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Designing of control chart of extended EWMA statistic using repetitive sampling scheme

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ABSTRACT

In this paper, we propose extended exponential weighted moving average (EEWMA) control chart (CC) for the surveillance the process mean using repetitive sampling scheme under the assumption that quality characteristic of a product is a quantitative variable following the normal distribution. Two control limits named as inner and outer control limits of the proposed CC are derived using the in-control process mean and variance. The necessary measures are derived to obtain the process in-control and out-of-control average run length (ARL) and tabulated for different process shifts and smoothing constants. Comparisons are made to check the capability of intended CC with the EEWMA CC and repetitive EWMA CC in term of ARL. It has been observed that the presented CC shows better results in term of early detection of divergence in process mean. A simulation study is carried out to illustrate the performance of the proposed CC. An industrial example has been incorporated for its practical purpose.

1. Introduction

There are so many brands of the product in a market but few of them are most popular with customers because of their quality and nature of economy. Quality is assessed by the important feature of the product and manufacturers are very careful about the standards of the product. Statistical process monitoring (SPM) is a major instrument in the manufacturing process to monitor these standards and provide signals to engineers if variation exists in manufacturing products. Variations are categories into two main types, one is recognized as a natural cause of variation and process is said to be in-control if these variations exist in the production process, other types of variations are the non-random/assignable cause of variation. With the existence of this type of variation process is said to out-of-control. In this situation, engineers have to take corrective action to avoid defective items. The CCs are the main instrument in SPM to examine the inconsistency in the ongoing process.

Walter Shewhart developed CC first time in around 1920. Shewhart CCs are not helpful for recognition of minor variations in the ongoing process, because it uses only current information and disregards the previous information lying in earlier samples. That is why we can say that these charts are not so much effective when we are concerned with the detection of minor shifts. To identify the early shift in the production process, the most accepted CCs in literature are exponential weighted moving average (EWMA) CCs.

The idea of these CCs was given by [35]. The reason behind the early detection of variations is that EWMA statistic utilized current as well as previous information along with their weights. Most of the researchers have used this statistic to enhance the capability of their CCs like [1] merged the cumulative sum and EWMA statistic and explained that after merging these chart the efficiency of the new chart is improved in term of smaller ARLs. Aslam et al. [13] used EWMA statistic in t-chart and prove the better results of intended CC in term of capturing the smaller variation. Abbas [2] suggested the mixture of regression estimator and EWMA statistic and showed that this estimator is performed better in

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recognition of minor shifts. Khan et al. [26] developed moving average CC using EWMA statistic when the quality feature follows the exponential distribution. Ahamad et al. [8] proposed the double moving average CC using EWMA statistic for exponentially distributed life. Recently Naveed et al. [31] suggested an Extended EWMA CC which has the ability to identify the smaller shifts more quickly in an ongoing process as compared to the EWMA CC. Recently Sansui et al. [37] proposed four different EWMA-type strategies for jointly observing the process location and dispersion under the assumption of Gaussian process. The authors used maximum likelihood estimator for watching the process parameters and combined them in one estimator using 'max' and 'distance' based combining function. The findings show the better performance of 'distance' type function as compared to the 'max' type function. Generally, the all proposed ideas are much better in searching the minor shifts very efficiently than the existing idea. More detail about the application and design of EWMA CC can be seen in [22,29,22,3,30,5,9,34,4,28,14,10].

Due to the advancement in CC, researchers have developed more effective CC using different sampling technique like sequential sampling, multiple dependent state sampling, repetitive sampling scheme (RSS). RSS has gained more attention to the researcher due to its ease and effectiveness. In RSS, two pairs of control limits are used nominated as inner and outer control limits. The operational procedure of repetitive CC is working such that if the value of statistic falls within the inner control limit we say that process is working in the control condition. The process is functioning in out-of-control condition if the value of statistic falls outside the control limits. If the value of statistic falls between inner and outer control limit, the procedure is repeated until the out-of-control or in-control decision is reached. Aslam et al. [15] suggested first time CC using RSS and compared the results with [36] which is based on single sampling and showed the outstanding performance of repetitive CC in term of smaller ARLs. Later on, many researchers have developed CC using RSS like [12,18,7,27,16,11,6,25,19,17]. Recently Huang et al. [24] proposed Generally weighted moving average (GWMA) CC using two parameters for identifying the minor variation in process mean using RSS. After that chen et al. [23] presented a RSS based nonparametric GWMA sign chart for searching the smaller alteration in process shifts. After that Nawaz et al. [33] proposed memory based CC using RSS for non-normal processes. Motivated by the performance of RSS, here we will construct an EEWMA CC using RSS.

Naveed et al. [32] proposed the control chart using EEWMA statistics for multiple dependent state sampling. The assumptions of the multiple dependent state sampling are different from RSS, see for example [21,20]. In this paper, we will concentrate on the development of CC using EEWMA statistic under RSS for monitoring process mean (known case) of phase-I. The detail discussion about the construction of the intended CC will be discussed in Section 2. In Section 2.1 advantages of the suggested CC are discussed. A simulation study is conducted to check the performance of the suggested CC in Section 3. In Section 4, an industrial example is incorporated to describe the execution of suggested CC. Finally, concluding statements are examined in Section 5.

