Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Cyclic normal fuzzy neutrosopic soft *G*-modular structures acting on a group

To cite this article: P Jayaraman 2018 *J. Phys.: Conf. Ser.* **1132** 012005

Cyclic normal fuzzy neutrosopic soft G-modular structures acting on a group

P Jayaraman

Department of Mathematics, Bharathiar University, Coimbatore - 641 046, India

E-mail: jrmsathya@gmail.com

Abstract. In this paper, we explain classical concept of the fuzzy soft sets to express the idea of cyclic normal fuzzy neutrosopic soft G-modular structures acting on a group. Neutrosopic soft set theory is studied as an effective parametric tool to discuss with uncertainities. We also investigate the relationship between cyclic fuzzy neutrosopic soft G-modules and classical modules. We study their concerned properties in terms of soft set operations, soft image, soft pre-image, soft anti image, α-inclusion of neutrosopic fuzzy soft sets and linear combinations of the vector spaces. Furthermore we show applications of this new G-modules on vector spaces with supporting proofs.

1. Introduction

The fuzzy set was introduced by Zadeh[19] in 1965 where each element had a degree of membership. The bifuzzy set on a universe was proposed by K.Atanasov[1] in 1983, as a generalization of fuzzy set, which discussed both the degree of membership and the degree of non membership of each element. The idea of Neutrosophic set was introduced by F.Smarandache[14] which is a parametric tool to deal with problems which involves vagueness, indeterminacy and inconsistent data. The theory of Neutrosophic set which is the modern set of the classical sets, conventional fuzzy set[19], intuitionistic fuzzy set[1] and interval valued fuzzy set[16] was proposed by F.Smarandache[14]. This idea was recently used in developing new approach in various field such as databases study, medical diagonses problems, decision making problem, topology, control theory and so on. The idea of neutrosophic set handles middle data where fuzzy theory and intuitionistic fuzzy set theory cannot be applied. The concept of cyclic fuzzy set is discussed in [12]. Our objective is to introduce the concept of cyclic fuzzy neutrosophic soft groups [CFNSG] and its properties.

2. Preliminaries

Definition 1. *Let U be a non-empty set. Then a fuzzy set on U is meant to be a function $A : U \longrightarrow [0, 1]$. A is called the graded function, $A(x)$ is called the membership grade of x in A. We also write $\{(x, A(x)) : x \in U\}$.*

Example 1. *Consider $U = \{a, b, c, d\}$ and $A : U \longrightarrow [0, 1]$ defined by $A(a) = 0, A(b) = 0.7, A(c) = 0.4, A(d) = 1$.*

Definition 2. *Let U be the initial universe set and E be the set of parameters. Let $P(U)$ denote the power set of U. Consider a non empty set $A, A \subset E$. A pair (F, A) is called a soft set over U, where $F : A \longrightarrow P(U)$.*

Example 2. *Suppose that U is the set of houses under consideration, say $U = \{h_1, h_2, \ldots, h_5\}$. Let E be the set of some attributes of such houses, say $E = \{e_1, e_2, \ldots, e_8\}$, where $e_1, e_2, \ldots, e_8$ stand for the attributes "expensive", "beautiful", "wooden", "cheap", "modern" and "in bad", "repair" respectively.*
In this problem, to discuss a soft set means to point out expensive houses, beautiful houses and so on. For example, the soft set (F, A) that explain the "attractiveness of the houses" in the opinion of a buyer, say Thomas, may be defined like this: $A = \{e_1, e_2, e_3, e_4, e_5\}$; $F(e_1) = \{h_2, h_3, h_5\}$, $F(e_2) = \{h_2, h_4\}$, $F(e_3) = \{h_1\}$, $F(e_4) = U$, $F(e_5) = \{h_3, h_5\}$.

Definition 3. *A Neutrosophic set 'A' on the universe of discourse X is defined as*

$$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$$

where $T_A(x) : X \longrightarrow [0^-, 1^+]$, $I_A(x) : X \longrightarrow [0^-, 1^+]$, $F_A(x) : X \longrightarrow [0^-, 1^+]$,
and $0^- \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+$, $T_A(x), I_A(x)$ and $F_A(x)$ are respectively truth membership, indeterminacy membership and falsity membership.

From certain point of view, the neutrosophic set takes from real standard and non-standard values $[0^-, 1^+]$. So instead of $[0^-, 1^+]$ we have to take the value $[0, 1]$ for technical applications, because $[0^-, 1^+]$ will be inconvenient to use in all main applications such as scientific and engineering problems.

If a Neutrosophic set 'A' is contained in another neutrosophic set B, i.e.$A \subseteq B$ then $T_A(x) \leq T_B(x)$, $I_A(x) \leq I_B(x)$, $F_A(x) \geq F_B(x)$, for all $x \in X$.

Definition 4. *A fuzzy neutrosophic set 'A' on the universe of discourse X is defined as*

$$A = \left\langle x, \Delta_{T_A(x)}, \Delta_{I_A(x)}, \Delta_{F_A(x)} \right\rangle, x \in X$$

where

$T : X \longrightarrow [0, 1]$, $I : X \longrightarrow [0, 1]$, $F : X \longrightarrow [0, 1]$, *and* $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 2$.

Example 3. *Consider the universe of discourse $U = \{x_1, x_2, x_3\}$, where x_1 characterizes the capability, x_2 characterizes the trustworthiness and x_3 indicates the prices of the objects. It may be further assumed that the values of x_1, x_2 and x_3 are in $[0, 1]$ and they are derived from doubtful of some experts. The experts may say their opinion in three components viz. the degree of goodness, the degree of indeterminacy and that of lowerness to explain the characteristics of the objects. Suppose A is a fuzzy neutrosophic set(FNS) of U, such that,*

$$A = \{\langle x_1, 0.3, 0.5I, 0.4\rangle, \langle x_2, 0.4I, 0.2, 0.6\rangle, \langle x_3, 0.7, 0.3, 0.5I\rangle, \},$$

where the membership of goodness of capability is 0.3, degree of indeterminacy of capability is 0.5I and degree of falsity of capability is 0.4 etc.

