symmetry

MDPI

Article

Decision-Making via Neutrosophic Support Soft Topological Spaces

Parimala Mani [1,*], Karthika Muthusamy [1], Saeid Jafari [2], Florentin Smarandache [3], Udhayakumar Ramalingam [4]

1 Department of Mathematics, Bannari Amman Institute of Technology, Sathyamangalam 638401, Tamil Nadu, India; karthikamuthusamy1991@gmail.com
2 College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark; jafaripersia@gmail.com
3 Mathematics & Science Department, University of New Maxico, 705 Gurley Ave, Gallup, NM 87301, USA; fsmarandache@gmail.com
4 Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore 632012, Tamil Nadu and India; udhayaram_v@yahoo.co.in
* Correspondence: rishwanthpari@gmail.com

Received: 29 April 2018; Accepted: 7 June 2018; Published: 13 June 2018

Abstract: The concept of interval neutrosophic sets has been studied and the introduction of a new kind of set in topological spaces called the interval valued neutrosophic support soft set has been suggested. We study some of its basic properties. The main purpose of this paper is to give the optimum solution to decision-making in real life problems the using interval valued neutrosophic support soft set.

Keywords: soft sets; support soft sets; interval valued neutrosophic support soft sets

2010 AMS Classification: 06D72; 54A05; 54A40; 54C10

1. Introduction

To deal with uncertainties, many theories have been recently developed, including the theory of probability, the theory of fuzzy sets, the theory of rough sets, and so on. However, difficulties are still arising due to the inadequacy of parameters. The concept of fuzzy sets, which deals with the nonprobabilistic uncertainty, was introduced by Zadeh [1] in 1965. Since then, many researchers have defined the concept of fuzzy topology that has been widely used in the fields of neural networks, artificial intelligence, transportation, etc.The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K. Atanaasov [2] in 1983 as a generalization of the fuzzy set in addition to the degree of membership and the degree of nonmembership of each element.

In 1999, Molodtsov [3] successfully proposed a completely new theory called soft set theory using classical sets. This theory is a relatively new mathematical model for dealing with uncertainty from a parametrization point of view. After Molodtsov, many researchers have shown interest in soft sets and their applications. Maji [4,5] introduced neutrosophic soft sets with operators, which are free from difficulties since neutrosophic sets [6–9] can handle indeterminate information. However, the neutrosophic sets and operators are hard to apply in real life applications. Therefore, Smarandache [10] proposed the concept of interval valued neutrosophic sets which can represent uncertain, imprecise, incomplete, and inconsistent information.

Nguyen [11] introduced the new concept in a type of soft computing, called the support-neutrosophic set. Deli [12] defined a generalized concept of the interval-valued neutrosophic soft set. In this paper, we combine interval-valued neutrosophic soft sets and support sets to yield the interval-valued neutrosophic support soft set, and we study some of its basic operations. Our main aim of this paper is to make decisions using interval-valued neutrosophic support soft topological spaces.

2. Preliminaries

In this paper, we provide the basic definitions of neutrosophic and soft sets. These are very useful for what follows.

Definition 1. *([13]) Let X be a non-empty set. A neutrosophic set, A, in X is of the form* $A = \{\langle x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x); x \in X\rangle\}$, *where* $\mu_A : X \to [0,1]$, $\sigma_A : X \to [0,1]$ *and* $\gamma_A : X \to [0,1]$ *represent the degree of membership function, degree of indeterminacy, and degree of non-membership function, respectively and* $0 \leq \sup \mu_A(x) + \sup \sigma_A(x) + \sup \gamma_A(x) \leq 3, \forall x \in X$.

Definition 2. *([5]) Let X be a non-empty set , let P(X) be the power set of X, and let E be a set of parameters, and* $A \subseteq E$. *The soft set function,* f_X, *is defined by*

$$f_X : A \to P(X) \text{ such that } f_X(x) = \emptyset \text{ if } x \notin X.$$

The function f_X *may be arbitrary. Some of them may be empty and may have non-empty intersections. A soft set over X can be represented as the set of order pairs* $F_X = \{(x, f_X(x)) : x \in X, f_X(x) \in P(X)\}$.

Example 1. *Consider the soft set* $\langle F, A\rangle$, *where X is a set of six mobile phone models under consideration to be purchased by decision makers, which is denoted by* $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$, *and A is the parameter set, where* $A = \{y_1, y_2, y_3, y_4, y_5\} = \{price, look, camera, efficiency, processsor\}$. *A soft set,* F_X, *can be constructed such that* $f_X(y_1) = \{x_1, x_2\}$, $f_X(y_2) = \{x_1, x_4, x_5, x_6\}$, $f_X(y_3) = \emptyset$, $f_X(y_4) = X$, *and* $f_X(y_5) = \{x_1, x_2, x_3, x_4, x_5\}$. *Then,*

$$F_X = \{(y_1, x_1, x_2), (y_2, x_1, x_4, x_5, x_6), (y_3, \emptyset), (y_4, X), (y_5, x_1, x_2, x_3, x_4, x_5)\}.$$