2. Designing of proposed control chart

Suppose that production process is working under control condition and quality feature of the manufactured goods follow the normal distribution with mean μ and variance σ^2. Furthermore, the EEWMA statistic developed by [31] is given as

$$Y_i = \lambda_1 \bar{X}_i - \lambda_2 \bar{X}_{i-1} + (1 - \lambda_1 + \lambda_2) Y_{i-1} \tag{1}$$

where λ_1 *and* λ_2 are smoothing constants (weights) such that $0 < \lambda_1 \le 1$ *and* $0 \le \lambda_2 < \lambda_1$ under the condition that sum of weights is unity. The quantity $\bar{X}_{i-1}$ depicts the prior value of study variable and Y_{i-1} symbolizes the past values of the statistic. The value of $\bar{X}_0$ *and* Y_0 are taken as equal to target mean value. The proposed statistic is transformed to standard EWMA statistic if $\lambda_2 = 0$. EWMA statistic is suffered from the inertia effect for smaller value of λ. ([38]). The behavior of EEWMA control chart is quite similar to EWMA control chart. EEWMA control chart is also suffered from the inertia effect for smaller value of λ_1. The mean and variance of EEWMA statistic are given as

$$E(Y_i) = \mu \tag{2}$$

$$var(Y_i) = \frac{\sigma^2}{n}\left[(\lambda_1^2 + \lambda_2^2)\left\{\frac{1-\theta^{2i}}{1-\theta^2}\right\} - 2\theta\lambda_1\lambda_2\left\{\frac{1-\theta^{2i-2}}{1-\theta^2}\right\}\right] \tag{3}$$

where $\theta = (1 - \lambda_1 + \lambda_2)$ *and* μ, σ^2 symbolizes the target mean and variance of X_i respectively. Hence the control limits termed as inner and outer control limits of EEWMA statistic using RSS are given as

$$LCL_1 = \mu_0 - k_1\sqrt{\frac{\sigma^2}{n}\left[(\lambda_1^2 + \lambda_2^2)\left\{\frac{1-\theta^{2i}}{1-\theta^2}\right\} - 2\theta\lambda_1\lambda_2\left\{\frac{1-\theta^{2i-2}}{1-\theta^2}\right\}\right]} \tag{4}$$

$$UCL_1 = \mu_0 + k_1\sqrt{\frac{\sigma^2}{n}\left[(\lambda_1^2 + \lambda_2^2)\left\{\frac{1-\theta^{2i}}{1-\theta^2}\right\} - 2\theta\lambda_1\lambda_2\left\{\frac{1-\theta^{2i-2}}{1-\theta^2}\right\}\right]} \tag{5}$$

$$LCL_2 = \mu_0 - k_2\sqrt{\frac{\sigma^2}{n}\left[(\lambda_1^2 + \lambda_2^2)\left\{\frac{1-\theta^{2i}}{1-\theta^2}\right\} - 2\theta\lambda_1\lambda_2\left\{\frac{1-\theta^{2i-2}}{1-\theta^2}\right\}\right]} \tag{6}$$

$$UCL_2 = \mu_0 + k_2\sqrt{\frac{\sigma^2}{n}\left[(\lambda_1^2 + \lambda_2^2)\left\{\frac{1-\theta^{2i}}{1-\theta^2}\right\} - 2\theta\lambda_1\lambda_2\left\{\frac{1-\theta^{2i-2}}{1-\theta^2}\right\}\right]} \tag{7}$$

Here LCL_1, UCL_1 are the outer control limits and LCL_2, UCL_2 are the inner control limits. Also k_1, k_2 are the control coefficients. The repetitive EEWMA CC is reduced to [31] if we use $k_1 = k_2 = k(say)$.The proposed CC is converted to EWMA repetitive CC when $\lambda_2 = 0$. The following algorithm is used to find the ARLs and CC coefficients of the proposed CC.

Algorithm1. Monte Carlo simulation of Extended EWMA CC using RSS for the in-control process.

Following are the steps that are used in the R program to explain the Monte Carlo simulation:**

1. Computation of EEWMA statistic Y_i.
   1.1. Mention the value of desired in-control ARL, designated by r_0 and smoothing constants λ_1, λ_2.
   1.2. Generate X, a random sample of size n at each subgroup from a normal distribution having specified parameters for the in-control process; that is, $X\ N(0,1)$. Generate 2000 such subgroups.
   1.3. Calculate the value of suggested EEWMA statistic Y_i.
2. Computation of the variable control limits.
   2.1 Select the appropriate values of k_1, k_2 using iterative method such that under control ARL of the repetitive EEWMA CC achieve the required value of r_0.

2.2. Calculate $LCL_{1(i)}, UCL_{1(i)}, LCL_{2(i)}$ and $UCL_{2(i)}$ from 2000 subgroups.

2.3. Keep in mind the working procedure of the suggested CC, whether the ongoing process should be acknowledged as a control condition, in the repeated situation, or out-of-control condition. If the running process is established as in-control, repeat steps 1.1–2.3. If the ongoing process is in the repetitive situation, count the number of repetition. Otherwise, define the run length to be the number of subgroups, together with the number of repetition, i.e. the time for which the running process was stated to be either in repeated mode or in-control before being announced to be out-of-control.

3. Compute the average run length (ARL) and average sample number (ASN).

3.1. Repeat steps 1.1–2.3 for a large number of times (say 10,000) to calculate the in-control ARL from run length and ASN from Resampling situation. If the calculated in-control ARL is equal to the desiredARL_0, then stop the process and go to Algorithm 2. Otherwise, adjust the values of the CC coefficients and repeat steps 1.1–3.1.

3.2. Settle the values of $k_1 and k_2$ such that $ARL_0 \geq r_0$.

Algorithm2. Monte Carlo simulation of EEWMA CC using RSS for shifted process mean$\mu_0 to \mu_1$. where $\mu_1 = \mu_0 + c\sigma$.

The following are the steps that are used in the R program to explain the Monte Carlo simulation:

1. Computation of intended EEWMA statistic Y_i.

1.1. Mention the values of smoothing constants λ_1, λ_2 and shift c.

1.2. Generate$X$, a random sample of size n at each subgroup from a normal distribution for the shifted process; that is, $X\ N(\mu_1, 1)$. Generate 2000 such subgroups.