Definition 5. *Let U be the initial universal set and E be a set of parameters. Let $P(U)$ denote the set of all fuzzy neutrosophic set of U. Consider a non-empty set $A, A \subset E$. The collection (F, A) is noted to be the fuzzy neutrosophic soft set(FNSS) over U, where $F : A \longrightarrow P(U)$.*

Example 4. *Let U be the set of blouses under consideration and E is the set of parameters (or qualities). Each parameter is a fuzzy neutrosophic word or sentence involving fuzzy neutrosophic words. Let $E = \{$Bright, Cheap, Costly, Very costly, Colorful, Cotton, Polystyrene, Long sleeve, Expensive $\}$. In this case, to define a fuzzy neutrosophic soft set means to point out in the universe $U = \{b_1, b_2, b_3, b_4, b_5\}$ and the set of constants $A = \{e_1, e_2, e_3, e_4\}$, where each e_i is a specific criterion for blouses:*

e_1 stands for 'Bright',

e_2 stands for 'Cheap',

e_3 stands for 'Costly',

e_4 stands for 'Colorful',

Suppose that,

$F(Bright) = \{\langle b_1, 0.5, 0.6I, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2I\rangle, \langle b_3, 0.6I, 0.2, 0.3\rangle, \langle b_4, 0.7I, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3I\rangle\}.$

$F(Cheap) = \{\langle b_1, 0.6I, 0.3, 0.5\rangle, \langle b_2, 0.7, 0.4I, 0.3\rangle, \langle b_3, 0.8I, 0.1, 0.2\rangle, \langle b_4, 0.7, 0.1, 0.3I\rangle, \langle b_5, 0.8I, 0.3, 0.4\rangle\}.$

$F(Costly) = \{\langle b_1, 0.7I, 0.4, 0.3\rangle, \langle b_2, 0.6, 0.1I, 0.2\rangle, \langle b_3, 0.7, 0.2, 0.5I\rangle, \langle b_4, 0.5I, 0.2, 0.6\rangle, \langle b_5, 0.7, 0.3I, 0.2\rangle\}.$

$F(Colorful) = \{\langle b_1, 0.8, 0.1I, 0.4\rangle, \langle b_2, 0.4, 0.2I, 0.6\rangle, \langle b_3, 0.3I, 0.6, 0.4\rangle, \langle b_4, 0.4, 0.8, 0.5I\rangle, \langle b_5, 0.3, 0.5I, 0.7\rangle\}.$

The fuzzy neutrosophic soft set(FNSS) (F, E) *is a parameterized family of all fuzzy neutrosophic sets of* U *and explain a collection of approximation of an element. The mapping* F *here is 'blouses(.)', where dot(.) is to be filled up by a parameter* $e_i \in E$. *Therefore,* $F(e_1)$ *means 'blouses(Bright)' whose functional-value is the fuzzy neutrosophic set*

$$\{\langle b_1, 0.5, 0.6I, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2I\rangle, \langle b_3, 0.6I, 0.2, 0.3\rangle, \langle b_4, 0.7I, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3I\rangle\}.$$

Thus we can see, (FNSS) (F, A) *as a collection of approximation as below:*

$(F, A) = \{Bright\ blouses = \{\langle b_1, 0.5, 0.6I, 0.3\rangle, \langle b_2, 0.4, 0.7, 0.2I\rangle, \langle b_3, 0.6I, 0.2, 0.3\rangle, \langle b_4, 0.7I, 0.3, 0.2\rangle, \langle b_5, 0.8, 0.2, 0.3I\rangle\}, Cheap\ blouses = \{\langle b_1, 0.6I, 0.3, 0.5\rangle, \langle b_2, 0.7, 0.4I, 0.3\rangle, \langle b_3, 0.8I, 0.1, 0.2\rangle, \langle b_4, 0.7, 0.1, 0.3I\rangle, \langle b_5, 0.8I, 0.3, 0.4\rangle\}, Costly\ blouses = \{\langle b_1, 0.7I, 0.4, 0.3\rangle, \langle b_2, 0.6, 0.1I, 0.2\rangle, \langle b_3, 0.7, 0.2, 0.5I\rangle, \langle b_4, 0.5I, 0.2, 0.6\rangle, \langle b_5, 0.7, 0.3I, 0.2\rangle\}, Colorful\ blouses = \{\langle b_1, 0.8, 0.1I, 0.4\rangle, \langle b_2, 0.4, 0.2I, 0.6\rangle, \langle b_3, 0.3I, 0.6, 0.4\rangle, \langle b_4, 0.4, 0.8, 0.5I\rangle, \langle b_5, 0.3, 0.5I, 0.7\rangle\}\}.$

In obtain a fuzzy neutrosophic soft set in a computer, we can express it in the form of a table as shown below (corresponding to the fuzzy neutrosophic soft set in the above example). In this table, the entries are c_{ij} *corresponding to the blouse* b_i *and the parameter* e_j, *where* c_{ij} *=(true-membership value of* b_i, *indeterminacy-membership value of* b_i, *falsity membership value of* b_i*) in* $F(e_j)$. *The fuzzy neutrosophic soft set* (F, A) *described as above is expressed as follows.*

$$\begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{matrix} \begin{pmatrix} (0.5, 0.6I, 0.3) & (0.6I, 0.3, 0.5) & (0.7I, 0.4, 0.3) & (0.8, 0.1I, 0.4) \\ (0.4, 0.7, 0.2I) & (0.7, 0.4I, 0.3) & (0.6, 0.1I, 0.2) & (0.4, 0.2I, 0.6) \\ (0.6I, 0.2, 0.3) & (0.8I, 0.1, 0.2) & (0.7, 0.2, 0.5I) & (0.3I, 0.6, 0.4) \\ (0.7I, 0.3, 0.2) & (0.7, 0.1, 0.3I) & (0.5I, 0.2, 0.6) & (0.4, 0.8, 0.5I) \\ (0.8, 0.2, 0.3I) & (0.8I, 0.3, 0.4) & (0.7, 0.3I, 0.2) & (0.3, 0.5I, 0.7) \end{pmatrix}$$

