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Decision-Making Method Based on Grey Relation Analysis and Trapezoidal Fuzzy Neutrosophic Numbers under Double Incomplete Information and Its Application in Typhoon Disaster Assessment

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ABSTRACT Multi-attribute decision-making problems under the trapezoidal fuzzy neutrosophic numbers environment are complex, particularly when the attribute value data are incomplete, and the attribute weight is completely unknown. As a solution, this study proposes a decision-making method based on information entropy and grey theory. First, regarding the problem of attribute value incompleteness in the decision matrix, a method of defining the missing information using trapezoidal fuzzy neutrosophic numbers is proposed, which is simpler and more reasonable than the traditional method of complements. Regarding the problem of completely unknown attribute weights, the definition of new trapezoidal fuzzy neutrosophic entropy is proposed and used to determine the attribute weight. Thereafter, the grey relation analysis method of grey theory is used to rank alternatives and select the best one. Finally, an illustrative example about typhoon disaster assessment is presented to show the feasibility and effectiveness of the proposed method. Finally, the advantages of the proposed method are illustrated by comparison with other methods from multiple aspects.

INDEX TERMS Decision making, double incomplete information, grey relation analysis (GRA), neutrosophic set, trapezoidal fuzzy neutrosophic number (TrFNN), trapezoidal fuzzy neutrosophic entropy, typhoon disaster assessment

I. INTRODUCTION

To deal with the vagueness and uncertainty of many practical problems in real life, Zadeh [1] proposed fuzzy set (FS), which is characterized by a membership degree with range in the unit interval [0, 1]. Atanassov [2] added in the definition of fuzzy set a new component that determines the degree of non-membership and introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the notion of FS. Because of the advantages of intuitionistic fuzzy set compared to fuzzy set, it has been widely studied and applied over the past few decades [3], [4], [5], [6], [7]. Cuong and Kreinovich [8] proposed the concept of picture fuzzy set (PFS) by adding the degree of neutral membership based on the two memberships of IFS. However, some situations that need to be addressed are beyond the capabilities of FS, IFS, and PFS. For example, consider voting results, where thirty percent vote "Yes," twenty percent vote "No," ten percent give up, and forty percent are undecided. Such a vote is beyond the scope of IFS to distinguish the information between "giving up" and "undecided" [9]. However, the neutrosophic set (NS) can handle this complex situation. A neutrosophic number (NN) <0.3, 04, 0.2> can be used to indicate the voting result; that is, the truth-membership degree (T) of the NN is 0.3, the falsity-membership degree (F) is 0.2, and the indeterminacy-membership degree (I) is 0.4. If

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the voting result is expressed as an intuitionistic fuzzy number (IFN), it is <0.3,0.2>; that is, the membership degree μ of the IFN is 0.3, the non-membership degree ν is 0.2, and the hesitancy degree is $\pi=1-\mu-\nu=0.5$, so the IFN cannot express the degree of "undecided." Similarly, if the voting result is expressed as a fuzzy number (FN), it is (0.3); that is, the membership degree of the FN is 0.3 and the non-membership degree is $\nu=1-\mu=0.7$, so FN cannot express the degree of "undecided." Similarly, if the voting result is expressed as a picture fuzzy number (PFN), it is (0.3, 0.1, 0.2); that is, the degree of positive membership μ of the PFN is 0.3, the degree of neutral membership η is 0.1, and the degree of negative membership ν is 0.2. The PFN cannot express the degree of "undecided" from the data form either. However, the degree of refusal membership $1-\mu-\eta-\nu=0.4$ can be obtained, which can express the degree of "undecided," so PFS has certain advantages, but the preferences of PFS and NS are different from the data form. For another example, in the online product review information, there is often an evaluation such as: "I think this dress looks good at first sight, but I don't think it looks good after a long time." If we make simple statistics on the online evaluation information of commodities, the support degree of the statistical results is 0.7, the negation degree is 0.3, and the uncertainty degree is 0.2, which is expressed as <0.7,0.2,0.3> by using the NN. However, 0.7+0.2+0.3 > 1, such information cannot be represented by IFS, FS, and PFS. Therefore, NS is a powerful tool to deal with incomplete, indeterminate, and inconsistent information in the real world, and is the latest theory of fuzzy fields.

The neutrosophic set, pioneered by Smarandache [10], is characterized by a truth-membership degree, an indeterminacy-membership degree, and a falsity-membership degree. It is a generalization of sets, such as crisp sets, fuzzy sets, intuitionistic fuzzy sets, etc. In recent years, this novel concept received attentions from many researches who proceeded to develop, improve, and expand the neutrosophic theory [11], [12]. Wang et al. [13]introduced the single-valued neutrosophic sets (SVNSs) for the practical application of neutrosophic set. Ye [14] defined the simplified neutrosophic sets (SNSs) and proposed a multi-criteria decision-making method using aggregation operators for SNSs. Wang et al. [15] and Yang et al. [16] defined the concept of multi-valued neutrosophic sets (MVNSs). Wang et al. [17] proposed interval neutrosophic sets (INSs). Deli et al. [18] proposed the concept of the bipolar neutrosophic sets (BNSs) and applied it to multi-attribute decision-making (MADM) problems. Pramanik et al. extended the VIKOR strategy [19] and TODIM method [20] to MADM with BNSs environment. Pramanik et al. [21] proposed the bipolar neutrosophic projection-based models and applied it to the MADM problem. Dey et al. [22] proposed an extension of TOPSIS for solving MADM problems under bi-polar a neutrosophic environment. Pramanik et al. [23] proposed interval bipolar neutrosophic sets (IBNSs) and studied the MADM method based on correlation coefficient measures of IBNSs. Deli et al. proposed neutrosophic refined sets (NRSs) [24] and bipolar neutrosophic refined sets (BNRSs) [25] and applied them to medical diagnosis. Mumtaz et al. [26] studied the bipolar neutrosophic soft sets (BNSSs) and their applications in decision making. Tian et al. [27] proposed the concept of the simplified neutrosophic linguistic sets (SNLSs) and applied it to multi-criteria decision-making problems. Biswas et al. [28], Ye [29], and Tan et al. [30] studied the trapezoidal fuzzy neutrosophic sets (TFNSs) and applied them to multiple-attribute decision making. Liang et al. [31] proposed a decision model based on the single-valued trapezoidal neutrosophic numbers (SVTNNs) and decision-making trial and evaluation laboratory (DEMATEL). Ji et al. [32] proposed a decision-making method based on QUALIFLEX-TODIM and SVTNNs and applied to the selection of treatment problems in the medical field. Pramanik et al. and Biswas et al. proposed a variety of decision-making methods based on trapezoidal neutrosophic numbers (TNNs), including extended TODIM strategy-based MADM in TNN environment [33], extended VIKOR-based MAGDM strategy with TNNs [34], extended TOPSIS strategy for MADM with TNNs [35], and the MADM method based on expected value in a TNNs environment [36]. Biswas et al. [37] defined the concept of interval trapezoidal neutrosophic numbers (ITNNs) and proposed an MADM strategy based on the distance measure of ITNNs. Jana et al. [38] proposed two interval trapezoidal neutrosophic aggregation operators and applied them to solve MADM problems. Broumi et al. and Tan et al. [30] combined the neutrosophic sets and graph theory to propose neutrosophic graphs (NGs), including single-valued neutrosophic graphs [39], bipolar single neutrosophic graphs [40], interval-valued neutrosophic graphs, trapezoidal fuzzy neutrosophic graphs [30], and used them for the shortest path solving problem. Liu et al. [41] studied linguistic neutrosophic sets (LNSs) and their application to MAGDM. Chen et al. [42] redefined linguistic neutrosophic number (LNN) operations based on Archimedean copulas and co-copulas to prevent information loss, and then developed a generalized weighted Choquet Heronian mean operator based on new operations. Mondal et al. [43] defined linguistic refined neutrosophic set (LRNS) and proposed a MAGDM based on LRNS strategy. Garg et al. [44] defined the linguistic single-valued neutrosophic prioritized aggregation operators and applied to MAGDM. Wang et al. [45] proposed some 2-tuple linguistic neutrosophic number (2TLNN) Muirhead mean operators and studied their applications to MADM. Broumi et al. [46] extended the TOPSIS method to interval neutrosophic uncertain linguistic numbers (INULNs). Ji et al. [47] proposed a decision model based on probability multivalued neutrosophic linguistic numbers (PMVNLNs) to characterize online reviews, thus providing good services for e-commerce consumers. Ji et al. [48] proposed a combined MABAC-ELECTRE method based on the single-valued neutrosophic linguistic sets (SVNLSs). Abdel-Basset et al. [49] defined the type-2 neutrosophic number set (T2NNS) and applied to developing supplier selection. Throughout the existing literature, there are not many studies on the trapezoidal fuzzy neutrosophic number [28], [29], [30], [38], and other studies on TNNs [34], [35], [36] differ from TFNNs. In addition, in some complicated situations, decision information is more appropriately expressed using TFNNs. For example, in our typhoon disaster assessment study, traffic conditions will be greatly affected after the typhoon disaster. If the assessment of traffic flow after disaster can get the maximum possible range and possible fluctuation range, TrFNNs can be generally used. Therefore, this study investigates a decision making method under the trapezoidal fuzzy neutrosophic number environment.

Incomplete information or missing information or unknown information often occurs in complex decision-making environments. For the research of decision-making method under incomplete information, scholars have done a lot of research from three aspects. The first one is to study a problem where the attribute weight is completely unknown or partially unknown, the second one is to study a problem where the decision-making expert's weight is completely unknown or partially unknown, and the third one is to study a problem where the attribute value information is incomplete, or the attribute value information is missing. Here, the first and third aspects are more important. First, for issues where the attribute weight is unknown or not completely known, Wang et al. [50] constructed the optimized linear programming of alternatives' integrated expected values, and obtained the optimized criteria weight by solving the linear programming. Chen et al. [51] established an objective programming model and solved the model using the Lagrange equation to obtain the weights for the case where the attribute weights are completely unknown. A linear programming model was also established to obtain the attribute weights in the case they are incomplete, in the multi-granular hesitant fuzzy linguistic term environment. Garg [52] proposed a linear programming based on these preferences and an improved score function to solve MADM problems with unknown attribute weights. Han et al. [53] used the deviation entropy weight method to determine the attribute weights based on hybrid multiple attributes containing both the quantitative index and the qualitative index. Liu et al. [54] used the entropy weight method to determine the weights of attributes for the hybrid multiple attribute decision-making under the risk of interval probability with unknown weights. Ye [55] proposed two weight models based on the improved similarity measures to derive the weights of the decision makers and the attributes from the decision matrices represented by the form of single-valued neutrosophic numbers. Tan et al. [56] used the entropy of neutrosophic linguistic sets to determine the attribute weights. Xiong et al. [57] presented a novel and simple nonlinear optimization model to determine the attribute weights by maximizing the total deviation of all attribute values, whether the attribute weights are partly known or completely unknown, with the single valued neutrosophic information. On the other hand, for the problem of incomplete attribute value information, existing research generally uses similarity function, or 0 value, or average value [58], or other methods to complete attribute values, so that the problems become decision-making problems with complete attribute information. However, in the process of supplementary information, when the amount of data itself is large or the amount of missing data is large, the overall amount of calculation will be very large [59]. In brief, there are few researches on the decision-making methods under the condition that the attribute weight information is unknown, and the attribute information is incomplete. In addition, in some complex real-world problems, the acquisition of decision information is sometimes difficult and incomplete. For example, after a typhoon disaster has occurred, we can use a web crawler to access real-time commentary by the local people in order to provide disaster information to the provincial government more quickly than the local government. In this case, some data information may not be available. In a certain city or county, for instance, the younger population, which is more active on social media, might be out to work. The elderly population, on the other hand, is less likely to publish disaster-related information online. Thus, there is little or no information in this area, which leads to incomplete decision matrices. In the case of information incompleteness, it is not appropriate to assume attribute weights; they should be objectively obtained based on the acquired information. Therefore, this paper studies a new decision-making method based on information entropy using a new definition of missing information to solve the double incomplete information problem under an TrFNNs environment.

