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Distance and Similarity Measures for Neutrosophic HyperSoft Set (NHSS) With Construction of NHSS-TOPSIS and Applications

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ABSTRACT Neutrosophic HyperSoft Set (NHSS) is a new approach towards computational intelligence and decision making under uncertainty. In this paper, we first consider distances for NHSS, and then propose similarity measures for NHSS. We also consider aggregated operation for aggregating NHSS decision matrix. TOPSIS (Technique for the order preference by similarity to ideal solution) is a strong approach for multi-criteria decision making (MCDM) which has been studied under various extensions of fuzzy sets. These approaches have drawbacks in depicting fuzzy decision-making information for handling MCDM situations under NHSS environment. To efficiently and accurately express fuzzy attribute values provided by decision-makers (DMs), we construct the TOPSIS based on the proposed distances and similarity measures of NHSS, called NHSS-TOPSIS. The proposed NHSS-TOPSIS provides the weights of DMs by utilizing similarity measures dependent on Hamming distance. We then aggregate the opinions of decision-makers using the proposed aggregated operation. Utilizing the relative closeness coefficient, we select the most ideal alternative in the proposed NHSS-TOPSIS procedures. To exhibit the relevance and adequacy of the proposed NHSS-TOPSIS, we apply it in a medical diagnosis and an optimal selection for the sustainable green security system. The proposed method reveals that the hypersoft set with the neutrosophic set theory can be very helpful to construct a connection between alternatives and attributes. It demonstrates that the proposed method is effective and useful in real applications.

INDEX TERMS Distance, similarity, aggregation operation, neutrosophic hypersoft set (NHSS), MCDM, TOPSIS, NHSS-TOPSIS.

I. INTRODUCTION

Fuzzy set was first proposed by Zadeh [1] in 1965 as an extension of crisp set in which it was widely used to handle fuzziness that is different from randomness in probability. Afterwards, various extensions of fuzzy set were proposed in the literature, such as type-2 fuzzy set [2], intuitionistic fuzzy set (IFS) [3], fuzzy multiset [4], hesitant fuzzy sets (HFS) [5], and pythagorean fuzzy set (PFS) [6]. They had also applied in multi-criteria decision making (MCDM), such as [7]–[10].

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On the other hand, Molodtsov [11] in 1999 proposed soft sets in first results, and then Maji *et al.* [12], [13] offered a hypothetical analysis of soft sets and upper sets of soft sets, equality of soft sets, and operation on soft sets, such as union, intersection, AND, and OR operations between different sets. Ali *et al.* [14] considered new operations in soft set theory which covers restricted union, intersection, and difference. Cagman and Egniloglu [15], [16] presented many results on the soft set theory which strengthens itself a very important measurement while looking after issues to make various choices. From Molodtsov [11] to present various useful applications identified with soft set theory have been presented and related to numerous fields of science and data innovation (see Refs. [12]–[17]).

Smarandache [18] in 2005 considered the idea of the Neutrosophic set which is a scientific gadget to solve issues like uncertain, indeterminant, and opposite information. The neutrosophic set shows truth membership value, indeterminacy membership value, and falsity membership value. This concept is important in so many applications because indeterminacy is checked extraordinarily and truth membership values, indeterminacy membership values, and falsity values are independent. The concept of soft set was first highlighted by Molodtsov [11] to handle issues of indefinite circumstances in which it is a parameterized family of subsets of a universal set. Soft sets are valuable in different ways like artificial insight, game hypothesis, and fundamental decision-making issues, it helps to determine different functions for various parameters and benefit values against established parameters. Over the last two years, the fundamentals of soft set theories have been pondered over by various scholars. Afterward, Maji [19] presented the Neutrosophic soft set shown by truth, indeterminacy, and falsity membership values which are independent. The neutrosophic soft set can handle inadequate, uncertain, and inconstant data while the intuitionistic fuzzy soft set can only deal with partial data. Smarandache [20] came up with a strategy to handle more uncertainty situations to extend soft set to hypersoft set (HSS) by changing functions into multi-decision functions. However, when attributes are more than one and further diverge, HSS cannot help to handle such type of issues. Thus, the neutrosophic hypersoft set (NHSS) was introduced with aggregate operators and tangent similarity measure and applied in MCDM by Saqlain *et al.* [21]–[23] and Zhou *et al.* [24].

In applications, distance and similarity measures are very important for giving degrees of difference and similarity between them. Various distance and similarity measures about various extensions of fuzzy set, soft set, IFS, PFS, HFS, and HSS had been studied and proposed in the literature (see Refs. [25]–[28]). However, there is less distance and similarity measures for NHSS. In this paper, we propose several extended distance and similarity measures for NHSSs. We consider a generalization of other distance measures, such as Hamming distance, Euclidean distance and their normalized types. We also give more aggregation operations for NHSS. These should be useful in applications of multi-criteria decision making (MCDM) under NHSS environment. MCDM is a branch of operations research that explicitly evaluates multiple conflicting criteria in decision making. The purpose of MCDM is to support decision-makers (DMs) facing problems in ranking feasible alternatives/objects. MCDM is a typical matter for everyone. For instance, while purchasing a product from the web, or picking an expert course in a training center, we need to think about different traits. MCDM is a procedure of finding an ideal alternative that has the very best degree of fulfillment from a lot of options associated with multiple clashing characteristics. Enormous procedures have been created for modeling uncertainties in MCDM issues, for example, PROMETHEE [29], TOPSIS (Technique for the order preference by similarity to ideal solution) [25], [30], [31], VIKOR [32], and many more.

In MCDM issues, assessment of attributes cannot be constantly communicated with crisp numbers as a result of the intricate nature of the attributes in real-life problems. Fuzzy set has their roots in membership value or the value that membership carries. Fuzzy MCDM with imprecise information can be used quite well by using the fuzzy set theory into the field of decision making [33]. However, fuzzy set can just concentrate on membership values, but it neglects to consider non-membership values, and so Atanassov [3] presented IFS under the enrollment of membership and non-membership values. Sometimes, there exist indeterminacy levels of uncertain boundaries in MCDM. In this way, IFS cannot deal with vulnerabilities appropriately in MCDM issues in which the issues include uncertain data as an autonomous part. On the other hand, TOPSIS strategy proposed by Hwang and Yoon [30] is one of the strategies providing appropriate solutions. The main idea of TOPSIS is that the best alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS) simultaneously. TOPSIS is very popular to deal with MCDM. Behzadian *et al.* [31] presented a detailed survey of TOPSIS applications in different fields.

Multi-attributive group decision-making problems consist of several attributes and indeterminacy. We mentioned that HSS cannot handle such type of issues. To deal with such types of multi-attributes with indeterminacy, NHSS proposed by Saqlain *et al.* [21] can be used because NHSS not only deals vagueness and uncertainty, but also deal multi-attributes with indeterminacy. In some real life problems, the selection of optimal choice becomes difficult due the dependence of the attributes and its further bifurcation. Fuzzy TOPSIS, neutrosophic TOPSIS and generalized fuzzy TOPSIS fails in such situations. To overcome with these issues, NHSS-TOPSIS has an ability to deal with such situations. We can fill the research gap with NHSS and NHSS-TOPSIS. We mention that NHSS is an extension of soft set that is a new topic. Although Saqlain *et al.* [21]–[23] and Zhou *et al.* [24] had studied and applied NHSS, they focus on generalization of TOPSIS for NHSS using accuracy function, tangent similarity, and aggregate operators. Up to now, no one considers distances for NHSS and similarity measures based on distance for NHSS that can be a new tool to construct the NHSS-TOPSIS method. Thus, the contributions of the paper can be summarized as follows:

1. To propose distances for NHSS, and then give similarity measures for NHSS.
2. To consider aggregated operation for aggregating NHSS decision matrix.
3. To construct the NHSS-TOPSIS based on the proposed distances and similarity measures of NHSS.
4. To exhibit the relevance and adequacy of the proposed NHSS-TOPSIS and then apply it in a medical diagnosis

TABLE 1. TOPSIS comparisons under different situations.

Methods	multi-attributive decision Problem	Multi-attributive further bifurcated problem using accuracy function	Multi-attributive further bifurcated problem without Using Accuracy Function
Extension of TOPSIS [32]	Yes	No	No
Generalized TOPSIS [22]	Yes	Yes	No
NHSS-TOPSIS [proposed]	Yes	Yes	Yes

and an optimal selection for the sustainable green security system.

To briefly present the comparison of the proposed method with existing methods, such as [22] and [32], we consider a simple case study on the selection of a Lecturer at University level. The post form is looked as (Ph.D., 4-year experience, 4 Q1 publications). The HR department has four candidates represented with C1, C2, C3 and C4 who have details as follows:

C1=(Master, 10-years experience, 10 Q1 Publications)
C2=(Ph.D., 10-year experience, 3 Q1 Publications)
C3=(Ph.D., 7-years experience, 3 Q1 Publications)
C4=(Ph.D., 2-year experience, 1 Q1 Publication)

The selection panel (decision-makers) wants to select the best candidate among above all with the help of existing methods. Thus, for more accurate and precise results in the selection of the candidate having variety of parameters which are further bifurcated, there is a research gap. By applying the existing methods [21], [32] and the proposed NHSS-TOPSIS, different status are shown in Table 1. In the status of multi-attributive further bifurcated problem without using accuracy function, only the proposed NHSS-TOPSIS can handle it.

The remainder of the paper is organized as follows. In Section II, we first review some basic definitions of neutrosophic soft set, hypersoft set, and neutrosophic hypersoft set with Neutrosophic hypersoft matrix. In Section III, we propose extended distance and similarity measures for NHSS. In Section IV, we construct the NHSS-TOPSIS for MCDM using our proposed distance and similarity measures. In Section V, we apply them in Sustainable Security System using the proposed NHSS-TOPSIS algorithm to show the effectiveness of the proposed strategy for the selection of the best security systems installed in the shopping mall. Finally, we make concluding remarks and future research in Section VI.

II. PRELIMINARIES

This section consists of some basic definitions that will be helpful in the rest of the article.

Definition 1 [19] (Neutrosophic Soft Set): Let $\mathbb{U}$ be the universal set and $\mathbb{E}$ be the set of attributes concerning $\mathbb{U}$. Let $\mathbb{P}(\mathbb{U})$ be the set of neutrosophic values of $\mathbb{U}$ and $\subseteq \mathbb{E}$. A pair $(\text{F}, \mathbb{A})$ is called a neutrosophic soft set over $\mathbb{U}$ where F is a mapping with $\text{F} : \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U})$.

Definition 2 [20] (HyperSoft Set): Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$ be the power set of $\mathbb{U}$. Let $l^1, l^2, l^3 \ldots l^n$, for $n \geq 1$, be n well-defined attributes, whose corresponding attributive values are the sets $L^1, L^2, L^3 \ldots L^n$, respectively, with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \ldots n\}$. A pair $(\text{F}, L^1 \times L^2 \times L^3 \ldots L^n)$ is said to be hypersoft set over $\mathbb{U}$ where F is a mapping with $\text{F} : L^1 \times L^2 \times L^3 \ldots L^n \rightarrow \mathbb{P}(\mathbb{U})$.

Definition 3 [22] (Neutrosophic Hypersoft Set): Let $\mathbb{U}$ be the universal set and $\mathbb{P}(\mathbb{U})$ be the power set of $\mathbb{U}$. Consider $l^1, l^2, l^3 \ldots l^n$ for $n \geq 1$ as n well-defined attributes, whose corresponding attributive values are the sets $L^1, L^2, L^3 \ldots L^n$, respectively, with $L^i \cap L^j = \emptyset$, for $i \neq j$ and $i, j \in \{1, 2, 3 \ldots n\}$ and the relation $L^1 \times L^2 \times L^3 \ldots L^n = \$$ The pair $(\text{F}, \$)$ is said to be Neutrosophic HyperSoft Set (NHSS) over $\mathbb{U}$ where F is a mapping given by $\text{F} : L^1 \times L^2 \times L^3 \ldots L^n \rightarrow \mathbb{P}(\mathbb{U})$ with $\text{F}\left(L^1 \times L^2 \times L^3 \ldots L^n\right) = \{< x, T(\text{F}(\$)), I(\text{F}(\$)), F(\text{F}(\$)) >, x \in \mathbb{U}\}$ where T is the membership value of truthness I is the membership value of indeterminacy and F is the membership value of falsity such that $T, I, F : \mathbb{U} \rightarrow [0, 1]$ and $0 \leq T(\text{F}(\$)) + I(\text{F}(\$)) + F(\text{F}(\$)) \leq 3$.