Definition 6. *Let* X *be a non-empty collection and*

IOP Conf. Series: Journal of Physics: Conf. Series 1132 (2018) 012005 doi:10.1088/1742-6596/1132/1/012005

$A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ *are fuzzy neutrosophic soft sets. Then union, intersection and difference sets defined as*

$T_{A\cup B}(x) = max\{T_A(x), T_B(x)\}, I_{A\cup B}(x) = max\{I_A(x), I_B(x)\}, F_{A\cup B}(x) = min\{F_A(x), F_B(x)\}.$

$T_{A\cap B}(x) = min\{T_A(x), T_B(x)\}, I_{A\cap B}(x) = min\{I_A(x), I_B(x)\}, F_{A\cap B}(x) = max\{F_A(x), F_B(x)\}.$

for all $x \in X$ *and*

$$\begin{aligned} A/B = T_{A/B}(x) = min\{T_A(x), T_B(x)\} &= I_{A/B}(x) = min\{I_A(x), 1 - I_B(x)\} \\ &= F_{A/B}(x) = max\{F_A(x), F_B(x)\} \end{aligned}$$

Definition 7. *A pair* (F, A) *is called Fuzzy neutrosophic soft group if the following conditions are satisfied:*
$(FNSG1) : T_A(xy) \geq \min\{T_A(x), T_A(y)\}, F_A(xy) \leq \max\{F_A(x), F_A(y)\},$
$I_A(xy) \leq \max\{I_A(x), I_A(y)\}$ *for all* $x, y \in X$.

$(FNSG2) : T_A(x^{-1}) \geq T_A(x), F_A(x^{-1}) \leq F_A(x), I_A(x^{-1}) \leq I_A(x)$ *for all* $x \in X$.

Example 5.

(i) Let the universe of discourse $X = \{x, y, z\}$. *Then*

$$A = \{\langle x, 0.1, 0.3I, 0.5\rangle, \langle y, 0.2, 0.5, 0.6I\rangle, \langle z, 0.3I, 0.4, 0.5\rangle\}$$

implies the degrees of goodness of capability is 0.1, degree of indeterminacy of capability is 0.3I and degree of falsity of capability is 0.5.

(ii) Let $X = \{DOG, CAT, RAT\}$. *A FNS 'A' of* X *could be*

$$A = \{\langle DOG, (0.3I, 0.2, 0.1)\rangle, \langle CAT, (0.3, 0.4I, 0.6)\rangle, \langle RAT, (0.1, 0.3, 0.4I)\rangle\}$$

Definition 8. *A fuzzy neutrosophic soft set is said to be zero FNS if* $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ *for all* $x \in X$. *It is denoted by* 0_N. *A fuzzy neutrosophic soft set is said to be unit FNS if* $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ *for all* $x \in X$. *It is known as* 1_N.

The below proposition is obvious.

Proposition 1. *Zero FNS and unit FNS of a group* X *are well known FNSG of* X.

Definition 9. *The $\alpha-$cut of the FNS. A is a classical subset A_α of the set X is given by*

$$A_\alpha = \{x, x \in X / T_A(x) \geq \alpha\}.$$

Proposition 2. *Let A be FNSG of a group X. Then for $\alpha \in [0,1], \alpha-$cut A_α is a classical subgroup of X.*

Proof. For all $x, y \in A_\alpha$. We have $T_A(x) \geq \alpha, T_A(y) \geq \alpha$.
Now $T_A(xy^{-1}) \geq \min\{T_A(x), T_A(y)\} = \alpha$.

□

3. Cyclic fuzzy neutrosophic soft groups

The main concepts on cyclic fuzzy neutrosophic soft group is related to cyclic group and fuzzy neutrosophic soft group. Before defining the cyclic fuzzy neutrosophic soft group, we will need the following well known definition.

Let 'a' be an element of a group A. Then the set $S = \{a^n / n \in \mathrm{Z}\}$ is a cyclic subgroup of A generated by a, and is denoted by $< a >$. Now we shall define a new class of fuzzy neutrosophic soft groups. Let $A = \langle a \rangle$ be a cyclic group. If

$$\tilde{A} = \{\langle a^n, (T_A(a^n)), (I_A(a^n)), (F_A(a^n))\rangle / n \in \mathrm{Z}\}$$

is fuzzy neutrosophic soft group, then $\tilde{A}$ is called a cyclic fuzzy neutrosophic soft group [CFNSG] powered by $(a, T_A(a), I_A(a), F_A(a))$ and will be denoted by $\langle a, T_A(a), I_A(a), F_A(a)\rangle$.

Definition 10. *Let e be the identity element of the group A. We define fuzzy neutrosophic soft group E by*

$$E = \{e, T_A(e), I_A(e), F_A(e) / T_A(e) = I_A(e) = F_A(e) = 1 | e \in A\}$$

Definition 11. *A negative fuzzy(N-Fuzzy) neutrosophic soft set $\mathcal{A}$ on the universe of discourse $\mathcal{X}$ is defined as*

$$\mathcal{A} = \left\langle x, \overline{\delta}_{\mathcal{A}}(x), \overline{I}_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(x) \right\rangle, x \in \mathcal{X}$$

$$\textit{where } \delta, I, \Delta : \mathcal{X} \longrightarrow [-1, 0] \textit{ and} -1 \leq \overline{\delta}_{\mathcal{A}}(x) + \overline{I}_{\mathcal{A}}(x) + \Delta_{\mathcal{A}}(x) \leq 0.$$

Definition 12. *A N-Fuzzy neutrosophic soft set $\mathcal{A}$ over the universe $\mathcal{X}$ is said to be null or empty N-Fuzzy neutrosophic soft set if*

$$\delta_{\mathcal{A}}(x) = -1, I_{\mathcal{A}}(x) = -1, \Delta_{\mathcal{A}}(x) = 0 \textit{ for all } x \in \mathcal{X}.$$

It is denoted by -1_N.

