Computers in Industry 108 (2019) 45–52

Contents lists available at ScienceDirect

Computers in Industry

journal homepage: www.elsevier.com/locate/compind

Dynamic interval valued neutrosophic set: Modeling decision making in dynamic environments

Nguyen Tho Thong[a,b], Luu Quoc Dat[c], Le Hoang Son[a,*], Nguyen Dinh Hoa[a], Mumtaz Ali[d], Florentin Smarandache[e]

[a] *VNU Information Technology Institute, Vietnam National University, Hanoi, Viet Nam*
[b] *VNU University of Engineering and Technology, Vietnam National University, Hanoi, Viet Nam*
[c] *VNU University of Economics and Business, Vietnam National University, Hanoi, Viet Nam*
[d] *University of Southern Queensland, 4300, QLD, Australia*
[e] *University of New Mexico, 705 Gurley Ave., Gallup, New Mexico 87301, USA*

ARTICLE INFO

Article history:
Received 24 October 2018
Received in revised form 31 January 2019
Accepted 10 February 2019
Available online xxx

Keywords:
Dynamic environment
Interval valued set
Neutrosophic set

ABSTRACT

Dynamic decision problems constrained by time are of highly-interested in many aspects of real life. This paper proposes a new concept called the Dynamic Interval-valued Neutrosophic Set (DIVNS) for such the dynamic decision-making applications. Firstly, we define the definitions and mathematical operations, properties and correlations of DIVNSs. Next, we develop a new TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) method based on the proposed DIVNS theory. Finally, a practical application of the method for evaluating lecturers' performance at the University of Languages and International Studies, Vietnam National University, Hanoi (ULIS-VNU) is given to illustrate the efficiency of our approach.

1. Introduction

Neutrosophic set (NS) [45] is able to handle indeterminacy information [51,52,58]. NS and its extensions have become widely applied in almost areas, such as decision-making [1,12,20,21,33,34,41,42,49,58–62], clustering analysis [56,59], image processing [27,28], etc. However, in some complex problems in real-life, data may be collected from different time intervals or multi-periods, which raises the need for dynamic decision making for such the situations. The term 'dynamic' can be regarded in term of criteria such as (a) a series of decisions required to reach a goal; (b) path dependent decision; (c) the state of decision. This research considers the 'dynamic' decision problems which are constrained by time, as seen, for example, in emergency management and patient care. Specifically, when the economic situation of a certain company is investigated, the economic growth level of product series should be investigated by changes of the trend of profit of all products through the periods. Another example can be found in medical diagnosis where clinicians have to exam patients by different time intervals.

Recently, Yan et al. [53] developed a dynamic multiple attribute decision making method with grey number (considering both attribute value aggregation of all periods and their fluctuation among periods) to calculate degree of every alternative. This model was also used in [32] to manage linguistic bipolar scales using transformation between bipolar and unipolar linguistic terms. Ye [57] proposed a dynamic neutrosophic multiset. For decision assistance in dynamic environments, some algorithms that used TOPSIS under neutrosophic linguistic environments were presented in [2,10,11,22,23,25,26,33–37,40,55]. There have been also some works that applied the Interval-Valued Neutrosophic Set (IVNS) with the TOPSIS method for decision making [6,11,29,33,49,54,62]. Other relevant decision making methods can be retrieved in [3–5,7–9,13–19]. However, the existing researches did not consider different time intervals as the objective of this research aims. To the best of our knowledge, fluctuation of alternative's attribute values within periods on NSs has not been examined. In many practical cases, there is not enough available information to judge complicated situations, indeed it often given approximate ranges.

In this paper, we propose a new TOPSIS method based on a new extension of NS called the Dynamic Interval-valued Neutrosophic Set (DIVSN) for dynamic decision-making problems. The main contribution includes:

(a) We define definitions and mathematical operations, properties and correlations of DIVNSs.

---

[*] Corresponding author at: 144 Xuan Thuy, Cau Giay, Hanoi, Viet Nam. Tel.: +84 904.171.284.

E-mail addresses: 17028003@vnu.edu.vn (N.T. Thong), datlq@vnu.edu.vn (L.Q. Dat), sonlh@vnu.edu.vn (L.H. Son), hoand@vnu.edu.vn (N.D. Hoa), Mumtaz.Ali@usq.edu.au (M. Ali), fsmarandache@gmail.com (F. Smarandache).

https://doi.org/10.1016/j.compind.2019.02.009
0166-3615/

(b) We develop a new TOPSIS method based on the proposed DIVNS theory.
(c) A practical application of the method for evaluating lecturers' performance at the Vietnam National University, Hanoi (ULIS-VNU) is given to illustrate the efficiency of our approach.

Section 2 defines the new concept of Dynamic Interval-valued Neutrosophic Set (DIVSN). Section 3 presents the TOPSIS method for DIVSN. Section 4 illustrates the proposed method in a practical application. Finally, Section 5 summarizes the findings.

2. Dynamic interval-valued neutrosophic set

2.1. Set definition

Definition 2.1. [45]: Let U be a universe of discourse. A neutrosophic set is:

$$A = \{\langle x : T_A(x), I_A(x), F_A(x)\rangle, x \in U\}$$

where $T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \le sup(T_A(x)) + sup(I_A(x)) + sup(F_A(x)) \le 3$.

Definition 2.2. [45]: A neutrosophic number is defined as $N = a + bI$, where a and b are real numbers, and I is the indeterminacy.

Definition 2.3. [57]: A Dynamic Single-Valued Neutrosophic Set (DSVNS) is: $A = \{x \in U; x(T_x(t), I_x(t), F_x(t))\}$ for all $x \in A$:

$$T_x, I_x, F_x : [0, \infty) \to [0,1]$$

where T_x, I_x, F_x are continuous functions whose argument is time (t).

Based on the definition of DSVNS above, we formulate the new definition as below.

Definition 2.4. A Dynamic Interval-Valued Neutrosophic Set (DIVNS) is in the form below:

$$x\left(\left[T_x^L(t), T_x^U(t)\right], \left[I_x^L(t), I_x^U(t)\right], \left[F_x^L(t), F_x^U(t)\right]\right)$$

where $t \ge 0$,

$$T_x^L(t) < T_x^U(t), I_x^L(t) < I_x^U(t), F_x^L(t) < F_x^U(t)$$

And

$$\left[T_x^L(t), T_x^U(t)\right], \left[I_x^L(t), I_x^U(t)\right], \left[F_x^L(t), F_x^U(t)\right] \subseteq [0,1]$$

In other words, a DIVNS is a neutrosophic set whose elements' neutrosophic components (membership, indeterminacy, non-membership) are all intervals changing with respect to time.

For a simplified notation, we denote:

$$T_x(t) = \left[T_x^L(t), T_x^U(t)\right], I_x(t) = \left[I_x^L(t), I_x^U(t)\right], F_x(t) = \left[F_x^L(t), F_x^U(t)\right]$$

where $T_x(t), I_x(t), F_x(t) : [0, \infty) \to P([0,1])$ with $P([0,1])$ been the power set of $[0,1]$.

We can also use the notation $A(t)$ and $x(t)$, meaning that each element x in A depends on t. Or $T_x(t), I_x(t), F_x(t)$ are interval – valued functions (a particular case of neutrosophic function [1]).