Definition 4 [34] (Neutrosophic hyperSoft Matrix, NHSM): Let $\mathbb{U} = \{u^1, u^2, \ldots u^a\}$ and $\mathbb{P}(\mathbb{U})$ be respectively the universal set and power set of the universal set $\mathbb{U}$. Let $\mathbb{L}_1, \mathbb{L}_2, \ldots \mathbb{L}_{\mathfrak{b}}$ for $\mathfrak{b} \geq 1$ be $\mathfrak{b}$ welldefined attributes, whose corresponding attributive values are respectively the sets $\mathbb{L}_1^a, \mathbb{L}_2^b, \ldots \mathbb{L}_{\mathfrak{b}}^z$ with the relation $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z$ where $a, b, c, \ldots z = 1, 2, \ldots n$. The NHSS $(\mathbb{F}, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z)$ over $\mathbb{U}$ with $\mathbb{F} : (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z) \rightarrow \mathbb{P}(\mathbb{U})$ can be represented as $\mathbb{F}\left(\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z\right) = \{< u, \mathbb{T}_\ell(u), \mathbb{I}_\ell(u), \mathbb{F}_\ell(u) >, u \in \mathbb{U}, \ell \in \left(\mathbb{L}_1^a \times \ldots \mathbb{L}_{\mathfrak{b}}^z\right)\}$. Let $\mathbb{R}_\ell = (\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z)$ be the relation with its characteristic function given by $\mathfrak{X}_{\mathbb{R}_\ell} : \left(\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z\right) \rightarrow \mathbb{P}(\mathbb{U})$. Then, it is defined as $\mathfrak{X}_{\mathbb{R}_\ell} = \{< u, \mathbb{T}_\ell(u), \mathbb{I}_\ell(u), \mathbb{F}_\ell(u) > u \in \mathbb{U}, \ell \in \left(\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z\right)\}$ and called neutrosophic hypersoft matrix (NHSM). The tabular representation of $\mathbb{R}_\ell$ is given in Table 2.

If $A_{ij} = \mathfrak{X}_{\mathbb{R}_\ell}\left(u^i, \mathbb{L}_j^{\mathfrak{k}}\right)$, where $i = 1, 2, 3 \ldots a$, $j = 1, 2, 3, \ldots \mathfrak{b}$, $\mathfrak{k} = a, b, c, \ldots z$, then a matrix is defined as

$$\left[A_{ij}\right]_{a\times \mathfrak{b}} = \begin{pmatrix} A_{11} & A_{12} & \ldots & A_{1\mathfrak{b}} \\ A_{21} & A_{22} & \ldots & A_{2\mathfrak{b}} \\ \vdots & \vdots & \ddots & \vdots \\ A_{a1} & A_{a2} & \ldots & A_{a\mathfrak{b}} \end{pmatrix}$$

where $A_{ij} = (\mathbb{T}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)$, $\mathbb{I}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)$, $\mathbb{F}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)$, $u_i \in \mathbb{U}$, $\mathbb{L}_j^{\mathfrak{k}} \in \left(\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \mathbb{L}_{\mathfrak{b}}^z\right)) =$

TABLE 2. The tabular representation of $\mathbb{R}_\ell$.

	$\mathbb{L}_1^a$	$\mathbb{L}_1^b$	...	$\mathbb{L}_6^z$
u^1	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^1, \mathbb{L}_6^z)$
u^2	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^2, \mathbb{L}_6^z)$
⋮	⋮	⋮	⋱	⋮
u^a	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^a)$	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_1^b)$	...	$\mathcal{X}_{\mathbb{R}_\ell}(u^a, \mathbb{L}_6^z)$

$\left(\mathbb{T}_{ijk}^A, \mathbb{I}_{ijk}^A, \mathbb{F}_{ijk}^A\right)$. Thus, we can represent an NHSS in term of an NHSM. It means that they are interchangeable. Its generalized form can be denoted by $\left[A_{ij}\right]_{a\times 6}$, as shown at the bottom of the page.

III. PROPOSED DISTANCE AND SIMILARITY MEASURES FOR NHSS

Definition 5 (Normalized Hamming Distance for NHSS): Let $A = A_i$ and $B = B_i$ be the two NHSSs where $A_i = \left(\mathbb{T}_i^A, \mathbb{I}_i^A, \mathbb{F}_i^A\right)$ and $B_i = \left(\mathbb{T}_i^B, \mathbb{I}_i^B, \mathbb{F}_i^B\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. A normalized Hamming distance between $A = A_i$ and $B = B_i$ is defined as

$$\mathbb{D}(A, B) = \frac{1}{3n}\sum\nolimits_i^n \left(\left|\mathbb{T}_i^A - \mathbb{T}_i^B\right| + \left|\mathbb{I}_i^A - \mathbb{I}_i^B\right| + \left|\mathbb{F}_i^A - \mathbb{F}_i^B\right|\right) \tag{1}$$

Example 6: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. Then the normalized Hamming distance is calculated as $\mathbb{D}(A, B) = \frac{1}{3(3)}(|0.7 - 0.8| + |0.5 - 0.1| + |0.6 - 0.2| + |0.7 - 0.6| + |0.2 - 0.1| + |0.3 - 0.2| + |0.8 - 0.3| + |0.2 - 0.6| + |0.1 - 0.4|) = 0.267$.

Proposition 7: Let $A = A_i, B = B_i$ and $C = C_i$ be the three NHSSs where $A_i = \left(\mathbb{T}_i^A, \mathbb{I}_i^A, \mathbb{F}_i^A\right)$, $B_i = \left(\mathbb{T}_i^B, \mathbb{I}_i^B, \mathbb{F}_i^B\right)$ and $C_i = \left(\mathbb{T}_i^C, \mathbb{I}_i^C, \mathbb{F}_i^C\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. Then it satisfies the following axioms:

1. $\mathbb{D}(A, B) \geq 0$
2. $\mathbb{D}(A, B) = \mathbb{D}(B, A)$
3. $\mathbb{D}(A, B) = 0$ *iff* $A = B$
4. $\mathbb{D}(A, B) + \mathbb{D}(B, C) \geq \mathbb{D}(A, C)$

Definition 8 (Normalized Euclidean Distance for NHSS): Let $A = A_i$ and $B = B_i$ be the two NHSSs where $A_i = \left(\mathbb{T}_i^A, \mathbb{I}_i^A, \mathbb{F}_i^A\right)$ and $B_i = \left(\mathbb{T}_i^B, \mathbb{I}_i^B, \mathbb{F}_i^B\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. A normalized Euclidean distance between $A = A_i$ and $B = B_i$ is defined as

$$\mathbb{D}(A, B) = \sqrt{\frac{\sum_i^n\left(\left|\mathbb{T}_i^A - \mathbb{T}_i^B\right|^2 + \left|\mathbb{I}_i^A - \mathbb{I}_i^B\right|^2 + \left|\mathbb{F}_i^A - \mathbb{F}_i^B\right|^2\right)}{3n}} \tag{2}$$

Example 9: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. Then the normalized Euclidean distance is calculated with $\mathbb{D}(A, B) = \frac{1}{3(3)}\left(|0.7 - 0.8|^2 + |0.5 - 0.1|^2 + |0.6 - 0.2|^2 + |0.7 - 0.6|^2 + |0.2 - 0.1|^2 + |0.3 - 0.2|^2 + |0.8 - 0.3|^2 + |0.2 - 0.6|^2 + |0.1 - 0.4|^2\right)$ and $\mathbb{D}(A, B) = 0.3091$.

Definition 10 (Generalized Weighted Distance for NHSS): Let $A = A_i$ and $B = B_i$ be the two NHSSs where $A_i = \left(\mathbb{T}_i^A, \mathbb{I}_i^A, \mathbb{F}_i^A\right)$ and $B_i = \left(\mathbb{T}_i^B, \mathbb{I}_i^B, \mathbb{F}_i^B\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. A generalized weighted distance between $A = A_i$ and $B = B_i$ is given as, for $\lambda > 0$

$$\mathbb{D}_\lambda(A, B) = \left[\frac{1}{3n}\sum\nolimits_i^n w_i\left(\left|\mathbb{T}_i^A - \mathbb{T}_i^B\right|^\lambda + \left|\mathbb{I}_i^A - \mathbb{I}_i^B\right|^\lambda + \left|\mathbb{F}_i^A - \mathbb{F}_i^B\right|^\lambda\right)\right]^{\frac{1}{\lambda}} \tag{3}$$

Definition 11 (Normalized Hamming Distance for NHSM): Let $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ be the two NHSMs of order $a \times 6$ with $A_{ij} = \left(\mathbb{T}_{ijk}^A, \mathbb{I}_{ijk}^A, \mathbb{F}_{ijk}^A\right)$ and $B_{ij} = \left(\mathbb{T}_{ijk}^B, \mathbb{I}_{ijk}^B, \mathbb{F}_{ijk}^B\right)$. The normalized Hamming distance between $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ is defined as

$$\mathbb{D}(A, B) = \frac{1}{3a6}\sum\nolimits_i^a\sum\nolimits_j^6\left(\left|\mathbb{T}_{ijk}^A - \mathbb{T}_{ijk}^B\right| + \left|\mathbb{I}_{ijk}^A - \mathbb{I}_{ijk}^B\right| + \left|\mathbb{F}_{ijk}^A - \mathbb{F}_{ijk}^B\right|\right) \tag{4}$$

Example 12: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. Then the normalized Hamming distance is calculated as $\mathbb{D}(A, B) = \frac{1}{3(1)(3)}(|0.7 - 0.8| + |0.5 - 0.1| + |0.6 - 0.2| + |0.7 - 0.6| + |0.2 - 0.1| + |0.3 - 0.2| + |0.8 - 0.3| + |0.2 - 0.6| + |0.1 - 0.4|)$. $\mathbb{D}(A, B) = 0.2667$.

Definition 13 (Normalized Euclidean Distance for NHSM): Let $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ be the two NHSMs of order $a \times 6$ with $A_{ij} = \left(\mathbb{T}_{ijk}^A, \mathbb{I}_{ijk}^A, \mathbb{F}_{ijk}^A\right)$ and $B_{ij} = \left(\mathbb{T}_{ijk}^B, \mathbb{I}_{ijk}^B, \mathbb{F}_{ijk}^B\right)$. The normalized Euclidean distance between

$$\left[A_{ij}\right]_{a\times 6} = \begin{bmatrix} \mathbb{T}_{\mathbb{L}_1^a}(u_1), \mathbb{I}_{\mathbb{L}_1^a}(u_1), \mathbb{F}_{\mathbb{L}_1^a}(u_1) & \mathbb{T}_{\mathbb{L}_2^b}(u_1), \mathbb{I}_{\mathbb{L}_2^b}(u_1), \mathbb{F}_{\mathbb{L}_2^b}(u_1) & \cdots & \mathbb{T}_{\mathbb{L}_6^z}(u_1), \mathbb{I}_{\mathbb{L}_6^z}(u_1), \mathbb{F}_{\mathbb{L}_6^z}(u_1) \\ \mathbb{T}_{\mathbb{L}_1^a}(u_2), \mathbb{I}_{\mathbb{L}_1^a}(u_2), \mathbb{F}_{\mathbb{L}_1^a}(u_2) & \mathbb{T}_{\mathbb{L}_2^b}(u_2), \mathbb{I}_{\mathbb{L}_2^b}(u_2), \mathbb{F}_{\mathbb{L}_2^b}(u_2) & \cdots & \mathbb{T}_{\mathbb{L}_6^z}(u_2), \mathbb{I}_{\mathbb{L}_6^z}(u_2), \mathbb{F}_{\mathbb{L}_6^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}_{\mathbb{L}_1^a}(u_a), \mathbb{I}_{\mathbb{L}_1^a}(u_a), \mathbb{F}_{\mathbb{L}_1^a}(u_a) & \mathbb{T}_{\mathbb{L}_2^b}(u_a), \mathbb{I}_{\mathbb{L}_2^b}(u_a), \mathbb{F}_{\mathbb{L}_2^b}(u_a) & \cdots & \mathbb{T}_{\mathbb{L}_6^z}(u_a), \mathbb{I}_{\mathbb{L}_6^z}(u_a), \mathbb{F}_{\mathbb{L}_6^z}(u_a) \end{bmatrix}$$

$A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ is given as (5), shown at the bottom of the page.

Example 14: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. The normalized Euclidean distance is calculated as $\mathbb{D}(A, B)$, shown at the bottom of the page, and $\mathbb{D}(A, B) = 0.3091$.

Definition 15 (Generalized Weighted Distance for NHSM): Let $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ be the two NHSMs of order $a \times b$ with $A_{ij} = \left(\mathbb{T}^A_{ijk}, \mathbb{I}^A_{ijk}, \mathbb{F}^A_{ijk}\right)$ and $B_{ij} = \left(\mathbb{T}^B_{ijk}, \mathbb{I}^B_{ijk}, \mathbb{F}^B_{ijk}\right)$. The generalized weighted distance between $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ is given as

$$\mathbb{D}_\lambda(A, B) = \left[\frac{1}{3ab}\sum_i^a\sum_j^b w_i\left(\left|\mathbb{T}^A_{ijk} - \mathbb{T}^B_{ijk}\right|^\lambda + \left|\mathbb{I}^A_{ijk} - \mathbb{I}^B_{ijk}\right|^\lambda + \left|\mathbb{F}^A_{ijk} - \mathbb{F}^B_{ijk}\right|^\lambda\right)\right]^{\frac{1}{\lambda}} \tag{6}$$

Definition 16 (Similarity Measure for NHSS): Let $A = A_i$ and $B = B_i$ be the two NHSs where $A_i = \left(\mathbb{T}^A_i, \mathbb{I}^A_i, \mathbb{F}^A_i\right)$ and $B_i = \left(\mathbb{T}^B_i, \mathbb{I}^B_i, \mathbb{F}^B_i\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. A similarity measure between $A = A_i$ and $B = B_i$ is defined as

$$\mathbb{S}(A, B) = 1 - \frac{1}{3n}\sum_i^n\left(\left|\mathbb{T}^A_i - \mathbb{T}^B_i\right| + \left|\mathbb{I}^A_i - \mathbb{I}^B_i\right| + \left|\mathbb{F}^A_i - \mathbb{F}^B_i\right|\right) \tag{7}$$

Example 17: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. Then the normalized Hamming distance is given as $\mathbb{S}(A, B) = 1 - \frac{1}{3(3)}(|0.7 - 0.8| + |0.5 - 0.1| + |0.6 - 0.2| + |0.7 - 0.6| + |0.2 - 0.1| + |0.3 - 0.2| + |0.8 - 0.3| + |0.2 - 0.6| + |0.1 - 0.4|)$. $\mathbb{D}(A, B) = 0.7333$.