Definition 13. *A N-Fuzzy neutrosophic soft set* $\mathcal{A}$ *over the universe* $\mathcal{X}$ *is said to be absolute(universe) N-Fuzzy neutrosophic soft collection if*

$$\delta_{\mathcal{A}}(x) = 0, I_{\mathcal{A}}(x) = 0, \Delta_{\mathcal{A}}(x) = -1 \text{ for all } x \in \mathcal{X}.$$

It is represented by 0_N.

Definition 14. *The complement of a N-Fuzzy neutrosophic soft set* $\mathcal{A}$ *is denoted by* $\mathcal{A}^C$ *and is defined as*

$$\mathcal{A}^C = \langle x, \delta_{\mathcal{A}^C}(x), I_{\mathcal{A}^C}(x), \Delta_{\mathcal{A}^C}(x)\rangle$$

where

$$\delta_{\mathcal{A}^C}(x) = \Delta_{\mathcal{A}}(x)$$

$$I_{\mathcal{A}^C}(x) = 1 - I_{\mathcal{A}}(x)$$

$$\Delta_{\mathcal{A}^C}(x) = \delta_{\mathcal{A}}(x).$$

The complement of a N-Fuzzy neutrosophic soft set $\mathcal{A}$ *can also be defined as* $0_N - \mathcal{A}$.

Definition 15. *Let* $(G, .)$ *be a groupoid and let* $-1_N \neq \mathcal{A} \in NFNS(G)$*, Then* $\mathcal{A}$ *is called N-Fuzzy neutrosophic soft subgroupoid (in short* $NFNSS$ *in G) if*

$$\delta_{\mathcal{A}}(xy) \leq max\,\{\delta_{\mathcal{A}}(x), \delta_{\mathcal{A}}(y)\}$$

$$I_{\mathcal{A}}(xy) \leq max\,\{I_{\mathcal{A}}(x), I_{\mathcal{A}}(y)\}$$

$$\Delta_{\mathcal{A}}(xy) \geq min\,\{\Delta_{\mathcal{A}}(x), \Delta_{\mathcal{A}}(y)\}$$

for all $x, y \in G$.

A N-Fuzzy neutrosophic soft set

$$\mathcal{A} = \left\{\left\langle x : \overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right\rangle, x \in \mathcal{X}\right\}$$

in $\mathcal{X}$ can be associated with an ordered pair $\left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ in

$$F\left(\mathcal{X}, [-1, 0]\right) \times F\left(\mathcal{X}, [-1, 0]\right) \times F\left(\mathcal{X}, [-1, 0]\right)$$

where $F\left(\mathcal{X}, [-1, 0]\right)$ explains the set of all collections from $\mathcal{X}$ to $[-1, 0]$. For simplicity, we write $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ instead of $\mathcal{A} = \left\{\left(x : \overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right) / x \in \mathcal{X}\right\}$.

Definition 16. *Let* $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ *be an* $NFNSS$ *in* $\mathcal{X}$*. Then the set*

$$N\left\{\left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right); (p, q, r)\right\} = \left\{x \in \mathcal{X}/\overline{\delta}_{\mathcal{A}}(x) \leq p, \overline{I}_{\mathcal{A}}(x) \leq q, \overline{\Delta}_{\mathcal{A}}(x) \geq r\right\}$$

where $p, q, r \in [-1, 0]$ *with* $p+q+r \geq -1$ *is said to be an* $N(p, q, r)$*-level set of* $\mathcal{A}$*. An* $N(p, p, p)$*-level set of* $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ *is said to be an* N*-level set of* $\mathcal{A}$*.*

Definition 17. *Let* $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ *and* $\mathcal{B} = \left(\overline{\delta}_{\mathcal{B}}, \overline{I}_{\mathcal{B}}, \overline{\Delta}_{\mathcal{B}}\right)$ *be two* $NFNSS$ *in* $\mathcal{X}$*. If for all* $x \in \mathcal{X}$*,* $\overline{\delta}_{\mathcal{A}}(x) \geq \overline{\delta}_{\mathcal{B}}(x)$*,* $\overline{I}_{\mathcal{A}}(x) \geq \overline{I}_{\mathcal{B}}(x)$ *and* $\overline{\Delta}_{\mathcal{A}}(x) \leq \overline{\Delta}_{\mathcal{B}}(x)$*, then* $\mathcal{A}$ *is said to be an* $NFNSS$ *subset of* $\mathcal{B}$ *and is denoted as* $\mathcal{A} \subseteq \mathcal{B}$*. We say that* $\mathcal{A} = \mathcal{B}$ *if and only if* $\mathcal{A} \overline{\subseteq} \mathcal{B}$ *and* $\mathcal{B} \overline{\supseteq} \mathcal{A}$*.*

Definition 18. *Let* $\mathcal{A} = \left(\overline{\Delta}_{\delta_{\mathcal{A}}}, \overline{\Delta}_{I_{\mathcal{A}}}, \overline{\Delta}_{\Delta_{\mathcal{A}}}\right)$ *and* $\mathcal{B} = \left(\overline{\delta}_{\mathcal{B}}, \overline{I}_{\mathcal{B}}, \overline{\Delta}_{\mathcal{B}}\right)$ *be two* $NFNSS$ *in* $\mathcal{X}$*. Then their collection and intersection are also* N*-Fuzzy neutrosophic soft set in* $\mathcal{X}$ *Where*

$$(\mathcal{A} \overline{\cup} \mathcal{B}) = \left\{x, min\left\{\overline{\delta}_{\mathcal{A}}(x), \overline{\delta}_{\mathcal{B}}(x)\right\}, min\left\{\overline{I}_{\mathcal{A}}(x), \overline{I}_{\mathcal{B}}(x)\right\}, max\left\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

and

$$(\mathcal{A} \overline{\cap} \mathcal{B}) = \left\{x, max\left\{\overline{\delta}_{\mathcal{A}}(x), \overline{\delta}_{\mathcal{B}}(x)\right\}, max\left\{\overline{I}_{\mathcal{A}}(x), \overline{I}_{\mathcal{B}}(x)\right\}, min\left\{\overline{\Delta}_{\mathcal{A}}(x), \overline{\Delta}_{\mathcal{B}}(x)\right\}\right\}$$