Grey relation analysis (GRA) is one of the most common analysis methods in the grey system proposed by Deng [60], [61]. GRA is a method to measure the relational degree of factors by using the similarity or difference degree of development trend among factors. The measure of the magnitude of correlation between the factors of two systems that change with time or different objects is called relation degree. In the process of system development, if the trends of the two factors are consistent (i.e., the degree of synchronous change is higher), the degree of correlation between the two factors is higher; on the contrary, the degree of correlation is lower. GRA has been used to solve multi-attribute decision-making (MADM) problems, and has been applied in the fields of economy, marketing, personal selection and agriculture. Throughout existing research, Zhang et al. [62] discussed GRA method for MADM for interval numbers. Wei et al. [63] studied the fuzzy MADM based on GRA method for triangular fuzzy number. Wei [64] proposed the MADM based on GRA method for intuitionistic fuzzy numbers. Fu et al. [65] proposed intuitionistic trapezoidal fuzzy MADM based on GRA method. Liang et al. [66] studied the grey relational analysis method for probabilistic linguistic MADM. Biswas et al. [67] proposed the entropy based GRA method for MADM under single-valued neutrosophic numbers. Mondal et al. [68] studied the rough neutrosophic MADM based on the GRA method. Pramanik et al. [69] studied the MADM based on grey relational analysis for interval neutrosophic numbers. Pramanik et al. [70] extended the GRA strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. However, the method proposed by Pramanik et al. has certain limitations. First, the decision matrix based on their method is complete, but our method considers the incomplete matrix. Second, their attribute weights were directly assumed but using our method they can be determined objectively. Third, their method converted the trapezoidal fuzzy neutrosophic numbers into exact numbers using the scoring function and the exact function, and then decisions were made based on the GRA method. This method prematurely converts the trapezoidal fuzzy wise numbers into exact numbers, resulting in a significant loss of information. However, our method converts the trapezoidal fuzzy neutrosophic numbers to exact numbers by calculating the grey relation coefficient, so the information is well preserved. In general, research on multi-attribute decision-making based on the GRA method in the neutrosophic numbers environment is still rare. In addition, GRA is simple, intuitive and effective. For example, in our typhoon disaster assessment, we can not only estimate the degree of disaster in each aspect of a city based on the positive and negative correlation coefficients, but also the overall degree of disaster of a city based on the closeness coefficient. This assessment method is more comprehensive and useful. Therefore, it is necessary to pay attention to this issue for neutrosophic environments. In this work, we study the MADM based on the GRA method in the trapezoidal fuzzy neutrosophic numbers environment.

Typhoons are powerful, destructive weather systems. They often bring stormy weather, causing huge waves in the sea and seriously threatening navigation safety. The increase in water brought about by a typhoon landing may destroy crops and various construction facilities, causing huge losses to people's lives and property. Typhoons are one of the biggest disasters facing humanity. Their destructive power exceeds that of an earthquake and cannot be avoided. Meteorological disasters such as typhoons account for more than 70% of natural disasters. In China, typhoons primarily impact the eastern coastal regions of the country, where the population is extremely dense, the economy is highly developed, and social wealth is notably concentrated. Fujian is one of the provinces with the most severe typhoon disasters. From 1961 to 2015, 385 typhoons made landfall and affected the Fujian province, with an average of 6.9 landfalls per year, including 1.6 landfalls and 5.3 impacts per year [71]. Once a typhoon comes, it causes huge property and economic losses, casualties, and environmental damage to this area. Therefore, the assessment of typhoon disasters is a very important issue that can provide important decision support for disaster relief and management to relevant departments. However, due to the strong destructive power of nature, the influencing factors of typhoon disasters are extremely hard to describe accurately. Economic losses, for example, include many aspects such as the collapse of buildings, extended damage to housing, the local economic conditions of the affected region, the degree of environmental damage, and the negative impact on society. The assessment information is usually expressed as hesitant, ambiguous, incomplete, inconsistent, and uncertain. Therefore, fuzzy sets (FSs), hesitant fuzzy sets (HFSs), and intuitionistic fuzzy sets (IFSs) have been used in typhoon disaster assessment in recent years. Shi et al. [72] proposed a MCDM hybrid approach to evaluate the damage level of typhoons based on AHP and TOPSIS with fuzzy numbers. He et al. [73] proposed a typhoon disaster assessment method based on Dombi aggregation operators with hesitant fuzzy information. Li et al. [74] proposed a method for evaluating typhoon disasters based on the TOPSIS method with intuitionistic fuzzy numbers. Yu [75] proposed typhoon disaster evaluation based on the generalized intuitionistic fuzzy aggregation operators with intuitionistic fuzzy numbers. Tang et al. [76] studied the nature disaster risk evaluation with a group decision-making method based on incomplete hesitant fuzzy linguistic preference relations. Tan et al. [77] studied the exponential aggregation operator of interval neutrosophic numbers and its application in typhoon disaster evaluation. Throughout the existing literature, there are not many studies on typhoon disaster assessment in the neutrosophic numbers environment. In addition, from the perspective of research issues, typhoon disaster assessment is a complex and uncertain problem. It is very appropriate to use neutrosophic sets to indicate the evaluation information. In addition, the acquisition of post-disaster data is very important for the government's disaster decision-making, but the actual lower-level reporting has a certain delay. In the current situation of network and mobile phone popularization, it is a wise study to obtain data from the network, and the acquisition of network data is sometimes incomplete due to privacy or other reasons. Therefore, in this study, we propose a decision-making method under double incomplete information that uses the information entropy, based on trapezoidal fuzzy neutrosophic numbers to get the attribute weights, and the extended grey relation analysis to sort the alternatives and

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obtain the best one(s). Then, the aforementioned method is applied to typhoon disaster assessment.

The remainder of this paper is organized as follows: Section 2 briefly introduces some basic concepts, including neutrosophic sets, trapezoidal fuzzy neutrosophic numbers, and so on. Section 3 gives the definition of the new distance measure, similarity measure and entropy of TrFNNs. Section 4 gives the processing methods of the double incomplete information in the TrFNNs environment. Section 5 proposes an MADM method based on the grey relation analysis and TrFNNs under double incomplete information. Section 6 uses a typhoon disaster evaluation example to illustrate the applicability of the proposed method and verifies the advantages of the proposed method by comparative analysis from multiple aspects. Finally, the conclusion is given in section 7.

II. PRELIMINARIES

In this section, we briefly outline various essential concepts, such as trapezoidal fuzzy neutrosophic sets, trapezoidal fuzzy neutrosophic numbers (TrFNNs), operational rules of TrFNN, and so on.

Definition 1. [29] Let X be a universe of discourse, then a trapezoidal fuzzy neutrosophic set $\tilde{N}$ in X has the following form:

$$\tilde{N} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\rangle | x \in X\}, \quad (1)$$

where $T_{\tilde{N}}(x) \subset [0,1]$, $I_{\tilde{N}}(x) \subset [0,1]$ and $F_{\tilde{N}}(x) \subset [0,1]$ are three trapezoidal fuzzy numbers,

$T_{\tilde{N}}(x) = \left(t_{\tilde{N}}^1(x), t_{\tilde{N}}^2(x), t_{\tilde{N}}^3(x), t_{\tilde{N}}^4(x)\right): X \to [0,1]$,

$I_{\tilde{N}}(x) = \left(i_{\tilde{N}}^1(x), i_{\tilde{N}}^2(x), i_{\tilde{N}}^3(x), i_{\tilde{N}}^4(x)\right): X \to [0,1]$,

and $F_{\tilde{N}}(x) = \left(f_{\tilde{N}}^1(x), f_{\tilde{N}}^2(x), f_{\tilde{N}}^3(x), f_{\tilde{N}}^4(x)\right): X \to [0,1]$

with the condition

$$0 \le t_{\tilde{N}}^4(x) + i_{\tilde{N}}^4(x) + f_{\tilde{N}}^4(x) \le 3, x \in X.$$

Definition 2. [28] A TrFNN $\tilde{n}$ is denoted by $\tilde{n} = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ in a universe of discourse X. The parameters satisfy the following relations: $a_1 \le a_2 \le a_3 \le a_4$, $b_1 \le b_2 \le b_3 \le b_4$ and $c_1 \le c_2 \le c_3 \le c_4$. Its truth-membership function, indeterminacy-membership function, and falsity-membership function are defined as follows:

$$T_{\tilde{N}}(x) = \left\langle \begin{matrix} \frac{x-a_1}{a_2-a_1} & a_1 \le x \le a_2 \\ 1 & a_2 \le x \le a_3 \\ \frac{a_4-x}{a_4-a_3} & a_3 \le x \le a_4 \\ 0 & otherwise \end{matrix} \right\rangle,$$

$$I_{\tilde{N}}(x) = \left\langle \begin{matrix} \frac{b_2-x}{b_2-b_1} & b_1 \le x \le b_2 \\ 0 & b_2 \le x \le b_3 \\ \frac{x-b_3}{b_4-b_3} & b_3 \le x \le b_4 \\ 1 & otherwise \end{matrix} \right\rangle,$$

$$F_{\tilde{N}}(x) = \left\langle \begin{matrix} \frac{c_2-x}{c_2-c_1} & c_1 \le x < c_2 \\ 0 & c_2 \le x \le c_3 \\ \frac{x-c_3}{c_4-c_3} & c_3 < x \le c_4 \\ 1 & otherwise \end{matrix} \right\rangle.$$

When $a_2 = a_3$, $b_2 = b_3$, and $c_2 = c_3$ in a TrFNN $\tilde{n}$, the number reduces to a triangular fuzzy neutrosop hic number, which is considered a special case of the trapezoidal fuzzy neutrosophic number.

Definition 3. [29] Let $\tilde{n}_1 = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4), (c_1, c_2, c_3, c_4) \rangle$ and $\tilde{n}_2 = \langle (e_1, e_2, e_3, e_4), (f_1, f_2, f_3, f_4), (g_1, g_2, g_3, g_4) \rangle$ be two TrFNNs. Then, the following operational rules apply:

(1) $$\tilde{n}_1 \oplus \tilde{n}_2 = \left\langle \begin{matrix} \left(\begin{matrix} a_1 + e_1 - a_1 e_1, a_2 + e_2 - a_2 e_2, \\ a_3 + e_3 - a_3 e_3, a_4 + e_4 - a_4 e_4 \end{matrix} \right), \\ (b_1 f_1, b_2 f_2, b_3 f_3, b_4 f_4), \\ (c_1 g_1, c_2 g_2, c_3 g_3, c_4 g_4) \end{matrix} \right\rangle; \quad (2)$$

(2) $$\tilde{n}_1 \otimes \tilde{n}_2 = \left\langle \begin{matrix} (a_1 e_1, a_2 e_2, a_3 e_3, a_4 e_4), \\ \left(\begin{matrix} b_1 + f_1 - b_1 f_1, b_2 + f_2 - b_2 f_2, \\ b_3 + f_3 - b_3 f_3, b_4 + f_4 - b_4 f_4 \end{matrix} \right), \\ \left(\begin{matrix} c_1 + g_1 - c_1 g_1, c_2 + g_2 - c_2 g_2, \\ c_3 + g_3 - c_3 g_3, c_4 + g_4 - c_4 g_4 \end{matrix} \right) \end{matrix} \right\rangle; \quad (3)$$

(3) $$\lambda \tilde{n}_1 = \left\langle \begin{matrix} \left(\begin{matrix} 1-(1-a_1)^\lambda, 1-(1-a_2)^\lambda, \\ 1-(1-a_3)^\lambda, 1-(1-a_4)^\lambda \end{matrix} \right), \\ (b_1^\lambda, b_2^\lambda, b_3^\lambda, b_4^\lambda), \\ (c_1^\lambda, c_2^\lambda, c_3^\lambda, c_4^\lambda) \end{matrix} \right\rangle, \lambda > 0; \quad (4)$$

(4) $$\tilde{n}_1^{\lambda}=\left\langle \begin{array}{l} \left(a_1^{\lambda},a_2^{\lambda},a_3^{\lambda},a_4^{\lambda}\right), \\ \left(\begin{array}{l} 1-(1-b_1)^{\lambda},1-(1-b_2)^{\lambda}, \\ 1-(1-b_3)^{\lambda},1-(1-b_4)^{\lambda} \end{array}\right), \\ \left(\begin{array}{l} 1-(1-c_1)^{\lambda},1-(1-c_2)^{\lambda}, \\ 1-(1-c_3)^{\lambda},1-(1-c_4)^{\lambda} \end{array}\right) \end{array}\right\rangle,\lambda>0; \quad (5)$$

(5) $\tilde{n}_1=\tilde{n}_2$ if $a_i=e_i$, $b_i=f_i$ and $c_i=g_i$ hold for $i=1,2,3,4$ i.e., $(a_1,a_2,a_3,a_4)=(e_1,e_2,e_3,e_4)$, $(b_1,b_2,b_3,b_4)=(f_1,f_2,f_3,f_4)$, $(c_1,c_2,c_3,c_4)=(g_1,g_2,g_3,g_4)$.

Definition 4. [29] Let $\tilde{n}=\left\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\right\rangle$ be a TrFNN. Then, the score function of a trapezoidal neutrosophic number can be defined as

$$sc(\tilde{n})=\frac{1}{3}\left(2+\frac{\sum_{i=1}^{4}a_i}{4}-\frac{\sum_{i=1}^{4}b_i}{4}-\frac{\sum_{i=1}^{4}c_i}{4}\right), \quad (6)$$

$$sc(\tilde{n})\in[0,1].$$

where the larger the value of $sc(\tilde{n})$, the bigger the trapezoidal fuzzy neutrosophic number $\tilde{n}$.