The difference of the new definition against the existing one in [57]:

We have extended Ye's DSVNS [57] to DIVNS by considering a time sequence: $t = \{t_1, t_2, \ldots, t_k\}$ then at each time t_l, $1 \le l \le m$, the neutrosophic components of the generic element $x \in A$ change as follow:

$$x(\langle T_x(t_1), I_x(t_1), F_x(t_1)\rangle; \langle T_x(t_2), I_x(t_2), F_x(t_2)\rangle; \ldots; \langle T_x(t_k), I_x(t_k), F_x(t_k)\rangle)$$

Example 2.1. A DIVNS in time sequence $t = \{t_1, t_2, t_3\}$ and a universal $NS = \{x_1, x_2, x_3\}$ is given:

$$A = \left\{ \begin{array}{l} \langle x_1, ([0.1, 0.25], [0.15, 0.2], [0.3, 0.6]), ([0.45, 0.5], [0.1, 0.3], [0.2, 0.4]), ([0.6, 0.7], [0.52, 0.6], [0.7, 0.9])\rangle, \\ \langle x_2, ([0.38, 0.4], [0.25, 0.4], [0.12, 0.3]), ([0.07, 0.1], [0.1, 0.2], [0.09, 0.1]), ([0.22, 0.3], [0.4, 0.5], [0.3, 0.43])\rangle, \\ \langle x_3, ([0.7, 0.9], [0.33, 0.45], [0.59, 0.6]), ([0.2, 0.22], [0.5, 0.6], [0.2, 0.3]), ([0.8, 0.9], [0.3, 0.41], [0.3, 0.33])\rangle \end{array} \right\}$$

2.2. Set theoretic operations of DIVNS

Let $A(t)$ and $B(t)$ be two DIVNSs included in U;

$$A(t) = \left\{\left(x(t), \left\langle T_x^A(t_l), I_x^A(t_l), F_x^A(t_l)\right\rangle\right), \forall t_l \in t, x \in U\right\}, B(t) = \left\{\left(x(t), \left\langle T_x^B(t_l), I_x^B(t_l), F_x^B(t_l)\right\rangle\right), \forall t_l \in t, x \in U\right\}$$

Definition 2.5. : DIVNS Intersection

$$A(t) \cap B(t) = \left\{\left(x(t), \left\langle T_x^A(t_l) \wedge T_x^B(t_l), I_x^A(t_l) \vee I_x^B(t_l), F_x^A(t_l) \vee F_x^B(t_l)\right\rangle\right), \forall t_l \in t, x \in U\right\}$$

Definition 2.6. DIVNS Union

$$A(t) \cup B(t) = \left\{\left(x(t), \left\langle T_x^A(t_l) \vee T_x^B(t_l), I_x^A(t_l) \wedge I_x^B(t_l), F_x^A(t_l) \wedge F_x^B(t_l)\right\rangle\right), \forall t_l \in t, x \in U\right\}$$

Definition 2.7. DIVNS Complement

$$A(t)^C = \left\{\left(x(t), \left\langle F_x^A(t_l), 1 - I_x^A(t_l), T_x^A(t_l)\right\rangle\right), \forall t_l \in t, x \in U\right\}$$

Definition 2.8. DIVNS inclusion

$$A(t) \subseteq B(t) \sim T_x^A(t_l) \le T_x^B(t_l), I_x^A(t_l) \ge I_x^B(t_l) \text{ and } F_x^A(t_l) \ge F_x^B(t_l).$$

Definition 2.9. DIVNS Equality

$$A(t) = B(t) \Leftrightarrow A(t) \subseteq B(t) \text{ and } A(t) \supseteq B(t).$$

In the above DIVNS aggregation operators by "$\wedge$" we meant the "t-norm" and by "$\vee$"

the t-conorm from the single–valued fuzzy sets

2.3. Operations on DIVNS numbers

Let us consider two DIVNS numbers:

$$a(t) = \left\{ \left\langle T_x^A(t_1), I_x^A(t_1), F_x^A(t_1) \right\rangle, ..., \left\langle T_x^A(t_k), I_x^A(t_k), F_x^A(t_k) \right\rangle \right\}$$

$$b(t) = \left\{ \left\langle T_x^B(t_1), I_x^B(t_1), F_x^B(t_1) \right\rangle, ..., \left\langle T_x^B(t_k), I_x^B(t_k), F_x^B(t_k) \right\rangle \right\}.$$

Definition 2.14. Correlation coefficient of DIVNSs

Let

$$A(t) = \left\{ \left(x(t), \left\langle T^A(x,t_l), I^A(x,t_l), F^A(x,t_l) \right\rangle \right), \forall t_l \in t, x \in U \right\},$$
$$B(t) = \left\{ \left(x(t), \left\langle T^B(x,t_l), I^B(x,t_l), F^B(x,t_l) \right\rangle \right), \forall t_l \in t, x \in U \right\}$$

be two DIVNs in $t = \{t_1, t_2, ..., t_k\}$ and $U = (x_1, x_2, ..., x_n)$.

A correlation coefficient is:

$$\rho(A(t),B(t)) = \frac{1}{k}\sum_{l=1}^{k} \frac{\sum_{i=1}^{n} \begin{pmatrix} \inf T^A(x_i,t_l) \times \inf T^B(x_i,t_l) + supT^A(x_i,t_l) \times supT^B(x_i,t_l) \\ +\inf I^A(t_l) \times \inf I^B(x_i,t_l) + supI^A(x_i,t_l) \times supI^B(x_i,t_l) \\ +\inf F^A(x_i,t_l) \times \inf F^B(x_i,t_l) + supF^A(x_i,t_l) \times supF^B(x_i,t_l) \end{pmatrix}}{\left(\begin{array}{l} \sqrt{\sum_{i=1}^{n} \left[\begin{array}{l} \left(\inf T^A(x_i,t_l)\right)^2 + \left(supT^A(x_i,t_l)\right)^2 + \left(\inf I^A(x_i,t_l)\right)^2 \\ +\left(supI^A(x_i,t_l)\right)^2 + \left(\inf F^A(x_i,t_l)\right)^2 + \left(supF^A(x_i,t_l)\right)^2 \end{array} \right]} \\ \times \sqrt{\sum_{i=1}^{n} \left[\begin{array}{l} \left(\inf T^B(x_i,t_l)\right)^2 + \left(supT^B(x_i,t_l)\right)^2 + \left(\inf I^B(x_i,t_l)\right)^2 \\ +\left(supI^B(x_i,t_l)\right)^2 + \left(\inf^B(x_i,t_l)\right)^2 + \left(supF^B(x_i,t_l)\right)^2 \end{array} \right]} \end{array} \right)} \tag{5}$$

Definition 2.10. Addition of DIVNS numbers

$$a(t) \oplus b(t) = \left\{ \begin{array}{l} \left\langle \begin{array}{l} T_x^A(t_1) + T_x^B(t_1) - T_x^A(t_1) \times T_x^B(t_1), \\ I_x^A(t_1) \times I_x^B(t_1), F_x^A(t_1) \times F_x^B(t_1) \end{array} \right\rangle, \\ ..., \\ \left\langle \begin{array}{l} T_x^A(t_k) + T_x^B(t_k) - T_x^A(t_k) \times T_x^B(t_k), \\ I_x^A(t_k) \times I_x^B(t_k), F_x^A(t_k) \times F_x^B(t_k) \end{array} \right\rangle \end{array} \right\} \tag{1}$$