1.3. Calculate the value of EEWMA statistic Y_i.

2. Computation of the variable control limits.

2.1. Take the values of CC coefficients k_1, k_2 from the result of Algorithm 1 with the condition that selected combination gives smaller ARL for shifted process.

2.2. Calculate $LCL_{1(i)}, UCL_{1(i)}, LCL_{2(i)}$ and $UCL_{2(i)}$ from 2000 subgroups.

3. Same working procedure is repeated as mentioned in algorithm 1 step 2.3

4. Compute the average run length (ARL_1) and ASN for the shifted process.

4.1. Repeat steps 1.1–2.3 for a large number of times (say 10,000) to calculate ARL_1 and ASN for the shifted process.

The values of ARL_1, ASN and Standard deviation of run length (SDRL) for various smoothing constants using various values of shifted constant $c$are given in Tables. 1–8.

Tables 1–8 represent the different levels of ARLs using various combinations of weights and different subgroup sizes. For the selection of λ_1 *and* λ_2,Naveed et al. [31] sguested that λ_1 should be greater thanλ_2. While selecting the value of λ_2, the performance of Naveed et al. [31] is much better than its competitor charts in term of ARL of all shifts in process mean when the value of λ_2 is close to zero. Wheras for smaller shifts in process mean Naveed et al. [31] gives better results in the form of ARL when the selected value of λ_2 is close to λ_1.

Aslam et al. [12] proposed first time EWMA based non parametric CC using RSS. They did not consider the ASN and used only the lower out-of –control ARL, named as ARL_1 using various process shifts. As we know that the cost of firm has increased for larger repetitive sampling. Later on [23] developed non parametric GWMA sign chart using RSS considered the lower value of ASN as well as lower ARL_1 based on the selection of its parameters $k_1 and k_2$. By motivating the concept of the selection of the combination of the parameters, two types of simulation have been carried out, one for lower ASN named as EEWMA-SN and other one for lower ARL named as EEWMA-RL. Tables 1–4 present the ARL values based on lower ARL and Tables 5–8 show the ARL values based on lower ASN when $r_o = 500, n = 5\ and\ 7$.

From these Tables we examine that when the value of shifted parameter $c = 0$ the ARL is neighboring to r_0. We note the declining tendency in the values of ARL by introducing shifts in process mean. We note that for same value of ARL_0 say 500, both types of ARL has decreased more sharply for larger value of sample size. For example when $n = 5, \lambda_1 = 0.10, \lambda_2 = 0.03, c = 0.03, EEWMA-LR = 355.40$ and for $n = 7, EEWMA - LR = 321.60$. Similarly when $n = 5, \lambda_1 = 0.20, \lambda_2 = 0.07, c = 0.04, EEWMA - SN = 369.95$ and for $n = 7, EEWMA - SN = 334$. We also note the smaller values of ARLs against the larger shifts. For example when $ARL_0 = 500, n = 5, c = 0.04, \lambda_1 = 0.10, \lambda_2 = 0.03, EEWMA - LR = 289.10$, and for shifted amount $c = 0.20, EEWMA - LR = 13.09$.Similarly for the case when sample size is 7 and $ARL_0 = 5000, c = 0.05, \lambda_1 = 0.30, \lambda_2 = 0.15$, $EEWMA - SN = 313.70$ and for $c = 0.40, EEWMA - SN = 9.46$.

We also see the fast-declining trend in the values of ARL using EEWMA-LR as compare to EEWMA-SN. For example when $ARL_0 = 500, n = 5, c = 0.05, \lambda_1 = 0.10, \lambda_2 = 0.03, EEWMA - LR = 228.60$, and for EEWMA-SN it is 268.81.

Moreover, the performance of offered chart is more effective for smaller value of smoothing constants. For example when $ARL_0 = 500, n = 7, c = 0.03, \lambda_1 = 0.10, \lambda_2 = 0.03$, $\mathrm{EEWMA - LR} = 321.60$, when we use the larger combination of weight say $\lambda_1 = 0.20$, $\lambda_2 = 0.07$, $\mathrm{EEWMA - LR} = 378.50, for \lambda_1 = 0.30, \lambda_2 = 0.15$, $\mathrm{EEWMA - LR} = 405.10, for \lambda_1 = 0.50, \lambda_2 = 0.25$, $\mathrm{EEWMA - LR} = 436.03$. We also notice that larger values of ASN for the case of EEWMA-LR and smaller values for the case of EEWMA-SN.

2.1. Advantages of proposed CC

In this section, advantages of repetitive EEWMA CC based on lower ARL (EEWMA-LR) as compared to EEWMA CC developed by Naveed et al. [31] and repetitive EWMA CC (EWMA-LR) have been discussed.

2.1.1. Comparison of repetitive EEWMA-LR CC versus Naveed et al. [31]

Here we discuss the superiority of the scheduled chart as compared to Naveed et al. [31]. The values of ARLs using EEWMA CC are reported in Table 9. We perceive the improved results in the form of quick detection of the proposed CC for different shifts. For example when $r_0 = 500, n = 5, \lambda_1 = 0.1, \lambda_2 = 0.03, k_1 = 2.964$, $k_2 = 0.978\ and\ c = 0.03$, the value of ARL for repetitive EEWMA-LR CC is 355.40 and for [31] it was 382.0 when $r_0 = 500, \lambda_1 = 0.1$, $\lambda_2 = 0.03, k = 2.8248, n = 5\ and\ c = 0.03$. This result indicates that the suggested CC is more suitable than Naveed et al. [31] in pointing out the early shifts in process mean. The comparability of these charts is also displayed in Fig. 1 with the help of ARL.