III. NEW DISTANCE MEASURE, SIMILARITY MEASURE, AND ENTROPY OF TrFNNs

A. NEW DISTANCE MEASURE AND SIMILARITY MEASURE OF TrFNNs

Definition 5. Let $\tilde{n}_1=\left\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\right\rangle$ and $\tilde{n}_2=\left\langle(e_1,e_2,e_3,e_4),(f_1,f_2,f_3,f_4),(g_1,g_2,g_3,g_4)\right\rangle$ be two TrFNNs. Then, the distance measure $D(\tilde{n}_1,\tilde{n}_2)$ between $\tilde{n}_1$ and $\tilde{n}_2$ is defined as follows:

$$D(\tilde{n}_1,\tilde{n}_2)=\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|^{\lambda}+\sum_{i=1}^{4}|b_i-f_i|^{\lambda}+\sum_{i=1}^{4}|c_i-g_i|^{\lambda}\right)\right\}^{1/\lambda}, \quad (7)$$

$\lambda\geq 0$.

If $\lambda=1$, then the distance formula (7) is reduced to the following Hamming distance:

$$D_H(\tilde{n}_1,\tilde{n}_2)=\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|+\sum_{i=1}^{4}|b_i-f_i|+\sum_{i=1}^{4}|c_i-g_i|\right). \quad (8)$$

If $\lambda=2$, then the distance formula (7) is reduced to the following Euclidean distance:

$$D_E(\tilde{n}_1,\tilde{n}_2)=\sqrt{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|^2+\sum_{i=1}^{4}|b_i-f_i|^2+\sum_{i=1}^{4}|c_i-g_i|^2\right)} \quad (9)$$

Theorem 1. The above defined distance $D(\tilde{n}_1,\tilde{n}_2)$ satisfies the following properties:

(P1) $0\leq D(\tilde{n}_1,\tilde{n}_2)\leq 1$;

(P2) $D(\tilde{n}_1,\tilde{n}_2)=0$ if and only if $\tilde{n}_1=\tilde{n}_2$;

(P3) $D(\tilde{n}_1,\tilde{n}_2)=D(\tilde{n}_2,\tilde{n}_1)$;

(P4) If $\tilde{n}_1\leq\tilde{n}_2\leq\tilde{n}_3$, then $D(\tilde{n}_1,\tilde{n}_2)\leq D(\tilde{n}_1,\tilde{n}_3)$ and $D(\tilde{n}_2,\tilde{n}_3)\leq D(\tilde{n}_1,\tilde{n}_3)$.

The proof process of Theorem 1 can be found in the Appendix.

Since $D_H(\tilde{n}_1,\tilde{n}_2)$ and $D_E(\tilde{n}_1,\tilde{n}_2)$ are special cases of $D(\tilde{n}_1,\tilde{n}_2)$, they also satisfy Theorem 1.

According to the relationship between distance and similarity, the similarity measure of the trapezoidal fuzzy neutrosophic numbers is:

$$S(\tilde{n}_1,\tilde{n}_2)=1-D(\tilde{n}_1,\tilde{n}_2)=1-\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|^{\lambda}+\sum_{i=1}^{4}|b_i-f_i|^{\lambda}+\sum_{i=1}^{4}|c_i-g_i|^{\lambda}\right)\right\}^{1/\lambda}. \quad (10)$$

If $\lambda=1$, then the similarity measure based on the Hamming distance is as follows:

$$S_H(\tilde{n}_1,\tilde{n}_2)=1-D_H(\tilde{n}_1,\tilde{n}_2)=1-\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|+\sum_{i=1}^{4}|b_i-f_i|+\sum_{i=1}^{4}|c_i-g_i|\right). \quad (11)$$

If $\lambda=2$, then the similarity measure based on Euclidean distance is as follows:

$$S_E(\tilde{n}_1,\tilde{n}_2)=1-D_E(\tilde{n}_1,\tilde{n}_2)=1-\sqrt{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|^2+\sum_{i=1}^{4}|b_i-f_i|^2+\sum_{i=1}^{4}|c_i-g_i|^2\right)}. \quad (12)$$

Theorem 2. The similarity measures above obviously satisfy the following properties:

(P1) $0\leq S(\tilde{n}_1,\tilde{n}_2)\leq 1$;

(P2) $S(\tilde{n}_1,\tilde{n}_2)=S(\tilde{n}_2,\tilde{n}_1)$;

(P3) $S(\tilde{n}_1,\tilde{n}_2)=1$ for $\tilde{n}_1=\tilde{n}_2$, i.e., $a_i=e_i$, $b_i=f_i$, $c_i=g_i$, for $i=1,2,3,4$.

B. NEW ENTROPY OF TrFNNs

Entropy represents the uncertainty of the attribute information. The greater the entropy value, the greater the uncertainty of the information. Inspired by the literature [78], this paper proposes the extended entropy measures of TrFNN and gives the following definition of entropy for TrFNN.

Definition 6. A real function $E_{TrFNN}:TrFNN \to [0,1]$ is called an entropy measure for an TrFNN, and $\tilde{n}=\langle(a_1,a_2,a_3,a_4),(b_1,b_2,b_3,b_4),(c_1,c_2,c_3,c_4)\rangle$ is an TrFNN. Then the entropy measure is $E_{TrFNN}(\tilde{n})=1\text{-}2D(\tilde{n},\tilde{n}')$, and $\tilde{n}'=\langle(0.5,0.5,0.5,0.5),(0.5,0.5,0.5,0.5),(0.5,0.5,0.5,0.5)\rangle$.

Then, according to the distance measure, the entropy becomes:

$$E_{TrFNN}(\tilde{n})=1-2D(\tilde{n},\tilde{n}')$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|^{\lambda}+\sum_{i=1}^{4}|b_i-0.5|^{\lambda}+\sum_{i=1}^{4}|c_i-0.5|^{\lambda}\right)\right\}^{1/\lambda}. \quad (13)$$

Theorem 3. The above defined entropy $E_{TrFNN}(\tilde{n})$ satisfies the following properties:

(P1) $E_{TrFNN}(\tilde{n})=0$ if $\tilde{n}$ is a crisp number;

(P2) $E_{TrFNN}(\tilde{n})=1$, if and only if
$$\tilde{n}=\tilde{n}'=\left\langle\begin{matrix}(0.5,0.5,0.5,0.5),(0.5,0.5,0.5,0.5),\\(0.5,0.5,0.5,0.5)\end{matrix}\right\rangle;$$

(P3) If $D(\tilde{n}_1,\tilde{n}')\geq D(\tilde{n}_2,\tilde{n}')$, then $E_{TrFNN}(\tilde{n}_1)\leq E_{TrFNN}(\tilde{n}_2)$ for $\tilde{n}_1$, $\tilde{n}_2$ are TrFNNs, where D is the distance of TrFNNs;

(P4) $E_{TrFNN}(\tilde{n})=E_{TrFNN}(\tilde{n}^c)$, where $\tilde{n}^c$ is the complement of $\tilde{n}$.

At the same time, according to the relationships $S(\tilde{n}_1,\tilde{n}_2)=1-D(\tilde{n}_1,\tilde{n}_2)$ and $E_{TrFNN}(\tilde{n})=1\text{-}2D(\tilde{n},\tilde{n}')$, we can get $E_{TrFNN}(\tilde{n})=2S(\tilde{n},\tilde{n}')-1$.

When $\lambda=1$, the Hamming distance is

$$D(\tilde{n},\tilde{n}')=\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|+\sum_{i=1}^{4}|b_i-0.5|+\sum_{i=1}^{4}|c_i-0.5|\right)$$

and the new entropy is

$$E_{TrFNN}(\tilde{n})$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|+\sum_{i=1}^{4}|b_i-0.5|+\sum_{i=1}^{4}|c_i-0.5|\right)\right\}. \quad (14)$$

When $\lambda=2$, the Euclidean distance is

$$D(\tilde{n},\tilde{n}')$$
$$=\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|^2+\sum_{i=1}^{4}|b_i-0.5|^2+\sum_{i=1}^{4}|c_i-0.5|^2\right)\right\}^{1/2}$$

and the new entropy is

$$E_{TrFNN}(\tilde{n})$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|^2+\sum_{i=1}^{4}|b_i-0.5|^2+\sum_{i=1}^{4}|c_i-0.5|^2\right)\right\}^{1/2}. \quad (15)$$

IV. PROCESSING METHOD OF THE DOUBLE INCOMPLETE INFORMATION IN THE TrFNNs ENVIRONMENT

A. DETERMINATION OF ATTRIBUTE WEIGHT-BASED ENTROPY

For the decision problem with unknown attribute weight, this study proposes a method based on TrFNN entropy to determine attribute weight.

In the trapezoidal neutrosophic number decision matrix $R(\tilde{n}_{ij})$ (where $\tilde{n}_{ij}$ represents the j-th attribute of the i-th scheme), for any attribute value, its trapezoidal fuzzy neutrosophic entropy can be calculated according to the definition of the above formula for $E_{TrFNN}(\tilde{n}_{ij})$. Entropy represents the uncertainty of the attribute information. The greater the entropy, the greater the uncertainty. Therefore, the attribute weight can be calculated by the following formula:

$$\omega_j=\frac{1-\frac{1}{m}\sum_{i=1}^{m}E_{TrFNN}(\tilde{n}_{ij})}{\sum_{j=1}^{n}\left(1-\frac{1}{m}\sum_{i=1}^{m}E_{TrFNN}(\tilde{n}_{ij})\right)}. \quad (16)$$

B. DETERMINATION OF THE MISSING ATTRIBUTE VALUE

For the solution of the attribute value information defect, the existing research mainly supplements the information by the complement method and then makes the decision. The shortcomings of this method are the computational complexity and misinterpretation of information representation. When the attribute value is missing, it means that the data are not acquired or is lost during the collection process, and the information available for decision making is empty. The value of this attribute is completely unknown, so its membership degree and non-membership degree should be 0, and the unknown degree is 1. Corresponding to TrFNN, fill truth-membership and falsity-membership to zero, indeterminacy-membership is 1. If the truth-membership and falsity-membership are added to a non-zero number, then the indeterminacy-membership is not 1, indicating that the attribute value information is not completely unknown. This does not match the meaning of the missing attribute value. Therefore, inspired by the literature [79], the representation of the missing attribute value information is given by us.

Definition 7. When the attribute value expressed as TrFNN is missing, the missing information is assigned $\langle (0,0,0,0),(1,1,1,1),(0,0,0,0)\rangle$; that is, the membership degree $T=(0,0,0,0)$, the non-membership $F=(0,0,0,0)$, and the indeterminacy-membership $I=(1,1,1,1)$.

It is not only simple and convenient to use Definition 7 to deal with the missing attribute values, but also to retain the feature of missing information, which is more reasonable.

V. MULTI-ATTRIBUTE DECISION-MAKING METHOD BASED ON GRA AND TrFNNs UNDER DOUBLE INCOMPLETE INFORMATION

Trapezoidal fuzzy neutrosophic numbers cannot only represent the range of values of the attribute, but also the most likely range of values of the attribute. In this work, we study multi-attribute decision making in the TrFNNs environment. To achieve our objectives, the proposed method consists of the following several steps, depicted graphically in Figure 1.

Consider a multi-attribute decision problem, let $X=\{x_1,x_2,\cdots x_m\}$ be a set of m alternatives, and $C=\{c_1,c_2,\cdots c_n\}$ a set of n attributes. $\omega=(\omega_1,\omega_2,\cdots,\omega_n)^T$ is the potential weighting vector of the attributes, where $\omega_j\in[0,1](j=1,2,\cdots,n)$, and $\sum_{j=1}^{n}\omega_j=1$. If the decision makers provide a trapezoidal fuzzy neutrosophic number to evaluate the alternative $x_i\,(i=1,2,\cdots,m)$ under the attribute $c_j\,(j=1,2,\cdots,n)$, it can be characterized by a trapezoidal fuzzy neutrosophic number

$\tilde{n}_{ij}=\langle (a_1^{ij},a_2^{ij},a_3^{ij},a_4^{ij}),(b_1^{ij},b_2^{ij},b_3^{ij},b_4^{ij}),(c_1^{ij},c_2^{ij},c_3^{ij},c_4^{ij})\rangle$, $(i=1,2,\cdots,m;\ j=1,2,\cdots,n)$. Suppose that $R=(\tilde{n}_{ij})_{m\times n}$ is the decision matrix, where $\tilde{n}_{ij}$ represents TrFNN. The current decision-making environment is more complicated when ω_j is unknown and the information in the $R=(\tilde{n}_{ij})_{m\times n}$ matrix is incomplete, so we study the MADM method in the case of double incomplete information. In the following, we propose a new MADM method based on GRA and TrFNNs. The steps are as follows:

Step 1: Give the decision matrix $R=(\tilde{n}_{ij})_{m\times n}$ provided by decision makers in the form of linguistic terms expressed easily by people, and then convert them into trapezoidal fuzzy neutrosophic numbers.

Step 2: Obtain the complete decision matrix $\bar{R}=(\bar{\tilde{n}}_{ij})_{m\times n}$ by supplementing the incomplete decision matrix with incomplete information definition based on Definition 7 with TrFNN form.