X	x_1	x_2	x_3	x_4	x_5	x_6
y_1	1	1	0	0	0	0
y_2	1	0	0	1	1	1
y_3	0	0	0	0	0	0
y_4	1	1	1	1	1	1
y_5	1	1	1	1	1	0

Definition 3. *([4]) Let X be a non-empty set, and* $A = \{y_1, y_2, y_3, ..., y_n\}$, *the subset of X and* F_X *is a soft set over X. For any* $y_i \in A$, $f_X(y_i)$ *is a subset of X. Then, the choice value of an object,* $x_i \in X$, *is* $C_{V_i} = \sum_j x_{ij}$, *where* x_{ij} *are the entries in the table of* F_X:

$$x_{ij} = \begin{cases} 1, & \text{if } x_i \in f_X(y_j) \\ 0, & \text{if } x_i \notin f_X(y_j). \end{cases}$$

Example 2. *Consider Example 2. Clearly,* $C_{V_1} = \sum_{j=1}^{5} x_{1j} = 4$, $C_{V_3} = C_{V_6} = \sum_{j=1}^{5} x_{3j} = \sum_{j=1}^{5} x_{6j} = 2$, $C_{V_2} = C_{V_4} = C_{V_5} = \sum_{j=1}^{5} x_{2j} = \sum_{j=1}^{5} x_{4j} = \sum_{j=1}^{5} x_{5j} = 3$.

Definition 4. *([13]) Let* F_X *and* F_Y *be two soft sets over X and Y. Then,*

(1) *The complement of* F_X *is defined by* $F_{X^c}(x) = X \setminus f_X(x)$ *for all* $x \in A$;
(2) *The union of two soft sets is defined by* $f_{X\cup Y}(x) = f_X(x) \cup f_Y(x)$ *for all* $x \in A$;
(3) *The intersection of two soft sets is defined by* $f_{X\cap Y}(x) = f_X(x) \cap f_Y(x)$ *for all* $x \in A$.

3. Interval Valued Neutrosophic Support Soft Set

In this paper, we provide the definition of a interval-valued neutrosophic support soft set and perform some operations along with an example.

Definition 5. *Let X be a non-empty fixed set with a generic element in X denoted by a. An interval-valued neutrosophic support set, A, in X is of the form*

$$A = \{\langle x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x)\rangle / a; a \in X\}.$$

For each point, $a \in X, x, \mu_A(x), \sigma_A(x), \omega_A(x),$ *and* $\gamma_A(x) \in [0,1]$.

Example 3. *Let* $X = \{a, b\}$ *be a non-empty set, where* $a, b \subseteq [0,1]$. *An interval valued neutrosophic support set,* $A \subseteq X$, *constructed according to the degree of membership function,* $(\mu_A(x))$, *indeterminacy* $(\sigma_A(x))$, *support function* $(\omega_A(x))$, *and non-membership function* $(\gamma_A(x))$ *is as follows:*
$A = \{\langle(0.2, 1.0), (0.2, 0.4), (0.1, 0.7), (0.5, 0.7)\rangle / a, \langle(0.6, 0.8), (0.8, 1.0), (0.4, 0.6), (0.4, 0.6)\rangle / b\}$.

Definition 6. *Let X be a non-empty set; the interval-valued neutrosophic support set A in X is of the form* $A = \{\langle x, \mu_A(x), \sigma_A(x), \omega_A(x), \gamma_A(x); x \in X\rangle\}$.

- *(i) An empty set A, denoted by* $A = \emptyset$, *is defined by*
  $\emptyset = \{\langle(0,0), (1,1), (0,0), (1,1)\rangle / x : x \in X\}$.
- *(ii) The universal set is defined by*
  $U = \{\langle(1,1), (0,0), (1,1), (0,0)\rangle / x : x \in X\}$.
- *(iii) The complement of A is defined by*
  $A^c = \{\langle(\inf \gamma_A(x), \sup \gamma_A(x)), (1 - \sup \sigma_A(x), 1 - \inf \sigma_A(x)), (1 - \sup \omega_A(x), 1 - \inf \omega_A(x)), (\inf \mu_A(x), \sup \mu_A(x))\rangle / x : x \in X\}$.
- *(iv) A and B are two interval-valued neutrosophic support sets of X. A is a subset of B if*
  $\mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \omega_A(x) \leq \omega_B(x), \gamma_A(x) \geq \gamma_B(x)$.
- *(v) Two interval-valued neutrosophic support sets A and B in X are said to be equal if* $A \subseteq B$ *and* $B \subseteq A$.