2.1.2. Comparison of repetitive EEWMA-LR CC versus repetitive EWMA-LR CC

Here we discuss the advantage of repetitive EEWMA-LR CC with repetitive EWMA-LR CC. The values of ARL using repetitive EWMA-LR CC are placed in Table 9. We noticed the early identification in switched process mean with proposed chart as equate it to repetitive EWMA-LR CC. For example when $\lambda_1 = 0.1$, $\lambda_2 = 0.03, n = 5, k_1 = 2.964, k_2 = 0.978, r_0 = 500,\ and\ c = 0.04$ the value of ARL for offered chart is 289.10 with ASN = 158.52 and for repetitive EWMA-LR CC the value of ARL = 322.10 with ASN = 174.33, when $r_0 = 500, n = 5, \lambda = 0.1, k_1 = 2.9658$,

Table 1
The ARLs, ASN and SDRL of EEWMA-RL repetitive CC when.$r_o = 500, n = 5$

	$\lambda_1 = 0.10, \lambda_2 = 0.03$			$\lambda_1 = 0.20, \lambda_2 = 0.07$		
	k_1		k_2	k_1		k_2
	2.964		0.978	3.092		1.062
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	501.90	505.5	228.38	503.91	496.5	196.48
0.03	355.40	356.7	180.24	402.90	406.40	166.39
0.04	289.10	297.2	158.52	355.10	357.80	151.40
0.05	228.60	233.90	134.76	302.50	302.30	133.44
0.07	142.20	145	101.01	207.00	207.50	104.84
0.10	71.34	72.47	66.38	120.10	119.30	73.35
0.12	46.26	46.38	52.02	81.35	81.15	57.81
0.15	26.69	26.12	38.27	48.42	48.75	42.83
0.18	17.94	15.92	28.82	30.26	29.03	32.42
0.20	13.09	12.15	24.13	22.54	20.88	28.08
0.23	9.40	8.532	19.28	15.26	13.76	22.78
0.25	7.87	6.934	17.09	12.20	10.90	19.73
0.30	5.20	4.54	12.42	7.71	6.48	14.32
0.35	3.90	3.127	9.51	5.19	4.12	11.04
0.40	3.11	2.447	7.66	3.86	2.96	8.62
1	1.12	0.37	1.40	1.15	0.40	1.61

Table 2
The ARLs, ASN and SDRL of EEWMA-RL repetitive CC when.$r_o = 500, n = 5$

	$\lambda_1 = 0.30, \lambda_2 = 0.15$			$\lambda_1 = 0.50, \lambda_2 = 0.25$		
	k_1		k_2	k_1		k_2
	3.1415		1.136	3.167		1.158
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	501.2	495.9	171.55	504.68	493.8499	162.95
0.03	424.60	417.80	149.90	458.00	454.84	152.61
0.04	381.80	384.50	142.27	429.25	426.56	142.86
0.05	333.90	328.50	126.38	405.61	400.42	141.42
0.07	253.00	250.50	101.64	337.90	340.33	120.91
0.10	152.60	153.80	73.74	241.54	241.54	96.73
0.12	111.80	111.80	59.75	194.13	190.61	79.84
0.15	71.13	69.62	44.84	134.08	133.15	63.90
0.18	44.69	41.84	34.99	94.03	93.90	50.06
0.20	33.85	31.01	30.29	73.82	72.36	44.09
0.23	23.46	21.05	24.33	52.32	51.41	36.08
0.25	18.97	16.54	21.35	42.07	39.88	31.42
0.30	11.53	9.59	15.63	24.60	22.40	23.17
0.35	7.72	5.88	11.95	15.18	13.31	17.55
0.40	5.41	3.96	9.70	9.94	8.32	13.99
1	1.23	0.50	1.96	1.27	0.54	2.20

Table 3
The ARLs, ASN and SDRL of EEWMA-RL repetitive CC when.$r_o = 500, n = 7$

	$\lambda_1 = 0.10, \lambda_2 = 0.03$			$\lambda_1 = 0.20, \lambda_2 = 0.07$		
	k_1		k_2	k_1		k_2
	2.964		0.978	3.092		1.062
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	502.20	501.40	234.46	502.40	494.20	195.78
0.03	321.60	324.80	165.22	378.50	383.00	158.45
0.04	242.60	243.60	138.96	312.60	312.40	135.47
0.05	181.40	191.10	115.95	254.00	249.80	118.93
0.07	103.70	103.00	84.24	159.60	156.10	88.38
0.10	47.56	47.96	52.38	83.83	83.49	59.72
0.12	30.31	29.69	41.05	55.10	53.37	47.36
0.15	17.51	16.63	29.06	31.44	30.15	33.40
0.18	11.23	10.17	22.00	18.90	17.06	25.63
0.20	8.79	7.90	18.52	14.13	12.63	21.75
0.23	6.48	5.61	14.67	9.73	8.43	16.98
0.25	5.32	4.48	12.92	7.76	6.52	14.82
0.30	3.82	3.08	9.23	5.10	4.00	10.72
0.35	3.02	2.29	7.25	3.71	2.73	8.15
0.40	2.44	1.75	5.79	2.88	2.05	6.38
1	1.05	0.22	0.94	1.07	0.27	1.09

Table 4
The ARLs, ASN and SDRL of EEWMA-RL repetitive CC when.$r_o = 500, n = 7$

	$\lambda_1 = 0.30, \lambda_2 = 0.15$			$\lambda_1 = 0.50, \lambda_2 = 0.25$		
	k_1 3.1415		k_2 1.136	k_1 3.167		k_2 1.158
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	505.60	502.80	168.19	502.88	492.55	161.55
0.03	405.10	406.50	143.23	436.03	433.73	148.60
0.04	348.40	347.80	128.33	415.61	410.61	140.71
0.05	300.00	298.60	113.89	376.25	371.27	129.79
0.07	202.70	202.70	87.14	298.38	293.31	109.82
0.10	115.70	113.30	62.13	197.62	197.63	82.82
0.12	79.61	77.80	48.82	149.20	150.15	68.14
0.15	46.93	44.47	36.32	98.25	99.18	52.32
0.18	29.01	26.45	27.93	62.22	61.41	40.89
0.20	21.94	19.50	23.53	48.85	47.40	34.26
0.23	14.82	12.62	18.65	32.85	30.52	27.84
0.25	11.95	10.03	15.98	25.83	23.97	23.67
0.30	7.40	5.69	11.95	14.66	12.87	17.32
0.35	5.09	3.63	9.06	9.15	7.40	12.83
0.40	3.77	2.56	7.36	6.04	4.66	9.97
1	1.10	0.32	1.38	1.12	0.34	1.54