Step 3: Obtain the normalized decision matrix $\bar{R}'=(\bar{\tilde{n}}_{ij}')_{m\times n}$. We need to standardize the decision information to ensure consistency of information. In general, attributes can be categorized into two types: benefit attributes and cost attributes. In this study, the normalization of benefit-type attribute values and cost-type attribute values is as follows:

$$\bar{\tilde{n}}_{ij}^{b}=\left\langle \begin{array}{l}\left(\frac{a_1^{ij}}{a_{4\max}},\frac{a_2^{ij}}{a_{4\max}},\frac{a_3^{ij}}{a_{4\max}},\frac{a_4^{ij}}{a_{4\max}}\right),\left(\frac{b_1^{ij}}{b_{4\max}},\frac{b_2^{ij}}{b_{4\max}},\frac{b_3^{ij}}{b_{4\max}},\frac{b_4^{ij}}{b_{4\max}}\right),\\ \left(\frac{c_1^{ij}}{c_{4\max}},\frac{c_2^{ij}}{c_{4\max}},\frac{c_3^{ij}}{c_{4\max}},\frac{c_4^{ij}}{c_{4\max}}\right)\end{array}\right\rangle,$$

$$\bar{\tilde{n}}_{ij}^{c}=\left\langle \begin{array}{l}\left(1-\frac{a_4^{ij}}{a_{4\max}},1-\frac{a_3^{ij}}{a_{4\max}},1-\frac{a_2^{ij}}{a_{4\max}},1-\frac{a_1^{ij}}{a_{4\max}}\right),\\ \left(1-\frac{b_4^{ij}}{b_{4\max}},1-\frac{b_3^{ij}}{b_{4\max}},1-\frac{b_2^{ij}}{b_{4\max}},1-\frac{b_1^{ij}}{b_{4\max}}\right),\\ \left(1-\frac{c_4^{ij}}{c_{4\max}},1-\frac{c_3^{ij}}{c_{4\max}},1-\frac{c_2^{ij}}{c_{4\max}},1-\frac{c_1^{ij}}{c_{4\max}}\right)\end{array}\right\rangle.$$

where $\bar{\tilde{n}}_{ij}^{b}$ represents the benefit-type attribute, $\bar{\tilde{n}}_{ij}^{c}$ represents the cost-type attribute, and $a_{4\max}=\max\{\max_{0\le i\le m}a_{4ij},\max_{0\le j\le n}a_{4ij}\}$, $b_{4\max}=\max\{\max_{0\le i\le m}b_{4ij},\max_{0\le j\le n}b_{4ij}\}$, $c_{4\max}=\max\{\max_{0\le i\le m}c_{4ij},\max_{0\le j\le n}c_{4ij}\}$.

Step 4: Determine the attribute weights using the trapezoidal fuzzy neutrosophic entropy based on Formula (13) and Formula (16).

Step 5: Calculate the weighted decision matrix $R'=(n_{ij})_{m\times n}=(\omega\bar{\tilde{n}}_{ij})_{m\times n}$ according to the operational rules of Definition 3, where

$$\omega\bar{\tilde{n}}_1=\left\langle \begin{array}{l}\left(\begin{array}{l}1-(1-a_1)^{\omega},1-(1-a_2)^{\omega},\\ 1-(1-a_3)^{\omega},1-(1-a_4)^{\omega}\end{array}\right),\\ (b_1^{\omega},b_2^{\omega},b_3^{\omega},b_4^{\omega}),\\ (c_1^{\omega},c_2^{\omega},c_3^{\omega},c_4^{\omega})\end{array}\right\rangle,\omega>0; \quad (17)$$

Step 6: Calculate the correlation coefficient matrix. Firstly, the positive ideal solution n_j^+ and the negative ideal solution n_j^- are determined, then the positive

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correlation coefficient matrix $\eta^{+}=\left(\xi_{ij}^{+}\right)_{m\times n}$ between the decision matrix R' and the positive ideal solution, and the negative correlation coefficient matrix $\eta^{-}=\left(\xi_{ij}^{-}\right)_{m\times n}$ between the decision matrix R' and the negative ideal solution are obtained using the grey correlation analysis method, where

$$\xi_{ij}^{\pm}=\frac{\min_i\min_j\Delta_{ij}^{\pm}+\rho\cdot\max_i\max_j\Delta_{ij}^{\pm}}{\Delta_{ij}^{\pm}+\rho\cdot\max_i\max_j\Delta_{ij}^{\pm}}. \quad (18)$$

Here $\Delta_{ij}^{\pm}=D\left(n_{ij}-n_j^{\pm}\right)\left(i=1,2,\cdots,m,\ j=1,2,\cdots,n\right)$ is the distance measure calculated by Formula (7). Here, $\rho\in[0,1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment and to control level of differences of the relation coefficients. When $\rho=1$, the comparison environment is unaltered; when $\rho=0$, the comparison environment disappears. Smaller values of the distinguishing coefficient will yield a large range of values for the grey relational coefficient. Generally, $\rho=0.5$ is considered for decision-making situations.

Step 7: Obtain the closeness coefficient U_i of each alternative A_i to the ideal solution, where

$$U_i=\frac{\frac{1}{n}\sum_{j=1}^{n}\xi_{ij}^{+}}{\frac{1}{n}\sum_{j=1}^{n}\xi_{ij}^{+}+\frac{1}{n}\sum_{j=1}^{n}\xi_{ij}^{-}}. \quad (19)$$

Step 8: Rank the alternatives, according to the closeness coefficients above; we can choose best alternative(s) or rank alternatives. In general, the greater the value of U_i, the better the alternative.

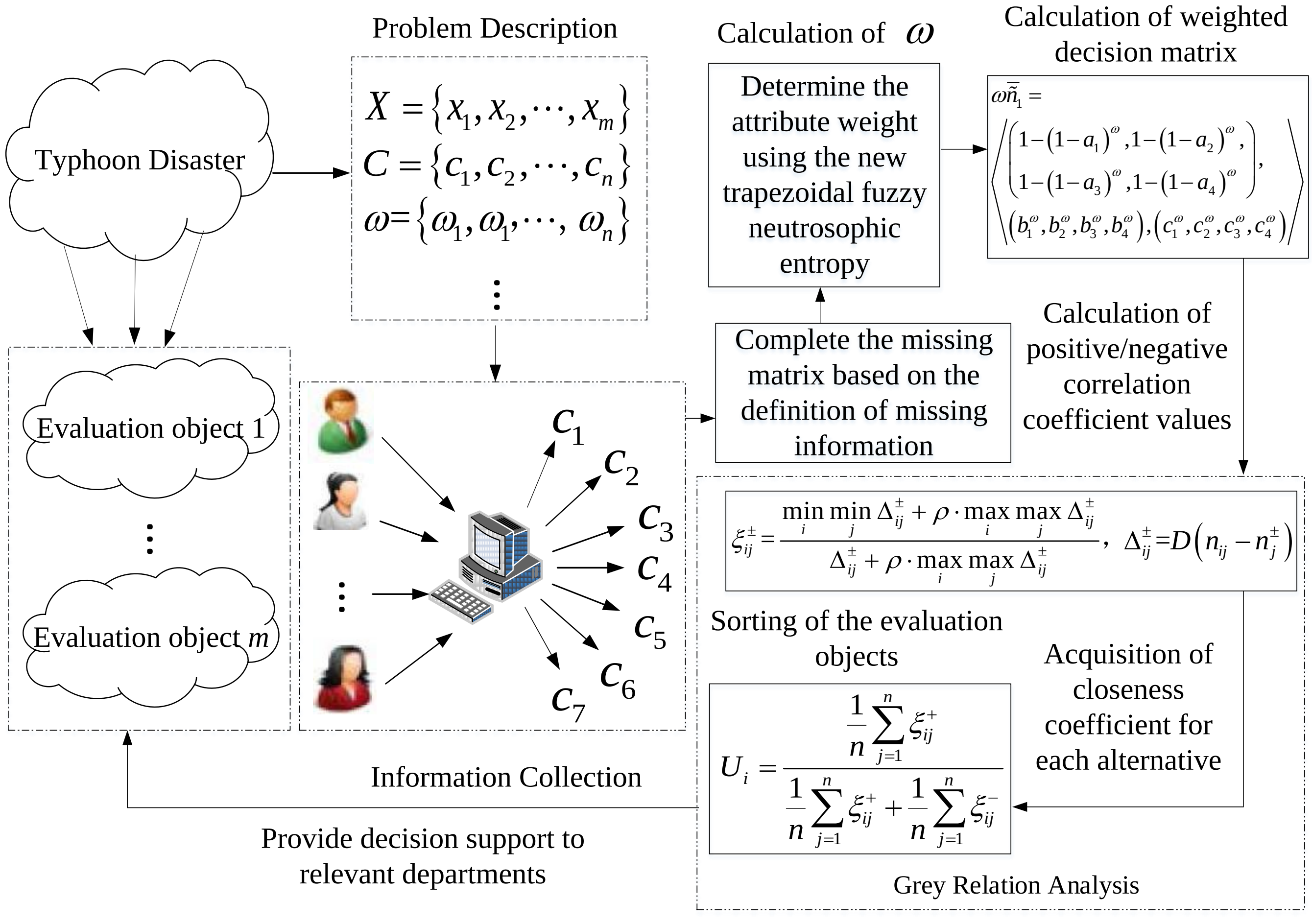

FIGURE 1. Framework of the proposed MADM based on double incomplete information and GRA

VI. CASE STUDY AND COMPARATIVE ANALYSIS

A. CASE STUDY

The Fujian province is located in the southeast coastal area of China and is one of the provinces with the most severe typhoon disasters. Typhoons bring many disasters, such as strong wind, heavy rain, storm surge, and huge waves,

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which often cause heavy casualties and huge economic losses. The typhoon landing in Fujian has a complex track, great changes in structure and intensity, strong rainfall intensity, and great destructive force of the coastal wind. It is the most important meteorological disaster in the Fujian province [71]. Therefore, we evaluate the typhoon disasters in Fujian province as a case study. After the disaster, we quickly obtain fragmented data from multiple sources for disaster assessment, which provides decision support for disaster relief in relevant departments. We constructed an evaluation indicator system based on literature [75], [80]. Yu [75] established the evaluation index system in the study of typhoon disaster assessment in the Zhejiang province, including four aspects: economic loss, social influence, environmental impact, and other impact. Shi et al. [80] studied typhoon disasters in 10 cities in China and constructed a two-level evaluation index system, including: affected population (affect, death, transaction), agricultural disaster (affected, disaster, damage), housing loss (collapse, damage), economic loss. Based on the actual situation of the Fujian province and the above indicators, we constructed the evaluation index system of this study. The assessment indicators $C=\{c_1,c_2,c_3,c_4,c_5,c_6,c_7\}$ include population death (c_1), population affected (c_2), agricultural damage (c_3), economic loss (c_4), environmental impact (c_5), social impact (c_6), and other impact (c_7). The nine assessment objects in $a_i (i=1,2,\cdots,9)$ are to be evaluated using the trapezoidal fuzzy neutrosophic numbers by decision makers or experts under the seven attributes of $c_j (j=1,2,\cdots,7)$. Therefore, the assessment matrix $R=(\tilde{n}_{ij})_{m\times n}$ is given in the form of TrFNNs. According to Section 5, typhoon disaster assessment using the MCDM model contains the following steps:

Step 1: Obtain evaluation data. First, we invite several experts to give the linguistic values of trapezoidal fuzzy neutrosophic numbers for linguistic terms shown in Table 1. Then, the incomplete decision matrix R provided by the decision makers would be as shown in Table 2.

TABLE I
CORRESPONDENCE BETWEEN LINGUISTIC TERMS AND TRFNNS

Linguistic terms	Trapezoidal fuzzy neutrosophic numbers
Absolutely low	$\langle(0.0,0.0,0.0,0.0),(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)\rangle$
Low	$\langle(0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$
Fairly low	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$
Medium	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$
Fairly high	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$
High	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$
Absolutely high	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$

TABLE 2
EVALUATION MATRIX R

Index / Cities	c_1	c_2	c_3	c_4
Nanping (NP)	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	×	$\langle(0.0,0.0,0.0,0.0),(1.0,1.0,1.0,1.0),(1.0,1.0,1.0,1.0)\rangle$
Ningde (ND)	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$	×	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$
Sanming (SM)	×	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	×

Fuzhou (FZ)	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$
Putian (PT)	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$
Longyan (LY)	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	×	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$
Quanzhou (QZ)	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	×	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$
Xiamen (XM)	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$	×
Zhangzhou (ZZ)	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$

Index / Cities	c_5	c_6	c_7
Nanping (NP)	×	×	$\langle(0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9)\rangle$
Ningde (ND)	$\langle(1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$
Sanming (SM)	×	$\langle(0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$	$\langle(0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0)\rangle$
Fuzhou (FZ)	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$	$\langle(0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7)\rangle$	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$
Putian (PT)	$\langle(0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$	$\langle(0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4)\rangle$

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Longyan (LY)	$\left\langle (0.3,0.4,0.5,0.6), (0.2,0.4,0.5,0.7), (0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\times$
Quanzhou (QZ)	$\times$	$\left\langle (0.3,0.4,0.5,0.6), (0.2,0.4,0.5,0.7), (0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$
Xiamen (XM)	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$
Zhangzhou (ZZ)	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$	$\times$	$\left\langle (0.7,0.8,0.9,1.0), (0.0,0.1,0.2,0.3), (0.0,0.1,0.2,0.3) \right\rangle$

Step 2: Complete the incomplete decision matrix based on Definition 7. The data are summarized in Table 3.