Definition 2.11. Multiplication of DIVNS numbers

$$a(t) \otimes b(t) = \left\{ \begin{array}{l} \left\langle \begin{array}{l} T_x^A(t_1) \times T_x^B(t_1), I_x^A(t_1) + I_x^B(t_1) - I_x^A(t_1) \times I_x^B(t_1), \\ F_x^A(t_1) + F_x^B(t_1) - F_x^A(t_1) \times F_x^B(t_1) \end{array} \right\rangle, \\ ..., \\ \left\langle \begin{array}{l} T_x^A(t_k) \times T_x^B(t_k), I_x^A(t_k) + I_x^B(t_k) - I_x^A(t_k) \times I_x^B(t_k), \\ F_x^A(t_k) + F_x^B(t_k) - F_x^A(t_k) \times F_x^B(t_k) \end{array} \right\rangle \end{array} \right\} \tag{2}$$

Definition 2.12. Scalar Multiplication of DIVNS numbers

$$\alpha \times a(t) = \left\{ \left\langle 1 - \left(1 - T_x^A(t_1)\right)^\alpha, I_x^A(t_1)^\alpha, F_x^A(t_1)^\alpha \right\rangle, ..., \left\langle 1 - \left(1 - T_x^A(t_k)\right)^\alpha, I_x^A(t_k)^\alpha, F_x^A(t_k)^\alpha \right\rangle \right\} \tag{3}$$

Definition 2.13. Power of the DIVNS numbers

$$a(t)^\alpha = \left\{ \begin{array}{l} \left\langle T_x^A(t_1)^\alpha, 1 - \left(1 - I_x^A(t_1)\right)^\alpha, 1 - \left(1 - F_x^A(t_1)\right)^\alpha \right\rangle, \\ ..., \\ \left\langle T_x^A(t_k)^\alpha, 1 - \left(1 - I_x^A(t_k)\right)^\alpha, 1 - \left(1 - F_x^A(t_k)\right)^\alpha \right\rangle \end{array} \right\} \tag{4}$$

Theorem 2.1. The correlation coefficient between A and B satisfies:

$(Pr1)$ $0 \le \rho(A(t),B(t)) \le 1$;
$(Pr2)$ $\rho(A(t),B(t)) = 1$ *if* $A(t) = B(t)$;
$(Pr3)$ $\rho(A(t),B(t)) = \rho(B(t),A(t))$

Proof.

(Pr1) It is obvious that $\rho(A(t),B(t)) \ge 0$. From Cauchy–Schwarz inequality, we have

$$\sum_{i=1}^{n} \begin{pmatrix} \inf T^A(x_i,t_l) \times \inf T^B(x_i,t_l) + supT^A(x_i,t_l) \times supT^B(x_i,t_l) \\ +\inf I^A(t_l) \times \inf I^B(x_i,t_l) + supI^A(x_i,t_l) \times supI^B(x_i,t_l) \\ +\inf F^A(x_i,t_l) \times \inf F^B(x_i,t_l) + supF^A(x_i,t_l) \times supF^B(x_i,t_l) \end{pmatrix} \le \left(\begin{array}{l} \sqrt{\sum_{i=1}^{n} \left[\begin{array}{l} \left(\inf T^A(x_i,t_l)\right)^2 + \left(supT^A(x_i,t_l)\right)^2 + \left(\inf I^A(x_i,t_l)\right)^2 \\ +\left(supI^A(x_i,t_l)\right)^2 + \left(\inf F^A(x_i,t_l)\right)^2 + \left(supF^A(x_i,t_l)\right)^2 \end{array} \right]} \\ \times \sqrt{\sum_{i=1}^{n} \left[\begin{array}{l} \left(\inf T^B(x_i,t_l)\right)^2 + \left(supT^B(x_i,t_l)\right)^2 + \left(\inf I^B(x_i,t_l)\right)^2 \\ +\left(supI^B(x_i,t_l)\right)^2 + \left(\inf^B(x_i,t_l)\right)^2 + \left(supF^B(x_i,t_l)\right)^2 \end{array} \right]} \end{array} \right)$$

for each $l \in \{1, 2, ., k\}$. Thus, $0 \le \rho(A(t),B(t)) \le 1$.

(Pr2) $A(t) = B(t)$. $\forall l \in \{1, 2, ..., k\}$. We have $\inf T^A(x_i,t_l) = \inf T^B(x_i,t_l)$; $supT^A(x_i,t_l) = supT^B(x_i,t_l)$; $\inf I^A(x_i,t_l) = \inf I^B(x_i,t_l)$; $supI^A(x_i,t_l) = supI^B(x_i,t_l)$; $\inf F^A(x_i,t_l) = \inf F^B(x_i,t_l)$; $supF^A(x_i,t_l) = supF^B(x_i,t_l)$; $\inf T^A(x_i,t_l) = \inf T^B(x_i,t_l) \Rightarrow \rho(A(t),B(t)) = 1$

(Pr3) It is easily observed.

Definition 2.15. Weighted Correlation Coefficient of DIVNSs

Different weights for $x_i (i = 1, ..., n)$ and $t_l (l = 1, ..., k)$ are integrated as follows.

$$\rho_W(A(t),B(t)) = \frac{1}{k}\sum_{l=1}^{k}\omega_l \times \frac{\sum_{i=1}^{n} w_i \times \begin{pmatrix} \inf T^A(x_i,t_l) \times \inf T^B(x_i,t_l) + supT^A(x_i,t_l) \times supT^B(x_i,t_l) \\ +\inf I^A(t_l) \times \inf I^B(x_i,t_l) + supI^A(x_i,t_l) \times supI^B(x_i,t_l) \\ +\inf F^A(x_i,t_l) \times \inf F^B(x_i,t_l) + supF^A(x_i,t_l) \times supF^B(x_i,t_l) \end{pmatrix}}{\begin{pmatrix} \sqrt{\sum_{i=1}^{n} w(x_i) \times \begin{pmatrix} \left(\inf T^A(x_i,t_l)\right)^2 + \left(supT^A(x_i,t_l)\right)^2 + \left(\inf I^A(x_i,t)\right)^2 \\ +\left(supI^A(x_i,t_j)\right)^2 + \left(\inf F^A(x_i,t_j)\right)^2 + \left(supF^A(x_i,t_j)\right)^2 \end{pmatrix}} \\ \times \sqrt{\sum_{i=1}^{n} w(x_i) \times \begin{pmatrix} \left(\inf T^B(x_i,t_l)\right)^2 + \left(supT^B(x_i,t_l)\right)^2 + \left(\inf I^B(x_i,t_l)\right)^2 \\ +\left(supI^B(x_i,t_l)\right)^2 + \left(\inf^B(x_i,t_l)\right)^2 + \left(supF^B(x_i,t_l)\right)^2 \end{pmatrix}} \end{pmatrix}} \tag{6}$$

where $w = (w_1, w_2, ..., w_n)^T$ and $\omega = (\omega_1, \omega_2, ..., \omega_m)_n^T$ are weighting vectors of $x_i (i = 1, ..., n)$ and $t_l (l = 1, ..., k)$ with $\sum_{i=1} w_i = 1$ and $\sum_{l=1}^{k} \omega_l = 1$.

When $w_i = {}^1/_n; i = 1, ..., n$ and $\omega_l = {}^1/_k; l = 1, ..., m$, Eq. (6) turns to (5).

The weighted correlation coefficient between A and B also satisfies the properties as in Theorem 2.1.