Definition 7. *Let A and B be two interval-valued neutrosophic support sets. Then, for every* $x \in X$

- *(i) The intersection of A and B is defined by*
  $A \cap B = \{\langle(\min[\inf \mu_A(x), \inf \mu_B(x)], \min[\sup \mu_A, \sup \mu_B(x)]), (\max[\inf \sigma_A(x), \inf \sigma_B(x)], \max[\sup \sigma_A(x), \sup \sigma_B(x)]), (\min[\inf \omega_A(x), \inf \omega_B(x)], \min[\sup \omega_A(x), \sup \omega_B(x)]), (\max[\inf \gamma_A(x), \inf \gamma_B(x)], \max[\sup \gamma_A(x), \sup \gamma_B(x)])\rangle / x : x \in X\}$.
- *(ii) The union of A and B is defined by*
  $A \cup B = \{\langle(\max[\inf \mu_A(x), \inf \mu_B(x)], \max[\sup \mu_A(x), \sup \mu_B(x)]), (\min[\inf \sigma_A(x), \inf \sigma_B(x)], \min[\sup \sigma_A(x), \sigma_B(x)]), (\max[\inf \omega_A(x), \inf \omega_B(x)], \max[\sup \omega_A(x), \sup \omega_B(x)]), (\min[\inf \gamma_A(x), \inf \gamma_B(x)], \min[\sup \gamma_A(x), \sup \gamma_B(x)])\rangle / x : x \in X\}$.
- *(iii) A difference, B, is defined by*
  $A \setminus B$=$\{\langle(\min[\inf \mu_A(x), \inf \gamma_B(x)], \min[\sup \mu_A(x), \sup \gamma_B(x)]), (\max[\inf \sigma_A(x), 1 - \sup \sigma_B(x)], \max[\sup \sigma_A(x), 1 - \inf \sigma_B(x)]), (\min[\inf \omega_A(x), 1 - \sup \omega_B(x)], \min[\sup \omega_A(x), 1 - \inf \omega_B(x)]), (\max[\inf \gamma_A(x), \inf \mu_B(x)], \max[\sup \gamma_B(x), \sup \mu_B(x)])\rangle / x : x \in X\}$.
- *(iv) Scalar multiplication of A is defined by*
  $A.a = \{\langle(\min[\inf \mu_A(x).a, 1], \min[\sup \mu_A(x).a, 1]), (\min[\inf \sigma_A(x).a, 1], \min[\sup \sigma_A(x).a, 1]), (\min[\inf \omega_A(x).a, 1], \min[\sup \omega_A(x).a, 1]), (\min[\inf \gamma_A(x).a, 1], \min[\sup \gamma_A(x).a, 1])\rangle / x : x \in X\}$.
- *(v) Scalar division of A is defined by*
  $A/a = \{\langle(\min[\inf \mu_A(x)/a, 1], \min[\sup \mu_A(x)/a, 1]), (\min[\inf \sigma_A(x)/a, 1], \min[\sup \sigma_A(x)/a, 1]), (\min[\inf \omega_A(x)/a, 1], \min[\sup \omega_A(x)/a, 1]), (\min[\inf \gamma_A(x)/a, 1], \min[\sup \gamma_A(x)/a, 1])\rangle / x : x \in X\}$.

Definition 8. *Let X be a non-empty set; IVNSS(X) denotes the set of all interval-valued neutrosophic support soft sets of X and a subset, A, of X . The soft set function is*

$$g_i : A \to IVNSS(x).$$

The interval valued neutrosophic support soft setover X can be represented by

$$G_i = \{(y, g_i(y)) : y \in A\}, \text{ such that } g_i(y) = \emptyset \; if x \notin X.$$

Example 4. *Consider the interval-valued neutrosophic support soft set, $\langle G_i, A\rangle$, where X is a set of two brands of mobile phones being considered by a decision maker to purchase, which is denoted by $X = \{a, b\}$, and A is a parameter set, where $A = \{y_1 = price,\ y_2 = camera\ specification,\ y_3 = Efficency,$ and $y_4 = size,$ $y_5 = processsor\}$. In this case, we define a set G_i over X as follows:*

G_i	a	b
y_1	[0.6,0.8],[0.8,0.9][0.5,0.6][0.1,0.5]	[0.6,0.8][0.1,0.8][0.3,0.7][0.1,0.7]
y_2	[0.2,0.4][0.5,0.8][0.4,0.3][0.3,0.8]	[0.2,0.8][0.6,0.9][0.5,0.8][0.2,0.3]
y_3	[0.1,0.9][0.2,0.5][0.5,0.7][0.6,0.8]	[0.4,0.9][0.2,0.6][0.5,0.6][0.5,0.7]
y_4	[0.6,0.8][0.8,0.9][0.1,0.9][0.8,0.9]	[0.5,0.7][0.6,0.8][0.7,0.9][0.1,0.8]
y_5	[0.0,0.9][1.0,0.1][1.0,0.9][1.0,1.0]	[0.0,0.9][0.8,1.0][0.3,0.5][0.2,0.5]

Clearly, we can see that the exact evaluation of each object on each parameter is unknown, while the lower limit and upper limit of such an evaluation are given. For instance, we cannot give the exact membership degree, support, indeterminacy and nonmembership degree of price 'a'; however, the price of model 'a' is at least on the membership degree of 0.6 and at most on the membership degree of 0.8.