Proposition 18: Let $A = A_i$, $B = B_i$ and $C = C_i$ be the three NHSS's where $A_i = \left(\mathbb{T}^A_i, \mathbb{I}^A_i, \mathbb{F}^A_i\right)$, $B_i = \left(\mathbb{T}^B_i, \mathbb{I}^B_i, \mathbb{F}^B_i\right)$ and $C_i = \left(\mathbb{T}^C_i, \mathbb{I}^C_i, \mathbb{F}^C_i\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. Then, it satisfies the following axioms:

1. $0 \le \mathbb{S}(A, B) \le 1$;
2. $\mathbb{S}(A, B) = 1$ *if and only if* $A = B$;
3. $\mathbb{S}(A, B) = \mathbb{S}(B, A)$;
4. If $A \subset B \subset \mathrm{C}$, then $\mathbb{S}(A, C) \le \mathbb{S}(A, B)$ and $(A, C) \le \mathbb{S}(B, C)$.

Definition 19 (Generalized Weighted Similarity Measure for NHSS): Let $A = A_i$ and $B = B_i$ be the two NHSs where $A_i = \left(\mathbb{T}^A_i, \mathbb{I}^A_i, \mathbb{F}^A_i\right)$ and $B_i = \left(\mathbb{T}^B_i, \mathbb{I}^B_i, \mathbb{F}^B_i\right)$ for $i = \{1, 2, 3 \ldots \mathrm{n}\}$. A generalized weighted similarity measure between $A = A_i$ and $B = B_i$ is given as

$$\mathbb{S}_\lambda(A, B) = 1 - \left[\frac{1}{3n}\sum_i^n w_i\left(\left|\mathbb{T}^A_i - \mathbb{T}^B_i\right|^\lambda + \left|\mathbb{I}^A_i - \mathbb{I}^B_i\right|^\lambda + \left|\mathbb{F}^A_i - \mathbb{F}^B_i\right|^\lambda\right)\right]^{\frac{1}{\lambda}}, \quad \text{where } \lambda > 0. \tag{8}$$

Definition 20 (Similarity Measure for NHSM): Let $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ be the two NHSMs of order $a \times b$, where $A_{ij} = \left(\mathbb{T}^A_{ijk}, \mathbb{I}^A_{ijk}, \mathbb{F}^A_{ijk}\right)$ and $B_{ij} = \left(\mathbb{T}^B_{ijk}, \mathbb{I}^B_{ijk}, \mathbb{F}^B_{ijk}\right)$. The similarity between $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ is given as

$$\mathbb{S}(A, B) = 1 - \frac{1}{3ab}\sum_i^a\sum_j^b\left(\left|\mathbb{T}^A_{ijk} - \mathbb{T}^B_{ijk}\right| + \left|\mathbb{I}^A_{ijk} - \mathbb{I}^B_{ijk}\right| + \left|\mathbb{F}^A_{ijk} - \mathbb{F}^B_{ijk}\right|\right). \tag{9}$$

Example 21: Let $A = \{< m^1, (samsung\,\{0.7, 0.5, 0.6\}, 6GB\,\{0.7, 0.2, 0.3\}, Dual\,\{0.8, 0.2, 0.1\}) >\}$ and $B = \{< m^4, (samsung\{0.8, 0.1, 0.2\}, 6GB\,\{0.6, 0.1, 0.2\}, Dual\,\{0.3, 0.6, 0.4\}) >\}$ be the two NHSSs. Then the normalized Hamming distance is given as $(A, B) = 1 - \frac{1}{3(1)(3)}(|0.7 - 0.8| + |0.5 - 0.1| + |0.6 - 0.2| + |0.7 - 0.6| + |0.2 - 0.1| + |0.3 - 0.2| + |0.8 - 0.3| + |0.2 - 0.6| + |0.1 - 0.4|)$. $\mathbb{D}(A, B) = 0.7333$.

Definition 22 (Generalized Weighted Similarity Measure for NHSM): Let $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ be the two NHSM of order $a \times b$, where $A_{ij} = \left(\mathbb{T}^A_{ijk}, \mathbb{I}^A_{ijk}, \mathbb{F}^A_{ijk}\right)$ and $B_{ij} = \left(\mathbb{T}^B_{ijk}, \mathbb{I}^B_{ijk}, \mathbb{F}^B_{ijk}\right)$. The generalized weighted similarity between $A = \left[A_{ij}\right]$ and $B = \left[B_{ij}\right]$ is given as

$$\mathbb{S}_\lambda(A, B) = 1 - \left[\frac{1}{3ab}\sum_i^a\sum_j^b w_i\left(\left|\mathbb{T}^A_{ijk} - \mathbb{T}^B_{ijk}\right|^\lambda + \left|\mathbb{I}^A_{ijk} - \mathbb{I}^B_{ijk}\right|^\lambda + \left|\mathbb{F}^A_{ijk} - \mathbb{F}^B_{ijk}\right|^\lambda\right)\right]^{\frac{1}{\lambda}}. \tag{10}$$

IV. ON CONSTRUCTION OF NHSS-TOPSIS FOR MCDM USING PROPOSED DISTANCE AND SIMILARITY MEASURES

TOPSIS (Technique for Order Preference by Similarly to Ideal Solution) is a suitable approach to deal with

$$\mathbb{D}(A, B) = \sqrt{\frac{\sum_i^a\sum_j^b\left(\left|\mathbb{T}^A_{ijk} - \mathbb{T}^B_{ijk}\right|^2 + \left|\mathbb{I}^A_{ijk} - \mathbb{I}^B_{ijk}\right|^2 + \left|\mathbb{F}^A_{ijk} - \mathbb{F}^B_{ijk}\right|^2\right)}{3ab}} \tag{5}$$

$$\mathbb{D}(A, B) = \sqrt{\frac{\left(|0.7 - 0.8|^2 + |0.5 - 0.1|^2 + |0.6 - 0.2|^2 + |0.7 - 0.6|^2 + |0.2 - 0.1|^2 + |0.3 - 0.2|^2 + |0.8 - 0.3|^2 + |0.2 - 0.6|^2 + |0.1 - 0.4|^2\right)}{3(1)(3)}},$$

multiattribute decision making problems. TOPSIS technique consists of the following steps:

1. Compose a decision matrix.
2. Normalizing decision matrix.
3. Determine the weighted normalized decision matrix.
4. Calculate the positive and negative ideal solution.
5. Calculate the distance of each alternative to the positive and negative ideal solution.
6. Calculate the relative closeness coefficients and ranking.

Consider a multiattribute decision making problem based on neutrosophic hypersoft sets (NHSSs) in which $\mathbb{U} = \{u^1, u^2, \ldots u^a\}$ be the set of alternatives and $\mathbb{L}_1, \mathbb{L}_2, \ldots \mathbb{L}_{\mathfrak{b}}$ be the sets of attributes and their corresponding attributive values are respectively the set $\mathbb{L}_1^a, \mathbb{L}_2^b, \ldots \mathbb{L}_{\mathfrak{b}}^z$ where $a, b, c, \ldots z = 1, 2, \ldots n$. Let w^j be the weight of attributes $\mathbb{L}_j^z, j = 1, 2 \ldots \mathfrak{b}$, where $0 \leq \mathrm{w}^j \leq 1$ and $\sum_{j=1}^{\mathfrak{b}} \mathrm{w}^j = 1$ Suppose that $\mathbb{D} = (\mathbb{D}_1, \mathbb{D}_2, \ldots \mathbb{D}_t)$ be the set of t decision makers and Δ^x be the weight of t decision-makers with $0 \leq \Delta^x \leq 1$ and $\sum_{x=1}^{t} \Delta^x = 1$ Let $\left[A_{ij}^x\right]_{a\times \mathfrak{b}}$ be the decision matrix where $A_{ij}^x = \left(\mathbb{T}_{ij\mathfrak{k}}^x, \mathbb{I}_{ij\mathfrak{k}}^x, \mathbb{F}_{ij\mathfrak{k}}^x\right)$, $i = 1, 2, 3 \ldots a, j = 1, 2, 3, \ldots \mathfrak{b}, \mathfrak{k} = a, b, c, \ldots z$ and $\mathbb{T}_{ij\mathfrak{k}}^x, \mathbb{I}_{ij\mathfrak{k}}^x, \mathbb{F}_{ij\mathfrak{k}}^x \in [0, 1], 0 \leq \mathbb{T}_{ij\mathfrak{k}}^x + \mathbb{I}_{ij\mathfrak{k}}^x + \mathbb{F}_{ij\mathfrak{k}}^x \leq 3$ Utilizing the following steps, the determination strategy for the selection of alternatives can be obntained.

Step 1 (Determine the Weight of Decision Makers): Let $\left[A_{ij}^x\right]_{a\times \mathfrak{b}}$ be the decision matrix where it is given as shown at the bottom of the page.

To find the ideal matrix we average all the individual decision matrix A_{ij}^x *where* $x = 1, 2 \ldots t$ with as shown at the bottom of the page, where

$$A_{ij}^{\star} = \left(\mathbb{T}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i), \mathbb{I}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i), \mathbb{F}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)$$
$$= \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{I}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\frac{1}{t}},\right.$$
$$\left.\prod_{x=1}^{t}\left(\mathbb{F}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\frac{1}{t}}\right) \quad \textit{for } i = 1, \ldots a, j = 1, \ldots \mathfrak{b},$$
$$\mathfrak{k} = a, \ldots z \quad \text{and } a, \ldots z = 1, 2, \ldots n.$$

To determine the weights of the decision-makers, first we find the similarity measure between each decision matrix and the ideal matrix as $\mathbb{S}\left(A_{ij}^x, A_{ij}^{\star}\right) = 1 - \frac{1}{3a\mathfrak{b}} \sum_i^a \sum_j^{\mathfrak{b}} \left(\left|\mathbb{T}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i) - \mathbb{T}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right| + \left|\mathbb{I}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i) - \mathbb{I}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right| + \left|\mathbb{F}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i) - \mathbb{F}^{\star}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right|\right)$. We now calculate the weight $\Delta^x (x = 1, 2, \ldots t)$ of t decision-makers using the above equation $\Delta^x = \mathbb{S}\left(A_{ij}^x, A_{ij}^{*}\right)$, where $0 \leq \Delta^x \leq 1$ and $\sum_{x=1}^{t} \Delta^x = 1$.

Step 2 (Aggregate NHSS Decision Matrices): By accumulating all the individual decision matrices, we construct an aggregated neutrosophic hypersoft decision matrix to obtain a decision. An aggregated neutrosophic hypersoft decision matrix is denoted by A_{ij} and it is given as at the bottom of the next page.

The elements of A_{ij} in the matrix $\left[A_{ij}\right]_{a\times \mathfrak{b}}$ is calculated as $\left[A_{ij}\right]_{a\times \mathfrak{b}} = \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^{x}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)^{\Delta^x}\right)$, $i = 1, 2, 3 \ldots a$, $j = 1, 2, 3, \ldots \mathfrak{b}$ *and* $x = 1, 2, \ldots t$.

Step 3 (Determine the Weight of Attributes): In the decision making procedure, decision-makers may perceive that all attributes are not similarly significant. In this manner, each decision maker may have their own opinion regarding attribute weights. To acquire the gathering assessment of the picked attributes, all the decision-makers opinions for the importance of each attribute need to be aggregated. For this purpose, weight w^j of attributes $\mathbb{L}_j^z, j = 1, 2 \ldots \mathfrak{b}$ is calculated as $\mathrm{w}^j = \left(\mathbb{T}_{\mathbb{L}_j}, \mathbb{I}_{\mathbb{L}_j}, \mathbb{F}_{\mathbb{L}_j}\right) = \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^{x}_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^{x}_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^{x}_{\mathbb{L}_j}\right)^{\Delta^x}\right)$.