Table 5
The ARLs, ASN and SDRL of EEWMA-SN repetitive CC when.$r_o = 500, n = 5$

	$\lambda_1 = 0.10, \lambda_2 = 0.03$			$\lambda_1 = 0.20, \lambda_2 = 0.07$		
	k_1 2.83		k_2 2.63	k_1 2.98		k_2 2.70
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	500.01	505.59	1.27	499.98	502.73	1.67
0.03	387.90	383.33	1.2139	415.84	413.68	1.60
0.04	321.61	318.17	1.18	369.95	368.40	1.58
0.05	268.81	266.73	1.15	322.11	315.34	1.52
0.07	182.54	178.16	1.10	233.63	230.48	1.41
0.10	106.86	100.24	1.02	148.45	145.99	1.33
0.12	79.26	71.91	0.96	111.44	104.34	1.26
0.15	53.63	44.99	0.91	74.01	69.24	1.20
0.18	38.23	31.05	0.84	51.94	45.19	1.11
0.20	31.51	25.20	0.79	42.22	36.54	1.05
0.23	24.34	18.43	0.73	31.72	26.46	0.98
0.25	20.90	15.36	0.73	26.76	21.88	0.96
0.30	15.12	10.47	0.63	18.83	14.09	0.88
0.35	11.56	7.71	0.57	14.00	9.78	0.78
0.40	9.32	6.09	0.51	10.77	7.09	0.72
1	2.19	1.06	0.19	2.34	1.12	0.27

Table 6
The ARLs, ASN and SDRL of EEWMA-SN repetitive CC when.$r_o = 500, n = 5$

	$\lambda_1 = 0.30, \lambda_2 = 0.15$			$\lambda_1 = 0.50, \lambda_2 = 0.25$		
	k_1 3.0455		k_2 2.7355	k_1 3.0842		k_2 2.7842
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	502.72	499.18	1.84	501.87	503.89	1.63
0.03	424.92	419.23	1.70	466.93	453.31	1.62
0.04	387.16	389.47	1.66	447.95	444.44	1.60
0.05	352.95	348.08	1.66	411.19	405.23	1.57
0.07	270.88	262.86	1.56	355.34	351.36	1.50
0.10	177.32	173.07	1.40	267.30	266.72	1.41
0.12	137.71	133.15	1.36	216.80	214.39	1.36
0.15	94.64	89.22	1.28	156.92	155.83	1.26
0.18	66.55	60.61	1.21	113.12	109.25	1.17
0.20	53.30	47.52	1.14	94.73	90.21	1.14
0.23	40.48	34.80	1.07	72.34	69.27	1.12
0.25	33.76	27.87	1.03	59.85	56.43	1.04
0.30	23.37	17.93	0.93	39.47	36.00	0.95
0.35	17.02	12.28	0.84	28.26	24.75	0.92
0.40	13.16	8.62	0.77	20.32	16.88	0.82
1	2.69	1.27	0.35	2.98	1.49	0.35

Table 7
The ARLs, ASN and SDRL of EEWMA-SN repetitive CC when.$r_o = 500, n = 7$

	$\lambda_1 = 0.10, \lambda_2 = 0.03$			$\lambda_1 = 0.20, \lambda_2 = 0.07$		
	k_1 2.83		k_2 2.63	k_1 2.98		k_2 2.70
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	503.43	500.92	1.26	497.21	499.19	1.69
0.03	346.63	351.53	1.19	392.83	392.83	1.60
0.04	284.40	285.26	1.17	334.00	334.77	1.54
0.05	221.85	218.07	1.11	280.32	276.57	1.47
0.07	144.17	136.96	1.04	191.67	188.40	1.38
0.10	80.92	74.31	0.95	114.49	109.98	1.25
0.12	58.97	51.39	0.94	83.96	78.26	1.23
0.15	39.49	32.17	0.83	53.99	48.33	1.10
0.18	28.31	21.74	0.7858	37.23	32.02	1.01
0.20	23.72	17.68	0.7547	30.24	25.24	1.00
0.23	18.37	13.06	0.6809	22.61	17.86	0.91
0.25	15.85	10.98	0.66	19.41	14.54	0.86
0.30	11.50	7.65	0.56	13.48	9.37	0.78
0.35	8.84	5.58	0.51	10.12	6.53	0.69
0.40	6.99	4.30	0.45	7.98	4.94	0.63
1	1.73	0.80	0.14	1.83	0.84	0.22
3	1.00	0.00	0.00	497.21	499.19	1.69