Definition 9. *Let G_i be a interval valued neutrosophic support soft set of X. Then, G_i is known as an empty interval valued neutrosophic support soft set, if $g_i(y) = \emptyset$.*

Definition 10. *Let G_i be a interval valued neutrosophic support soft set of X. Then, G_i is known as the universal interval valued neutrosophic support soft set, if $g_i(y) = X$.*

Definition 11. *Let G_i ,G_j be two interval valued neutrosophic support soft set of X. Then, G_i is said to be subset of G_j, if $g_i(y) \subseteq g_j(y)$.*

Example 5. *Two interval-valued neutrosophic support soft sets, G_i and G_j, are constructed as follows:*

G_i	a	b
y_1	[0.6,0.8],[0.8,0.9][0.5,0.6][0.1,0.5]	[0.6,0.8][0.1,0.8][0.3,0.7][0.1,0.7]
y_2	[0.2,0.4][0.5,0.8][0.4,0.3][0.3,0.8]	[0.2,0.8][0.6,0.9][0.5,0.8][0.2,0.3]
y_3	[0.1,0.9][0.2,0.5][0.5,0.7][0.6,0.8]	[0.4,0.9][0.2,0.6][0.5,0.6][0.5,0.7]
y_4	[0.6,0.8][0.8,0.9][0.1,0.9][0.8,0.9]	[0.5,0.7][0.6,0.8][0.7,0.9][0.1,0.8]
y_5	[0.0,0.9][1.0,0.1][1.0,0.9][1.0,1.0]	[0.0,0.9][0.8,1.0][0.3,0.5][0.2,0.5]

G_j	a	b
y_1	[0.7,0.8],[0.7,0.9][0.6,0.6][0.1,0.5]	[0.7,0.9][0.0,0.8][0.4,0.8][0.1,0.6]
y_2	[0.3,0.6][0.5,0.5][0.5,0.3][0.2,0.6]	[0.4,0.8][0.6,0.9][0.5,0.8][0.1,0.2]
y_3	[0.2,1.0][0.2,0.5][0.5,0.7][0.5,0.7]	[0.5,0.9][0.2,0.6][0.6,0.6][0.5,0.5]
y_4	[0.6,0.8][0.8,0.9][0.1,0.7][0.8,0.9]	[0.6,0.8][0.6,0.8][0.9,0.9][0.1,0.4]
y_5	[0.1,1.0][0.9,0.1][1.0,1.0][0.9,0.8]	[0.2,0.9][0.7,0.9][0.3,0.5][0.2,0.5]

Following Definition 11, G_i is a subset of G_j.

Definition 12. *The two interval valued neutrosophic support soft sets, G_i, G_j, such that $G_i \subseteq G_j$, is said to be classical subset of X where every element of G_i does not need to be an element of G_j*

Proposition 1. *Let G_i, G_j, G_k be an interval valued neutrosophic support soft set of X. Then,*

(1) *Each G_n is a subset of G_X, where n= i,j,k;*
(2) *Each G_n is a superset of $G_\emptyset$, where n= i,j,k;*
(3) *If G_i is a subset of G_j and G_j is a subset of G_k, then, G_i is a subset of G_k.*

Proof. The proof of this proposition is obvious. □

Definition 13. *The two interval valued neutrosophic support soft sets of X are said to be equal, if and only if $g_i = g_j$, for all $i, j \in X$*

Proposition 2. *Let X be a non-empty set and G_i, G_j be an interval valued neutrosophic support soft set of X. G_i is a subset of G_j, and G_j is a subset of G_i, if and only if G_i is equal to G_j*

Definition 14. *The complement of the interval valued neutrosophic support soft set, G_i, of X is denoted by G_{i^c}, for all $i \in A$*

(i) *The complement of the empty interval valued neutrosophic support soft set of X is the universal interval valued neutrosophic support soft setof X.*
(ii) *The complement of the universal interval valued neutrosophic support soft set of X is the empty interval valued neutrosophic support soft set of X.*

Theorem 1. *Let G_i, G_j be an interval valued neutrosophic support soft set of X. Then, G_i is a subset of G_j and the complement of G_j is a subset of the complement of G_i.*

Proof. Let G_i, and G_j be an interval valued neutrosophic support soft set of X. By definition, 3.7 G_i is a subset of G_j if $g_i(y) \subseteq g_j(y)$. Then, the complement of $g_i(y) \subseteq g_j(y)$ is $g_i^c(y) \supseteq g_j^c(y)$. Hence, the complement of G_j is a subset of the complement of G_i. □

Example 6. *From Example 4, the complement of G_i is constructed as follows:*

G_{i^c}	a	b
y_1	[0.1,0.5],[0.1,0.2][0.4,0.5][0.6,0.8]	[0.1,0.7][0.2,0.9][0.3,0.7][0.6,0.8]
y_2	[0.3,0.8][0.2,0.5][0.6,0.7][0.2,0.4]	[0.2,0.3][0.1,0.4][0.2,0.5][0.2,0.8]
y_3	[0.6,0.8][0.5,0.8][0.3,0.5][0.1,0.9]	[0.5,0.7][0.4,0.8][0.4,0.5][0.4,0.9]
y_4	[0.8,0.9][0.1,0.2][0.3,0.9][0.6,0.8]	[0.1,0.8][0.2,0.4][0.1,0.3][0.5,0.7]
y_5	[1.0,1.0]0.0,0.9][0.0,0.1][0.0,0.9]	[0.2,0.5][0.0,0.2][0.5,0.7][0.0,0.8]