Example 6. *Let* $\mathcal{X}$ *be as in example 1 and*

$$\mathcal{A} = \{\langle w, -0.9, -0.8, -0.7\rangle, \langle x, -0.5, -0.4, -0.5\rangle, \langle y, -0.7, -0.6, -0.6\rangle, \langle z, -0.6, -0.3, -0.2\rangle\}$$

$$\mathcal{B} = \{\langle w, -0.8, -0.4, -0.6\rangle, \langle x, -0.9, -0.6, -0.7\rangle, \langle y, -0.7, -0.4, -0.5\rangle, \langle z, -0.8, -0.4, -0.7\rangle\}$$

then $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ *and* $\mathcal{B} = \left(\overline{\delta}_{\mathcal{B}}, \overline{I}_{\mathcal{B}}, \overline{\Delta}_{\mathcal{B}}\right)$ *are* N*-Fuzzy neutrosophic soft set in* $\mathcal{X}$*. Easily we can verify that* $\mathcal{A} \overline{\subseteq} \mathcal{B}$*.*

Example 7. *Let* $\mathcal{X}$ *be as in example 2 and*

$$\mathcal{A} = \{\langle w, -0.7, -0.4, -0.5\rangle, \langle x, -0.8, -0.5, -0.7\rangle, \langle y, -0.9, -0.5, -0.6\rangle, \langle z, -0.4, -0.2, -0.3\rangle\}$$

$$\mathcal{B} = \{\langle w, -0.9, -0.4, -0.5\rangle, \langle x, -0.7, -0.5, -0.4\rangle, \langle y, -0.6, -0.4, -0.5\rangle, \langle z, -0.8, -0.5, -0.6\rangle\}$$

then $\mathcal{A} = \left(\overline{\delta}_{\mathcal{A}}, \overline{I}_{\mathcal{A}}, \overline{\Delta}_{\mathcal{A}}\right)$ *and* $\mathcal{B} = \left(\overline{\delta}_{\mathcal{B}}, \overline{I}_{\mathcal{B}}, \overline{\Delta}_{\mathcal{B}}\right)$ *are* N*-Fuzzy neutrosophic soft set in* $\mathcal{X}$*.*

$$\mathcal{A} \overline{\cup} \mathcal{B} = \{\langle w, -0.9, -0.4, -0.5\rangle, \langle x, -0.8, -0.5, -0.4\rangle, \langle y, -0.9, -0.5, -0.5\rangle, \langle z, -0.8, -0.5, -0.3\rangle\}$$

$$\mathcal{A} \overline{\cap} \mathcal{B} = \{\langle w, -0.7, -0.4, -0.5\rangle, \langle x, -0.7, -0.5, -0.7\rangle, \langle y, -0.6, -0.4, -0.6\rangle, \langle z, -0.4, -0.2, -0.6\rangle\}$$

Obviously,$\mathcal{A} \overline{\cup} \mathcal{B}$ *and* $\mathcal{A} \overline{\cap} \mathcal{B}$ *are N-Fuzzy neutrosophic soft set in* $\mathcal{X}$*.*

Definition 19. *Let C be a non empty subset of X. Then N-Fuzzy neutrosophic soft membership function of C is a function $\bar{\Psi}_C = (\overline{\delta}_{\bar{\Psi}_C}, \overline{I}_{\bar{\Psi}_C}, \overline{\Delta}_{\bar{\Psi}_C})$ defined as, for any $x \in X$,*

$$\overline{\delta}_{\bar{\Psi}_C}(x) = \begin{cases} -1 & if \quad x \in C \\ 0 & if \quad x \notin C \end{cases}$$

$$\overline{I}_{\bar{\Psi}_C}(x) = \begin{cases} 0 & if \quad x \in C \\ 0.5 & if \quad x \notin C \end{cases}$$

$$\overline{\Delta}_{\bar{\Psi}_C}(x) = \begin{cases} 0.5 & if \quad x \in C \\ -1 & if \quad x \notin C \end{cases}$$

We denote N-Fuzzy neutrosophic soft characteristic function of X by $\overline{\Psi}_x = (\overline{\delta}_x, \overline{I}_x, \overline{\Delta}_x)$.

Theorem 1. *If A is a cyclic fuzzy neutrosophic soft group, then*

$$A^m = \{(a^n, (T_A(a^n))^m, (I_A(a^n)), (F_A(a^n)))/n \in \mathrm{Z}, m \in \mathrm{N}\}$$

is also a cyclic fuzzy neutrosophic soft group.

Theorem 2. *If A^i and A^j are cyclic fuzzy neutrosophic soft groups, then $A^i \cup A^j$ is also a cyclic fuzzy neutrosophic soft group, for some $i, j \in \mathrm{N}$.*

Proof. We can show only grade of function. Let $m \leq n$.
In this case (FNSG1), since $A^i \supset A^j$,

$$\begin{aligned} T_{A^i \cup A^j}(a^n a^m) &= \max\{T_{A^i}(a^n a^m), T_{A^j}(a^n a^m)\} \\ &\geq \max\left\{(T_A(a^n a^m))^i, (T_A(a^n a^m))^j\right\} \\ &\geq \max\left\{\min\left\{(T_A(a^n))^i, (T_A(a^m))^i\right\}, \min\left\{(T_A(a^n))^j, (T_A(a^m))^j\right\}\right\} \\ &\geq \min\left\{\max\left\{T_{A^i}(a^n), T_{A^i}(a^m)\right\}, \max\left\{T_{A^j}(a^n), T_{A^j}(a^m)\right\}\right\} \\ &\geq \min\left\{\max\left\{T_{A^i}(a^n), T_{A^j}(a^n)\right\}, \max\left\{T_{A^i}(a^m), T_{A^j}(a^m)\right\}\right\} \\ &\geq \min\left\{T_{A^i \cup A^j}(a^n), T_{A^i \cup A^j}(a^m)\right\} \end{aligned}$$