Table 8
The ARLs, ASN and SDRL of EEWMA-SN repetitive CC when.$r_o = 500, n = 7$

	$\lambda_1 = 0.30, \lambda_2 = 0.15$			$\lambda_1 = 0.50, \lambda_2 = 0.25$		
	k_1 3.0455		k_2 2.7355	k_1 3.0842		k_2 2.7842
c	ARL	SDRL	ASN	ARL	SDRL	ASN
0	504.43	502.92	1.76	502.72	499.43	1.63
0.03	408.06	406.58	1.71	454.94	446.29	1.59
0.04	363.02	364.65	1.65	419.85	413.19	1.54
0.05	313.70	311.87	1.58	394.39	387.43	1.58
0.07	229.62	226.59	1.51	310.80	303.69	1.44
0.10	140.00	135.55	1.38	220.00	218.20	1.36
0.12	102.49	96.44	1.27	169.72	167.31	1.27
0.15	68.78	63.90	1.18	120.52	117.26	1.19
0.18	47.52	41.31	1.11	84.03	80.10	1.11
0.20	38.01	32.40	1.05	68.77	64.87	1.07
0.23	28.67	23.16	0.97	50.56	46.78	1.03
0.25	24.05	18.51	0.94	41.98	38.02	0.99
0.30	16.42	11.63	0.84	27.13	23.68	0.88
0.35	12.25	7.95	0.77	18.68	15.21	0.79
0.40	9.46	5.76	0.69	13.43	10.14	0.71
1	2.08	0.96	0.29	2.22	1.07	0.29

Table 9
Comparisons of ARLs, ASN and SDRL when.$r_o = 500, n = 5$

	Proposed EEWMA-LR CC			Naveed et al. [31]		Repetitive EWMA-LR CC		
	$\lambda_1 = 0.10$	$\lambda_2 = 0.03$		$\lambda_1 = 0.10$	$\lambda_2 = 0.03$	$\lambda = 0.10$		
	k_1 2.964	k_2 0.978		k 2.8248		k_1 2.9658	k_2 0.9789	
c	ARL	SDRL	ASN	ARL	SDRL	ARL	SDRL	ASN
0	501.90	505.5	228.38	504.70	502.80	505.70	501.10	237.57
0.03	355.40	356.7	180.24	382.00	384.00	388.20	399.00	195.76
0.04	289.10	297.2	158.52	325.20	323.30	322.10	327.20	174.33
0.05	228.60	233.90	134.76	274.30	270.00	259.80	262.80	146.37
0.07	142.20	145	101.01	182.80	178.70	164.40	168.20	109.87
0.10	71.34	72.47	66.38	108.80	98.74	85.72	87.25	74.72
0.12	46.26	46.38	52.02	81.31	75.38	57.09	57.48	59.63
0.15	26.69	26.12	38.27	53.94	45.51	31.79	32.12	43.23
0.18	17.94	15.92	28.82	39.27	32.25	19.87	19.59	32.50
0.20	13.09	12.15	24.13	32.16	25.35	14.76	14.32	26.95
0.23	9.40	8.53	19.28	25.21	18.88	10.07	9.52	21.24
0.25	7.87	6.93	17.09	21.67	15.55	8.28	7.61	18.47
0.30	5.20	4.54	12.42	15.81	11.03	5.36	4.79	13.58
0.35	3.90	3.12	9.51	12.18	7.76	3.96	3.39	10.23
0.40	3.11	2.44	7.66	9.84	6.25	3.17	2.51	7.93
1	1.12	0.37	1.40	2.25	1.11	1.13	0.36	1.26

Table 10
Simulated data.

Sample#	$\bar{X}_i$	Y_i	LCL_1	UCL_1	LCL_2	UCL_2
1	−0.3546	−0.0355	−0.1384	0.1384	−0.0457	0.0457
2	−0.5514	−0.0775	−0.1610	0.1610	−0.0531	0.0531
3	0.3440	−0.0211	−0.1782	0.1782	−0.0588	0.0588
4	0.0749	−0.0225	−0.1919	0.1919	−0.0633	0.0633
5	0.4660	0.0235	−0.2030	0.2030	−0.0670	0.0670
6	0.2987	0.0377	−0.2121	0.2121	−0.0700	0.0700
7	0.9638	0.1225	−0.2196	0.2196	−0.0725	0.0725
8	−0.3796	0.0470	−0.2260	0.2260	−0.0746	0.0746
9	−0.6469	−0.0096	−0.2313	0.2313	−0.0763	0.0763
10	−0.4551	−0.0350	−0.2359	0.2359	−0.0778	0.0778
11	1.0034	0.0815	−0.2397	0.2397	−0.0791	0.0791
12	−0.2025	0.0254	−0.2430	0.2430	−0.0802	0.0802
13	0.0940	0.0391	−0.2458	0.2458	−0.0811	0.0811
14	0.1993	0.0535	−0.2482	0.2482	−0.0819	0.0819
15	−0.3443	0.0093	−0.2503	0.2503	−0.0826	0.0826
16	0.3728	0.0563	−0.2520	0.2520	−0.0832	0.0832
17	0.3892	0.0801	−0.2535	0.2535	−0.0837	0.0837
18	−0.1896	0.0438	−0.2548	0.2548	−0.0841	0.0841
19	0.0792	0.0544	−0.2560	0.2560	−0.0845	0.0845
20	0.8092	0.1291	−0.2569	0.2569	−0.0848	0.0848
21	−0.0327	0.0925	−0.2578	0.2578	−0.0851	0.0851
22	−0.1813	0.0689	−0.2585	0.2585	−0.0853	0.0853
23	0.2735	0.0969	−0.2591	0.2591	−0.0855	0.0855
24	0.5595	0.1378	−0.2596	0.2596	−0.0857	0.0857
25	−0.8176	0.0296	−0.2601	0.2601	−0.0858	0.0858
26	0.5377	0.1059	−0.2605	0.2605	−0.0860	0.0860
27	−0.0702	0.0753	−0.2608	0.2608	−0.0861	0.0861
28	0.6178	0.1339	−0.2611	0.2611	−0.0862	0.0862
29	−0.0883	0.0972	−0.2614	0.2614	−0.0863	0.0863
30	0.5658	0.1496	−0.2616	0.2616	−0.0863	0.0863
31	0.5520	0.1774	−0.2618	0.2618	−0.0864	0.0864
32	−0.0584	0.1425	−0.2620	0.2620	−0.0864	0.0864
33	−0.1038	0.1239	−0.2621	0.2621	−0.0865	0.0865
34	0.4874	0.1671	−0.2622	0.2622	−0.0865	0.0865
35	0.0846	0.1493	−0.2624	0.2624	−0.0866	0.0866
36	0.1521	0.1515	−0.2624	0.2624	−0.0866	0.0866
37	−0.0325	0.1331	−0.2625	0.2625	−0.0866	0.0866
38	−0.7726	0.0475	−0.2626	0.2626	−0.0867	0.0867
39	0.6447	0.1318	−0.2627	0.2627	−0.0867	0.0867
40	0.0727	0.1105	−0.2627	0.2627	−0.0867	0.0867
41	0.0652	0.1071	−0.2628	0.2628	−0.0867	0.0867
42	0.9216	0.1898	−0.2628	0.2628	−0.0867	0.0867
43	1.3511	0.2840	−0.2628	0.2628	−0.0867	0.0867
44	0.2746	0.2510	−0.2629	0.2629	−0.0867	0.0867
45	0.3298	0.2582	−0.2629	0.2629	−0.0867	0.0867
46	0.2680	0.2570	−0.2629	0.2629	−0.0868	0.0868
47	−0.0499	0.2260	−0.2629	0.2629	−0.0868	0.0868
48	−0.2678	0.1849	−0.2629	0.2629	−0.0868	0.0868
49	−0.6312	0.1169	−0.2630	0.2630	−0.0868	0.0868
50	−0.3941	0.0882	−0.2630	0.2630	−0.0868	0.0868