Definition 15. *The union of the interval valued neutrosophic support soft set of X is denoted by $G_i \cup G_j$ and is defined by $g_i(y) \cup g_j(y) = g_j(y) \cup g_i(y)$ for all $y \in A$.*

Proposition 3. *Let G_i,G_j,G_k be an interval valued neutrosophic support soft set of X. Then,*

(i) $G_i \cup G_\emptyset = G_i$.
(ii) $G_i \cup G_X = G_X$.
(iii) $G_i \cup G_j = G_j \cup G_i$.
(iv) $(G_i \cup G_j) \cup G_k = G_i \cup (G_j \cup G_k)$.

Example 7. *From Example 4, the union of two sets is represented as follows:*

$G_i \cup G_j$	a	b
y_1	[0.7,0.8],[0.7,0.9][0.6,0.6][0.1,0.5]	[0.7,0.9][0.0,0.8][0.4,0.8][0.1,0.6]
y_2	[0.3,0.6][0.5,0.5][0.5,0.3][0.2,0.6]	[0.4,0.8][0.6,0.9][0.5,0.8][0.1,0.2]
y_3	[0.2,1.0][0.2,0.5][0.5,0.7][0.5,0.7]	[0.5,0.9][0.2,0.6][0.6,0.6][0.5,0.5]
y_4	[0.6,0.8][0.8,0.9][0.1,0.7][0.8,0.9]	[0.6,0.8][0.6,0.8][0.9,0.9][0.1,0.4]
y_5	[0.1,1.0]0.1,0.9][1.0,1.0][0.8,0.9]	[0.2,0.9][0.7,0.9][0.3,0.5][0.2,0.5]

Definition 16. *Let G_i,G_j be an interval valued neutrosophic support soft set of X. Then, the intersection of two sets denoted by $G_i \cap G_j$ is defined as $g_i(y) \cap g_j(y) = g_j(y) \cap g_i(y)$ for all $y \in A$.*

Proposition 4. *Let G_i,G_j,G_k be an interval valued neutrosophic support soft set of X. Then,*

(i) $G_i \cap G_\emptyset = G_\emptyset$.
(ii) $G_i \cap G_X = G_i$.
(iii) $G_i \cap G_j = G_j \cap G_i$.
(iv) $(G_i \cap G_j) \cap G_k = G_i \cap (G_j \cap G_k)$.

Proof. The proof is obvious. □

Example 8. *In accordance with Example 4, the intersection operation is performed as follows:*

$G_i \cap G_j$	a	b
y_1	[0.6,0.8],[0.8,0.9][0.5,0.6][0.1,0.5]	[0.6,0.8][0.1,0.8][0.3,0.7][0.1,0.7]
y_2	[0.2,0.4][0.5,0.8][0.3,0.4][0.3,0.8]	[0.2,0.8][0.6,0.9][0.5,0.8][0.2,0.3]
y_3	[0.1,0.9][0.2,0.5][0.5,0.7][0.6,0.8]	[0.4,0.9][0.2,0.6][0.5,0.6][0.5,0.7]
y_4	[0.6,0.8][0.8,0.9][0.1,0.7][0.8,0.9]	[0.5,0.7][0.6,0.8][0.7,0.9][0.1,0.8]
y_5	[0.0,0.9]0.1,0.9][0.9,1.0][1.0,1.0]	[0.0,0.8][0.8,1.0][0.3,0.5][0.2,0.5]

Definition 17. *Let G_i be an interval valued neutrosophic support soft set of X. Then, the union of interval valued neutrosophic support soft setand its complement is not a universal set and it is not mutually disjoint.*

Proposition 5. *Let G_i, G_j be an interval valued neutrosophic support soft set of X. Then, the D'Margan Laws hold.*

(i) $(G_i \cup G_j)^c = G_i^c \cap G_j^c$.
(ii) $(G_i \cap G_j)^c = G_i^c \cup G_j^c$.

Proposition 6. *Let G_i,G_j, G_k be an interval valued neutrosophic support soft set of X. Then, the following hold.*

(i) $G_i \cup (G_j \cap G_k) = (G_i \cup G_j) \cap (G_i \cap G_k)$.
(ii) $G_i \cap (G_i \cup G_j) = (G_i \cap G_j) \cup (G_i \cap G_k)$

Definition 18. *Let G_i, G_j be an interval valued neutrosophic support soft set of X. Then, the difference between two sets is denoted by G_i / G_j and is defined by*

$$g_{i/j}(y) = g_i(y)/g_j(y)$$

for all $y \in A$.