3. A topsis method for divns

Assume $A = \{A_1, A_2, ..., A_v\}$ and $C = \{C_1, C_2, ..., C_n\}$ and $D = \{D_1, D_2, ..., D_h\}$ are sets of alternatives, attributes, and decision makers. For a decision maker $D_q; q = 1, ..., h$, the evaluation characteristic of an alternatives $A_m; m = 1, ..., v$, on an attribute $C_p; p = 1, ..., n$, in time sequence $t = \{t_1, t_2, ..., t_k\}$ is represented by the decision matrix $D^q(t_l) = \left(d_{mp}^q(t)\right)_{v\times n}; l = 1, 2, ..., k$. where $d_{mp}^q(t) = \left\langle x_{d_{mp}}^q(t), (T^q(d_{mp},t), I^q(d_{mp},t), F^q(d_{mp},t))\right\rangle; t = \{t_1, t_2, ..., t_k\}$ taken by DIVNSs evaluated by decision maker D_q.

3.1. Aggregate ratings

Let $x_{mpq}(t_l) = \{ \left[T_{mpq}^L(x_{t_l}), T_{mpq}^U(x_{t_l})\right], \left[I_{mpq}^L(x_{t_l}), I_{mpq}^U(x_{t_l})\right], \left[F_{mpq}^L(x_{t_l}), F_{mpq}^U(x_{t_l})\right]\}$ be the suitability rating of alternative A_m for criterion C_p by decision-maker D_q in time sequence t_l, where: $m = 1, ..., v; p = 1, ..., n; q = 1, ..., h; l = 1, ..., k$. The averaged suitability rating $\overline{x_{mp}} = \left\{ \left[\overline{T_{mp}^L(x)}, \overline{T_{mp}^U(x)}\right], \left[\overline{I_{mp}^L(x)}, \overline{I_{mp}^U(x)}\right], \left[\overline{F_{mp}^L(x)}, \overline{F_{mp}^U(x)}\right]\right\}$ can be evaluated as:

$$\overline{x_{mp}} = \frac{1}{h*k} \otimes \left\langle \begin{array}{c} \left\{ \left[T_{mpq}^L(x_{t_1}), T_{mpq}^U(x_{t_1})\right], \left[I_{mpq}^L(x_{t_1}), I_{mpq}^U(x_{t_1})\right], \left[F_{mpq}^L(x_{t_1}), F_{mpq}^U(x_{t_1})\right] \right\} + ...+ \\ \left\{ \left[T_{mpq}^L(x_{t_k}), T_{mpq}^U(x_{t_k})\right], \left[I_{mpq}^L(x_{t_k}), I_{mpq}^U(x_{t_k})\right], \left[F_{mpq}^L(x_{t_k}), F_{mpq}^U(x_{t_k})\right] \right\} \end{array} \right\rangle, \tag{7}$$

where,

$$\overline{T_{mp}(x)} = \left[\left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^{h} T_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle, \left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^{h} T_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle \right]$$

$$\overline{I_{mp}(x)} = \left[\left(\sum_{q=1}^{h} I_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h*k}}, \left(\sum_{q=1}^{h} I_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h*k}} \right]$$

$$\overline{F_{mp}(x)} = \left[\left(\sum_{q=1}^{h} F_{pmq}^L(x_{t_l}) \right)^{\frac{1}{h*k}}, \left(\sum_{q=1}^{h} F_{pmq}^U(x_{t_l}) \right)^{\frac{1}{h*k}} \right]$$

3.2. Importance weight aggregation

Let $x_{pq}(t_l) = \left\{ \left[T_{pq}^L(x_{t_l}), T_{pq}^U(x_{t_l})\right], \left[I_{pq}^L(x_{t_l}), I_{pq}^U(x_{t_l})\right], [F_{pq}^L(x_{t_l}), F_{pq}^U(x_{t_l})]\right\}$ be weight of D_q to criterion C_p in time sequence t_l, where: $p = 1, ..., n; q = 1, ..., h; l = 1, ..., k$. The average weight $\overline{w_p} = \left\{ \left[\overline{T_p^L(x)}, \overline{T_p^U(x)}\right], \left[\overline{I_p^L(x)}, \overline{I_p^U(x)}\right], \left[\overline{F_p^L(x)}, \overline{F_p^U(x)}\right]\right\}$ can be evaluated as:

$$\overline{w_p} = \frac{1}{h*k} \otimes \left\langle \begin{array}{c} \left\{ \left[T_{p1}^L(x_{t_1}), T_{p1}^U(x_{t_1})\right], \left[I_{p1}^L(x_{t_1}), I_{p1}^U(x_{t_1})\right], \left[F_{p1}^L(x_{t_1}), F_{p1}^U(x_{t_1})\right] \right\} + ...+ \\ \left\{ \left[T_{ph}^L(x_{t_h}), T_{ph}^U(x_{t_h})\right], \left[I_{ph}^L(x_{t_h}), I_{ph}^U(x_{t_h})\right], \left[F_{ph}^L(x_{t_h}), F_{ph}^U(x_{t_h})\right] \right\} \end{array} \right\rangle, \tag{8}$$

where,

$$\overline{T_p(x)} = \left[\left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^{h} T_{pq}^{L}(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle, \left\langle 1 - \left\{ 1 - \left(1 - \sum_{q=1}^{h} T_{pq}^{U}(x_{t_l}) \right)^{\frac{1}{h}} \right\}^{\frac{1}{k}} \right\rangle \right]$$

$$\overline{I_p(x)} = \left[\left(\sum_{q=1}^{h} I_{pq}^{L}(x_{t_l}) \right)^{\frac{1}{h*k}}, \left(\sum_{q=1}^{h} I_{pq}^{U}(x_{t_l}) \right)^{\frac{1}{h*k}} \right]$$

$$\overline{F_p(x)} = \left[\left(\sum_{q=1}^{h} F_{pq}^{L}(x_{t_l}) \right)^{\frac{1}{h*k}}, \left(\sum_{q=1}^{h} F_{pq}^{U}(x_{t_l}) \right)^{\frac{1}{h*k}} \right]$$

3.3. Compute the average weighted ratings

Average weighted ratings of alternatives in t_l, are:

$$\overline{G_m} = \frac{1}{n} \sum_{p=1}^{n} \overline{x_{mp}} * \overline{w_p}; m = 1, ..., v; p = 1, ..., n; \tag{9}$$

3.4. Determination of A^+, A^-, d_i^+ and d_i^-

Interval neutrosophic positive and negative ideal solutions namely (PIS, A^+) and (NIS, A^-) are:

$$A^+ = \{x, ([1, 1], [0, 0], [0, 0])\} \tag{10}$$

$$A^- = \{x, ([0, 0], [1, 1], [1, 1])\} \tag{11}$$

The distances of each alternative $A_m, m = 1, \ldots, t$ from A^+ and A^- in time sequence t_l, are calculated as:

$$\overline{d_m^+} = \sqrt{\left(\overline{G_m} - A^+ \right)^2} \tag{12}$$

$$\overline{d_m^-} = \sqrt{\left(\overline{G_m} - A^- \right)^2} \tag{13}$$

where d_m^+ and d_m^- represents the shortest and farthest distances of A_m.