$k_2 = 0.9789$ *and* $c = 0.04$.This shows that suggested CC has capability to identify slighter alteration in process location earlier than repetitive EWMA-LR CC. The analogy of these charts is exhibited in Fig. 2 with the assist of ARL.

3. Simulation study

In this section, a simulation study is conducted to test the achievement of presented CC based on EEWMA-LR using RSS. Assuming that working process is under control and data $X_i$follows the standard normal distribution i.e. $X_i\ N(0,1)$. Firstly, we draw twenty five observations of size five from the IC process then we generated twenty five observation of size five from the changed process presuming that process mean has switched to amount $c = 0.18$. The values of statisticY_i are computed using smoothing constants $\lambda_1 = 0.1, \lambda_2 = 0.03$. The simulation data and values of statistic Y_i are reported in Table 10 and plotted values of statistic under RSS with CC parameters $k_1 = 2.964, k_2 = 0.978$, $n = 5$ *and* $ARL_0 = 500$are shown in Fig. 3. From Fig. 3 we can see that process is OOC at 25 + 18 = 43th observation (same value was reported in Table 1). We also plot the simulation data by Naveed et al. [31] with CC parameter $\lambda_1 = 0.1, \lambda_2 = 0.03$, $k = 2.8248, n = 5$ *and* $ARL_0 = 500$ in Fig. 4. From Fig. 4, we observe that manufacturing process is in control using Naveed et al. [31]. So we can say that EEWMA-LR CC under RSS has ability to identify the smaller variation more quickly as compare to Naveed et al. [31].

4. Industrial application

In this section, industrial data is applied to the proposed CC which is taken from Montgomery (2013) pp 280–281. The data set comprise the 24 observation of size 5 which are obtained from the internal mensuration of the diameter of bearing are enlisted in

Table 11
Inside diameter of bearing.

Sample#	$\bar{X}_i$	Y_i	LCL_1	UCL_1	LCL_2	UCL_2
1	0.50354	0.503418	0.503382	0.503427	0.503397	0.503412
2	0.50342	0.503414	0.503378	0.50343	0.503396	0.503413
3	0.50316	0.503389	0.503375	0.503433	0.503395	0.503414
4	0.50315	0.503372	0.503373	0.503435	0.503394	0.503414
5	0.5035	0.503391	0.503371	0.503437	0.503393	0.503415
6	0.50341	0.50339	0.50337	0.503439	0.503393	0.503416
7	0.50326	0.503376	0.503368	0.50344	0.503392	0.503416
8	0.50338	0.50338	0.503367	0.503441	0.503392	0.503416
9	0.50348	0.50339	0.503366	0.503442	0.503392	0.503417
10	0.50336	0.503384	0.503366	0.503443	0.503392	0.503417
11	0.50319	0.503366	0.503365	0.503443	0.503391	0.503417
12	0.50386	0.50342	0.503365	0.503444	0.503391	0.503417
13	0.50354	0.503419	0.503364	0.503444	0.503391	0.503417
14	0.5034	0.503414	0.503364	0.503445	0.503391	0.503418
15	0.50371	0.503444	0.503363	0.503445	0.503391	0.503418
16	0.50349	0.50344	0.503363	0.503445	0.503391	0.503418
17	0.50335	0.50343	0.503363	0.503446	0.503391	0.503418
18	0.50317	0.503406	0.503363	0.503446	0.503391	0.503418
19	0.5034	0.503413	0.503362	0.503446	0.50339	0.503418
20	0.50351	0.503423	0.503362	0.503446	0.50339	0.503418
21	0.50337	0.503415	0.503362	0.503446	0.50339	0.503418
22	0.50328	0.503403	0.503362	0.503446	0.50339	0.503418
23	0.50335	0.503401	0.503362	0.503446	0.50339	0.503418
24	0.50342	0.503405	0.503362	0.503447	0.50339	0.503418