Definition 19. *Let G_i, G_j be an interval valued neutrosophic support soft set of X. Then the addition of two sets are denoted by $G_i + G_j$ and is defined by*

$$g_{i+j}(y) = g_i(y) + g_j(y)$$

for all $y \in A$.

Definition 20. *Let G_i be an interval valued neutrosophic support soft set of X. Then, the scalar division of G_I is denoted by G_i/a and is defined by*

$$g_{i/a}(y) = g_i(y)/a$$

for all $y \in A$.

4. Decision-Making

In this paper, we provide the definition of relationship between the interval valued neutrosophic support soft set, the average interval valued neutrosophic support soft setand the algorithm to get the optimum decision.

Definition 21. *Let G_i be an interval valued neutrosophic support soft set of X. Then, the relationship, R, for G_i is defined by*

$$R_{G_i} = \{r_{G_i}(y,a) : r_{G_i}(y,a) \in \ interval\ valued\ neutrosophic\ support\ set.\ y \in A, a \in X\}$$

where $r_{G_i} : A \setminus X \Rightarrow\ interval\ valued\ neutrosophic\ support\ soft\ set\ (X)$ and $r_{G_i}(y,a) = g_{i(y)}(a)$ for all $y \in A$ and $a \in X$

Example 9. *From Example 4, the relationship for the interval valued neutrosophic support soft set of X is given below.*
$g_{i(y_1)}(a) = \langle [0.6, 0.8], [0.8, 0.9], [0.5, 0.6], [0.1, 0.5] \rangle$,
$g_{i(y_1)}(b) = \langle [0.6, 0.8], [0.1, 0.8], [0.3, 0.7], [0.1, 0.7] \rangle$,
$g_{i(y_2)}(a) = \langle [0.2, 0.4], [0.5, 0.8], [0.4, 0.3], [0.3, 0.8] \rangle$,
$g_{i(y_2)}(b) = \langle [0.2, 0.8], [0.6, 0.9], [0.5, 0.8], [0.4, 0.3] \rangle$,
$g_{i(y_3)}(a) = \langle [0.1, 0.9], [0.2, 0.5], [0.5, 0.7], [0.6, 0.8] \rangle$,
$g_{i(y_3)}(b) = \langle [0.4, 0.9], [0.2, 0.6], [0.5, 0.6], [0.5, 0.7] \rangle$,
$g_{i(y_4)}(a) = \langle [0.6, 0.8], [0.8, 0.9], [0.1, 0.7], [0.8, 0.9] \rangle$,
$g_{i(y_4)}(b) = \langle [0.5, 0.7], [0.6, 0.8], [0.7, 0.9], [0.1, 0.8] \rangle$,
$g_{i(y_5)}(a) = \langle [0.0, 0.9], [1.0, 0.1], [1.0, 0.9], [1.0, 1.0] \rangle$,
$g_{i(y_5)}(b) = \langle [0.0, 0.8], [0.8, 1.0], [0.3, 0.5], [0.2, 0.5] \rangle$.

Definition 22. *Let G_i be an interval valued neutrosophic support soft set of X. For $\mu, \sigma, \omega, \gamma \subseteq [0,1]$, the $(\mu, \sigma, \omega, \gamma)$-level support soft set of G_i defined by $\langle G_i; (\mu, \sigma, \omega, \gamma) \rangle = \{(y_i, \{a_{ij}: a_{ij} \in X, \mu(a_{ij}) = 1\}) : y \in A\}$, where*

$$\mu(a_{ij}) = \begin{cases} 1, & if\ \ (\mu, \sigma, \omega, \gamma) \leq g_i(y_i)(a_j) \\ 0, & if\ \ otherwise \end{cases}. \text{ For all } a_j \in X.$$

Definition 23. *Let G_i be an interval valued neutrosophic support soft set of X. The average interval valued neutrosophic support soft set is defined by $\langle \mu, \sigma, \omega, \gamma \rangle Avg_{G_i}(y_i) = \sum_{a \in X} g_{i(y_i)}(a)/|X|$ for all $y \in A$*

Example 10. *Considering Example 4, the average interval valued neutrosophic support soft set is calculated as follows:*

$$\langle \mu, \sigma, \omega, \gamma \rangle Avg_{G_i}(y_1) = \sum_{i=1}^{2} g_{i(y_1)}(a)/|X| = \langle [0.6, 0.8], [0.45, 0.85], [0.4, 0.65], [0.1, 0.6] \rangle$$

$$\langle \mu, \sigma, \omega, \gamma \rangle Avg_{G_i}(y_2) = \sum_{i=1}^{2} g_{i(y_2)}(a)/|X| = \langle [0.2, 0.6], [0.55, 0.85], [0.45, 0.55], [0.25, 0.55] \rangle$$

$$\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}(y_3) = \sum_{i=1}^{2} g_{i(y_3)}(a)/|X| = \langle[0.25, 0.9], [0.2, 0.55], [0.5, 0.65], [0.55, 0.75]\rangle$$

$$\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}(y_4) = \sum_{i=1}^{2} g_{i(y_4)}(a)/|X| = \langle[0.55, 0.75], [0.7, 0.85], [0.4, 0.8], [0.45, 0.85]\rangle$$

$$\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}(y_5) = \sum_{i=1}^{2} g_{i(y_5)}(a)/|X| = \langle[0.0, 0.85], [0.9, 0.55], [0.65, 0.7], [0.6, 0.75]\rangle$$