TABLE 3
COMPLETE EVALUATION MATRIX $\bar{R}$

Index / Cities	c_1	c_2	c_3	c_4
Nanping (NP)	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0,0,0,0), (1,1,1,1), (0,0,0,0) \right\rangle$	$\left\langle (0.0,0.0,0.0,0.0), (1.0,1.0,1.0,1.0), (1.0,1.0,1.0,1.0) \right\rangle$
Ningde (ND)	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$	$\left\langle (0,0,0,0), (1,1,1,1), (0,0,0,0) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0), (0.0,0.1,0.2,0.3), (0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$
Sanming (SM)	$\left\langle (0,0,0,0), (1,1,1,1), (0,0,0,0) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.0,0.1,0.2,0.3), (0.7,0.8,0.9,1.0), (0.7,0.8,0.9,1.0) \right\rangle$	$\left\langle (0,0,0,0), (1,1,1,1), (0,0,0,0) \right\rangle$
Fuzhou (FZ)	$\left\langle (0.7,0.8,0.9,1.0), (0.0,0.1,0.2,0.3), (0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (0.5,0.6,0.7,0.8), (0.0,0.2,0.3,0.4), (0.0,0.2,0.3,0.4) \right\rangle$	$\left\langle (0.5,0.6,0.7,0.8), (0.0,0.2,0.3,0.4), (0.0,0.2,0.3,0.4) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0), (0.0,0.1,0.2,0.3), (0.0,0.1,0.2,0.3) \right\rangle$
Putian (PT)	$\left\langle (0.3,0.4,0.5,0.6), (0.2,0.4,0.5,0.7), (0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0), (0.0,0.0,0.0,0.0), (0.0,0.0,0.0,0.0) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0), (0.0,0.1,0.2,0.3), (0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (0.5,0.6,0.7,0.8), (0.0,0.2,0.3,0.4), (0.0,0.2,0.3,0.4) \right\rangle$
Longyan (LY)	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0,0,0,0), (1,1,1,1), (0,0,0,0) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4), (0.4,0.6,0.7,0.9), (0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.3,0.4,0.5,0.6), (0.2,0.4,0.5,0.7), (0.2,0.4,0.5,0.7) \right\rangle$

Quanzhou (QZ)	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$
Xiamen (XM)	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$
Zhangzhou (ZZ)	$\left\langle (0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \right\rangle$

Index / Cities	c_5	c_6	c_7
Nanping (NP)	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$
Ningde (ND)	$\left\langle (1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$
Sanming (SM)	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$	$\left\langle (0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0) \right\rangle$	$\left\langle (0.0,0.1,0.2,0.3),(0.7,0.8,0.9,1.0),(0.7,0.8,0.9,1.0) \right\rangle$
Fuzhou (FZ)	$\left\langle (0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4) \right\rangle$	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \right\rangle$
Putian (PT)	$\left\langle (0.7,0.8,0.9,1.0),(0.0,0.1,0.2,0.3),(0.0,0.1,0.2,0.3) \right\rangle$	$\left\langle (0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4) \right\rangle$	$\left\langle (0.5,0.6,0.7,0.8),(0.0,0.2,0.3,0.4),(0.0,0.2,0.3,0.4) \right\rangle$
Longyan (LY)	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$
Quanzhou (QZ)	$\left\langle (0,0,0,0),(1,1,1,1),(0,0,0,0) \right\rangle$	$\left\langle (0.3,0.4,0.5,0.6),(0.2,0.4,0.5,0.7),(0.2,0.4,0.5,0.7) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0) \right\rangle$
Xiamen (XM)	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (0.1,0.2,0.3,0.4),(0.4,0.6,0.7,0.9),(0.4,0.6,0.7,0.9) \right\rangle$	$\left\langle (1.0,1.0,1.0,1.0),(0.0,0.0,0.0,0.0),(0.0,0.0,0.0,0.0) \right\rangle$

Zhangzhou (ZZ)	$\left\langle \begin{array}{l}(1.0,1.0,1.0,1.0),\\(0.0,0.0,0.0,0.0),\\(0.0,0.0,0.0,0.0)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0,0,0,0),\\(1,1,1,1),\\(0,0,0,0)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.7,0.8,0.9,1.0),\\(0.0,0.1,0.2,0.3),\\(0.0,0.1,0.2,0.3)\end{array}\right\rangle$

Step 3: Obtain the normalized evaluation matrix $\overline{R}$. Because all the attributes in this article are of the same type, they do not need to be standardized.

Step 4: Determine the attribute weights. First, the entropy value of evaluation information is calculated according to Formula (14), and the data are summarized in Table 4.

TABLE 4
ENTROPY VALUE MATRIX

Cities \ Index	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Nanping (NP)	0.5667	0.5667	0.0000	0.0000	0.0000	0.0000	0.5667
Ningde (ND)	0.0000	0.0000	0.3000	0.0000	0.0000	0.3000	0.7333
Sanming (SM)	0.0000	0.5667	0.3000	0.0000	0.0000	0.3000	0.3000
Fuzhou (FZ)	0.3000	0.5333	0.5333	0.3000	0.5333	0.7333	0.3000
Putian (PT)	0.7333	0.0000	0.3000	0.5333	0.3000	0.5333	0.5333
Longyan (LY)	0.5667	0.0000	0.5667	0.7333	0.7333	0.5667	0.0000
Quanzhou (QZ)	0.7333	0.5667	0.0000	0.5667	0.0000	0.7333	0.0000
Xiamen (XM)	0.5667	0.5667	0.7333	0.0000	0.5667	0.5667	0.0000
Zhangzhou (ZZ)	0.5333	0.3000	0.0000	0.3000	0.0000	0.0000	0.3000

Then, determine the attribute weights according to formula (16). The attribute weights are as follows: $\omega_1 = 0.1917$, $\omega_2 = 0.1486$, $\omega_3 = 0.1310$, $\omega_4 = 0.1166$, $\omega_5 = 0.1022$, $\omega_6 = 0.1789$, $\omega_7 = 0.1310$.

Step 5: Calculate the weighted decision matrix R' according to Formula (17), and the data are summarized in Table 5.

TABLE 5
WEIGHTED DECISION MATRIX R'

Cities \ Index	c_1	c_2	c_3	c_4
Nanping (NP)	$\left\langle \begin{array}{l}(0.020,0.042,0.066,0.093),\\(0.839,0.907,0.934,0.980),\\(0.839,0.907,0.934,0.980)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.016,0.033,0.052,0.073),\\(0.873,0.927,0.948,0.984),\\(0.873,0.927,0.948,0.984)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(1.000,1.000,1.000,1.000)\end{array}\right\rangle$
Ningde (ND)	$\left\langle \begin{array}{l}(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.146,0.190,0.260,1.000),\\(0.000,0.740,0.810,0.854),\\(0.000,0.740,0.810,0.854)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$
Sanming (SM)	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.016,0.033,0.052,0.073),\\(0.873,0.927,0.948,0.984),\\(0.873,0.927,0.948,0.984)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.014,0.029,0.046),\\(0.954,0.971,0.986,1.000),\\(0.954,0.971,0.986,1.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$
Fuzhou (FZ)	$\left\langle \begin{array}{l}(0.206,0.265,0.357,1.000),\\(0.000,0.643,0.735,0.794),\\(0.000,0.643,0.735,0.794)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.098,0.127,0.164,0.213),\\(0.000,0.787,0.836,0.873),\\(0.000,0.787,0.836,0.873)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.087,0.113,0.146,0.190),\\(0.000,0.810,0.854,0.887),\\(0.000,0.810,0.854,0.887)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.131,0.171,0.235,1.000),\\(0.000,0.765,0.829,0.869),\\(0.000,0.765,0.829,0.869)\end{array}\right\rangle$
Putian(PT)	$\left\langle \begin{array}{l}(0.066,0.093,0.124,0.161),\\(0.735,0.839,0.876,0.934),\\(0.735,0.839,0.876,0.934)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.146,0.190,0.260,1.000),\\(0.000,0.740,0.810,0.854),\\(0.000,0.740,0.810,0.854)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.078,0.101,0.131,0.171),\\(0.000,0.829,0.869,0.899),\\(0.000,0.829,0.869,0.899)\end{array}\right\rangle$
Longyan (LY)	$\left\langle \begin{array}{l}(0.020,0.042,0.066,0.093),\\(0.839,0.907,0.934,0.980),\\(0.839,0.907,0.934,0.980)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.000,0.000,0.000,0.000),\\(1.000,1.000,1.000,1.000),\\(0.000,0.000,0.000,0.000)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.014,0.029,0.046,0.065),\\(0.887,0.935,0.954,0.986),\\(0.887,0.935,0.954,0.986)\end{array}\right\rangle$	$\left\langle \begin{array}{l}(0.041,0.058,0.078,0.101),\\(0.829,0.899,0.922,0.959),\\(0.829,0.899,0.922,0.959)\end{array}\right\rangle$

Quanzhou (QZ)	$\langle(0.066,0.093,0.124,0.161),(0.735,0.839,0.876,0.934),(0.735,0.839,0.876,0.934)\rangle$	$\langle(0.016,0.033,0.052,0.073),(0.873,0.927,0.948,0.984),(0.873,0.927,0.948,0.984)\rangle$	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.012,0.026,0.041,0.058),(0.899,0.942,0.959,0.988),(0.899,0.942,0.959,0.988)\rangle$
Xiamen (XM)	$\langle(0.020,0.042,0.066,0.093),(0.839,0.907,0.934,0.980),(0.839,0.907,0.934,0.980)\rangle$	$\langle(0.016,0.033,0.052,0.073),(0.873,0.927,0.948,0.984),(0.873,0.927,0.948,0.984)\rangle$	$\langle(0.046,0.065,0.087,0.113),(0.810,0.887,0.913,0.954),(0.810,0.887,0.913,0.954)\rangle$	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$
Zhangzhou (ZZ)	$\langle(0.124,0.161,0.206,0.265),(0.000,0.735,0.794,0.839),(0.000,0.735,0.794,0.839)\rangle$	$\langle(0.164,0.213,0.290,1.000),(0.000,0.710,0.787,0.836),(0.000,0.710,0.787,0.836)\rangle$	$\langle(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.131,0.171,0.235,1.000),(0.000,0.765,0.829,0.869),(0.000,0.765,0.829,0.869)\rangle$

Cities \ Index	c_5	c_6	c_7
Nanping (NP)	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.014,0.029,0.046,0.065),(0.887,0.935,0.954,0.986),(0.887,0.935,0.954,0.986)\rangle$
Ningde (ND)	$\langle(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.194,0.250,0.338,1.000),(0.000,0.662,0.750,0.806),(0.000,0.662,0.750,0.806)\rangle$	$\langle(0.046,0.065,0.087,0.113),(0.810,0.887,0.913,0.954),(0.810,0.887,0.913,0.954)\rangle$
Sanming (SM)	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.000,0.019,0.039,0.062),(0.938,0.961,0.981,1.000),(0.938,0.961,0.981,1.000)\rangle$	$\langle(0.000,0.014,0.029,0.046),(0.954,0.971,0.986,1.000),(0.954,0.971,0.986,1.000)\rangle$
Fuzhou (FZ)	$\langle(0.068,0.089,0.116,0.152),(0.000,0.848,0.884,0.911),(0.000,0.848,0.884,0.911)\rangle$	$\langle(0.062,0.087,0.117,0.151),(0.750,0.849,0.883,0.938),(0.750,0.849,0.883,0.938)\rangle$	$\langle(0.146,0.190,0.260,1.000),(0.000,0.740,0.810,0.854),(0.000,0.740,0.810,0.854)\rangle$
Putian (PT)	$\langle(0.116,0.152,0.210,1.000),(0.000,0.790,0.848,0.884),(0.000,0.790,0.848,0.884)\rangle$	$\langle(0.117,0.151,0.194,0.250),(0.000,0.750,0.806,0.849),(0.000,0.750,0.806,0.849)\rangle$	$\langle(0.087,0.113,0.146,0.190),(0.000,0.810,0.854,0.887),(0.000,0.810,0.854,0.887)\rangle$
Longyan (LY)	$\langle(0.036,0.051,0.068,0.089),(0.848,0.911,0.932,0.964),(0.848,0.911,0.932,0.964)\rangle$	$\langle(0.019,0.039,0.062,0.087),(0.849,0.913,0.938,0.981),(0.849,0.913,0.938,0.981)\rangle$	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$
Quanzhou (QZ)	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.062,0.087,0.117,0.151),(0.750,0.849,0.883,0.938),(0.750,0.849,0.883,0.938)\rangle$	$\langle(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000),(0.000,0.000,0.000,0.000)\rangle$
Xiamen (XM)	$\langle(0.011,0.023,0.036,0.051),(0.911,0.949,0.964,0.989),(0.911,0.949,0.964,0.989)\rangle$	$\langle(0.019,0.039,0.062,0.087),(0.849,0.913,0.938,0.981),(0.849,0.913,0.938,0.981)\rangle$	$\langle(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000),(0.000,0.000,0.000,0.000)\rangle$
Zhangzhou (ZZ)	$\langle(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.000,0.000,0.000,0.000),(1.000,1.000,1.000,1.000),(0.000,0.000,0.000,0.000)\rangle$	$\langle(0.146,0.190,0.260,1.000),(0.000,0.740,0.810,0.854),(0.000,0.740,0.810,0.854)\rangle$

Step 6: Calculate the correlation coefficient matrices based on Formula (18). Here the positive ideal solution

$$n_j^+ = \left\langle \begin{matrix} (1.000, 1.000, 1.000, 1.000), \\ (0.000, 0.000, 0.000, 0.000), \\ (0.000, 0.000, 0.000, 0.000) \end{matrix} \right\rangle$$

and the negative ideal solution

$$n_j^- = \left\langle \begin{matrix} (0.000, 0.000, 0.000, 0.000), \\ (1.000, 1.000, 1.000, 1.000), \\ (1.000, 1.000, 1.000, 1.000) \end{matrix} \right\rangle,$$

then the positive correlation coefficient matrix η^+ and the negative correlation coefficient matrix η^- are listed in Table 6 and Table 7, respectively.