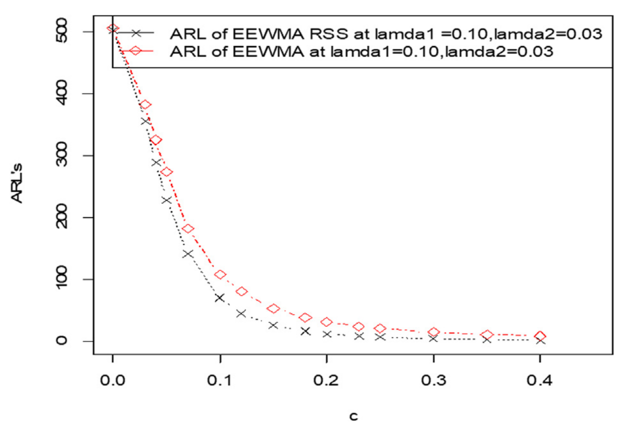

Fig. 1. Graph of comparison of ARLs when.$r_o = 500, n = 5$

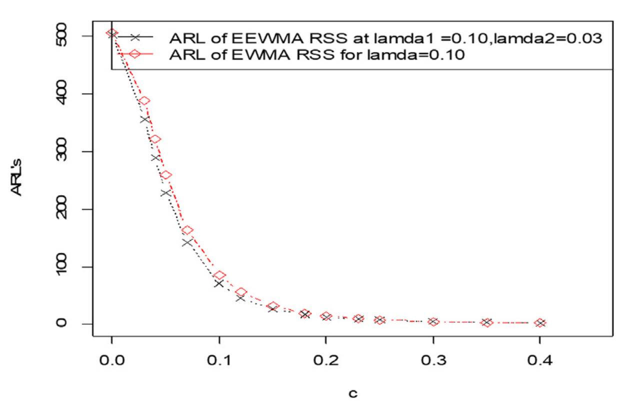

Fig. 2. Graph of comparison of ARLs when.$r_o = 500, n = 5$

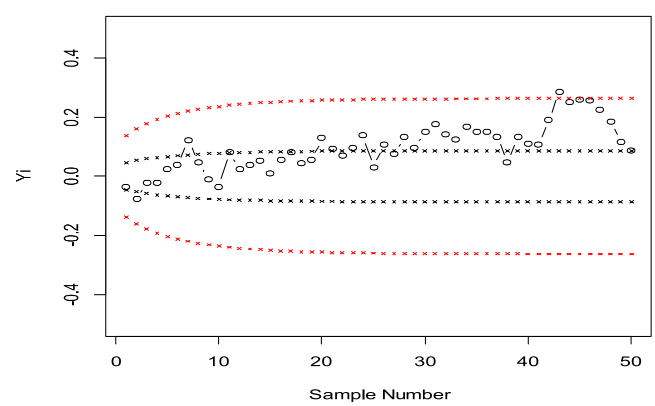

Fig. 3. Simulated data of proposed CC.

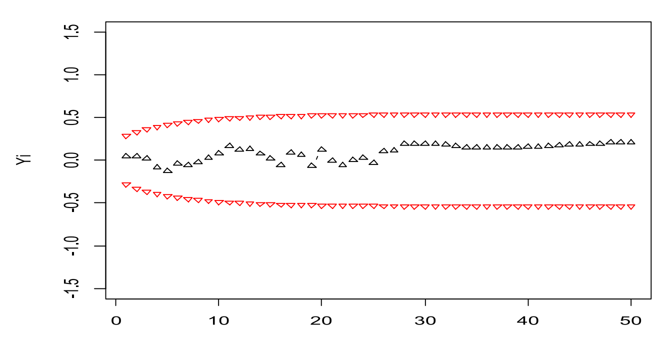

Fig. 4. Simulated data [31].

Table 11. Assume that diameter is normally distributed Montgomery (2013). We calculate the value of statistic Y_i using $\lambda_1 = 0.1, \lambda_2 = 0.03$ are shown Table 11and plotted in Fig. 5 under RSS using CC parameter $\lambda_1 = 0.1, \lambda_2 = 0.03, k_1 = 2.89, k_2 = 0.95, n = 5$ *and* $ARL_0 = 370$.From Fig. 5 we can see the manufacturing process is under control. However, the observations 4 and 11 are close to outer lower control limit and observations 16 and 17 are close to external upper limit. These observations are lie within the repetitive area. Therefore, the industrial engineering should repeat the process to bring the process in in-control state.

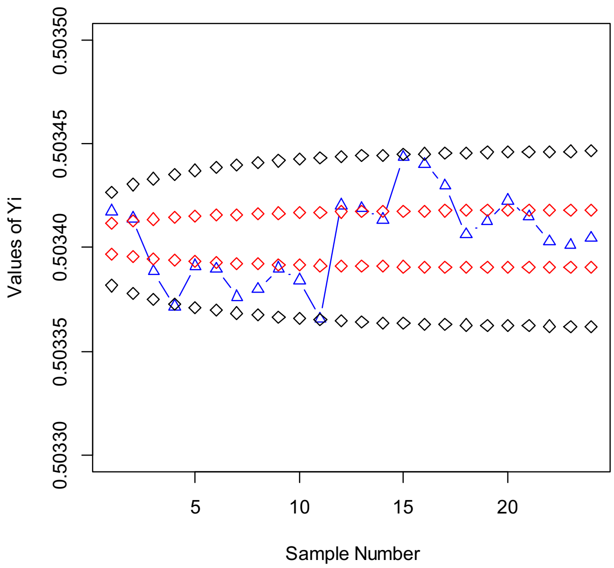

Fig. 5. Industrial data of proposed CC.

5. Concluding remarks

In this paper, we have proposed an EEWMA CC using RSS based on lower ARL as well as lower ASN when the quality features of the product follow the normal distribution. The performance of the proposed CC has been assessed in term of ARL. Comparison of the suggested CC has been made with the EEWMA CC using single sampling and repetitive EWMA-LR CC in term of ARL, which demonstrate the capability of the proposed CC in term of early detection of variation in the process mean. A simulation study has been carried out to check the capability of the proposed CC. In addition, an application of the proposed CC in the field of industry has been demonstrated with the help of internal mensuration of the diameter of bearing data. The proposed chart can be extended for joint monitoring the process mean and variance using RSS for further research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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