Step 4 (Calculate Weighted Aggregated Decision Matrix): After finding the weights of individual attributes, we use the weights to each row of the aggregated decision matrix with

$$\left[A_{ij}^{\omega}\right]_{a\times \mathfrak{b}} = \left(\mathbb{T}^{\omega}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i), \mathbb{I}^{\omega}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i), \mathbb{F}^{\omega}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i)\right)$$
$$= \left((\mathbb{T}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i).\mathbb{T}_{\mathbb{L}_j}), \left(\mathbb{I}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i) + \mathbb{I}_{\mathbb{L}_j} - \mathbb{I}_{\mathbb{L}_j^{\mathfrak{k}}}(u_i).\mathbb{I}_{\mathbb{L}_j}\right),\right.$$

$$\left[A_{ij}^{x}\right]_{a\times \mathfrak{b}} = \begin{bmatrix} \mathbb{T}^{x}_{\mathbb{L}_1^a}(u_1), \mathbb{I}^{x}_{\mathbb{L}_1^a}(u_1), \mathbb{F}^{x}_{\mathbb{L}_1^a}(u_1) & \mathbb{T}^{x}_{\mathbb{L}_2^b}(u_1), \mathbb{I}^{x}_{\mathbb{L}_2^b}(u_1), \mathbb{F}^{x}_{\mathbb{L}_2^b}(u_1) & \cdots & \mathbb{T}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1), \mathbb{I}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1), \mathbb{F}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1) \\ \mathbb{T}^{x}_{\mathbb{L}_1^a}(u_2), \mathbb{I}^{x}_{\mathbb{L}_1^a}(u_2), \mathbb{F}^{x}_{\mathbb{L}_1^a}(u_2) & \mathbb{T}^{x}_{\mathbb{L}_2^b}(u_2), \mathbb{I}^{x}_{\mathbb{L}_2^b}(u_2), \mathbb{F}^{x}_{\mathbb{L}_2^b}(u_2) & \cdots & \mathbb{T}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2), \mathbb{I}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2), \mathbb{F}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}^{x}_{\mathbb{L}_1^a}(u_a), \mathbb{I}^{x}_{\mathbb{L}_1^a}(u_a), \mathbb{F}^{x}_{\mathbb{L}_1^a}(u_a) & \mathbb{T}^{x}_{\mathbb{L}_2^b}(u_a), \mathbb{I}^{x}_{\mathbb{L}_2^b}(u_a), \mathbb{F}^{x}_{\mathbb{L}_2^b}(u_a) & \cdots & \mathbb{T}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_a), \mathbb{I}^{x}_{\mathbb{L}_{\mathfrak{b}}^z}(u_a), \mathbb{F}^{x}_{\mathbb{L}_a^z}(u_a) \end{bmatrix}$$

$$\left[A_{ij}^{\star`}\right]_{a\times \mathfrak{b}} = \begin{bmatrix} \mathbb{T}^{\star}_{\mathbb{L}_1^a}(u_1), \mathbb{I}^{\star}_{\mathbb{L}_1^a}(u_1), \mathbb{F}^{\star}_{\mathbb{L}_1^a}(u_1) & \mathbb{T}^{\star}_{\mathbb{L}_2^b}(u_1), \mathbb{I}^{\star}_{\mathbb{L}_2^b}(u_1), \mathbb{F}^{\star}_{\mathbb{L}_2^b}(u_1) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1), \mathbb{I}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1), \mathbb{F}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_1) \\ \mathbb{T}^{\star}_{\mathbb{L}_1^a}(u_2), \mathbb{I}^{\star}_{\mathbb{L}_1^a}(u_2), \mathbb{F}^{\star}_{\mathbb{L}_1^a}(u_2) & \mathbb{T}^{\star}_{\mathbb{L}_2^b}(u_2), \mathbb{I}^{\star}_{\mathbb{L}_2^b}(u_2), \mathbb{F}^{\star}_{\mathbb{L}_2^b}(u_2) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2), \mathbb{I}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2), \mathbb{F}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}^{\star}_{\mathbb{L}_1^a}(u_a), \mathbb{I}^{\star}_{\mathbb{L}_1^a}(u_a), \mathbb{F}^{\star}_{\mathbb{L}_1^a}(u_a) & \mathbb{T}^{\star}_{\mathbb{L}_2^b}(u_a), \mathbb{I}^{\star}_{\mathbb{L}_2^b}(u_a), \mathbb{F}^{\star}_{\mathbb{L}_2^b}(u_a) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_a), \mathbb{I}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_a), \mathbb{F}^{\star}_{\mathbb{L}_{\mathfrak{b}}^z}(u_a) \end{bmatrix},$$

$\left(\mathbb{F}_{\mathbb{L}_j^{k}}(u_i)+\mathbb{F}_{\mathbb{L}_j}-\mathbb{F}_{\mathbb{L}_j^{k}}(u_i)\,.\mathbb{F}_{\mathbb{L}_j}\right)\Big)$. Then we get a weighted aggregated decision matrix.

Step 5 (Determine the Ideal Solution): In real life we deal with two types of attributes, one is benefit type attributes and the other is cost type attributes In our MAGDM problem we also deal with these two types of attributes. Let $\mathbb{C}_1$ be the benefit type attributes and $\mathbb{C}_2$ be the cost type attributes. The neutrosophic hypersoft positive ideal solution is given as $A_j^{\omega^+}=\left(\mathbb{T}^{\omega+}_{\mathbb{L}_j^{k}}(u_i),\mathbb{I}^{\omega+}_{\mathbb{L}_j^{k}}(u_i),\mathbb{F}^{\omega+}_{\mathbb{L}_j^{k}}(u_i)\right)=$

$$\begin{cases}\overset{i}{\max}\left\{\mathbb{T}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{I}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{F}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},j\in\mathbb{C}_1\\ \overset{i}{\min}\left\{\mathbb{T}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{I}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{F}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},j\in\mathbb{C}_2\end{cases}.$$

Similarly, the neutrosophic hypersoft negative ideal solution is given as $A_j^{\omega^-}=\left(\mathbb{T}^{\omega-}_{\mathbb{L}_j^{k}}(u_j),\mathbb{I}^{\omega-}_{\mathbb{L}_j^{k}}(u_i),\mathbb{F}^{\omega-}_{\mathbb{L}_j^{k}}(u_i)\right)=$

$$\begin{cases}\overset{i}{\min}\left\{\mathbb{T}^{\omega}_{\mathbb{L}_j^{k}}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{I}^{k\ \omega}_{\mathbb{L}_j}(u_i)\right\},\overset{i}{\max}\left\{\mathbb{F}^{k\ \omega}_{\mathbb{L}_j}(u_i)\right\},j\in\mathbb{C}_1\\ \overset{i}{\max}\left\{\mathbb{T}^{k\ \omega}_{\mathbb{L}_j}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{I}^{k\ \omega}_{\mathbb{L}_j}(u_i)\right\},\overset{i}{\min}\left\{\mathbb{F}^{k\ \omega}_{\mathbb{L}_j}(u_i)\right\},j\in\mathbb{C}_2\end{cases}.$$

Step 6 (Calculate the Distances): Now we should find the normalized Hamming distance between the alternatives and positive ideal solution with $\mathbb{D}^{i+}\left(A_{ij}^{\omega},A_j^{\omega^+}\right)=\frac{1}{3b}\sum_{j=1}^{b}\left(\left|\mathbb{T}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{T}^{\omega+}_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{I}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{I}^{\omega+}_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{F}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{F}^{\omega+}_{\mathbb{L}_j^{k}}(u_i)\right|\right)$. Similarly, we find the normalized Hamming distance between the alternatives and positive ideal solution as $\mathbb{D}^{i-}\left(A_{ij}^{\omega},A_j^{\omega^-}\right)=\frac{1}{3b}\sum_{j=1}^{b}\{\left|\mathbb{T}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{T}^{\omega-}_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{I}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{I}^{\omega-}_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{F}^{\omega}_{\mathbb{L}_j^{k}}(u_i)-\mathbb{F}^{\omega-}_{\mathbb{L}_j^{k}}(u_i)\right|\}$.

Step 7 (Calculate the Relative Closeness Coefficient): Relative closeness index is used to rank the alternatives and it is calculated with, $i=1,\ldots,a$,

$$\mathbb{RC}^i=\frac{\mathbb{D}^{i-}\left(A_{ij}^{\omega},A_j^{\omega^-}\right)}{\overset{i}{\max}\left\{\mathbb{D}^{i-}\left(A_{ij}^{\omega},A_j^{\omega^-}\right)\right\}}-\frac{\mathbb{D}^{i+}\left(A_{ij}^{\omega},A_j^{\omega^+}\right)}{\overset{i}{\min}\left\{\mathbb{D}^{i+}\left(A_{ij}^{\omega},A_j^{\omega^+}\right)\right\}}. \tag{11}$$

The set of selected alternatives are ranked according to the descending order of relative closeness index.

V. APPLICATION USING THE PROPOSED NHSS-TOPSIS

In this section, we apply the proposed NHSS-TOPSIS method to the two cases study. One is a medical diagnosis. Another is to solve an optimal selection for the sustainable green security system.

Example 23 (A Medical Diagnosis): Consider four patients $\mathcal{P}=\{\wp_1,\wp_2,\wp_3,\wp_4\}$ suffering from different diseases, $\mathcal{D}=\left\{d^1=\text{Covid}-19,\,d^2=\text{Typhoid},\,d^3=\text{Malaria}\right\}$ which are further bifurcated. Doctors (decision-makers) $(\mathfrak{M}^1,\mathfrak{M}^2,\mathfrak{M}^3)$ want the proper medical diagnosis to know which one is suffering from COVID-19. The tool of NHSS-TOPSIS under similarity measures will be used for analyzing patients. For this purpose, the set of the symptoms of the COVID-19 is considered in the form of NHSS. Consider, $\mathcal{P}=\{\wp_1,\wp_2,\wp_3,\wp_4\}$, $\mathcal{D}=\left\{d^1=\text{Covid}-19,\,d^2=\text{Typhoid},\,d^3=\text{Malaria}\right\}$, and $\mathcal{S}=\{\mathfrak{s}^1=\text{Sense of Taste},\,\mathfrak{s}^2=\text{Temperature},\,\mathfrak{s}^3=\text{Chest Pain},\,\mathfrak{s}^4=\text{flu}\}$ be the set of patients, disease, and symptoms respectively. The set of symptoms is further classified into further-bifurcated values as; $\mathfrak{s}^1=\text{Sense of Taste}=\{\text{No Taste, can Taste}\}$ $\mathfrak{s}^2=\text{Temperature}=\{97.5°\text{F}-98.5°\text{F},98.6°\text{F}-99.5°\text{F},99.6°\text{F}-101.5,101.6°\text{F}-102.5\}$ $\mathfrak{s}^3=\text{Chest Pain}=\{\text{Short ness of breath, no pain, normal pain, Angina}\}$ $\mathfrak{s}^4=\text{flu}=\{\text{Sore Throat, Cough, Strep Throat}\}$ Now, let us define the relation for the function; $f: d^1\times d^2\times d^3\rightarrow P(\mathcal{P})$ as, $f\left(d^1\times d^2\times d^3\right)=(\mathfrak{B}=\textit{shortness of breath},\ \mathfrak{T}=101.3,\ \mathfrak{S}=\textit{Sore Throat},\ \mathfrak{y}=\textit{No taste})$ is the actual sample of the patient for the disease confirmation. Three patients $\{\wp_1,\wp_2,\wp_3\}$ are selected based on sample. The panel of three doctors (decision-maker) $\left\{\mathfrak{M}^1,\mathfrak{M}^2,\mathfrak{M}^3\right\}$ will examine the sample and select the most relevant diseases. These decision-makers give their valuable opinion in the form of neutrosophic number based on their experience and knowledge, and are presented in NHSM, separately, as shown at the bottom of the page.

Step 1 (Determine the Weights of Decision Makers): To determine the weights of the decision-makers, first, we find the similarity measure between each decision matrix $\left\{\mathfrak{M}^1,\mathfrak{M}^2,\mathfrak{M}^3\right\}$ and the ideal matrix $\mathcal{S}^\star$ using $\mathbb{S}\left(A_{ij}^x,A_{ij}^\star\right)=1-\frac{1}{3ab}\sum_i^a\sum_j^b\{\left|\mathbb{T}^x_{\mathbb{L}_j^{k}}(u_i)-\mathbb{T}^\star_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{I}^x_{\mathbb{L}_j^{k}}(u_i)-\mathbb{I}^\star_{\mathbb{L}_j^{k}}(u_i)\right|+\left|\mathbb{F}^x_{\mathbb{L}_j^{k}}(u_i)-\mathbb{F}^\star_{\mathbb{L}_j^{k}}(u_i)\right|\}$. So, $\mathbb{S}(\wp_1,\wp_*)=0.5641$, $\mathbb{S}(\wp_1,\wp_*)=0.1224$, $\mathbb{S}(\wp_1,\wp_*)=0.1046$. Now we calculate the weight Δ^x *for* $(x=1,2,3)$ of each decision-makers using $\Delta^x=\frac{\mathbb{S}\left(A_{ij}^x,A_{ij}^*\right)}{\sum_{x=1}^t\mathbb{S}\left(A_{ij}^x,A_{ij}^*\right)}$. We have

$$\Delta^1=\frac{0.5641}{(0.5641+0.1224+0.1046)}=0.7130$$

$$[A_{ij}]_{a\times b}=\begin{bmatrix}\mathbb{T}_{\mathbb{L}_1^a}(u_1),\mathbb{I}_{\mathbb{L}_1^a}(u_1),\mathbb{F}_{\mathbb{L}_1^a}(u_1) & \mathbb{T}_{\mathbb{L}_2^b}(u_1),\mathbb{I}_{\mathbb{L}_2^b}(u_1),\mathbb{F}_{\mathbb{L}_2^b}(u_1) & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_1),\mathbb{I}_{\mathbb{L}_b^z}(u_1),\mathbb{F}_{\mathbb{L}_b^z}(u_1)\\ \mathbb{T}_{\mathbb{L}_1^a}(u_2),\mathbb{I}_{\mathbb{L}_1^a}(u_2),\mathbb{F}_{\mathbb{L}_1^a}(u_2) & \mathbb{T}_{\mathbb{L}_2^b}(u_2),\mathbb{I}_{\mathbb{L}_2^b}(u_2),\mathbb{F}_{\mathbb{L}_2^b}(u_2) & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_2),\mathbb{I}_{\mathbb{L}_b^z}(u_2),\mathbb{F}_{\mathbb{L}_b^z}(u_2)\\ \vdots & \vdots & \ddots & \vdots\\ \mathbb{T}_{\mathbb{L}_1^a}(u_a),\mathbb{I}_{\mathbb{L}_1^a}(u_a),\mathbb{F}_{\mathbb{L}_1^a}(u_a) & \mathbb{T}_{\mathbb{L}_2^b}(u_a),\mathbb{I}_{\mathbb{L}_2^b}(u_a),\mathbb{F}_{\mathbb{L}_2^b}(u_a)\,s & \cdots & \mathbb{T}_{\mathbb{L}_b^z}(u_a),\mathbb{I}_{\mathbb{L}_b^z}(u_a),\mathbb{F}_{\mathbb{L}_b^z}(u_a)\end{bmatrix}$$

$$\Delta^2 = \frac{0.1224}{(0.5641 + 0.1224 + 0.1046)} = 0.1547$$
$$\Delta^3 = \frac{0.1046}{(0.5641 + 0.1224 + 0.1046)} = 0.1322$$

Step 2 (Aggregate Neutrosophic Hypersoft Decision Matrices): Now we construct an aggregated neutrosophic hypersoft decision matrix NHSM, to obtain group decision. We obtain, as shown at the bottom of the page.