Theorem 2. *Let X be a non-empty set and G_i, G_j be an interval valued neutrosophic support soft set of X. $\{G_i;\langle\mu_1,\sigma_1,\omega_1,\gamma_1\rangle\}$ and $\{G_i;\langle\mu_2,\sigma_2,\omega_2,\gamma_2\rangle\}$ are level support soft sets if $\langle\mu_1,\sigma_1,\omega_1,\gamma_1\rangle \leq \langle\mu_2,\sigma_2,\omega_2,\gamma_2\rangle$. Then, $\{G_i;\langle\mu_1,\sigma_1,\omega_1,\gamma_1\rangle\} \leq \{G_i;\langle\mu_2,\sigma_2,\omega_2,\gamma_2\rangle\}$.*

Proof. Let G_i and G_j be an interval valued neutrosophic support soft set of X. In accordance with Definition 3.2 (iv), each function is $\mu_1 \leq \mu_2,\ \sigma_1 \leq \sigma_2,\ \omega_1 \leq \omega_2,\ \gamma_1 \geq \gamma_2$. Thus, the corresponding interval valued neutrosophic support soft set is $\{G_i;\langle\mu_1,\sigma_1,\omega_1,\gamma_1\rangle\} \leq \{G_j;\langle\mu_2,\sigma_2,\omega_2,\gamma_2\rangle\}$. Hence, the proof. □

The following algorithm is used to make decisions in an interval-valued neutrosophic support soft set.

Algorithm 1:

(1) Enter the interval valued neutrosophic support soft set, G_i;
(2) Enter the average interval valued neutrosophic support soft set, $\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}$, using average-level decision rules to make decisions;
(3) Determine the average-level support soft set, $G_i;\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}$;
(4) Present the level support soft set in tabular form;
(5) Determine the choice value, C_{v_i}, of a_i for any $a \in X$;
(6) Select the optimum value for the optimum decision, $C_{v_i} = \max_{a_i \in X} C_{v_i}$.

Example 11. *People who are affected by cancer, have a combination of treatments, such as surgery with chemotherapy and/or radiation therapy, hormone therapy, and immunotherapy. Our main objective is to find the best treatment from the above mentioned therapies. However, all the treatments can cause side effects. Our goal is to find the best treatment which cause the least side effects, reduce the cost of the treatment, extend the patient's life, cure the cancer and control its growth using an interval-valued neutrosophic support soft set.*

G_i	*a*	*b*
y_1	[0.4,0.7][0.8,0.8][0.4,0.8][0.3,0.5]	[0.3,0.6][0.3,0.8][0.3,0.7][0.3,0.8]
y_2	[0.1,0.3][0.6,0.7][0.2,0.3][0.3,0.8]	[0.2,0.7][0.7,0.9][0.3,0.6][0.3,0.4]
y_3	[0.2,0.6][0.4,0.5][0.1,0.5][0.7,0.8]	[0.4,0.9][0.1,0.6][0.3,0.8][0.5,0.7]
y_4	[0.6,0.9][0.6,0.9][0.6,0.9][0.6,0.9]	[0.5,0.9][0.6,0.8][0.2,0.8][0.1,0.7]
y_5	[0.0,0.9]1.0,1.0][1.0,1.0][1.0,1.0]	[0.0,0.9][0.8,1.0][0.1,0.4][0.2,0.5]

G_i	*c*	*d*
y_1	[0.5,0.7][0.8,0.9][0.4,0.8][0.2,0.5]	[0.3,0.6][0.3,0.9][0.2,0.8][0.2,0.8]
y_2	[0.0,0.3][0.6,0.8][0.1,0.4][0.3,0.9]	[0.1,0.8][0.8,0.9][0.2,0.9][0.3,0.5]
y_3	[0.1,0.7][0.4,0.5][0.2,0.8][0.8,0.9]	[0.2,0.5][0.5,07][0.3,0.6][0.6,0.8]
y_4	[0.2,0.4][0.7,0.9][0.6,0.8][0.6,0.9]	[0.3,0.9][0.6,0.9][0.2,0.8][0.3,0.9]
y_5	[0.0,0.2][1.0,1.0][1.0,1.0][1.0,1.0]	[0.0,0.1][0.9,1.0][0.2,0.2][0.2,0.9]

1. *The average interval valued neutrosophic support soft set is determined as follows:*

$$\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i} = \{\langle(0.375,0.65),(0.55,0.85),(0.325,0.775),(0.25,0.6),\rangle/y_1,\langle(0.125,0.575),(0.675,0.825),(0.2,0.5),(0.3,0.65)\rangle/y_2,\langle(0.225,0.675),(0.35,0.575),(0.225,0.675),(0.65,0.8)\rangle/y_3,\langle(0.4,0.775),(0.625,0.875),(0.4,0.825),(0.4,0.85)\rangle/y_4,\langle(0.0,0.525),(0.825,1.0),(0.575,0.625),(0.6,0.85)\rangle/y_5\};$$