3.4. Obtain best coefficient

The best coefficient in time sequence t_l, is shown below where high value indicates closer to interval neutrosophic PIS and farther from interval neutrosophic NIS:

$$\overline{CC_m} = \frac{\overline{d_m^-}}{\overline{d_m^+} + \overline{d_m^-}} \tag{14}$$

4. Applications

This section applies the proposed method to evaluate lecturers' performance in the case study of ULIS-VNU having 11 Faculties, 11 Departments, 09 Functional departments, 05 Centers and 01 Foreign Language Specializing High School with over 700 lecturers and 8000 high school, undergraduate and graduate students. Assume that ULIS-VNU needs to evaluate the lecturers' performance. After preliminary screening, five lecturers, i.e. $A_1, \ldots, A_5$, and three decision makers, i.e. $D_1, \ldots, D_3$, are chosen. Ratings of five lecturers are done by criteria as total of publications (C_1), teaching student evaluations (C_2), personality characteristics (C_3), professional society (C_4), teaching experience (C_5), fluency of foreign language (C_6).

4.1. Aggregate ratings

Suitability ratings *S*= {*Ve_Po*,*Po*, *Me*, *Go*, *Ve_Go*} in *t* = {t_1, t_2, t_3} is,
Ve_Po = Very_Poor = ([0.1, 0.2], [0.6, 0.7], [0.7, 0.8]),
Po = Poor = ([0.2, 0.3], [0.5, 0.6], [0.6, 0.7]),
Me = Medium = ([0.3, 0.5], [0.4, 0.6], [0.4, 0.5]),
Go = Good = ([0.5, 0.6], [0.4, 0.5], [0.3, 0.4]),
Ve_Go = Very_Good = ([0.6, 0.7], [0.2, 0.3], [0.2, 0.3]),

Table 1 presents suitability ratings where the aggregated ratings of lecturers versus criteria are shown at the last column of Table 1.

4.2. Importance weight aggregation

The importance *V* = {U_ IPA, O_ IPA, IPA, V_ IPA, A_IPA} in *t* = {t_1, t_2, t_3} is:
U_ IPA = ([0.1, 0.2], [0.4, 0.5], [0.6, 0.7]) = Unimportant,
O_IPA = ([0.2, 0.4], [0.5, 0.6], [0.4, 0.5]) = Ordinary_Important,
IPA = ([0.4, 0.6], [0.4, 0.5], [0.3, 0.4]) = Important,
V_IPA = ([0.6, 0.8], [0.3, 0.4], [0.2, 0.3]) = Very_Important,
A_IPA = ([0.7, 0.9], [0.2, 0.3], [0.1, 0.2]) = Absolutely_Important (Tables 2–4),

4.3. Weighted ratings

A^+, A^-, d_i^+ and d_i^-

4.4. Determine the lecturer

Table 5 shows the ranking order is $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$. Thus, the best lecturer is A_2.

5. Comparison

This section compares the proposed TOPSIS method for DIVSN with the similarity measures between INSs proposed by Ye [62] to illustrate the advantages and applicability of the proposed method. Using Ye's [62] method and the data in Table 3, the score function, the accuracy function and the certainty function of the lecturers are shown in Table 6.

Table 1
Aggregated ratings.

Criteria	Lecturers	Decision makers									Aggregated ratings
		t_1			t_2			t_3			
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	
C_1	A_1	Me	Go	Go	Go	Go	Go	Go	Ve_Go	Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_2	Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.558, 0.659], [0.272, 0.4], [0.239, 0.3])
	A_3	Me	Go	Go	Go	Go	Go	Go	Go	Ve_Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Go	Me	Go	Go	Go	Go	Go	Go	Go	([0.481, 0.590], [0.400, 0.5], [0.310, 0.4])
	A_5	Me	Go	Me	Go	Go	Me	Go	Go	Go	([0.441, 0.569], [0.400, 0.5], [0.330, 0.4])
C_2	A_1	Go	Go	Go	Ve_Go	Go	Go	Go	Go	Go	([0.512, 0.613], [0.370, 0.5], [0.287, 0.4])
	A_2	Ve_Go	Go	Ve_Go	Me	Go	Go	Ve_Go	Go	Go	([0.518, 0.627], [0.317, 0.4], [0.271, 0.4])
	A_3	Ve_Go	Go	Go	Go	Me	Go	Go	Me	Go	([0.474, 0.593], [0.370, 0.5], [0.306, 0.4])
	A_4	Go	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Me	([0.506, 0.615], [0.343, 0.5], [0.283, 0.4])
C_3	A_1	Ve_Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Me	Go	([0.518, 0.627], [0.317, 0.4], [0.271, 0.4])
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Ve_Go	Go	Go	Ve_Go	([0.547, 0.648], [0.294, 0.4], [0.251, 0.4])
	A_3	Go	Ve_Go	Ve_Go	Go	Go	Go	Go	Ve_Go	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_4	Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Ve_Go	Go	Go	Go	Ve_Go	Go	Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
C_4	A_1	Me	Go	Me	Go	Go	Me	Me	Go	Me	([0.397, 0.547], [0.400, 0.6], [0.352, 0.5])
	A_2	Go	Me	Go	Go	Me	Go	Go	Me	Go	([0.441, 0.569], [0.400, 0.5], [0.330, 0.4])
	A_3	Go	Go	Go	Go	Go	Me	Go	Go	Ve_Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Me	Po	Me	Go	Me	Me	Go	Go	Me	([0.365, 0.518], [0.410, 0.6], [0.380, 0.5])
	A_5	Me	Me	Po	Me	Me	Me	Me	Go	Me	([0.316, 0.494], [0.410, 0.6], [0.405, 0.5])
C_5	A_1	Me	Go	Me	Me	Go	Go	Go	Me	Go	([0.419, 0.558], [0.400, 0.5], [0.341, 0.4])
	A_2	Go	Ve_Go	Go	Ve_Go	Go	Go	Go	V_G	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_3	Go	Go	Me	Go	Go	Go	Go	Ve_Go	Go	([0.494, 0.603], [0.370, 0.5], [0.296, 0.4])
	A_4	Ve_Go	Go	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	([0.536, 0.637], [0.317, 0.4], [0.262, 0.4])
	A_5	Go	Go	Go	Go	Go	Go	Go	Ve_Go	Go	([0.512, 0.613], [0.370, 0.5], [0.287, 0.4])
C_6	A_1	Ve_Go	Go	Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.558, 0.659], [0.272, 0.4], [0.239, 0.3])
	A_2	Go	Go	Go	Go	Ve_Go	Ge	Go	Go	Ve_Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_3	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	Ve_Go	Go	Ve_Go	([0.569, 0.670], [0.252, 0.4], [0.229, 0.3])
	A_4	Go	Ve_Go	Go	Go	Ve_Go	Go	Go	Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])
	A_5	Go	Go	Go	Ve_Go	Go	Go	Go	Ve_Go	Go	([0.524, 0.625], [0.343, 0.4], [0.274, 0.4])

Table 2
Aggregated weights.

Criteria	Decision-makers									Aggregated weights
	t_1			t_2			t_3			
	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	
C_1	IPA	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	V_IPA	([0.476, 0.683], [0.363, 0.5], [0.262, 0.4])
C_2	V_IPA	V_IPA	IPA	V_IPA	V_IPA	V_IPA	A_IPA	V_IPA	V_IPA	([0.595, 0.800], [0.296, 0.4], [0.194, 0.3])
C_3	IPA	IPA	V_IPA	IPA	IPA	V_IPA	V_IPA	IPA	V_IPA	([0.499, 0.706], [0.352, 0.5], [0.251, 0.4])
C_4	IPA	V_IPA	IPA	IPA	O_IPA	IPA	IPA	IPA	IPA	([0.408, 0.613], [0.397, 0.5], [0.296, 0.4])
C_5	IPA	IPA	IPA	V_IPA	IPA	V_IPA	IPA	IPA	IPA	([0.452, 0.657], [0.375, 0.5], [0.274, 0.4])
C_6	V_IPA	V_IPA	IPA	IPA	IPA	IPA	V_IPA	V_IPA	IPA	([0.499, 0.706], [0.352, 0.5], [0.251, 0.4])

Table 3
Weighted ratings.