Similarly we can prove $F_{A^i \cup A^j}$ and $I_{A^i \cup A^j}$.

$$\begin{aligned}(FNSG2) \quad T_{A^i \cup A^j}(x^{-1}) &= \max\left\{T_{A^i}(x^{-1}), T_{A^j}(x^{-1})\right\} \\ &\geq \max\left\{(T_A(x^{-1}))^i, (T_A(x^{-1}))^j\right\} \\ &\geq \max\left\{(T_A(x))^i, (T_A(x))^j\right\} \\ &\geq \max\left\{T_{A^i}(x), T_{A^j}(x)\right\} \\ &\geq T_{A^i \cup A^j}(x).\end{aligned}$$

Similarly we can prove $F_{A^i \cup A^j}(x^{-1})$ and $I_{A^i \cup A^j}(x^{-1})$. □

Theorem 3. *If A^i and A^j are cyclic fuzzy neutrosophic soft groups, then $A^i \cap A^j$ is also a cyclic fuzzy neutrosophic soft group, for some $i, j \in$ N.*

Proof. We can show only grade of function. Let $m \geq n$.
In this case (FNSG1), since $A^i \subset A^j$,

$$\begin{aligned}T_{A^i \cap A^j}(a^n a^m) &= \min\left\{T_{A^i}(a^n a^m), T_{A^j}(a^n a^m)\right\} \\ &\leq \min\left\{(T_A(a^n a^m))^i, (T_A(a^n a^m))^j\right\} \\ &\leq \min\left\{\max\left\{(T_A(a^n))^i, (T_A(a^m))^i\right\}, \max\left\{(T_A(a^n))^j, (T_A(a^m))^j\right\}\right\} \\ &\leq \max\left\{\min\left\{T_{A^i}(a^n), T_{A^i}(a^m)\right\}, \min\left\{T_{A^j}(a^n), T_{A^j}(a^m)\right\}\right\} \\ &\leq \max\left\{\min\left\{T_{A^i}(a^n), T_{A^j}(a^n)\right\}, \min\left\{T_{A^i}(a^m), T_{A^j}(a^m)\right\}\right\} \\ &\leq \max\left\{T_{A^i \cap A^j}(a^n), T_{A^i \cap A^j}(a^m)\right\}\end{aligned}$$

Similarly we can prove $F_{A^i \cap A^j}$ and $I_{A^i \cap A^j}$.

$$\begin{aligned}(FNSG2) \quad T_{A^i \cap A^j}(x^{-1}) &= \min\left\{T_{A^i}(x^{-1}), T_{A^j}(x^{-1})\right\} \\ &\leq \min\left\{(T_A(x^{-1}))^i, (T_A(x^{-1}))^j\right\} \\ &\leq \min\left\{(T_A(x))^i, (T_A(x))^j\right\} \\ &\leq \min\left\{T_{A^i}(x), T_{A^j}(x)\right\} \\ &\leq T_{A^i \cap A^j}(x).\end{aligned}$$

Similarly we can prove $F_{A^i \cap A^j}(x^{-1})$ and $I_{A^i \cap A^j}(x^{-1})$. □

Definition 20. *Let A be a cyclic fuzzy neutrosophic soft group. Then the given set of the cyclic neutrosophic soft group $\left\{A, A^2, A^3, \ldots, A^m, \ldots, E\right\}$ is called cyclic fuzzy neutrosophic soft group powered by A. It will be represented by $\langle A \rangle$.*

Theorem 4. *Consider collection* $\langle A\rangle = \{A, A^2, A^3, \ldots, A^m, \ldots, E\}$. *Then* $\bigcup_{n=1}^{\infty} A^n = A$ *and* $\bigcap_{n=1}^{\infty} E$.

Proof. The proof is clear. □

4. Fuzzy Characteristic Neutrosophic Soft Group

In this section, we define fuzzy characteristic neutrosophic soft group (FCNSG) and discuss their properties. First we deal with the notations $T_A^\theta, I_A^\theta, F_A^\theta$ which is applicable in the following section.

Definition 21. *Let A be a fuzzy neutrosophic soft set of a group G. Let* $\theta : G \longrightarrow G$ *be a map. Define the maps* $T_A^\theta : G \longrightarrow [0,1], I_A^\theta : G \longrightarrow [0,1], F_A^\theta : G \longrightarrow [0,1]$ *given by, respectively* $T_A^\theta(x) = T_A(\theta(x)), I_A^\theta(x) = I_A(\theta(x)), F_A^\theta(x) = F_A(\theta(x))$ *for all* $x \in X$.

Definition 22. *A FNSG 'A' of a group G is called fuzzy characteristic neutrosophic soft group(FCNSG) of G if* $T_A^\theta = T_A, I_A^\theta = I_A, F_A^\theta = F_A$ *for every automorphism* θ *of G.*

We now show the following results

Proposition 3. *If A is FNSG of a group G and* θ *is a homomorphism of G, then the fuzzy neutrosophic soft set* A^θ *of G given by*

$$A^\theta = \left\{ \left\langle x, T_A^\theta, I_A^\theta, F_A^\theta \right\rangle / x \in G \right\},$$

is also FNSG of G.