Step 3 (Determine the Weight of Attributes): Weight $\mathbb{w}^j$of attributes $\mathbb{L}_j$, $j = 1, 2 \ldots b$ is calculated using $\mathbb{w}^j = (\mathbb{T}_{\mathbb{L}_j}, \mathbb{I}_{\mathbb{L}_j}, \mathbb{F}_{\mathbb{L}_j}) = \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^x_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^x_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^x_{\mathbb{L}_j}\right)^{\Delta^x}\right)$. we get $\mathbb{w}^1 = (0.7224, 0.6938, 0.2346)$, $\mathbb{w}^2 = (0.6755, 0.1340, 0.1004)$, $\mathbb{w}^3 = (0.2821, 0.1269, 0.0992)$

Step 4 (Calculate the Weighted Aggregated Decision Matrix): After finding the weights of attributes, we apply these weights to each row of aggregated decision matrix using $\left[A^{\omega}_{ij}\right]_{a\times b} = \left(\mathbb{T}^{\omega}_{\mathbb{L}^k_j}(u_i), \mathbb{I}^{\omega}_{\mathbb{L}^k_j}(u_i), \mathbb{F}^{\omega}_{\mathbb{L}^k_j}(u_i)\right) = \left((\mathbb{T}_{\mathbb{L}^k_j}(u_i).\mathbb{T}_{\mathbb{L}_j}), \left(\mathbb{I}_{\mathbb{L}^k_j}(u_i) + \mathbb{I}_{\mathbb{L}_j} - \mathbb{I}_{\mathbb{L}^k_j}(u_i).\mathbb{I}_{\mathbb{L}_j}\right), \left(\mathbb{F}_{\mathbb{L}^k_j}(u_i) + \mathbb{F}_{\mathbb{L}_j} - \mathbb{F}_{\mathbb{L}^k_j}(u_i).\mathbb{F}_{\mathbb{L}_j}\right)\right)$. We get a weighted aggregated decision matrix $[\mathcal{S}^{\omega}]$ as shown at the bottom of the page.

Step 5 (Determine the Ideal Solution): Neutrosophic hypersoft positive ideal solution is calculated using $\mathcal{S}^{\omega^+} = [(\mathfrak{B}, (0.23, 0.53, 013))\ (\mathfrak{T}, (0.95, 0.16, 0.62))\ (\mathfrak{S}, (0.75, 0.85, 0.75))\ (\mathfrak{h}, (0.42, 0.85, 0.13))]$. Similarly, the neutrosophic hypersoft negative ideal solution is given as $\mathcal{S}^{\omega^-} = [(\mathfrak{B}, (0.34, 0.52, 0.77))\ (\mathfrak{T}, (0.23, 0.32, 0.21))\ (\mathfrak{S}, (0.86, 0.23, 0.11))\ (\mathfrak{h}, (0.12, 0.09, 0.03))$).

Step 6 (Calculate the Distance Measure): Now we find the normalized Hamming distance between the alternatives and positive ideal solution using $\mathbb{D}^{i+}\left(A^{\omega}_{ij}, A^{\omega^+}_j\right) = \frac{1}{3b}\sum_{j=1}^{b}\left(\left|\mathbb{T}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{T}^{\omega+}_{\mathbb{L}^k_j}(u_i)\right| + \left|\mathbb{I}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{I}^{\omega+}_{\mathbb{L}^k_j}(u_i)\right| + \left|\mathbb{F}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{F}^{\omega+}_{\mathbb{L}^k_j}(u_i)\right|\right)$. We get $\mathbb{D}^{1+}\left(\mathcal{S}^{\omega}_1, \mathcal{S}^{\omega^+}\right) = 0.342$, $\mathbb{D}^{2+}\left(\mathcal{S}^{\omega}_2, \mathcal{S}^{\omega^+}\right) = 0.127$, $\mathbb{D}^{3+}\left(\mathcal{S}^{\omega}_3, \mathcal{S}^{\omega^+}\right) = 0.985$.

Similarly, we find the normalized hamming distance between the alternatives and negative ideal solution using $\mathbb{D}^{i-}\left(A^{\omega}_{ij}, A^{\omega^-}_j\right) = \frac{1}{3b}\sum_{j=1}^{b}\left(\left|\mathbb{T}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{T}^{\omega-}_{\mathbb{L}^k_j}(u_i)\right| + \left|\mathbb{I}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{I}^{\omega-}_{\mathbb{L}^k_j}(u_i)\right| + \left|\mathbb{F}^{\omega}_{\mathbb{L}^k_j}(u_i) - \mathbb{F}^{\omega-}_{\mathbb{L}^k_j}(u_i)\right|\right)$. We get, $\mathbb{D}^{1-}\left(\mathcal{S}^{\omega}_1, \mathcal{S}^{\omega^-}\right) = 0.741$, $\mathbb{D}^{2-}\left(\mathcal{S}^{\omega}_2, \mathcal{S}^{\omega^-}\right) = 0.443$, $\mathbb{D}^{3-}\left(\mathcal{S}^{\omega}_3, \mathcal{S}^{\omega^-}\right) = 0.332$.

Step 7 (Calculate the Relative Closeness Coefficient): Now we calculate the relative closeness index using $\mathbb{R}\wp_i = \frac{\mathbb{D}^{i-}\left(A^{\omega}_{ij}, A^{\omega^-}_j\right)}{\max\limits_{i}\left\{\mathbb{D}^{i-}\left(A^{\omega}_{ij}, A^{\omega^-}_j\right)\right\}} - \frac{\mathbb{D}^{i+}\left(A^{\omega}_{ij}, A^{\omega^+}_j\right)}{\min\limits_{i}\left\{\mathbb{D}^{i+}\left(A^{\omega}_{ij}, A^{\omega^+}_j\right)\right\}}$. We get

$$\mathbb{R}\wp_1 = \frac{0.741}{0.741} - \frac{0.342}{0.005} = -67.40$$
$$\mathbb{R}\wp_2 = \frac{0.443}{0.741} - \frac{0.127}{0.005} = -24.80$$
$$\mathbb{R}\wp_3 = \frac{0.332}{0.741} - \frac{0.985}{0.005} = -196.5$$

By using the proposed NHSS-TOPSIS for neutrosophic hypersoft sets, we can decide that which patient is suffering from the following disease by considering the values of relative closeness coefficient in descending order. We rank the selected alternatives as shown in Figure 1 according to the descending order of relative closeness index as $\wp_2 > \wp_1 > \wp_3$. This shows that $\wp_2$ is the best alternative which is suffered with COVID-19. The proposed NHSS-TOPSIS algorithm is used under the NHSS environment for the ranking of alternatives and the results are compared with some existing decision-making methods, such as Zhang and Xu [25],

$$\left[\mathfrak{M}^1\right]_{3\times 4} = \begin{bmatrix} (\mathfrak{B}\,(0.7, 0.4, 0.3)) & (\mathfrak{T}\,(0.5, 0.3, 0.4)) & (\mathfrak{S}\,(0.3, 0.2, 0.1)) & (\mathfrak{h}\,(0.9, 0.1, 0.0)) \\ (\mathfrak{B}\,(0.3, 0.1, 0.1)) & (\mathfrak{T}\,(1.0, 0.2, 0.2)) & (\mathfrak{S}\,(1.0, 0.3, 0.1)) & (\mathfrak{h}\,(0.1, 0.2, 0.8)) \\ (\mathfrak{B}\,(0.8, 0.7, 0.6)) & (\mathfrak{T}\,(0.7, 0.5, 0.11)) & (\mathfrak{S}\,(0.6, 0.3, 0.21)) & (\mathfrak{h}\,(0.68, 0.31, 0.38)) \end{bmatrix}$$

$$\left[\mathfrak{M}^2\right]_{3\times 4} = \begin{bmatrix} (\mathfrak{B}\,(0.1, 0.5, 0.3)) & (\mathfrak{T}\,(0.7, 0.5, 0.4)) & (\mathfrak{S}\,(0.8, 0.3, 0.7)) & (\mathfrak{h}\,(0.6, 0.4, 0.7)) \\ (\mathfrak{B}\,(0.2, 0.4, 0.9)) & (\mathfrak{T}\,(0.3, 0.210.0)) & (\mathfrak{S}\,(1.0, 0.1, 0.9)) & (\mathfrak{h}\,(0.4, 0.1, 0.6)) \\ (\mathfrak{B}\,(0.6, 0.5, 0.9)) & (\mathfrak{T}\,(0.4, 0.7, 0.91)) & (\mathfrak{S}\,(0.3, 0.6, 0.41)) & (\mathfrak{h}\,(0.34, 0.16, 0.19)) \end{bmatrix}$$

$$\left[\mathfrak{M}^3\right]_{3\times 4} = \begin{bmatrix} (\mathfrak{B}\,(0.8, 0.2, 0.1)) & (\mathfrak{T}\,(0.8, 0.4, 0.2)) & (\mathfrak{S}\,(0.9, 0.3, 0.5)) & (\mathfrak{h}\,(1.0, 0.3, 0.1)) \\ (\mathfrak{B}\,(0.2, 0.1, 0.3)) & (\mathfrak{T}\,(3.0, 012, 0.12)) & (\mathfrak{S}\,(0.13, 0.23, 0.41)) & (\mathfrak{h}\,(0.51, 0.22, 0.1)) \\ (\mathfrak{B}\,(0.9, 0.4, 0.2)) & (\mathfrak{T}\,(0.8, 0.2, 0.71)) & (\mathfrak{S}\,(0.6, 05., 0.23)) & (\mathfrak{h}\,(0.8, 0.1, 0.3)) \end{bmatrix}$$

$$[\wp]_{3\times 4} = \begin{bmatrix} (\mathfrak{B}\,(0.116, 0.341, 0.312)) & (\mathfrak{T}\,(0.896.0.341, 0.334)) & (\mathfrak{S}\,(0.234, 0.241, 0.210)) & (\mathfrak{h}\,(0.352, 0.112, 0.007)) \\ (\mathfrak{B}\,(0.621, 0.241, 0653)) & (\mathfrak{T}\,(0.342, 0.121, 0.732)) & (\mathfrak{S}\,(0.234, 0.466, 0.369)) & (\mathfrak{h}\,(0.251, 0.144, 0.330)) \\ (\mathfrak{B}\,(0.871, 0.636, 0.346)) & (\mathfrak{T}\,(0.212, 0.1111, 0.203)) & (\mathfrak{S}\,(0.223, 0.761, 0.474)) & (\mathfrak{h}\,(0.467, 0.831, 0.120)) \end{bmatrix}$$

$$[\mathcal{S}^{\omega}]_{3\times 4} = \begin{bmatrix} (\mathfrak{B}\,(0.342, 0.121, 0.732)) & (\mathfrak{T}\,(0.754, 0.466, 0.369)) & (\mathfrak{S}\,(0.871, 0.636, 0.346)) & (\mathfrak{h}\,(0.812, 0.1111, 0.203)) \\ (\mathfrak{B}\,(0.467, 0.831, 0.120)) & (\mathfrak{T}\,(0.6251, 0.144, 0.330)) & (\mathfrak{S}\,(0.734, 0.466, 0.369)) & (\mathfrak{h}\,(0.223, 0.761, 0.474)) \\ (\mathfrak{B}\,(0.422, 0.974, 0.146)) & (\mathfrak{T}\,(0.612, 0.171, 0.403)) & (\mathfrak{S}\,(0.143, 0.861, 0.474)) & (\mathfrak{h}\,(0.897, 0.831, 0.120)) \end{bmatrix}$$

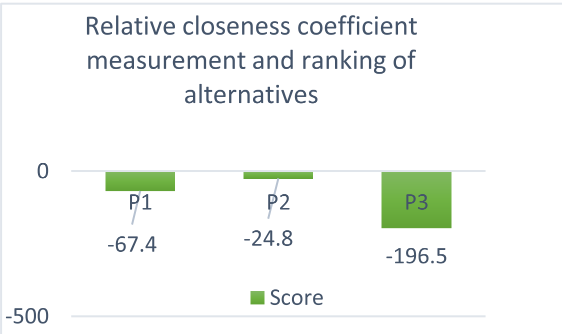

FIGURE 1. Relative closeness coefficient measurement and ranking of alternatives.