Lecturers	Aggregated weights
A_1	([0.170, 0.397], [0.648, 0.8], [0.545, 0.6])
A_2	([0.190, 0.436], [0.617, 0.7], [0.519, 0.6])
A_3	([0.187, 0.419], [0.642, 0.8], [0.535, 0.6])
A_4	([0.178, 0.400], [0.643, 0.8], [0.538, 0.6])
A_5	([0.173, 0.395], [0.649, 0.8], [0.549, 0.6])

Table 4
The distance of each lecturer from A^+ and A^-.

Lecturers	d^+	d^-
A_1	0.346	0.675
A_2	0.375	0.647
A_3	0.359	0.662
A_4	0.352	0.668
A_5	0.345	0.676

Table 5
Closeness coefficient.

Lecturers	Closeness coefficient	Ranking
A_1	0.339	4
A_2	0.367	1
A_3	0.351	2
A_4	0.345	3
A_5	0.338	5

Table 6
Modified score, accuracy and certainty function of each lecturer.

Lecturers	Score function	Accuracy function	Certainty function	Ranking
A_1	0,332	−0,297	0,283	4
A_2	0,361	−0,241	0,313	1
A_3	0,345	−0,267	0,303	2
A_4	0,339	−0,284	0,289	3
A_5	0,331	−0,300	0,284	5

Table 6 shows that the ranking order of the five lecturers is $A_2 \succ A_3 \succ A_4 \succ A_1 \succ A_5$. Thus, the best lecturer is A_2. The result is the same as that of the proposed method. This means that our method in the simplest form can procedure the results of the best method for this problem. Moreover, it is more generalized and flexible than the Ye's [62] method in dynamic environments.

6. Conclusion

This paper proposed a new concept of Dynamic Interval Valued Neutrosophic Set (DIVNS) where all the factors in DIVNSs such as truth, indeterminacy and falsity degrees are in different ranges of time. Mathematical operations associated with DIVNSs and correlation coefficients have also been defined. In addition, we have proposed a new TOPSIS method based on the DIVNSs and their application to evaluate lecturers' performance in the ULIS-VNU. This shows the feasibility and applications of Neutrosophic Theory in Industry.

In the future, we will use DIVNSs as well as the TOPSIS method to express dynamic information, and develop additional extention theories for DIVNSs such as operators, similarity measure. In addition, we extended this method to predictive problems such as in [24, 30, 31, 38, 39, 43, 44,46, 47, 48, 50, 63–92].

Acknowledgement

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 502.01-2015.16.