TABLE 6
POSITIVE CORRELATION COEFFICIENT MATRIX η^+

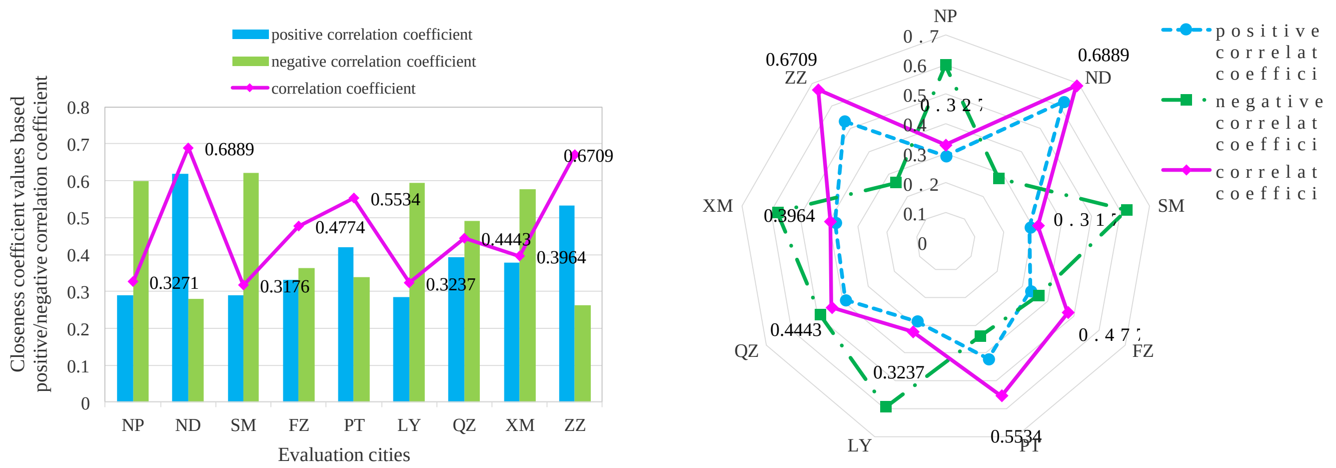
Cities \ Index	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Nanping (NP)	0.265	0.262	0.333	0.250	0.333	0.333	0.260
Ningde (ND)	1.000	0.333	0.357	1.000	1.000	0.375	0.270
Sanming (SM)	0.333	0.262	0.254	0.333	0.333	0.256	0.254
Fuzhou (FZ)	0.380	0.323	0.318	0.351	0.311	0.277	0.357
Putian (PT)	0.279	1.000	0.357	0.315	0.346	0.331	0.318
Longyan (LY)	0.265	0.333	0.260	0.268	0.266	0.264	0.333
Quanzhou (QZ)	0.279	0.262	0.333	0.259	0.333	0.277	1.000
Xiamen (XM)	0.265	0.262	0.270	0.333	0.258	0.264	1.000
Zhangzhou (ZZ)	0.334	0.364	1.000	0.351	1.000	0.333	0.357

TABLE 7
NEGATIVE CORRELATION COEFFICIENT MATRIX η^-

Cities \ Index	c_1	c_2	c_3	c_4	c_5	c_6	c_7
Nanping (NP)	0.689	0.739	0.333	1.000	0.333	0.333	0.761
Ningde (ND)	0.143	0.333	0.295	0.143	0.143	0.272	0.629
Sanming (SM)	0.333	0.739	0.883	0.333	0.333	0.848	0.883
Fuzhou (FZ)	0.267	0.357	0.368	0.302	0.389	0.559	0.295
Putian (PT)	0.544	0.143	0.295	0.378	0.311	0.339	0.368
Longyan (LY)	0.689	0.333	0.761	0.654	0.681	0.703	0.333
Quanzhou (QZ)	0.544	0.739	0.333	0.781	0.333	0.559	0.143
Xiamen (XM)	0.689	0.739	0.629	0.333	0.802	0.703	0.143
Zhangzhou (ZZ)	0.332	0.286	0.143	0.302	0.143	0.333	0.295

Step 7: Obtain the closeness coefficient U_i of each alternative based on Formula (19), and the data is as follows:

U_{NP}=0.3271, U_{ND}=0.6889, U_{SM}=0.3176, U_{FZ}=0.4774, U_{PT}=0.5534, U_{LY}=0.3237, U_{QZ}=0.4443, U_{XM}=0.3964, U_{ZZ}=0.6709.



(a) (b)

FIGURE 2. **Ranking results of the evaluation objects**

Step 8: Rank the alternatives based on U_i. The bigger U_i, the more the city is affected by the disaster. From the calculation results in Step 7 and Figure 2, we can see that $U_{ND} \succ U_{ZZ} \succ U_{PT} \succ U_{FZ} \succ U_{QZ} \succ U_{XM} \succ U_{NP} \succ U_{LY} \succ U_{SM}$. Therefore, the ranking of disaster severity in nine cities is as follows: $ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$. The results of this decision can be provided to relevant departments for effective disaster relief and material distribution to provide security for the people, establish the government's reputation for action, and maintain social stability and well-being.

B. SENSITIVITY ANALYSIS

To illustrate the robustness of the algorithm, sensitivity analysis is performed on the parameter ρ in the grey relation analysis under different values, and the results are summarized in Table 8 and Figure 3. Then, sensitivity analysis is performed on the parameter λ in the distance measure under different values, and the results are summarized in Table 9 and Figure 4.

TABLE 8
CLOSENESS COEFFICIENT U_i AND RANKING RESULT R_i FOR ρ WITH DIFFERENT PARAMETER VALUES

ρ / Cities	ρ=0.1		ρ=0.2		ρ=0.3		ρ=0.4		ρ=0.5		ρ=0.6		ρ=0.7		ρ=0.8		ρ=0.9		ρ=1	
	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i	U_i	R_i
Nanping (NP)	0.187	8	0.245	8	0.281	8	0.307	7	0.327	7	0.343	7	0.356	7	0.367	7	0.377	7	0.391	7
Ningde (ND)	0.851	1	0.781	1	0.738	1	0.710	1	0.689	1	0.673	1	0.660	1	0.649	1	0.640	1	0.645	1
Sanming (SM)	0.183	9	0.234	9	0.270	9	0.297	9	0.318	9	0.334	9	0.348	9	0.360	9	0.370	8	0.384	8
Fuzhou (FZ)	0.460	5	0.466	4	0.471	4	0.475	4	0.477	4	0.480	4	0.482	4	0.483	4	0.484	4	0.498	4
Putian (PT)	0.689	3	0.614	3	0.582	3	0.564	3	0.553	3	0.546	3	0.541	3	0.537	3	0.533	3	0.544	3
Longyan (LY)	0.226	7	0.260	7	0.286	7	0.307	8	0.324	8	0.338	8	0.350	8	0.360	8	0.369	9	0.383	9
Quanzhou (QZ)	0.512	4	0.459	5	0.446	5	0.444	5	0.444	5	0.446	5	0.449	5	0.452	5	0.454	5	0.465	5
Xiamen (XM)	0.440	6	0.396	6	0.389	6	0.391	6	0.396	6	0.402	6	0.408	6	0.413	6	0.418	6	0.429	6
Zhangzhou (ZZ)	0.839	2	0.765	2	0.721	2	0.692	2	0.671	2	0.655	2	0.642	2	0.632	2	0.623	2	0.630	2

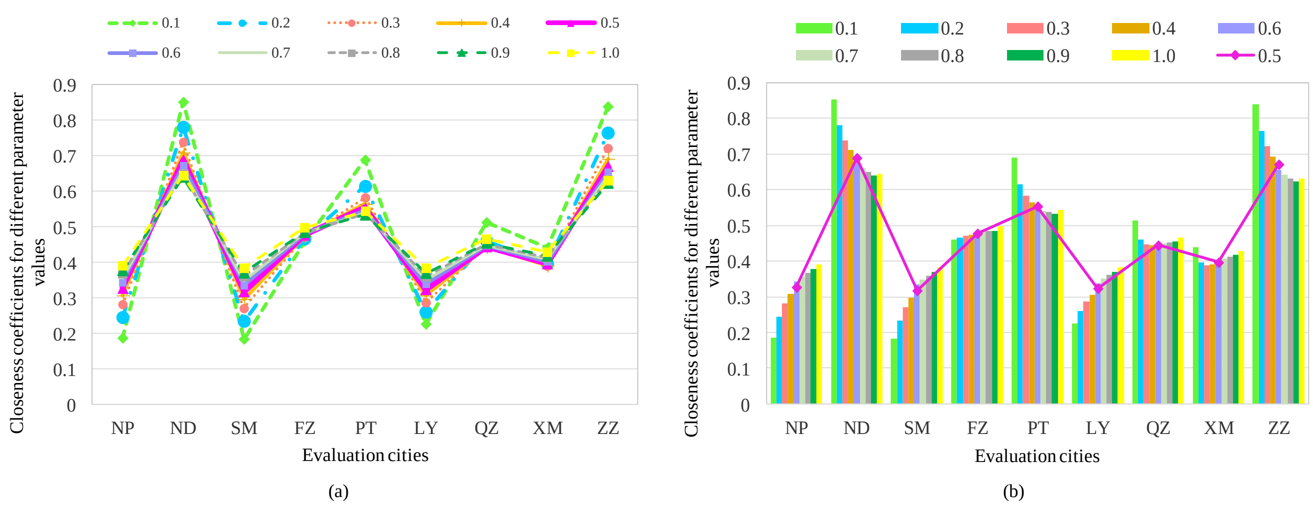


FIGURE 3. **Ranking results of the evaluation objects for parameter ρ with different values**

From the Table 8 and Figure 3, it can be seen that the parameter ρ of grey correlation analysis has little influence on the evaluation results in our study, and the algorithm has certain stability and robustness. Under different parameters, the evaluation results are slightly different, and the ranking of the top evaluation objects is consistent.