TABLE 3. Comparison analysis of final ranking with existing methods.

Methods	Ranking of alternatives	The optimal alternatives
Zhang and Xu [25]	$\wp_2 > \wp_1 > \wp_3$	$\wp_2$
Naeem et al. [35]	$\wp_2 > \wp_1 > \wp_3$	$\wp_2$
Eraslan and Karaaslan [36]	$\wp_2 > \wp_3 > \wp_1$	$\wp_2$
Garg and Kumar [37]	$\wp_2 > \wp_1 > \wp_3$	$\wp_2$
Peng and Dai [38]	$\wp_2 > \wp_1 > \wp_3$	$\wp_2$
The proposed NHSS-TOPSIS	$\wp_2 > \wp_1 > \wp_3$	$\wp_2$

Naeem *et al.* [35], Eraslan and Karaaslan [36], Garg and Kumar [37] and Peng and Dai [38], as indicated in Table 3, in which we give the ranking of the top three alternatives with the optimal alternative from each method.

Example 24 (Application to the Sustainable Green Security System): One of the most popular hubs in most of countries is shopping malls where people in large number travel are regular for various reasons. Shopping malls have numerous shops, dining halls, public parking lots, bathrooms, cinemas etc. These could be exposed by many illegal activities such as robbery and other destructions without adequate safety measures and guards. It is essential to ensure the protection of the valuable items in the mall outlets as well as regular visitors and staffs. There are several different types of security systems that can provide the flow of peoples with security. For this purpose, let $\mathbb{S} = \{\mathbb{S}^1, \mathbb{S}^2, \mathbb{S}^3, \mathbb{S}^4, \mathbb{S}^5, \mathbb{S}^6, \mathbb{S}^7, \mathbb{S}^8\}$ be the set of different setups for security systems from different companies that need to be installed in the respective shopping mall.

The attributes for respective security systems are as follows: $\mathbb{D}_1 = \textit{Digital Monitoring}$, $\mathbb{D}_2 = \textit{Fire safety measures}$, $\mathbb{D}_3 = \textit{Healthcare safety}$, $\mathbb{D}_4 = \textit{Mall events security}$.

These attributes are further characterized as

$$\begin{aligned}\mathbb{D}_1^a &= \textit{Digital Monitoring}\\ &= \left\{\begin{matrix}\textit{Web services, Biometric, Smart cards,}\\ \textit{Antivirus softwares}\end{matrix}\right\},\\ &\quad a = 1, 2, 3, 4.\end{aligned}$$

$$\begin{aligned}\mathbb{D}_2^b &= \textit{Fire safety measures}\\ &= \{\textit{Fire alarms, Smoke detectors, Emergency exits,}\\ &\quad \textit{Water fire extinguishers}\}, \quad b = 1, 2, 3, 4.\\ \mathbb{D}_3^c &= \textit{Healthcare safety} = \{\textit{Ambulance service,}\\ &\quad \textit{Paramedic staff, First aid kits}\}, \quad c = 1, 2, 3.\\ \mathbb{D}_4^d &= \textit{Mallevent security}\{\textit{Extra camers, Walk through}\\ &\quad \textit{gates, Bomb disposal teams}\} \quad d = 1, 2, 3.\end{aligned}$$

Assume that the relation for the function $\mathcal{F} : \mathbb{D}_1^a \times \mathbb{D}_2^b \times \mathbb{D}_3^c \times \mathbb{D}_4^d \rightarrow P(\mathbb{S})$ as $\mathcal{F}\left(\mathbb{D}_1^a \times \mathbb{D}_2^b \times \mathbb{D}_3^c \times \mathbb{D}_4^d\right) = \left(\mathbb{D}_1^2, \mathbb{D}_2^2, \mathbb{D}_3^1, \mathbb{D}_4^2\right) = \left(\begin{matrix}\textit{Biometric}\,(B)\,,\ \textit{Smoke detectors}\,(SD)\,,\\ \textit{Ambulance service}\,(AS)\,,\ \textit{Walk through gates}\,(WTG)\end{matrix}\right)$ is the actual requirement of the shopping mall for the security system. Four company security systems $\{\mathbb{S}^2, \mathbb{S}^4, \mathbb{S}^5, \mathbb{S}^6\}$ are selected based on assumed relation These are that *Biometric* (B), *Smoke detectors* (SD), *Ambul* our decision-makers $\left\{\mathbb{M}^1, \mathbb{M}^2, \mathbb{M}^3, \mathbb{M}^4\right\}$ are intended to select the most suitable security system for the respective shopping mall. These decision-makers give their valuable opinion in the form of NHSM separately, as shown at the bottom of the next page. The importance of selected attributes by each decisionmaker is given as shown at the bottom of the next page.

Step 1 (Determine the Weights of Decision Makers): To find the ideal matrix we average all the individual decision matrix $\mathbb{M}^1, \mathbb{M}^2, \mathbb{M}^3, \mathbb{M}^4$ using as shown at the bottom of the next page, where $A_{ij}^{\star} = \left(\mathbb{T}^{\star}_{\mathbb{L}_j^{k}}(u_i), \mathbb{I}^{\star}_{\mathbb{L}_j^{k}}(u_i), \mathbb{F}^{\star}_{\mathbb{L}_j^{k}}(u_i)\right)$ $= \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^{x}_{\mathbb{L}_j^{k}}(u_i)\right)^{\frac{1}{t}}, \prod_{x=1}^{t}\left(\mathbb{I}^{x}_{\mathbb{L}_j^{k}}(u_i)\right)^{\frac{1}{t}},\right.$ $\left.\prod_{x=1}^{t}\left(\mathbb{F}^{x}_{\mathbb{L}_j^{k}}(u_i)\right)^{\frac{1}{t}}\right)$, *for* $i = 1, 2, 3, 4$, $j = 1, 2, 3, 4$, $k = a, b, c, d$ and $a = 2$, $b = 2$, $c = 1$, $d = 2$. By averaging all decision matrices, we get as shown at the bottom of the next page.

One calculation is provided for the convenience of the reader. For $i = 1, j = 1, k = a = 2$, we have

$$\mathbb{S}_{11}^{\star} = \left(\mathbb{T}^{\star}_{\mathbb{L}_1^2}(\wp_1), \mathbb{I}^{\star}_{\mathbb{L}_1^2}(\wp_1), \mathbb{F}^{\star}_{\mathbb{L}_1^2}(\wp_1)\right)$$

$$= \left(\begin{matrix}\left(1 - \left(1 - \mathbb{T}^1_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(1 - \mathbb{T}^2_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\right.\\ \left.\left(1 - \mathbb{T}^3_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(1 - \mathbb{T}^4_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\right),\\ \left(\mathbb{I}^1_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^2_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\\ \left(\mathbb{I}^3_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(\mathbb{I}^4_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}},\\ \left(\mathbb{F}^1_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^2_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\\ \left(\mathbb{F}^3_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\left(\mathbb{F}^4_{\mathbb{L}_1^2}(\wp_1)\right)^{\frac{1}{4}}\end{matrix}\right).$$

Thus, we have

$$\mathcal{S}^{\star}_{11} = \left(\begin{array}{c} \left(1-(1-0.9)^{\frac{1}{4}}\,(1-0.3)^{\frac{1}{4}}\,(1-0.6)^{\frac{1}{4}}\,(1-0.9)^{\frac{1}{4}}\right), \\ (0.2)^{\frac{1}{4}}\,(0.3)^{\frac{1}{4}}\,(0.3)^{\frac{1}{4}}\,(0.1)^{\frac{1}{4}}, \\ (0.1)^{\frac{1}{4}}\,(0.7)^{\frac{1}{4}}\,(0.4)^{\frac{1}{4}}\,(0.1)^{\frac{1}{4}} \end{array} \right),$$

and then $\mathcal{S}^{\star}_{11} = (0.7700, 0.2060, 0.2300)$.

To determine the weights of the decision-makers, first we find the similarity measure between each decision matrix $\mathbb{M}^1, \mathbb{M}^2, \mathbb{M}^3, \mathbb{M}^4$ and the ideal matrix $\mathcal{S}^{\star}$ using $\mathbb{S}\left(A^x_{ij}, A^{\star}_{ij}\right) = 1 - \frac{1}{3a\mathfrak{b}} \sum_i^a \sum_j^{\mathfrak{b}} \{ \left| \mathbb{T}^x_{\mathbb{L}^{k}_j}(u_i) - \mathbb{T}^{\star}_{\mathbb{L}^{k}_j}(u_i) \right| + \left| \mathbb{I}^x_{\mathbb{L}^{k}_j}(u_i) - \mathbb{I}^{\star}_{\mathbb{L}^{k}_j}(u_i) \right| + \left| \mathbb{F}^x_{\mathbb{L}^{k}_j}(u_i) - \mathbb{F}^{\star}_{\mathbb{L}^{k}_j}(u_i) \right| \}$. So, $\mathbb{S}\left(\mathcal{S}^1, \mathcal{S}^{\star}\right) = 0.8728$, $\mathbb{S}\left(\mathcal{S}^2, \mathcal{S}^{\star}\right) = 0.9004$, $\mathbb{S}\left(\mathcal{S}^3, \mathcal{S}^{\star}\right) = 0.8690$, $\mathbb{S}\left(\mathcal{S}^4, \mathcal{S}^{\star}\right) = 0.8916$. One calculation is provided for the convenience of the reader. For $i = 1, 2, 3, 4$, when $j = 1$, then $k = a = 2$; When $j = 2$, then $k = b = 2$; When $j = 3$, then $k = c = 1$; When $j = 4$, then $k = d = 2$. We have $\mathbb{S}\left(\mathcal{S}^1, \mathcal{S}^{\star}\right) = 1 - \frac{1}{3(4)(4)}((|0.90 - 0.77| + |0.20 - 0.206| + |0.10 - 0.23|) + (|0.8 - 0.8318| + |0.3 - 0.1861| + |0.2 - 0.1189|)$
$+ (|0.6 - 0.7172| + |0.1 - 0.1732| + |0.3 - 0.2213|)$
$+ (|0.90 - .8373| + |0.10 - .1316| + |0.10 - 0.23|)$
$+ (|0.3 - 0.7264| + |0.3 - 0.206| + |0.7 - 0.3253|)$
$+ (|0.60 - 0.8| + |0.20 - 0.1861| + |0.6 - 0.1861|)$
$+ (|0.8 - 0.8318| + |0.1 - 0.1189| + |0.2 - 0.1682|)$
$+ (|0.9 - 0.8066| + |0.1 - 0.1316| + |0.1 - 0.1934|)$
$+ (|0.6 - 0.6131| + |0.4 - 0.2632| + |0.2 - 0.1861|)$
$+ (|0.8 - 0.8318| + |0.3 - 0.1732| + ||0.1 - 0.1414)$
$+ (|0.80 - 0.8811| + |0.20 - 0.1189| + |0.10 - 0.1|)$
$+ (|0.80 - 0.7551| + |0.1 - 0.1316| + |0.1 - 0.206|)$
$+ (|0.7 - 0.4336| + |0.1 - 0.3834| + |0.3 - 0.2213|)$
$+ (|0.20 - 0.7172| + |0.60 - 0.3568| + |0.8 - 0.2|)$
$+ (|0.6 - 0.3494| + |0.3 - 0.3568| + |0.4 - 0.4757|)$
$+ (|0.90 - 0.8| + |0.10 - 0.1732| + |0.2 - 0.2378|))$ Thus, we obtain $\mathbb{S}\left(\mathcal{S}^1, \mathcal{S}^{\star}\right) = 0.8728$.