References

[1] M. Abdel-Basset, M. Gunasekaran, M. Mohamed, N. Chilamkurti, Three-way decisions based on neutrosophic sets and AHP-QFD framework for supplier selection problem, Future Gener. Comput. Syst. 89 (2018) 19–30.
[2] M. Abdel-Basset, G. Manogaran, A. Gamal, F. Smarandache, A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria, Des. Autom. Embed. Syst. (2018) 1–22.
[3] M. Abdel-Basset, M. Mohamed, A.N. Hussien, A.K. Sangaiah, A novel group decision-making model based on triangular neutrosophic numbers, Soft Comput. 22 (20) (2018) 6629–6643.
[4] M. Abdel-Basset, M. Mohamed, F. Smarandache, An extension of neutrosophic AHP–SWOT analysis for strategic planning and decision-making, Symmetry 10 (4) (2018) 116.
[5] M. Abdel-Basset, Y. Zhou, M. Mohamed, V. Chang, A group decision making framework based on neutrosophic VIKOR approach for e-government website evaluation, J. Intell. Fuzzy Syst. 34 (6) (2018) 4213–4224.
[6] Z. Aiwu, D. Jianguo, G. Hongjun, Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator, J. Intell. Fuzzy Syst. 29 (6) (2015) 2697–2706.
[7] M. Ali, L.H. Son, I. Deli, N.D. Tien, Bipolar neutrosophic soft sets and applications in decision making, J. Intell. Fuzzy Syst. 33 (6) (2017) 4077–4087.
[8] M. Ali, L.H. Son, N.D. Thanh, N. Van Minh, A neutrosophic recommender system for medical diagnosis based on algebraic neutrosophic measures, Appl. Soft Comput. 71 (2018) 1054–1071.
[9] M.A. Basset, M. Mohamed, A.K. Sangaiah, V. Jain, An integrated neutrosophic AHP and SWOT method for strategic planning methodology selection, Benchmarking Int. J. 25 (7) (2018) 2546–2564.
[10] P. Biswas, S. Pramanik, B.C. Giri, TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment, Neural Comput. Appl. 27 (3) (2016) 727–737.
[11] S. Broumi, J. Ye, F. Smarandache, An extended TOPSIS method for multiple attribute decision making based on interval neutrosophic uncertain linguistic variables, Infinite Study (2015).
[12] S. Broumi, I. Deli, Correlation measure for neutrosophic refined sets and its application in medical diagnosis, Palestine J. Math. 5 (2015) 135–143.
[13] S. Broumi, A. Dey, A. Bakali, M. Talea, F. Smarandache, L.H. Son, D. Koley, Uniform single valued neutrosophic graphs, Neutrosophic Sets Syst. 17 (2017) 42–49.
[14] S. Broumi, L.H. Son, A. Bakali, M. Talea, F. Smarandache, G. Selvachandran, Computing operational matrices in neutrosophic environments: a matlab toolbox, Neutrosophic Sets Syst. 18 (2017).
[15] S. Broumi, J. Ye, F. Smarandache, An Extended TOPSIS Method for Multiple Attribute Decision Making based on Interval Neutrosophic Uncertain Linguistic Variables, Neutrosophic Sets Syst. 8 (2015) 22–31.
[16] O. Castillo, L. Amador-Angulo, J.R. Castro, M. Garcia-Valdez, A comparative study of type-1 fuzzy logic systems, interval type-2 fuzzy logic systems and generalized type-2 fuzzy logic systems in control problems, Inf. Sci. 354 (2016) 257–274.
[17] O. Castillo, L. Cervantes, J. Soria, M. Sanchez, J.R. Castro, A generalized type-2 fuzzy granular approach with applications to aerospace, Inf. Sci. 354 (2016) 165–177.
[18] O. Castillo, P. Melin, Intelligent systems with interval type-2 fuzzy logic, Int. J. Innov. Comput. Inf. Control 4 (4) (2008) 771–783.
[19] L. Cervantes, O. Castillo, Type-2 fuzzy logic aggregation of multiple fuzzy controllers for airplane flight control, Inf. Sci. 324 (2015) 247–256.
[20] P. Chi, P. Liu, An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set, Neutrosophic Sets Syst. 1 (1) (2013) 63–70.
[21] I. Deli, S. Broumi, F. Smarandache, On neutrosophic refined sets and their applications in medical diagnosis, J. New Theory 6 (2015) 88–98.
[22] I. Deli, Y. Şubaş, A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems, Int. J. Mach. Learn. Cybern. 8 (4) (2017) 1309–1322.
[23] İ. Deli, Y. Şubaş, Bipolar neutrosophic refined sets and their applications in medical diagnosis, Proceedings of the International Conference on Natural Science and Engineering (ICNASE'16) (2016).
[24] A. Dey, S. Broumi, L.H. Son, A. Bakali, M. Talea, F. Smarandache, A new algorithm for finding minimum spanning trees with undirected neutrosophic graphs, Granul. Comput. (2018) 1–7, doi:http://dx.doi.org/10.1007/s41066-018-0084-7.
[25] W. Edwards, Dynamic decision theory and probabilistic information processings, Hum. Factors 4 (2) (1962) 59–74.
[26] C. Fan, E. Fan, J. Ye, The cosine measure of single-valued neutrosophic multisets for multiple attribute decision-making, Symmetry 10 (5) (2018) 154.
[27] Y. Guo, H.D. Cheng, New neutrosophic approach to image segmentation, Pattern Recognit. 42 (5) (2009) 587–595.
[28] Y. Guo, A. Şengür, J. Ye, A novel image thresholding algorithm based on neutrosophic similarity score, Measurement 58 (2014) 175–186.
[29] S.A.I. Hussain, S.P. Mondal, U.K. Mandal, VIKOR method for decision making problems in interval valued neutrosophic environment, Fuzzy Multi-criteria Decision-Making Using Neutrosophic Sets, Springer, Cham, 2019, pp. 587–602.
[30] S. Jha, R. Kumar, L.H. Son, J.M. Chatterjee, M. Khari, N. Yadav, F. Smarandache, Neutrosophic soft set decision making for stock trending analysis, Evol. Syst. (2018) 1–7, doi:http://dx.doi.org/10.1007/s12530-018-9247-7.
[31] M. Khan, L. Son, M. Ali, H. Chau, N. Na, F. Smarandache, Systematic review of decision making algorithms in extended neutrosophic sets, Symmetry 10 (8) (2018) 314.
[32] H. Liu, L. Jiang, L. Martínez, A dynamic multi-criteria decision making model with bipolar linguistic term sets, Expert Syst. Appl. 95 (2018) 104–112.
[33] P. Liu, G. Tang, Some power generalized aggregation operators based on the interval neutrosophic sets and their application to decision making, J. Intell. Fuzzy Syst. 30 (5) (2016) 2517–2528.
[34] P. Liu, Y. Wang, Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making, J. Syst. Sci. Complex. 29 (3) (2016) 681–697.
[35] S. Liu, T.A. Moughal, A novel method for dynamic multicriteria decision making with hybrid evaluation information, J. Appl. Math. (2014).
[36] R. Lourenzutti, R.A. Krohling, A generalized TOPSIS method for group decision making with heterogeneous information in a dynamic environment, Inf. Sci. 330 (2016) 1–18.
[37] R. Lourenzutti, R.A. Krohling, M.Z. Reformat, Choquet based TOPSIS and TODIM for dynamic and heterogeneous decision making with criteria interaction, Inf. Sci. 408 (2017) 41–69.
[38] G.N. Nguyen, L.H. Son, A.S. Ashour, N. Dey, A survey of the state-of-the-arts on neutrosophic sets in biomedical diagnoses, Int. J. Mach. Learn. Cybern. (2018) 1–13, doi:http://dx.doi.org/10.1007/s13042-017-0691-7.
[39] E. Ontiveros-Robles, P. Melin, O. Castillo, Comparative analysis of noise robustness of type 2 fuzzy logic controllers, Kybernetika 54 (1) (2018) 175–201.
[40] J.J. Peng, J.Q. Wang, X.H. Wu, J. Wang, X.H. Chen, Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems, Int. J. Comput. Intell. Syst. 8 (2) (2015) 345–363.
[41] J.J. Peng, J.Q. Wang, J. Wang, H.Y. Zhang, X.H. Chen, Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems, Int. J. Syst. Sci. 47 (10) (2016) 2342–2358.
[42] R. Şahin, P. Liu, Possibility-induced simplified neutrosophic aggregation operators and their application to multi-criteria group decision-making, J. Exp. Theor. Artif. Intell. 29 (4) (2017) 769–785.
[43] M.A. Sanchez, O. Castillo, J.R. Castro, Information granule formation via the concept of uncertainty-based information with Interval Type-2 Fuzzy sets representation and Takagi–Sugeno–Kang consequents optimized with Cuckoo search, Appl. Soft Comput. 27 (2015) 602–609.
[44] M.A. Sanchez, O. Castillo, J.R. Castro, Generalized type-2 fuzzy systems for controlling a mobile robot and a performance comparison with interval type-2 and type-1 fuzzy systems, Expert Syst. Appl. 42 (14) (2015) 5904–5914.
[45] F. Smarandache, Neutrosophy. Neutrosophic Probability, Set, and Logic, ProQuest Information & Learning, Ann Arbor, Michigan, USA, 1998, pp. 105.
[46] N.D. Thanh, M. Ali, A novel clustering algorithm in a neutrosophic recommender system for medical diagnosis, Cognit. Comput. 9 (4) (2017) 526–544.

[47] N.D. Thanh, L.H. Son, M. Ali, Neutrosophic recommender system for medical diagnosis based on algebraic similarity measure and clustering, 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE) (2017) 1–6.

[48] N.X. Thao, B.C. Cuong, M. Ali, L.H. Lan, Fuzzy equivalence on standard and rough neutrosophic sets and applications to clustering analysis, Information Systems Design and Intelligent Applications, Springer, Singapore, 2018, pp. 834–842.

[49] Z.P. Tian, H.Y. Zhang, J. Wang, J.Q. Wang, X.H. Chen, Multi-criteria decision-making method based on a cross-entropy with interval neutrosophic sets, Int. J. Syst. Sci. 47 (15) (2016) 3598–3608.

[50] T.M. Tuan, P.M. Chuan, M. Ali, T.T. Ngan, M. Mittal, L.H. Son, Fuzzy and neutrosophic modeling for link prediction in social networks, Evol. Syst. (2018) 1–6, doi:http://dx.doi.org/10.1007/s12530-018-9251-y.

[51] H. Wang, F. Smarandache, R. Sunderraman, Y.Q. Zhang, Interval Neutrosophic Sets and Logic: Theory and Applications in Computing: Theory and Applications in Computing, (2005) , pp. 5.

[52] H. Wang, F. Smarandache, Y.Q. Zhang, R. Sunderraman, Single Valued Neutrosophic Sets. Multispace and Multistructure, 4, Kalyan Mondal, and Surapati Pramanik, 2010, pp. 410–413.

[53] S. Yan, S. Liu, J. Liu, L. Wu, Dynamic grey target decision making method with grey numbers based on existing state and future development trend of alternatives, J. Intell. Fuzzy Syst. 28 (5) (2015) 2159–2168.

[54] W. Yang, Y. Pang, New multiple attribute decision making method based on DEMATEL and TOPSIS for multi-valued interval neutrosophic sets, Symmetry 10 (4) (2018) 115.

[55] J. Ye, An extended TOPSIS method for multiple attribute group decision making based on single valued neutrosophic linguistic numbers, J. Intell. Fuzzy Syst. 28 (1) (2015) 247–255.

[56] J. Ye, Clustering methods using distance-based similarity measures of single-valued neutrosophic sets, J. Intell. Syst. 23 (4) (2014) 379–389.

[57] J. Ye, Correlation coefficient between dynamic single valued neutrosophic multisets and its multiple attribute decision-making method, Information 8 (2) (2017) 41.

[58] J. Ye, Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, Int. J. Gen. Syst. 42 (4) (2013) 386–394.