Proof. Let $x, y \in G$.Then

$$\begin{aligned}
(FNSG1): T_A^\theta(xy) &= T_A(\theta(xy)) = T_A(\theta(x)\theta(y)) \\
&\geq \min\{T_A(\theta(x)), T_A(\theta(y))\} \\
&= \min\left\{T_A^\theta(x), T_A^\theta(y)\right\}. \\
I_A^\theta(xy) &= I_A(\theta(xy)) = I_A(\theta(x)\theta(y)) \\
&\leq \max\{I_A(\theta(x)), I_A(\theta(y))\} \\
&= \max\left\{I_A^\theta(x), I_A^\theta(y)\right\}. \\
F_A^\theta(xy) &= F_A(\theta(xy)) = F_A(\theta(x)\theta(y)) \\
&\leq \max\{F_A(\theta(x)), F_A(\theta(y))\} \\
&= \max\left\{F_A^\theta(x), F_A^\theta(y)\right\}. \\
(FNSG2): T_A^\theta(x^{-1}) &= T_A(\theta(x^{-1})) \geq T_A(\theta(x)) = T_A(\theta(x)).
\end{aligned}$$

Also it is easy to prove $I_A^\theta(x^{-1}) = I_A(\theta(x))$ and $F_A^\theta(x^{-1}) = F_A(\theta(x))$.
Therefore A^θ is FNSG of G. □

5. Main results

Theorem 5. *Let A and B be fuzzy neutrosophic soft groups in X, then so is $A \cup B$.*

Proof. Since A and B be fuzzy neutrosophic soft groups in X. Then clearly FNSG1 and FNSG2 are satisfied.
Now, let $x, y \in X$. Then

$$\begin{aligned}
(FNSG1): T_{A\cup B}(xy) &= \max\{T_A(xy), T_B(xy)\} \\
&\geq \max\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\}\} \\
&\geq \min\{\max\{T_A(x), T_A(y)\}, \max\{T_B(x), T_B(y)\}\} \\
&\geq \min\{\max\{T_A(x), T_B(x)\}, \max\{T_A(y), T_B(y)\}\} \\
&\geq \min\{T_{A\cup B}(x), T_{A\cup B}(y)\}.
\end{aligned}$$

$$\begin{aligned}
I_{A\cup B}(xy) &= \max\{I_A(xy), I_B(xy)\} \\
&\leq \max\{\max\{I_A(x), I_A(y)\}, \min\{I_B(x), I_B(y)\}\} \\
&\leq \max\{\max\{I_A(x), I_B(x)\}, \min\{I_A(y), I_B(y)\}\} \\
&\leq \max\{I_{A\cup B}(x), I_{A\cup B}(y)\}.
\end{aligned}$$

$$\begin{aligned} F_{A\cup B}(xy) &= \min\{F_A(xy), F_B(xy)\} \\ &\leq \min\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\}\} \\ &\leq \max\{\min\{F_A(x), F_A(y)\}, \min\{F_B(x), F_B(y)\}\} \\ &\leq \max\{\min\{F_A(x), F_B(x)\}, \min\{F_A(y), F_B(y)\}\} \\ &\leq \max\{F_{A\cup B}(x), F_{A\cup B}(y)\}. \end{aligned}$$

$$\begin{aligned} (FNSG2): T_{A\cup B}(x^{-1}) &= \max\left\{T_A(x^{-1}), T_B(x^{-1})\right\} \\ &\geq \max\{T_A(x), T_B(x)\} \\ &\geq T_{A\cup B}(x). \end{aligned}$$

$$\begin{aligned} I_{A\cup B}(x^{-1}) &= \max\left\{I_A(x^{-1}), I_B(x^{-1})\right\} \\ &\leq \max\{I_A(x), I_B(x)\} \\ &\leq I_{A\cup B}(x). \end{aligned}$$

$$\begin{aligned} F_{A\cup B}(x^{-1}) &= \min\left\{F_A(x^{-1}), F_B(x^{-1})\right\} \\ &\leq \min\{F_A(x), F_B(x)\} \\ &\leq F_{A\cup B}(x). \end{aligned}$$

□

Theorem 6. *If A and B be fuzzy neutrosophic soft group in X, then $A\cap B$ is also a fuzzy neutrosophic soft group in X.*

Proof. Since A and B be fuzzy neutrosophic soft groups in X. Then clearly FNSG1 and FNSG2 are satisfied.
Now, let $x, y \in X$. Then

$$\begin{aligned} (FNSG1): T_{A\cap B}(xy) &= \max\{T_A(xy), T_B(xy)\} \\ &\geq \max\{\min\{T_A(x), T_A(y)\}, \min\{T_B(x), T_B(y)\}\} \\ &\geq \min\{\max\{T_A(x), T_A(y)\}, \max\{T_B(x), T_B(y)\}\} \\ &\geq \max\{\max\{T_A(x), T_B(x)\}, \max\{T_A(y), T_B(y)\}\} \\ &\geq \min\{T_{A\cap B}(x), T_{A\cap B}(y)\}. \end{aligned}$$

$$\begin{aligned} I_{A\cap B}(xy) &= \min\{I_A(xy), I_B(xy)\} \\ &\geq \min\{\min\{I_A(x), I_A(y)\}, \max\{I_B(x), I_B(y)\}\} \\ &\geq \min\{\min\{I_A(x), I_B(x)\}, \max\{I_A(y), I_B(y)\}\} \\ &\geq \min\{I_{A\cap B}(x), I_{A\cap B}(y)\}. \end{aligned}$$

$$\begin{aligned} F_{A\cap B}(xy) &= \max\{F_A(xy), F_B(xy)\} \\ &\geq \max\{\min\{F_A(x), F_A(y)\}, \min\{F_B(x), F_B(y)\}\} \\ &\geq \min\{\max\{F_A(x), F_A(y)\}, \max\{F_B(x), F_B(y)\}\} \\ &\geq \min\{\max\{F_A(x), F_B(x)\}, \max\{F_A(y), F_B(y)\}\} \\ &\geq \min\{F_{A\cap B}(x), F_{A\cap B}(y)\}. \end{aligned}$$

$$\begin{aligned} (FNSG2): T_{A\cap B}(x^{-1}) &= \min\left\{T_A(x^{-1}), T_B(x^{-1})\right\} \\ &\leq \min\{T_A(x), T_B(x)\} \\ &\leq T_{A\cap B}(x). \end{aligned}$$

$$\begin{aligned} I_{A\cap B}(x^{-1}) &= \min\left\{I_A(x^{-1}), I_B(x^{-1})\right\} \\ &\geq \min\{I_A(x), I_B(x)\} \\ &\geq I_{A\cap B}(x). \end{aligned}$$

$$\begin{aligned} F_{A\cap B}(x^{-1}) &= \max\left\{F_A(x^{-1}), F_B(x^{-1})\right\} \\ &\geq \max\{F_A(x), F_B(x)\} \\ &\geq F_{A\cap B}(x). \end{aligned}$$

□

Theorem 7. *If A and B be fuzzy neutrosophic soft group in X, then A/B also fuzzy neutrosophic soft group in X.*

Proof. Let $x, y \in X$.