2. $\{G_i;\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}\} = \{(y_2,b),(y_3,b),(y_4,a),(y_5,b)\};$
3. *The average-level support soft set, $\{G_i;\langle\mu,\sigma,\omega,\gamma\rangle Avg_{G_i}\}$ is represented in tabular form.*

X	a	b	c	d
y_1	0	0	0	0
y_2	0	1	0	0
y_3	0	1	0	0
y_4	1	0	0	0
y_5	0	1	0	0

4. *Compute the choice value, C_{v_i}, of a_i for all $a_i \in X$ as*

$$C_{v_3} = C_{v_4} = \sum_{j=1}^{4} a_{3j} = \sum_{j=1}^{4} a_{4j} = 0,\ C_{v_1} = \sum_{j=1}^{4} a_{1j} = 1,\ C_{v_2} = \sum_{j=1}^{4} a_{2j} = 3;$$

5. *C_{v_2} gives the maximum value. Therefore b is the optimum choice.*

   Now, we conclude that there are a few ways to get rid of cancer, but surgery chemotherapy is preferred by most of the physicians with respect to the cost of treatment and extending the life of the patient with the least side effects. Moreover, side effects will be reduced or vanish completely after finished chemotherapy, and the cancer and its growth will be controlled.

5. Conclusions and Future Work

Fuzzy sets are inadequate for representing some parameters. Therefore, intuitionistic fuzzy sets were introduced to overcome this inadequacy. Further, neutrosophic sets were introduced to represent the indeterminacy. In order to make decisions efficiently, we offer this new research work which does not violate the basic definitions of neutrosophic sets and their properties. In this paper, we add one more function called the support function in interval-valued neutrosophic soft set, and we also provide the basic definition of interval valued neutrosophic support soft set and some of its properties. Further, we framed an algorithm for making decisions in medical science with a real-life problem. Here, we found the best treatment for cancer under some constraints using interval valued neutrosophic support soft set. In the future, motivated by the interval valued neutrosophic support soft set, we aim to develop interval valued neutrosophic support soft set in ideal topological spaces. In addition, weaker forms of open sets, different types of functions and theorems can be developed using interval valued neutrosophic support soft set to allow continuous function. This concept may be applied in operations research, data analytics, medical sciences, etc. Industry may adopt this technique to minimize the cost of investment and maximize the profit.

Author Contributions: All authors have contributed equally to this paper. The individual responsibilities and contributions of all authors can be described as follows: the idea of this whole paper was put forward by M.P. and M.K. R.U. and M.K. completed the preparatory work of the paper. F.S. and S.J. analyzed the existing work. The revision and submission of this paper was completed by M.P. and F.S.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control* **1965**, *8*, 338–353. [CrossRef]
2. Atanassov, K.T. Intuitionstic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
3. Molodtsov, D. Soft set theory—First results. *Comput. Math. Appl.* **1999**, *37*, 19–31. [CrossRef]
4. Maji, P.K. Neutrosophic soft sets. *Ann. Fuzzy Math. Inf.* **2013**, *5*, 157–168.
5. Maji, P. K.; Roy, A.R.; Biswass, R. An application of soft sets in a decision making problem. *Comput. Math. Appl.* **2002**, *44*, 1077–1083. [CrossRef]
6. Parimala, M.; Smarandache, F.; Jafari, S.; Udhayakumar, R. On Neutrosophic $\alpha\psi$ -Closed Sets. *Information* **2018**, *9*, 103. [CrossRef]
7. Parimala, M.; Karthika, M.; Dhavaseelan, R.; Jafari, S. On neutrosophic supra pre-continuous functions in neutrosophic topological spaces. In *New Trends in Neutrosophic Theory and Applications*; European Union: Brussels, Belgium, 2018; Volume 2, pp. 371–383.
8. Smarandache, F. *Neutrosophy. Neutrosophic Probability, Set, and Logic*; ProQuest Information & Learning: Ann Arbor, MI, USA, 1998; 105p.
9. Broumi, S.; Smarandache, F. Intuitionistic neutrosophic soft set. *J. Inf. Comput. Sc.* **2013**, *8*, 130–140.
10. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*; Neutrosophic Book Series, No. 5; Hexis: Staffordshire, UK, 2005.
11. Thao, N.X.; Smarandache, F.; Dinh, N.V. Support-Neutrosophic Set: A New Concept in Soft Computing. *Neutrosophic Sets Syst.* **2017**, *16*, 93–98.
12. Deli, I. Interval-valued neutrosophic soft sets and its decision making. *Int. J. Mach. Learn. Cyber.* **2017**, *8*, 665–676. [CrossRef]
13. Cagman, N.; Citak, F.; Enginoglu, S. FP-soft set theory and its applications. *Ann. Fuzzy Math. Inform.* **2011**, *2*, 219–226.