[59] J. Ye, Single-valued neutrosophic minimum spanning tree and its clustering method, J. Intell. Syst. 23 (3) (2014) 311–324.

[60] S. Ye, J. Fu, J. Ye, Medical diagnosis using distance-based similarity measures of single valued neutrosophic multisets, Neutrosophic Sets Syst. 7 (2015) 47–52.

[61] J. Ye, Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making, J. Intell. Fuzzy Syst. 27 (2014) 2231–2241, doi:http://dx.doi.org/10.3233/IFS-141187.

[62] H. Zhang, J. Wang, X. Chen, An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets, Neural Comput. Appl. 27 (3) (2016) 615–627.

[63] J. Hemanth, J. Anitha, A. Naaji, O. Geman, D. Popescu, L.H. Son, A modified deep convolutional neural network for abnormal brain image classification, IEEE Access 7 (1) (2018) 4275–4283.

[64] L.H. Son, H. Fujita, Neural-fuzzy with representative sets for prediction of student performance, Appl. Intell. 49 (1) (2019) 172–187.

[65] D.J. Hemanth, J. Anitha, L.H. Son, M. Mittal, Diabetic retinopathy diagnosis from retinal images using modified hopfield neural network, J. Med. Syst. 42 (12) (2018) 247.

[66] D.J. Hemanth, J. Anitha, L.H. Son, Brain signal based human emotion analysis by circular back propagation and Deep Kohonen Neural Networks, Comput. Electr. Eng. 68 (2018) 170–180.

[67] C.N. Giap, L.H. Son, F. Chiclana, Dynamic structural neural network, J. Intell. Fuzzy Syst. 34 (2018) 2479–2490.

[68] S. Doss, A. Nayyar, G. Suseendran, S. Tanwar, A. Khanna, L.H. Son, P.H. Thong, APD-JFAD: accurate prevention and detection of jelly fish attack in MANET, IEEE Access 6 (2018) 56954–56965.

[69] D.T. Hai, H. Son, L.T. Vinh, Novel fuzzy clustering scheme for 3D wireless sensor networks, Appl. Soft Comput. 54 (2017) 141–149.

[70] R. Kapoor, R. Gupta, R. Kumar, L.H. Son, S. Jha, New scheme for underwater acoustically wireless transmission using direct sequence code division multiple access in MIMO systems, Wirel. Netw. (2019) 1–13, doi:http://dx.doi.org/10.1007/s11276-018-1750-z.

[71] R. Kapoor, R. Gupta, L.H. Son, S. Jha, R. Kumar, Detection of power quality event using histogram of oriented gradients and support vector machine, Measurement 120 (2018) 52–75.

[72] R. Kapoor, R. Gupta, L.H. Son, S. Jha, R. Kumar, Boosting performance of power quality event identification with KL Divergence measure and standard deviation, Measurement 126 (2018) 134–142.

[73] H.V. Long, M. Ali, M. Khan, D.N. Tu, A novel approach for fuzzy clustering based on neutrosophic association matrix, Comput. Ind. Eng. (2019), doi:http://dx.doi.org/10.1016/j.cie.2018.11.007.

[74] P.T.M. Phuong, P.H. Thong, L.H. Son, Theoretical analysis of picture fuzzy clustering: convergence and property, J. Comput. Sci. Cybern. 34 (1) (2018) 17–32.

[75] Y.H. Robinson, E.G. Julie, K. Saravanan, R. Kumar, L.H. Son, FD-AOMDV: fault-tolerant disjoint ad-hoc on-demand multipath distance vector routing algorithm in mobile ad-hoc networks, J. Ambient Intell. Humaniz. Comput. (2019) 1–18, doi:http://dx.doi.org/10.1007/s12652-018-1126-3.

[76] K. Saravanan, E. Anusuya, R. Kumar, L.H. Son, Real-time water quality monitoring using Internet of Things in SCADA, Environ. Monit. Assess. 190 (9) (2018) 556.

[77] K. Saravanan, S. Aswini, R. Kumar, L.H. Son, How to prevent maritime border collision for fisheries?-A design of Real-Time Automatic Identification System, Earth Sci. Inform. (2019) 1–12, doi:http://dx.doi.org/10.1007/s12145-018-0371-5.

[78] K. Singh, K. Singh, L.H. Son, A. Aziz, Congestion control in wireless sensor networks by hybrid multi-objective optimization algorithm, Comput. Netw. 138 (2018) 90–107.

[79] N. Singh, L.H. Son, F. Chiclana, M. Jean-Pierre, A new fusion of salp swarm with sine cosine for optimization of non-linear functions, Eng. Comput. (2019), doi:http://dx.doi.org/10.1007/s00366-018-00696-8.

[80] L.H. Son, A novel kernel fuzzy clustering algorithm for geo-demographic analysis, Inf. Sci.—Inf. Comput. Sci. Intell. Syst. Appl.: Int. J. 317 (C) (2015) 202–223.

[81] L.H. Son, Generalized picture distance measure and applications to picture fuzzy clustering, Appl. Soft Comput. 46 (C) (2016) 284–295.

[82] L.H. Son, P.V. Hai, A novel multiple fuzzy clustering method based on internal clustering validation measures with gradient descent, Int. J. Fuzzy Syst. 18 (5) (2016) 894–903.

[83] L.H. Son, S. Jha, R. Kumar, J.M. Chatterjee, M. Khari, Collaborative handshaking approaches between internet of computing and internet of things towards a smart world: a review from 2009–2017, Telecommun. Syst. (2019) 1–18, doi:http://dx.doi.org/10.1007/s11235-018-0481-x.

[84] L.H. Son, N.D. Tien, Tune up fuzzy C-means for big data: some novel hybrid clustering algorithms based on initial selection and incremental clustering, Int. J. Fuzzy Syst. 19 (5) (2017) 1585–1602.

[85] L.H. Son, T.M. Tuan, A cooperative semi-supervised fuzzy clustering framework for dental X-ray image segmentation, Expert Syst. Appl. 46 (2016) 380–393.

[86] L.H. Son, P.H. Thong, Some novel hybrid forecast methods based on picture fuzzy clustering for weather nowcasting from satellite image sequences, Appl. Intell. 46 (1) (2017) 1–15.

[87] L.H. Son, T.M. Tuan, Dental segmentation from X-ray images using semi-supervised fuzzy clustering with spatial constraints, Eng. Appl. Artif. Intell. 59 (2017) 186–195.

[88] N.T. Tam, D.T. Hai, L.H. Son, L.T. Vinh, Improving lifetime and network connections of 3D wireless sensor networks based on fuzzy clustering and particle swarm optimization, Wirel. Netw. 24 (5) (2018) 1477–1490.

[89] P.H. Thong, L.H. Son, Picture fuzzy clustering: a new computational intelligence method, Soft Comput. 20 (9) (2016) 3549–3562.

[90] P.H. Thong, L.H. Son, A novel automatic picture fuzzy clustering method based on particle swarm optimization and picture composite cardinality, Knowl. Based Syst. 109 (2016) 48–60.

[91] P.H. Thong, L.H. Son, Picture fuzzy clustering for complex data, Eng. Appl. Artif. Intell. 56 (2016) 121–130.

[92] T.M. Tuan, T.T. Ngan, L.H. Son, A novel semi-supervised fuzzy clustering method based on interactive fuzzy satisficing for dental X-ray image segmentation, Appl. Intell. 45 (2) (2016) 402–428.