ set is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. Let $U_1^{\otimes}$ is $BN_{eu}(W)C_S$. Let $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus} BN_{eu}(S)O_S$ in B $N_{u_Y^{\otimes}}$.

Since every $BN_{eu}(S)O_S$ is $BN_{eu}(g)O_S J_1^{\oplus}$ is $BN_{eu}(g)O_S$.

Using definition $BN_{eu}(W)C_S BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

But $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$. We have $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq J_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(S)O_S$ in $BN_{u_Y^{\otimes}}$. Therefore $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Theorem 3.2. Every $BN_{eu}\left(g^{\otimes}\right)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. Let $U_1^{\otimes}$ is $BN_{eu}\left(g^{\otimes}\right)C_S$. Let $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $N_{u_Y^{\otimes}}$. using definition $BN_{eu}\left(g^{\otimes}\right)C_S BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.But $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$. We have $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq J_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $N_{u_Y^{\otimes}}$. Therefore $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Theorem 3.3. Every $BN_{eu}(g)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. $U_1^{\otimes}$ is $BN_{eu}(g)C_S$. $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus} BN_{eu} O_S$ in $BN_{u_Y^{\otimes}}$. Since every $BN_{eu} O_S$ is $BN_{eu}(g)O_S J_1^{\oplus}$ is $BN_{eu}(g)O_S$.

Presently using definition $BN_{eu}(g)C_S BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

But $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$. We have $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq J_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $BN_{u_Y^{\otimes}}$. Therefore $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$ set.

Theorem 3.4. Every $BN_{eu}(\alpha g)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. Let $U_1^{\otimes}$ is $BN_{eu}(\alpha g)C_S$. Let $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus} BN_{eu}O_S$ in $BN_{u_Y^{\otimes}}$. Since every

$BN_{eu}O_S$ $BN_{eu}(g)O_S J_1^{\oplus}$ is $BN_{eu}(g)O_S$.

Presently using definition $BN_{eu}(\alpha g)C_S$, $BN_{eu}(\alpha) \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

But $BN_{eu} \approx (\alpha) BCL\left(U_1^{\otimes}\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right)$

Therefore $BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq U_1^{\otimes}$. Now $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

We have $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq J_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $BN_{u_Y^{\otimes}}$. Therefore $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Theorem 3.5. Every $BN_{eu}(\alpha g)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. From theorem 3.4 we get every $BN_{eu}(\alpha g)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$

Theorem 3.6 Every $BN_{eu}(gP)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. Let $U_1^{\otimes}$ is $BN_{eu}(gP)C_S$. Let $U_1^{\otimes} \subseteq J_1^{\oplus}$ $J_1^{\oplus} BN_{eu}O_S$ in $BN_{u_Y^{\otimes}}$. Since every

$BN_{eu}O_S$ is $BN_{eu}(g)O_S J_1^{\oplus}$ is $BN_{eu}(g)O_S$.

Presently using definition $BN_{eu}(Pg)C_S BN_{eu}(P) \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

But $BN_{eu} \approx (P) BCL\left(U_1^{\otimes}\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right)$

Therefore $BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq U_1^{\otimes}$.

Now $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(U_1^{\otimes}\right) \subseteq J_1^{\oplus}$.

We have $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq J_1^{\oplus}$ whenever $U_1^{\otimes} \subseteq J_1^{\oplus}$ and $J_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $BN_{u_Y^{\otimes}}$. Therefore $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$

Corollary 3.1.

(1) Every $BN_{eu}C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

(2) Every $BN_{eu}(\alpha)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

(3) Every $BN_{eu}(P)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

(4) Every $BN_{eu}(R)C_S$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Proof. Obvious.

Theorem 3.7 Let $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$ is a BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ and $U_1^{\otimes} \subseteq U_2^{\otimes} \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$. Then $U_2^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$ in $BN_{u_Y^{\otimes}}$.

Proof.

Let $P_1^{\oplus}$ is $BN_{eu}(g)O_S$ in $BN_{u_Y^{\otimes}}$ such that $U_2^{\otimes} \subseteq P_1^{\oplus}$. Then $U_1^{\otimes} \subseteq P_1^{\oplus}$ and since $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$, $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq P_1^{\oplus}$.

Now $U_2^{\otimes} \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$

$$\Rightarrow BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_2^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(BN_{eu} \approx CL\left(BN_{eu} \approx INT\left(U_1^{\otimes}\right)\right)\right)\right) = BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$$

, $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_2^{\otimes}\right)\right) \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq P_1^{\oplus}$.

Consequently $U_2^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$.

Definition 3.8 A Bipolar Neutrosophic set $U_1^{\otimes}$ of a BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ is called $BN_{eu}\left(g^{\otimes}\right)O_S$ iff $U_1^{\otimes C}$ is $BN_{eu}\left(g^{\otimes}\right)C_S$.

Remark 3.9 Every $BN_{eu}(W)O_S$ is $BN_{eu}\left(Wg^{\otimes}\right)O_S$.

Theorem 3.8. A Bipolar Neutrosophic set $U_1^{\otimes}$ of a BNTS$\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$, $BN_{eu}\left(W_{g^{\otimes}}\right)O_S$ if $M_1^{\otimes} \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)$ whenever $M_1^{\otimes}$ is $BN_{eu}\left(g\right)C_S$ and $M_1^{\otimes} \subseteq U_1^{\otimes}$.

Proof: Follows from Definition 3.8.

Theorem 3.9. $U_1^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)O_S$ of a BNTS $\left(BN_{u_Y^{\otimes}}, BN_{S\zeta}\right)$ and $BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq U_2^{\otimes} \subseteq U_1^{\otimes}$. Then $U_2^{\otimes}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)O_S$.

Proof. Suppose $U_1^{\otimes}$ is a $BN_{eu}\left(W_{g^{\otimes}}\right)O_S$ in $BN_{u_Y^{\otimes}}$ and

$$BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right) \subseteq U_2^{\otimes} \subseteq U_1^{\otimes}$$

$$\Rightarrow U_1^{\otimes C} \subseteq U_2^{\otimes C} \subseteq \left(BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes}\right)\right)\right)^C$$

$\Rightarrow U_1^{\otimes C} \subseteq U_2^{\otimes C} \subseteq BN_{eu} \approx BCL\left(BN_{eu} \approx BINT\left(U_1^{\otimes C}\right)\right)$ and $U_1^{\otimes C}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$ it follows from theorem 3.8 that $U_2^{\otimes C}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)C_S$. Hence $U_2^{\otimes C}$ is $BN_{eu}\left(W_{g^{\otimes}}\right)O_S$.

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