Now,

$$\begin{aligned} T_{A/B}(xy) &= \min\{T_A(xy), F_B(xy)\} \\ &\geq \min\{\min\{T_A(x), T_A(y)\}, 1 - F_B^c(xy)\} \\ &= \min\{\min\{T_A(x), T_A(y)\}, 1 - \max\{F_B^c(x), F_B^c(y)\}\} \\ &= \min\{\min\{T_A(x), T_A(y)\}, \min\{F_B(x), F_B(y)\}\} \\ &= \min\{\min\{T_A(x), F_B(x)\}, \min\{T_A(y), F_B(y)\}\} \\ &= \min\left\{T_{A/B}(x), T_{A/B}(y)\right\}. \end{aligned}$$

$$\text{Also} \quad \begin{aligned} I_{A/B}(xy) &= \min\{I_A(xy), 1 - I_B(xy)\} \\ &\leq \min\{\max\{I_A(x), I_A(y)\}, \max\{1 - I_B(y), 1 - I_B(x)\}\} \\ &\leq \max\{\min\{I_A(x), 1 - I_B(x)\}, \min\{I_A(y), 1 - I_B(y)\}\} \\ &\leq \max\left\{I_{A/B}(x), I_{A/B}(y)\right\}. \end{aligned}$$

$$\text{Also} \quad \begin{aligned} F_{A/B}(xy) &= \max\{F_A(xy), T_B(xy)\} \\ &\leq \max\{\max\{F_A(x), F_A(y)\}, 1 - T_B^c(xy)\} \\ &\leq \max\{\max\{F_A(x), F_A(y)\}, \max\{T_B(x), T_B(y)\}\} \\ &\leq \max\{\max\{F_A(x), T_B(x)\}, \max\{F_A(y), T_B(y)\}\} \\ &\leq \max\left\{F_{A/B}(x), F_{A/B}(y)\right\} \end{aligned}$$

Therefore, A/B is also fuzzy neutrosophic soft group in X. □

6. Conclusion and Future work

In this paper, the notion of Fuzzy neutrosophic normal soft subgroups [FNNSG] is introduced and their basic properties are presented. Union, intersection and difference of Fuzzy neutrosophic soft groups are defined. Further we defined cyclic fuzzy neutrosophic normal soft group [CFNNSG] and studied some related properties with supporting proofs. These ideas can be extended to ring and ideal theory.

References

[1] Atanassov K 1986 Intuitionistic fuzzy sets, Fuzzy sets and Systems **20** 87-96.

[2] Arokiarani L, Sumathi J R and Martina J J 2013 Fuzzy Neutrosophic soft topological spaces *IJMA***4**(10) 225-238.

[3] Aygunoglu A and Aygun H 2009 Introduction to fuzzy soft groups *Comp. Math. Appl* **58** 1279-1286.

[4] Aktas H and Cagman N 2007 Soft sets and Soft groups *Inf. Sci* **177**(13) 2726-2735.

[5] Broumi S and Smarandache F 2013 Intuitionstic Neutrosophic Soft Set *Journal of Information and Computing Science***8**(2) 130-140.

[6] Irfan M A, Feng F, Liu X, Min W K and Shabir M 2009 On some new operations in soft set theory *Compt. Math. Appl***57** 1547-1553.

[7] Maji P K Neutrosophic soft set *Annals of fuzzy mathematics and Informatics* **5**(1) ISSN:2093-9310, ISSN:2287-623.

[8] Molodtsov D A 1999 Soft set theory: First results *Comput. Math. Appli***37** 19-31.

[9] Rosenfield A 1971 Fuzzy groups *J. Math. Anal. Appl***35** 512-517.

[10] Smarandache F 1999 A unifying Field in logics *Neutrosophy; Neutrosophic probability set and logic, Rehoboth; American Research press* (1999).

[11] Salama A A and Alblowi S A 2012 Generalized neutrosophic set and Generalized neutrosophic *Topological Spaces, Computer science and Engineering* p-ISSN 2163-1484, e-ISSN 2163-1492, DOI:10.5923/j.computer.20120207.01, **2**(7) 129-132.

[12] Subbiah G, Balakrishnan R and Nagarajan R 2013 Constructions on cyclic M-fuzzy groups family *International Journal fuzzy Mathematics and systems* **3**(4) 295-302.

[13] Salama A A and Alblowi S A 2012 Neutrosophic topological spaces *IOSR Journal of Math.***13**(42) 31-35.

[14] Smarandache F 2005 Neutrosophic set, a generalization of the intuitionistic fuzzy sets *Inter. J. Pure. Appl. Math***24** 287-297.

[15] Sezgin A and Atagun A O 2011 Soft groups and Normalistic soft groups *Computers and Mathematics with Applications* **62** 685-698.

[16] Tursksen 1968 Interval valued fuzzy sets based on normal forms, *Fuzzy Sets and System* **20** 191-210.

[17] Wang H, Smarandache F, Zhang Y Q and Sunderramman R 2010 Single valued neutrosophic sets *Multispace and Multi structures* **4** 410-413.

[18] Yang H L 2011 Notes on Generalized fuzzy soft sets *Journal of Mathematics Research and Exposition* **31**(3)567-570.

[19] Zadeh L A 1965 Fuzzy sets *Information and Control* **8** 338-353.