$$\left[\mathbb{M}^1\right]_{4\times4} = \begin{bmatrix} (B,(0.9,0.2,0.1)) & (SD,(0.3,0.3,0.7)) & (AS,(0.6,0.4,0.2)) & (WTG,(0.7,0.1,0.3)) \\ (B,(0.8,0.3,0.2)) & (SD,(0.6,0.2,0.6)) & (AS,(0.8,0.3,0.1)) & (WTG,(0.2,0.6,0.8)) \\ (B,(0.6,0.1,0.3)) & (SD,(0.8,0.1,0.2)) & (AS,(0.8,0.2,0.1)) & (WTG,(0.6,0.3,0.4)) \\ (B,(0.9,0.1,0.1)) & (SD,(0.9,0.1,0.1)) & (AS,(0.8,0.1,0.1)) & (WTG,(0.9,0.1,0.2)) \end{bmatrix}$$

$$\left[\mathbb{M}^2\right]_{4\times4} = \begin{bmatrix} (B,(0.3,0.3,0.7)) & (SD,(0.9,0.2,0.1)) & (AS,(0.6,0.1,0.3)) & (WTG,(0.3,0.6,0.2)) \\ (B,(0.8,0.2,0.1)) & (SD,(0.8,0.3,0.2)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.8,0.3,0.1)) \\ (B,(0.6,0.3,0.4)) & (SD,(0.8,0.1,0.2)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.2,0.3,0.8)) \\ (B,(0.9,0.1,0.2)) & (SD,(0.8,0.1,0.1)) & (AS,(0.7,0.1,0.3)) & (WTG,(0.6,0.3,0.4)) \end{bmatrix}$$

$$\left[\mathbb{M}^3\right]_{4\times4} = \begin{bmatrix} (B,(0.6,0.3,0.4)) & (SD,(0.2,0.3,0.8)) & (AS,(0.3,0.6,0.2)) & (WTG,(0.3,0.6,0.2)) \\ (B,(0.9,0.1,0.1)) & (SD,(0.9,0.1,0.1)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.8,0.3,0.1)) \\ (B,(0.8,0.3,0.2)) & (SD,(0.9,0.2,0.1)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.2,0.3,0.8)) \\ (B,(0.3,0.3,0.7)) & (SD,(0.9,0.1,0.2)) & (AS,(0.7,0.1,0.3)) & (WTG,(0.6,0.3,0.4)) \end{bmatrix}$$

$$\left[\mathbb{M}^4\right]_{4\times4} = \begin{bmatrix} (B,(0.9,0.1,0.1)) & (SD,(0.9,0.1,0.2)) & (AS,(0.8,0.2,0.1)) & (WTG,(0.3,0.6,0.2)) \\ (B,(0.8,0.2,0.1)) & (SD,(0.8,0.2,0.1)) & (AS,(0.6,0.3,0.4)) & (WTG,(0.8,0.3,0.2)) \\ (B,(0.8,0.1,0.1)) & (SD,(0.8,0.1,0.2)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.3,0.6,0.2)) \\ (B,(0.9,0.1,0.2)) & (SD,(0.3,0.3,0.7)) & (AS,(0.8,0.3,0.2)) & (WTG,(0.9,0.1,0.1)) \end{bmatrix}$$

$$\begin{array}{lllll} \mathbb{M}^1 \longrightarrow & (B,(0.9,0.2,0.1)) & (SD,(0.7,0.2,0.2)) & (AS,(0.8,0.1,0.1)) & (WTG,(0.6,0.3,0.5)) \\ \mathbb{M}^2 \longrightarrow & (B,(0.8,0.1,0.1)) & (SD,(0.6,0.3,0.4)) & (AS,(0.5,0.2,0.4)) & (WTG,(0.7,0.2,0.2)) \\ \mathbb{M}^3 \longrightarrow & (B,(0.5,0.2,0.4)) & (SD,(0.8,0.1,0.1)) & (AS,(0.9,0.1,0.1)) & (WTG,(0.9,0.1,0.2)) \\ \mathbb{M}^4 \longrightarrow & (B,(0.9,0.1,0.1)) & (SD,(0.9,0.2,0.1)) & (AS,(0.9,0.2,0.1)) & (WTG,(0.7,0.1,0.3)) \end{array}$$

$$\left[A^{\star}_{ij}\right]_{a\times\mathfrak{b}} = \begin{bmatrix} \mathbb{T}^{\star}_{\mathbb{L}^a_1}(u_1), \mathbb{I}^{\star}_{\mathbb{L}^a_1}(u_1), \mathbb{F}^{\star}_{\mathbb{L}^a_1}(u_1) & \mathbb{T}^{\star}_{\mathbb{L}^b_2}(u_1), \mathbb{I}^{\star}_{\mathbb{L}^b_2}(u_1), \mathbb{F}^{\star}_{\mathbb{L}^b_2}(u_1) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_1), \mathbb{I}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_1), \mathbb{F}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_1) \\ \mathbb{T}^{\star}_{\mathbb{L}^a_1}(u_2), \mathbb{I}^{\star}_{\mathbb{L}^a_1}(u_2), \mathbb{F}^{\star}_{\mathbb{L}^a_1}(u_2) & \mathbb{T}^{\star}_{\mathbb{L}^b_2}(u_2), \mathbb{I}^{\star}_{\mathbb{L}^b_2}(u_2), \mathbb{F}^{\star}_{\mathbb{L}^b_2}(u_2) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_2), \mathbb{I}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_2), \mathbb{F}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}^{\star}_{\mathbb{L}^a_1}(u_a), \mathbb{I}^{\star}_{\mathbb{L}^a_1}(u_a), \mathbb{F}^{\star}_{\mathbb{L}^a_1}(u_a) & \mathbb{T}^{\star}_{\mathbb{L}^b_2}(u_a), \mathbb{I}^{\star}_{\mathbb{L}^b_2}(u_a), \mathbb{F}^{\star}_{\mathbb{L}^b_2}(u_a) & \cdots & \mathbb{T}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_a), \mathbb{I}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_a), \mathbb{F}^{\star}_{\mathbb{L}^z_{\mathfrak{b}}}(u_i) \end{bmatrix}$$

$$\left[\mathcal{S}^{\star}\right] = \begin{bmatrix} (B,(0.7700,0.2060,0.2300)) & (SD,(0.7264,0.2060,0.3253)) & (AS,(0.6131,0.2632,0.1861)) & (WTG,(0.4336,0.3834,0.2213)) \\ (B,(0.8318,0.1861,0.1189)) & (SD,(0.800,0.1861,0.1861)) & (AS,(0.8318,0.1732,0.1414)) & (WTG,(0.7172,0.3568,0.200)) \\ (B,(0.7172,0.1732,0.2213)) & (SD,(0.8318,0.1189,0.1682)) & (AS,(0.8811,0.1189,0.100)) & (WTG,(0.3494,0.3568,0.4757)) \\ (B,(0.8373,0.1316,0.2300)) & (SD,(0.8066,0.1316,0.1934)) & (AS,(0.7551,0.1316,0.2060)) & (WTG,(0.800,0.1732,0.2378)) \end{bmatrix}$$

Now we calculate the weight Δ^x *for* $(x = 1, 2, 3, 4)$ of each decision-makers using $\Delta^x = \frac{\mathbb{S}\left(A^x_{ij}, A^*_{ij}\right)}{\sum_{x=1}^{t} \mathbb{S}\left(A^x_{ij}, A^*_{ij}\right)}$. We have

$$\Delta^1 = \frac{0.8728}{(0.8728 + 0.9004 + 0.8690 + 0.8916} = 0.2470$$
$$\Delta^2 = \frac{0.9004}{(0.8728 + 0.9004 + 0.8690 + 0.8916)} = 0.2548$$
$$\Delta^3 = \frac{0.8690}{(0.8728 + 0.9004 + 0.8690 + 0.8916)} = 0.2459$$
$$\Delta^4 = \frac{0.8916}{(0.8728 + 0.9004 + 0.8690 + 0.8916)} = 0.2532$$

Step 2 (Aggregate Neutrosophic Hypersoft Decision Matrices): Now we construct an aggregated neutrosophic hypersoft decision matrix to obtain one group decision. An aggregated neutrosophic hypersoft decision matrix is denoted as A_{ij} and it is given as shown at the bottom of the page.

The elements of A_{ij} in the matrix $\left[A_{ij}\right]_{a\times b}$ is calculated as $\left[A_{ij}\right]_{a\times b} = \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^x_{\mathbb{L}^k_j}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^x_{\mathbb{L}^k_j}(u_i)\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^x_{\mathbb{L}^k_j}(u_i)\right)^{\Delta^x}\right)$.

After calculations, the aggregated neutrosophic hypersoft decision matrices is as shown at the bottom of the page.

One calculation is provided for the convenience of the reader. For $i = 1$, $j = 1$, $k = a = 2$, $\mathbb{S}_{11} = \begin{pmatrix} \left(1 - (1-0.9)^{0.2470}(1-0.3)^{0.2548}(1-0.6)^{0.2459}\right. \\ \left.(1-0.9)^{0.2523}\right), \\ (0.2)^{0.2470}(0.3)^{0.2548}(0.3)^{0.2459}(0.1)^{0.2523}, \\ (0.1)^{0.2470}(0.7)^{0.2548}(0.4)^{0.2459}(0.1)^{0.2523} \end{pmatrix}$.

We obtain that $\mathbb{S}_{11} = (0.7691, 0.2057, 0.2309)$.

Step 3 (Determine the Weight of Attributes): Weight $\mathbb{w}^j$ of attributes $\mathbb{L}_j, j = 1, 2 \ldots b$ is calculated using $\mathbb{w}^j = \left(\mathbb{T}_{\mathbb{L}_j}, \mathbb{I}_{\mathbb{L}_j}, \mathbb{F}_{\mathbb{L}_j}\right) = \left(1 - \prod_{x=1}^{t}\left(1 - \mathbb{T}^x_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{I}^x_{\mathbb{L}_j}\right)^{\Delta^x}, \prod_{x=1}^{t}\left(\mathbb{F}^x_{\mathbb{L}_j}\right)^{\Delta^x}\right)$. To calculate the weight of attributes, we use the importance of selected attributes by each decisionmaker as shown at the bottom of the page.

Using this importance of attributes, we get $\mathbb{w}^1 = (0.8228, 0.1407, 0.1406)$, $\mathbb{w}^2 = (0.7785, 0.1870, 0.1424)$, $\mathbb{w}^3 = (0.8212, 0.1421, 0.1424)$, $\mathbb{w}^4 = (0.7542, 0.1565, 0.2778)$. One calculation is provided for the reader. For $j = 1$,

$$\mathbb{w}^1 = \begin{pmatrix} \left(1-(1-0.9)^{0.2470}(1-0.8)^{0.2548}(1-0.5)^{0.2459}\right. \\ \left.(1-0.9)^{0.2523}\right), \\ (0.2)^{0.2470}(0.1)^{0.2548}(0.2)^{0.2459}(0.1)^{0.2523}, \\ (0.1)^{0.2470}(0.1)^{0.2548}(0.4)^{0.2459}(0.1)^{0.2523} \end{pmatrix},$$

and then, $\mathbb{w}^1 = (0.8228, 0.1407, 0.1406)$

Step 4 (Calculate the Weighted Aggregated Decision Matrix): After finding the weights of attributes, we apply these weights to each row of aggregated decision matrix using $\left[A^{\omega}_{ij}\right]_{a\times b} = \left(\mathbb{T}^{\omega}_{\mathbb{L}^k_j}(u_i), \mathbb{I}^{\omega}_{\mathbb{L}^k_j}(u_i), \mathbb{F}^{\omega}_{\mathbb{L}^k_j}(u_i)\right) = \left(\left(\mathbb{T}_{\mathbb{L}^k_j}(u_i).\mathbb{T}_{\mathbb{L}_j}\right), \left(\mathbb{I}_{\mathbb{L}^k_j}(u_i) + \mathbb{I}_{\mathbb{L}_j} - \mathbb{I}_{\mathbb{L}^k_j}(u_i).\mathbb{I}_{\mathbb{L}_j}\right), \left(\mathbb{F}_{\mathbb{L}^k_j}(u_i) + \mathbb{F}_{\mathbb{L}_j} - \mathbb{F}_{\mathbb{L}^k_j}(u_i).\mathbb{F}_{\mathbb{L}_j}\right)\right)$. We get a weighted aggregated decision matrix $[\mathbb{S}^{\omega}]$ as shown at the top of the page.

One calculation is provided for the convenience of the reader. For $i = 1, j = 1, k = a = 2$, we have $\mathbb{S}^{\omega}_{11} = (((0.7691)(0.8228)), ((0.2057 + 0.1407 - (0.2057)())), 0.1407 \quad ((0.2309 + 0.1406 - (0.2309)(0.1406))))$. We obtain $\mathbb{S}^{\omega}_{11} = (0.6328, 0.3175, 0.3390)$.