TABLE 9
RANKING RESULT R AND ATTRIBUTE WEIGHT ω FOR λ WITH DIFFERENT PARAMETER VALUES

Parameter value		Calculation results
λ=1	ω	$\omega_1=0.1917$, $\omega_2=0.1486$, $\omega_3=0.1310$, $\omega_4=0.1166$, $\omega_5=0.1022$, $\omega_6=0.1789$, $\omega_7=0.1310$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=2	ω	$\omega_1=0.1926$, $\omega_2=0.1479$, $\omega_3=0.1307$, $\omega_4=0.1163$, $\omega_5=0.1020$, $\omega_6=0.1797$, $\omega_7=0.1307$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=3	ω	$\omega_1=0.1940$, $\omega_2=0.1463$, $\omega_3=0.1303$, $\omega_4=0.1161$, $\omega_5=0.1019$, $\omega_6=0.1809$, $\omega_7=0.1303$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=4	ω	$\omega_1=0.1957$, $\omega_2=0.1448$, $\omega_3=0.1297$, $\omega_4=0.1159$, $\omega_5=0.1020$, $\omega_6=0.1823$, $\omega_7=0.1297$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=5	ω	$\omega_1=0.1975$, $\omega_2=0.1436$, $\omega_3=0.1290$, $\omega_4=0.1155$, $\omega_5=0.1020$, $\omega_6=0.1835$, $\omega_7=0.1290$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=7	ω	$\omega_1=0.2010$, $\omega_2=0.1419$, $\omega_3=0.1273$, $\omega_4=0.1146$, $\omega_5=0.1020$, $\omega_6=0.1859$, $\omega_7=0.1273$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=10	ω	$\omega_1=0.2059$, $\omega_2=0.1406$, $\omega_3=0.1247$, $\omega_4=0.1134$, $\omega_5=0.1020$, $\omega_6=0.1887$, $\omega_7=0.1247$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=12	ω	$\omega_1=0.2087$, $\omega_2=0.1402$, $\omega_3=0.1231$, $\omega_4=0.1126$, $\omega_5=0.1021$, $\omega_6=0.1902$, $\omega_7=0.1231$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=15	ω	$\omega_1=0.2123$, $\omega_2=0.1397$, $\omega_3=0.1210$, $\omega_4=0.1116$, $\omega_5=0.1022$, $\omega_6=0.1921$, $\omega_7=0.1210$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$
λ=20	ω	$\omega_1=0.2169$, $\omega_2=0.1393$, $\omega_3=0.1183$, $\omega_4=0.1103$, $\omega_5=0.1024$, $\omega_6=0.1945$, $\omega_7=0.1183$
	R	$ND \succ ZZ \succ PT \succ FZ \succ QZ \succ XM \succ NP \succ LY \succ SM$

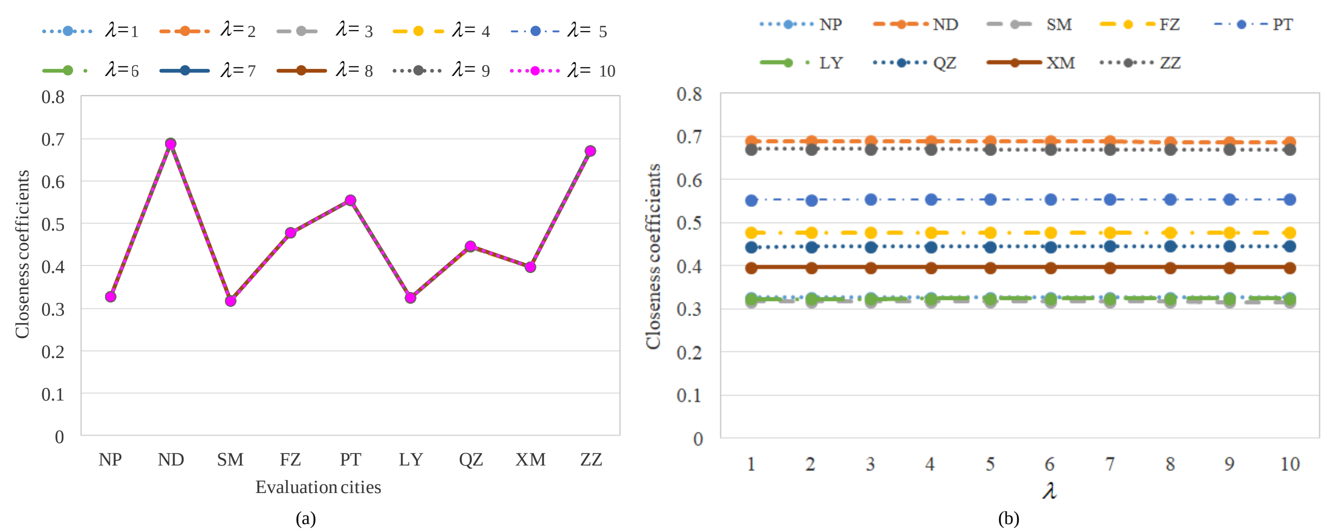


FIGURE 4. **Ranking results of the evaluation objects for parameter λ with different values**

As it can be seen from Figure 4 (a) , when λ takes different values, the display lines of the closeness coefficients of the evaluation object are substantially completely coincident, that is, the sorting results are basically the same. At the same time, it can be seen from Figure 4. (b), when λ takes different values, the relative positions between the evaluation objects are basically unchanged and remain parallel, that is, the sorting results are ranked in the same order. Therefore, it can be seen that the parameter λ of distance measure has little influence on the evaluation results in our study, and the algorithm has certain stability and robustness.

C. COMPARATIVE ANALYSIS WITH DIFFERENT METHODS

To illustrate the validity and rationality of the algorithm, the proposed method is compared with other methods from different aspects. This paper makes a comprehensive comparative analysis from the processing method of missing information, the ranking method, and the distance measurement method based on TrFNNs.

(1) Comparative analysis of different missing information processing methods

In this section, we compare our method of completing missing information with other methods (that is, methods of complementing zero or complementing the mean value [58]) to illustrate the advantages of our approach.

TABLE 10
RANKING RESULTS USING DIFFERENT MISSING INFORMATION PROCESSING METHODS

Cities \ Different methods	Missing information fills 0		Missing information fills average value		Our proposed method for missing information processing	
	Closeness coefficient	Ranking results	Closeness coefficient	Ranking results	Closeness coefficient	Ranking results
Nanping (NP)	0.3803	7	0.3940	8	0.3271	7
Ningde (ND)	0.6701	1	0.8095	2	0.6889	1
Sanming (SM)	0.3713	8	0.3859	9	0.3176	9
Fuzhou (FZ)	0.4316	5	0.7240	4	0.4774	4
Putian (PT)	0.5067	3	0.7663	3	0.5534	3
Longyan (LY)	0.3498	9	0.4517	7	0.3237	8
Quanzhou (QZ)	0.4622	4	0.5648	5	0.4443	5
Xiamen (XM)	0.3957	6	0.5361	6	0.3964	6
Zhangzhou (ZZ)	0.6475	2	0.8185	1	0.6709	2

The methods of completing the missing information with 0 or the mean are commonly used for dealing with imperfect decision matrices. It can be seen from the results in Table 10 that the methods based on the three complements are roughly the same, but not identical. This is because the method of perfecting the evaluation matrix has a greater impact on the evaluation results. The missing typhoon disaster assessment data indicates that no assessment information has been obtained or is uncertain, and 0 means nothing. If the evaluation data are expressed as 0, it is intuitively indicated that the evaluation object is affected by the typhoon as 0, that is, there is no loss, which is not consistent with the actual meaning of the typhoon disaster assessment information. If the evaluation data are expressed as 0.5, it means that the degree of support, negation and uncertainty of the assessment object affected by the typhoon disaster is 0.5, that is, the degree of influence is not small, which is not consistent with the actual meaning of the lack of information in typhoon disaster assessment. In short, the representation of the missing information in this paper is more reasonable and consistent with the actual meaning. Therefore, the decision-making result is more reasonable and effective.

(2) Comparative analysis of different ranking methods

In this section, we compare our sorting method with the other two most widely used sorting methods (that is, the TOPSIS method and the sorting method based on the exact function and the scoring function) to illustrate the advantages of the proposed method based on the gray correlation analysis.

TABLE 11
RANKING RESULTS USING DIFFERENT SORTING METHODS

Cities \ Different methods	Our proposed method based on GRA		TOPSIS method		Traditional function method		
	Closeness coefficients	Ranking results	Closeness coefficients	Ranking results	Score function	Accuracy function	Ranking results
Nanping (NP)	0.3271	7	0.4404	8	0.4404	0.1292	8
Ningde (ND)	0.6889	1	1.0000	1	1.0000	1.0000	1
Sanming (SM)	0.3176	9	0.4137	9	0.4137	0.1101	9
Fuzhou (FZ)	0.4774	4	0.7827	6	0.7827	0.5560	6
Putian (PT)	0.5534	3	1.0000	1	1.0000	1.0000	1
Longyan (LY)	0.3237	8	0.5229	7	0.5229	0.2369	7
Quanzhou (QZ)	0.4443	5	1.0000	1	1.0000	1.0000	1
Xiamen (XM)	0.3964	6	1.0000	1	1.0000	1.0000	1
Zhangzhou (ZZ)	0.6709	2	1.0000	1	1.0000	1.0000	1

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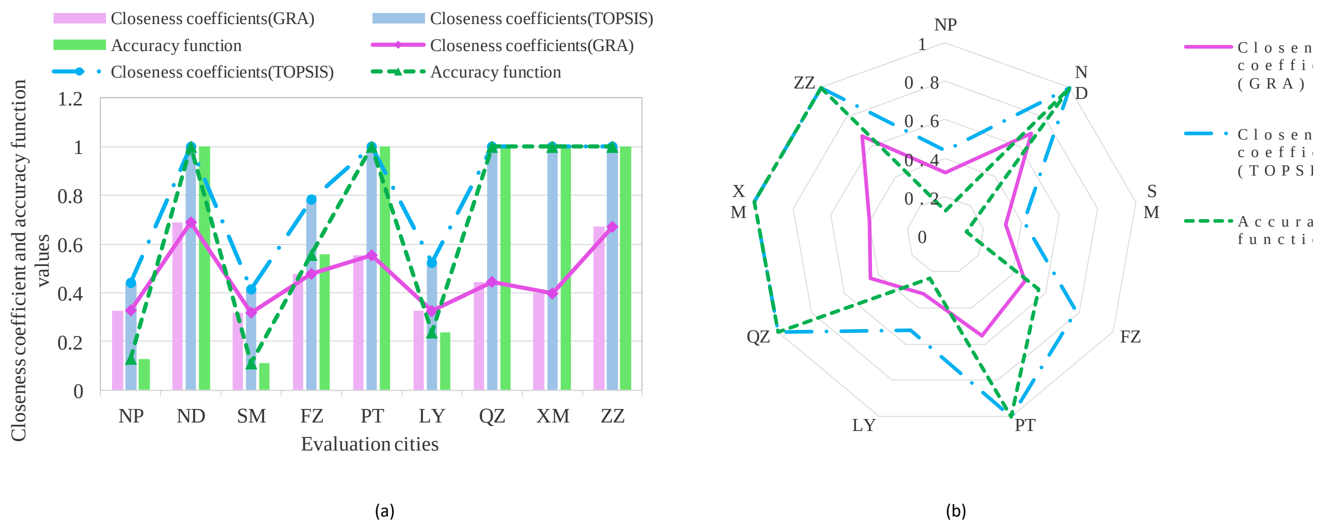

FIGURE 5. Ranking results of the evaluation objects using different sorting methods

It can be seen from Table 11 and Figure 5 that the ranking results using the three different sorting methods are not identical, but the optimal scheme is the same. In addition, because the TOPSIS and traditional function methods are based on the TNNWAA operator, which is sensitive to data 0, information is lost during the information aggregation process. Therefore, the two methods sometimes cannot completely sort the evaluation objects. For example, the comprehensive evaluation values of cities such as ND, PT, QZ, XM and ZZ are the same, so they cannot be distinguished. However, this indistinguishable phenomenon will not occur in our method based on grey relational analysis.

(3) Comparative analysis of different distance measurement methods

TABLE 12
RANKING RESULTS USING DIFFERENT DISTANCE MEASUREMENT METHODS

Different methods / Cities	Our proposed distance measurement method		Cosine similarity-based distance measurement method [3]	
	Closeness coefficients	Ranking results	Closeness coefficients	Ranking results
Nanping (NP)	0.3271	7	0.3294	7
Ningde (ND)	0.6889	1	0.6907	1
Sanming (SM)	0.3176	9	0.3209	9
Fuzhou (FZ)	0.4774	4	0.4809	4
Putian (PT)	0.5534	3	0.5552	3
Longyan (LY)	0.3237	8	0.3281	8
Quanzhou (QZ)	0.4443	5	0.4449	5
Xiamen (XM)	0.3964	6	0.3975	6
Zhangzhou (ZZ)	0.6709	2	0.6724	2

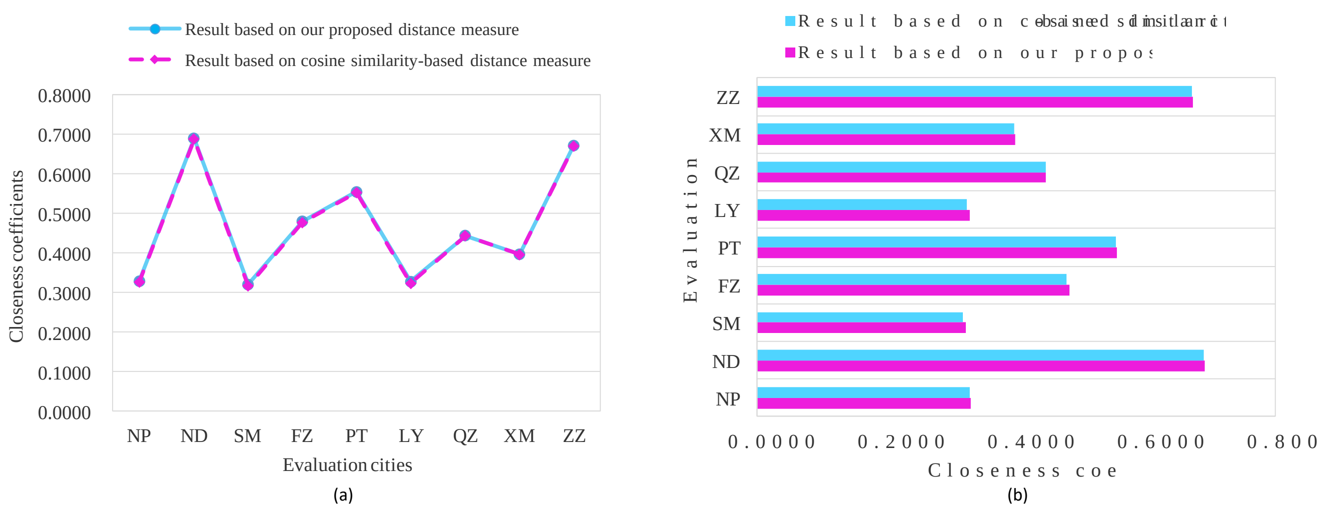

FIGURE 6. Ranking results of the evaluation objects using different distance measurement methods

It can be seen from Table 12 and Figure 6 that the ranking results of the two different distance measurement methods are exactly the same; therefore, the distance formula proposed in this paper is reasonable and robust.