Step 5 (Determine the Ideal Solution): Since we are dealing with benefits type $(\mathbb{C}_1)$ attributes so Neutrosophic

$$\left[A_{ij}\right]_{a\times b} = \begin{bmatrix} \mathbb{T}_{\mathbb{L}^a_1}(u_1), \mathbb{I}_{\mathbb{L}^a_1}(u_1), \mathbb{F}_{\mathbb{L}^a_1}(u_1) & \mathbb{T}_{\mathbb{L}^b_2}(u_1), \mathbb{I}_{\mathbb{L}^b_2}(u_1), \mathbb{F}_{\mathbb{L}^b_2}(u_1) & \cdots & \mathbb{T}_{\mathbb{L}^z_b}(u_1), \mathbb{I}_{\mathbb{L}^z_b}(u_1), \mathbb{F}_{\mathbb{L}^z_b}(u_1) \\ \mathbb{T}_{\mathbb{L}^a_1}(u_2), \mathbb{I}_{\mathbb{L}^a_1}(u_2), \mathbb{F}_{\mathbb{L}^a_1}(u_2) & \mathbb{T}_{\mathbb{L}^b_2}(u_2), \mathbb{I}_{\mathbb{L}^b_2}(u_2), \mathbb{F}_{\mathbb{L}^b_2}(u_2) & \cdots & \mathbb{T}_{\mathbb{L}^z_b}(u_2), \mathbb{I}_{\mathbb{L}^z_b}(u_2), \mathbb{F}_{\mathbb{L}^z_b}(u_2) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{T}_{\mathbb{L}^a_1}(u_a), \mathbb{I}_{\mathbb{L}^a_1}(u_a), \mathbb{F}_{\mathbb{L}^a_1}(u_a) & \mathbb{T}_{\mathbb{L}^b_2}(u_a), \mathbb{I}_{\mathbb{L}^b_2}(u_a), \mathbb{F}_{\mathbb{L}^b_2}(u_a) & \cdots & \mathbb{T}_{\mathbb{L}^z_b}(u_a), \mathbb{I}_{\mathbb{L}^z_b}(u_b), \mathbb{F}_{\mathbb{L}^z_b}(u_a) \end{bmatrix}$$

$$[\mathbb{S}] = \begin{bmatrix} (B, (0.7691, 0.2057, 0.2309)) & (SD, (0.7303, 0.2051, 0.3212)) & (AS, (0.6146, 0.2606, 0.1862)) & (WTG, (0.4322, 0.3854, 0.2211)) \\ (B, (0.8313, 0.1864, 0.1187)) & (SD, (0.7998, 0.1870, 0.1857)) & (AS, (0.8316, 0.1731, 0.1419)) & (WTG, (0.7183, 0.3560, 0.1991)) \\ (B, (0.7168, 0.1733, 0.2215)) & (SD, (0.8318, 0.1186, 0.1687)) & (AS, (0.8813, 0.1187, 0.100)) & (WTG, (0.3482, 0.3573, 0.4752)) \\ (B, (0.8386, 0.1310, 0.2293)) & (SD, (0.8051, 0.1319, 0.1937)) & (AS, (0.7550, 0.1319, 0.2065)) & (WTG, (0.7998, 0.1733, 0.2376)) \end{bmatrix}$$

$$\begin{array}{lllll} \mathbb{M}^1 \longrightarrow & (B, (0.9, 0.2, 0.1)) & (SD, (0.7, 0.2, 0.2)) & (AS, (0.8, 0.1, 0.1)) & (WTG, (0.6, 0.3, 0.5)) \\ \mathbb{M}^2 \longrightarrow & (B, (0.8, 0.1, 0.1)) & (SD, (0.6, 0.3, 0.4)) & (AS, (0.5, 0.2, 0.4)) & (WTG, (0.7, 0.2, 0.2)) \\ \mathbb{M}^3 \longrightarrow & (B, (0.5, 0.2, 0.4)) & (SD, (0.8, 0.1, 0.1)) & (AS, (0.9, 0.1, 0.1)) & (WTG, (0.9, 0.1, 0.2)) \\ \mathbb{M}^4 \longrightarrow & (B, (0.9, 0.1, 0.1)) & (SD, (0.9, 0.2, 0.1)) & (AS, (0.9, 0.2, 0.1)) & (WTG, (0.7, 0.1, 0.3)) \end{array}$$

$$[\mathbb{S}^{\omega}] = \begin{bmatrix} (B, (0.6328, 0.3175, 0.3390)) & (SD, (0.5685, 0.3537, 0.4179)) & (AS, (0.5047, 0.3657, 0.3021)) & (WTG, (0.326, 0.4816, 0.4375)) \\ (B, (0.6840, 0.3009, 0.2426)) & (SD, (0.6226, 0.3390, 0.3017)) & (AS, (0.6829, 0.2906, 0.2641)) & (WTG, (0.5417, 0.4568, 0.4216)) \\ (B, (0.5898, 0.2896, 0.331)) & (SD, (0.6476, 0.2834, 0.2871)) & (AS, (0.7237, 0.2439, 0.2282)) & (WTG, (0.2626, 0.4579, 0.621)) \\ (B, (0.69, 0.2533, 0.3377)) & (SD, (0.6268, 0.2942, 0.3085)) & (AS, (0.62, 0.2551, 0.3195)) & (WTG, (0.6032, 0.3027, 0.4494)) \end{bmatrix}$$

hypersoft positive ideal solution is calculated using $\mathbb{S}^{\omega^+} = [(B, (0.69, 0.2533, 0.2426))\ (SD, (0.6476, 0.2834, 0.2871))\ (AS, (0.7237, 0.3439, 0.2282))\ (WTG, (0.6032, 0.3027, 0.4216))]$ One calculation is provided for the convenience of the reader. For $i = 1, 2, 3, 4, j = 1, \Bbbk = a = 2$ we have $\mathbb{S}_1^{\omega^+} = max\{0.6328, 0.6840, 0.5898, 0.69\}$, $min\{0.3175, 0.3009, 0.2896, 0.2533\}$, $min\{0.3390, 0.2426, 0.331, 0.3377\}$. Thus, we obtain $\mathbb{S}_1^{\omega^+} = (B, (0.69, 0.2533, 0.2426))$ Similarly, the neutrosophic hypersoft negative ideal solution is given as $\mathbb{S}^{\omega^-} = [(B, (0.5898, 0.3175, 0.3390))\ (SD, (0.5685, 0.3537, 0.4179))$. One calculation is provided for the convenience of the reader. For $i = 1, 2, 3, 4, j = 1, \Bbbk = a = 2$ we have $\mathbb{S}_1^{\omega^+} = min\{0.6328, 0.6840, 0.5898, 0.69\}$, $max\ \{0.3175, 0.3009, 0.2896, 0.2533\}$, $max\{0.3390, 0.2426, 0.331, 0.3377\}$. $\mathbb{S}_1^{\omega^-} = (B, (0.5898, 0.3175, 0.3390))$.

Step 6 (Calculate the Distance Measure): Now we find the normalized hamming distance between the alternatives and positive ideal solution using $\mathbb{D}^{i+}\left(A_{ij}^{\omega}, A_j^{\omega^+}\right) = \frac{1}{3\mathfrak{b}} \sum_{j=1}^{\mathfrak{b}} \left(\left| \mathbb{T}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{T}^{\omega+}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| + \left| \mathbb{I}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{I}^{\omega+}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| + \left| \mathbb{F}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{F}^{\omega+}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| \right)$. We get $\mathbb{D}^{1+}\left(\mathbb{S}_1^{\omega}, \mathbb{S}^{\omega^+}\right) = 0.1154$, $\mathbb{D}^{2+}\left(\mathbb{S}_2^{\omega}, \mathbb{S}^{\omega^+}\right) = 0.0407$, $\mathbb{D}^{3+}\left(\mathbb{S}_3^{\omega}, \mathbb{S}^{\omega^+}\right) = 0.0767$, $\mathbb{D}^{4+}\left(\mathbb{S}_4^{\omega}, \mathbb{S}^{\omega^+}\right) = 0.0318$.

One calculation is provided for the convenience of the reader. For $i = 1$, we obtain $\mathbb{D}^{1+}\left(\mathbb{S}_1^{\omega}, \mathbb{S}^{\omega^+}\right) = \frac{1}{12}((|0.6328-0.69| + |0.69-0.2533| + |0.3390-0.2426|) + (|0.5685-0.6476| + |0.3537-0.2834| + |0.4179-0.2871|) + (|0.5047-0.7237| + |0.3657-0.2439| + |0.3021-0.2282|) + (|0.326-0.6032| + |0.4816-0.3027| + |0.4375-0.4216|))$. $\mathbb{D}^{1+}\left(\mathbb{S}_1^{\omega}, \mathbb{S}^{\omega^+}\right) = 0.1154$.

Similarly, we will find the normalized hamming distance between the alternatives and negative ideal solution using $\mathbb{D}^{i-}\left(A_{ij}^{\omega}, A_j^{\omega^-}\right) = \frac{1}{3\mathfrak{b}} \sum_{j=1}^{\mathfrak{b}} \left(\left| \mathbb{T}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{T}^{\omega-}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| + \left| \mathbb{I}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{I}^{\omega-}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| + \left| \mathbb{F}^{\omega}_{\mathbb{L}_j^{\Bbbk}}(u_i) - \mathbb{F}^{\omega-}_{\mathbb{L}_j^{\Bbbk}}(u_i) \right| \right)$. We get, $\mathbb{D}^{1-}\left(\mathbb{S}_1^{\omega}, \mathbb{S}^{\omega^-}\right) = 0.0131$, $\mathbb{D}^{2-}\left(\mathbb{S}_2^{\omega}, \mathbb{S}^{\omega^-}\right) = 0.1111$, $\mathbb{D}^{3-}\left(\mathbb{S}_3^{\omega}, \mathbb{S}^{\omega^-}\right) = 0.0753$, $\mathbb{D}^{4-}\left(\mathbb{S}_4^{\omega}, \mathbb{S}^{\omega^-}\right) = 0.0943$.

One calculation is provided for the convenience of the reader, for $i = 1$, $\mathbb{D}^{1-}\left(\mathbb{S}_1^{\omega}, \mathbb{S}^{\omega^-}\right) = \frac{1}{12}\{(|0.6328 - 0.5898| + |0.3175 - 0.3175| + |0.3390 - 0.3390|) + (|0.5685 - 0.5685| + |0.3537 - 0.3537| + |0.4179 - 0.4179|) + (|0.5047 - 0.5047| + |0.3657 - 0.3657| + |0.3021 - 0.3195|) + (|0.326-0.2626| + |0.4816 - 0.4816| + |0.4375-0.621|)\}$

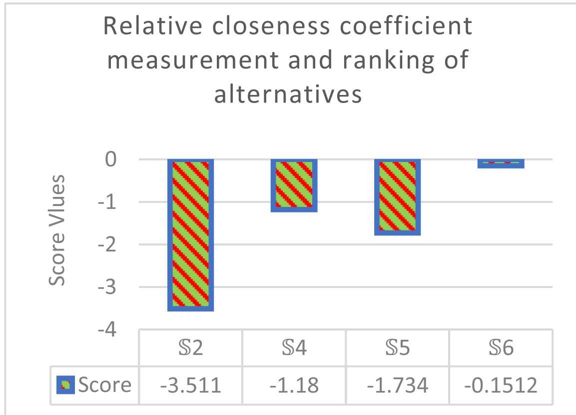

FIGURE 2. **Relative closeness coefficient measurement and ranking of alternatives.**

Step 7 (Calculate the Relative Closeness Coefficient): Now we will calculate the relative closeness index using $\mathbb{RC}^i = \frac{\mathbb{D}^{i-}\left(A_{ij}^{\omega}, A_j^{\omega^-}\right)}{\max\limits_i\left\{\mathbb{D}^{i-}\left(A_{ij}^{\omega}, A_j^{\omega^-}\right)\right\}} - \frac{\mathbb{D}^{i+}\left(A_{ij}^{\omega}, A_j^{\omega^+}\right)}{\min\limits_i\left\{\mathbb{D}^{i+}\left(A_{ij}^{\omega}, A_j^{\omega^+}\right)\right\}}$. We get

$$\mathbb{RC}^1 = \frac{0.0131}{0.1111} - \frac{0.1154}{0.0318} = -3.5110$$
$$\mathbb{RC}^2 = \frac{0.1111}{0.1111} - \frac{0.0407}{0.0318} = -1.18$$
$$\mathbb{RC}^3 = \frac{0.0753}{0.1111} - \frac{0.0767}{0.0318} = -1.7342$$
$$\mathbb{RC}^4 = \frac{0.0943}{0.1111} - \frac{0.0318}{0.0318} = -0.1512$$

Since we know that the 4-companies selected security system set is $\{\mathbb{S}^2, \mathbb{S}^4, \mathbb{S}^5, \mathbb{S}^6\}$ for $i = 1, 2, 3, 4$ respectively, we rank the selected alternatives as shown in Figure 2 according to the descending order of relative closeness index as $\mathbb{S}^6 > \mathbb{S}^4 > \mathbb{S}^5 > \mathbb{S}^2$ This shows that $\mathbb{S}^6$ is the best alternative for the security system. The proposed NHSS-TOPSIS algorithm is used under the NHSS environment for the ranking of alternatives with a good result.

VI. CONCLUSION

Neutrosophic hypersoft set (NHSS) an extension of soft set is a new topic NHSS can be a strong mathematical model to deal with incomplete, indeterminate uncertain and vague information. Generally, NHSS is more efficient to deal with uncertain and vague information than fuzzy sets and intuitionistic fuzzy sets. However, no one had considered distances and similarity for NHSS In this paper, we first propose distance and similarity measures for NHSS. By using the proposed distance and similarity measures, we make an extension of TOPSIS technique to the NHSSTOPSIS for MCDM. The Hamming distance measure is used to calculate the distance of alternatives from the positive ideal and negative ideal. We then rank the alternatives based on the relative closeness index. At last, we solve illustrative cases of a medical diagnosis problem and security system selection to confirm the reliability and adequacy of the proposed NHSS-TOPSIS technique. The proposed NHSS-TOPSIS has gigantic chances for MCDM issues in different fields such as supplier determination, manufacturing frameworks, and numerous other regions of management frameworks. In expansions, the proposed methodology can be amplified in several directions to include an extensive range of decision-making issues in different neutrosophic hypersoft situations. On the other hand, measuring the uncertainty/fuzziness of NHSS is an important step in NHSS applied systems. Since entropy is an important measure of uncertainty/fuzziness in our future works, we will define some entropies for NHSS. We will also give axioms for these entropy measures and then give their applications

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