VII. CONCLUSION

Regarding the double incomplete information environment, this study proposes a decision-making method based on grey relation analysis and trapezoidal fuzzy neutrosophic numbers. We give the definition of trapezoidal fuzzy neutrosophic numbers for missing information, which can fill the defect decision matrix. Compared with other complement methods, our method of dealing with missing information is more reasonable and effective. At the same time, we define a new distance measure formula and a new similarity measure formula based on trapezoidal fuzzy neutrosophic numbers, thus defining the new trapezoidal fuzzy neutrosophic entropy and discussing the relationship between the three. Based on the new information entropy, we can objectively calculate the attribute weights. Then, we use grey relational analysis in grey theory to rank alternatives and select the best one. Finally, an illustrative example about typhoon disaster assessment is presented to show the feasibility and effectiveness of the proposed method, and the advantages of the proposed method are illustrated by comparison with other methods from multiple aspects. In future work, the decision-making methods based on the neutrosophic numbers, and their application in typhoon disaster assessment will be further studied.

APPENDIX

We prove the four properties of Theorem 1 and Theorem 3.
(1) Proof of the four properties of Theorem 1:

Proof

(P1) Because $0 \le a_i, e_i, b_i, f_i, b_i, f_i \le 1$,

$0 \le |a_i - e_i| \le 1$, $0 \le |b_i - f_i| \le 1$, $0 \le |c_i - g_i| \le 1$,

$0 \le |a_i - e_i|^\lambda \le 1$, $0 \le |b_i - f_i|^\lambda \le 1$,

$0 \le |c_i - g_i|^\lambda \le 1$, $0 \le \sum_{i=1}^{4} |a_i - e_i|^\lambda \le 1$,

$0 \le \sum_{i=1}^{4} |b_i - f_i|^\lambda \le 1$, $0 \le \sum_{i=1}^{4} |c_i - g_i|^\lambda \le 1$, so

$$0 \le \sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda \le 12,$$

$$0 \le \frac{1}{12}\left(\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda \right) \le 1,$$

$$0 \le \left\{ \frac{1}{12}\left(\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda \right) \right\}^{1/\lambda} \le 1,$$

thus $0 \le D(\tilde{n}_1, \tilde{n}_2) \le 1$ is established.

(P2) Because $\tilde{n}_1 = \tilde{n}_2$, $|a_i - e_i| = 0$, $|b_i - f_i| = 0$, $|c_i - g_i| = 0$, $|a_i - e_i|^\lambda = 0$, $|b_i - f_i|^\lambda = 0$, $|c_i - g_i|^\lambda = 0$,

$$\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda = 0,$$

$$\left\{ \frac{1}{12}\left(\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda \right) \right\}^{1/\lambda} = 0,$$

$D(\tilde{n}_1, \tilde{n}_2) = 0$.

Conversely, when $D(\tilde{n}_1, \tilde{n}_2) = 0$, it is easy to get $\tilde{n}_1 = \tilde{n}_2$, thus the property (P2) is established.

(P3) Because $|a_i - e_i| = |e_i - a_i|$, $|b_i - f_i| = |f_i - b_i|$, $|c_i - g_i| = |g_i - c_i|$, $|a_i - e_i|^\lambda = |e_i - a_i|^\lambda$, $|b_i - f_i|^\lambda = |f_i - b_i|^\lambda$, $|c_i - g_i|^\lambda = |g_i - c_i|^\lambda$,

$$\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda = \sum_{i=1}^{4} |e_i - a_i|^\lambda + \sum_{i=1}^{4} |f - b_{ii}|^\lambda + \sum_{i=1}^{4} |g_i - c_i|^\lambda,$$

$$\left\{ \frac{1}{12}\left(\sum_{i=1}^{4} |a_i - e_i|^\lambda + \sum_{i=1}^{4} |b_i - f_i|^\lambda + \sum_{i=1}^{4} |c_i - g_i|^\lambda \right) \right\}^{1/\lambda} = \left\{ \frac{1}{12}\left(\sum_{i=1}^{4} |e_i - a_i|^\lambda + \sum_{i=1}^{4} |f_i - b_i|^\lambda + \sum_{i=1}^{4} |g_i - c_i|^\lambda \right) \right\}^{1/\lambda},$$

thus $D(\tilde{n}_1, \tilde{n}_2) = D(\tilde{n}_2, \tilde{n}_1)$ is established.

(P4) Suppose $\tilde{n}_3 = \langle (h_1, h_2, h_3, h_4), (i_1, i_2, i_3, i_4), (j_1, j_2, j_3, j_4) \rangle$, because $\tilde{n}_1 \le \tilde{n}_2 \le \tilde{n}_3$, $|a_i - e_i| \le |a_i - h_i|$, $|b_i - f_i| \le |b_i - i_i|$, $|c_i - g_i| \le |c_i - j_i|$, $|a_i - e_i|^\lambda \le |a_i - h_i|^\lambda$, $|b_i - f_i|^\lambda \le |b_i - i_i|^\lambda$, $|c_i - g_i|^\lambda \le |c_i - j_i|^\lambda$, $\sum_{i=1}^{4} |a_i - e_i|^\lambda \le \sum_{i=1}^{4} |a_i - h_i|^\lambda$,

$\sum_{i=1}^{4} |b_i - f_i|^\lambda \le \sum_{i=1}^{4} |b_i - i_i|^\lambda$,

$\sum_{i=1}^{4} |c_i - g_i|^\lambda \le \sum_{i=1}^{4} |c_i - j_i|^\lambda$,

then

$$\left(\sum_{i=1}^{4}|a_i-e_i|^{\lambda}+\sum_{i=1}^{4}|b_i-f_i|^{\lambda}+\sum_{i=1}^{4}|c_i-g_i|^{\lambda}\right)$$
$$\le\left(\sum_{i=1}^{4}|a_i-h_i|^{\lambda}+\sum_{i=1}^{4}|b_i-i_i|^{\lambda}+\sum_{i=1}^{4}|c_i-j_i|^{\lambda}\right),$$
$$\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-e_i|^{\lambda}+\sum_{i=1}^{4}|b_i-f_i|^{\lambda}+\sum_{i=1}^{4}|c_i-g_i|^{\lambda}\right)\right\}^{1/\lambda}$$
$$\le\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-h_i|^{\lambda}+\sum_{i=1}^{4}|b_i-i_i|^{\lambda}+\sum_{i=1}^{4}|c_i-j_i|^{\lambda}\right)\right\}^{1/\lambda},$$
so $D(\tilde{n}_1,\tilde{n}_2)\le D(\tilde{n}_1,\tilde{n}_3)$.

Because $\tilde{n}_1\le\tilde{n}_2\le\tilde{n}_3$, $|e_i-h_i|\le|a_i-h_i|$, $|f_i-i_i|\le|b_i-i_i|$, $|g_i-j_i|\le|c_i-j_i|$, $|e_i-h_i|^{\lambda}\le|a_i-h_i|^{\lambda}$, $|f_i-i_i|^{\lambda}\le|b_i-i_i|^{\lambda}$, $|g_i-j_i|^{\lambda}\le|c_i-j_i|^{\lambda}$, $\sum_{i=1}^{4}|e_i-h_i|^{\lambda}\le\sum_{i=1}^{4}|a_i-h_i|^{\lambda}$, $\sum_{i=1}^{4}|f_i-i_i|^{\lambda}\le\sum_{i=1}^{4}|b_i-i_i|^{\lambda}$, $\sum_{i=1}^{4}|g_i-j_i|^{\lambda}\le\sum_{i=1}^{4}|c_i-j_i|^{\lambda}$,
then

$$\left(\sum_{i=1}^{4}|e_i-h_i|^{\lambda}+\sum_{i=1}^{4}|f_i-i_i|^{\lambda}+\sum_{i=1}^{4}|g_i-j_i|^{\lambda}\right)$$
$$\le\left(\sum_{i=1}^{4}|a_i-h_i|^{\lambda}+\sum_{i=1}^{4}|b_i-i_i|^{\lambda}+\sum_{i=1}^{4}|c_i-j_i|^{\lambda}\right),$$
$$\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|e_i-h_i|^{\lambda}+\sum_{i=1}^{4}|f_i-i_i|^{\lambda}+\sum_{i=1}^{4}|g_i-j_i|^{\lambda}\right)\right\}^{1/\lambda}$$
$$\le\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-h_i|^{\lambda}+\sum_{i=1}^{4}|b_i-i_i|^{\lambda}+\sum_{i=1}^{4}|c_i-j_i|^{\lambda}\right)\right\}^{1/\lambda},$$

Therefore, synthesizing the above proof process, the property (P4) is established.

(2) Proof of the four properties of Theorem 3:

Proof

(P1) If $\tilde{n}$ is a crisp number, then $\tilde{n}$ is not fuzzy, so its entropy is 0. In this case,

$\tilde{n}=\langle(1,1,1,1),(0,0,0,0),(0,0,0,0)\rangle$, or

$\tilde{n}=\langle(0,0,0,0),(0,0,0,0),(1,1,1,1)\rangle$.

Thus

$$E_{TrFNN}(\tilde{n})=1\text{-}2D(\tilde{n},\tilde{n}')$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}0.5^{\lambda}+\sum_{i=1}^{4}0.5^{\lambda}+\sum_{i=1}^{4}0.5^{\lambda}\right)\right\}^{1/\lambda}$$
$$=1-2\times\left(\frac{1}{12}\right)^{1/\lambda}\times\left(12\times0.5^{\lambda}\right)^{1/\lambda}=0.$$

Then, the property (P1) is established.

(P2) Because $E_{TrFNN}(\tilde{n})=1$, that is $1\text{-}2D(\tilde{n},\tilde{n}')=1$, then $D(\tilde{n},\tilde{n}')=0$, based on the property (P2) of Theorem 1, we can get

$$\tilde{n}=\tilde{n}'=\left\langle\begin{matrix}(0.5,0.5,0.5,0.5),(0.5,0.5,0.5,0.5),\\(0.5,0.5,0.5,0.5)\end{matrix}\right\rangle.$$

Conversely, if

$$\tilde{n}=\tilde{n}'=\left\langle\begin{matrix}(0.5,0.5,0.5,0.5),(0.5,0.5,0.5,0.5),\\(0.5,0.5,0.5,0.5)\end{matrix}\right\rangle,$$

then $D(\tilde{n},\tilde{n}')=0$, $1\text{-}2D(\tilde{n},\tilde{n}')=1$, so $E_{TrFNN}(\tilde{n})=1$, thus, the property (P2) is established.

(P3) Because $D(\tilde{n}_1,\tilde{n}')\ge D(\tilde{n}_2,\tilde{n}')$, $2D(\tilde{n}_1,\tilde{n}')\ge 2D(\tilde{n}_2,\tilde{n}')$, $1\text{-}2D(\tilde{n}_1,\tilde{n}')\le 1\text{-}2D(\tilde{n}_1,\tilde{n}')$, then

$$E_{TrFNN}(\tilde{n}_1)=1\text{-}2D(\tilde{n}_1,\tilde{n}')\le E_{TrFNN}(\tilde{n}_2)=1\text{-}2D(\tilde{n}_1,\tilde{n}').$$

The property (P3) is established.

(P4) Because

$$\tilde{n}^c=\langle(c_1,c_2,c_3,c_4),(b_1,b_2,b_3,b_4),(a_1,a_2,a_3,a_4)\rangle,$$
$$E_{TrFNN}(\tilde{n})=1\text{-}2D(\tilde{n},\tilde{n}')$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|a_i-0.5|^{\lambda}+\sum_{i=1}^{4}|b_i-0.5|^{\lambda}+\sum_{i=1}^{4}|c_i-0.5|^{\lambda}\right)\right\}^{1/\lambda},$$
$$E_{TrFNN}(\tilde{n}^c)=1\text{-}2D(\tilde{n}^c,\tilde{n}')$$
$$=1-2\left\{\frac{1}{12}\left(\sum_{i=1}^{4}|c_i-0.5|^{\lambda}+\sum_{i=1}^{4}|b_i-0.5|^{\lambda}+\sum_{i=1}^{4}|a_i-0.5|^{\lambda}\right)\right\}^{1/\lambda},$$

$E_{TrFNN}(\tilde{n})=E_{TrFNN}(\tilde{n}^c)$. The property (P4) is established.

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