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Effect of variable carbon emission in a multi-objective transportation-*p*-facility location problem under neutrosophic environment

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ABSTRACT

Several industries locate a pre-assigned number of facilities in order to determine a transportation way for optimizing the objective functions simultaneously. The multi-objective transportation-*p*-facility location problem is an optimization based model to integrate the facility location problem and the transportation problem under the multi-objective environment. This study delineates the stated formulation in which we need to seek the locations of *p*-facilities in the Euclidean plane, and the amounts of transported products so that the total transportation cost, transportation time, and carbon emission cost from existing sites to *p*-facilities will be minimized. In fact, variable carbon emission under carbon tax, cap and trade regulation is considered due to the locations of *p*-facilities and the amounts of transported flow. Thereafter, a hybrid approach is improved based on an alternating locate-allocate heuristic and the neutrosophic compromise programming to obtain the non-dominated solution. Additionally, the performance of our findings are evaluated by an application example. Furthermore, a sensitivity analysis is incorporated to explore the resiliency of the designed model. Finally, conclusions and further research areas conclude the paper.

1. Introduction

The *facility location problem* (FLP) is a crucial integrant of strategic planning for a wide spectrum of the public as well as the private sector. In fact, it deals with locating facilities among existing sites with the goal of optimizing the economic criteria (e.g., transportation cost, transportation time, carbon emission cost and good service). The traditional FLP is described by four given sets, (i) a set of existing sites with capacity, (ii) a set of weights associated with the existing site, (iii) a set of potential facility sites with demand, and (iv) a set of objective functions. It can be cataloged into different categories depending on the assumptions. Industrial organizations locate assembly plants and depots. Warehouses are situated by the retailers. The performance of the manufacturing, productivity, and marketing of goods is dependent on the location of the facilities. Moreover, the government also selects the location of hospitals, offices, schools, fire stations, etc. Everywhere, the quality of service is dependent on the location of the facilities. The FLP was studied by several researchers. A few of them are depicted here. Farahani, SteadieSeifi, and Asgari (2010) made a comprehensive survey of the facility location problems in a multi-criteria environment. Then, Bieniek (2015) presented a note on the FLP where the demands follow the arbitrary distribution. Later, Chen, He, and Wu (2016) solved a single FLP with random weights. Moreover, the FLP can be applied in a broad area of transportation networks, supply chain management, plant location problem, and green logistics such as Mišković, Stanimirović, and Grujičić (2017), Melo, Nickel, and Saldanha-da-Gama (2009), Amin and Baki (2017), Saif and Elhedhli (2016), and Harris, Mumford, and Naima (2014).

In the real scenario, the *transportation problem* (TP) plays a vital role in global competition for minimizing transportation cost, time and providing service. Generally, the classical TP consists of three major components: (a) a set of all sources, (b) a set of all destinations, and (c) single-objective function as total transportation cost. Mainly, in the TP, homogeneous goods are sent from sources to destinations, and the total transportation cost is directly proportional to the amount of goods to be transported. It was the first introduced by Hitchcock (1941). However, the traditional TP is not sufficient for handling real-life application problems. Due to this reason, the multi-objective environment is introduced here on the TP in which the objectives are conflicting and non-commensurable in nature. In fact, the *multi-objective TP* (MOTP) was analyzed by so many researchers in different environments. Some works are annexed here. Mahapatra, Roy, and Biswal (2013) solved a multi-choice stochastic TP where the supply and demand parameters follow extreme value distribution. Thereafter, Sabbagh, Ghafari, and Mousavi (2015) proposed a hybrid approach for the balanced TP. Maity, Roy, and Verdegay (2016) discussed a MOTP with cost reliability

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in an uncertain environment. Later on, Roy, Maity, Weber, and Gök (2017) described a MOTP where cost, demand, and supply parameters are in multi-choice nature. And they solved the problem using two approaches multi-choice goal programming and conic scalarizing function.

The FLP and TP are the core components of a tactical transportation planning system. Determining the best locations for the facilities (i.e., plants, depots, warehouses, offices, fire stations, railway stations, etc.) and minimizing the total transportation cost from existing sites to facilities can significantly affect the transportation planning system. Cooper (1972) first made a connection between the FLP and TP, and was also known as the transportation-location problem. Later, he (1978) studied the problem under stochastic environment. Recently, Carlo, David, and Salvat (2017) extended the problem with an unknown number of facilities. Afterward, several researchers made connections among the FLP and TP in many different ways. Klibi, Lasalle, Martel, and Ichoua (2010) studied a location-transportation problem delineated by multiple demand periods, multiple transportation options, and a stochastic demand. Gabrel, Lacroix, Murat, and Remli (2014) illustrated a robust location transportation problems under uncertain demands. Recently, Jaafari and Delage (2017) presented a capacitated fixed-charge multi-period location-transportation problem.

A fast-flowing of transportation emerges tremendous amounts of carbon, which is the fundamental explanation for global warming. To control carbon emanations, the government endorses several policies among all *tax, cap and trade policy* (TCTP) is widely accepted. Under TCTP, the companies are firstly allowed some emission cap with the usual tax basis from the government, and subsequently, they can also trade (i.e., buy or sell) the emission cap in the carbon trading market. This type of study was implemented by many scholars such as Benjaafar, Li, and Daskin (2013), Wu, Jin, Shi, and Shyu (2017), Dua, Tang, and Song (2016), Cao, Xu, Wu, and Zhang (2017), Turken, Carrillo, and Verter (2017) and Elhedhli and Merrick (2012). Here, we consider variable carbon emission as it depends on the locations of facilities as well as the amounts of transported items. This concept is totally new which did not incorporate by the researcher(s).

From Table 1, we trace a gap for making a connection among the FLP, MOTP and carbon emission under TCTP. To fill the gap concretely, here, we flourish a formulation by integrating the FLP and TP in the light of a multi-objective optimization environment. Therefore, we refer to the proposed problem as the *multi-objective transportation-p-facility location problem* (MOT-*p*-FLP). In the MOT-*p*-FLP, one has to ask the locations of *p*-facilities in the Euclidean plane and the amounts of transported goods simultaneously with three objective functions. We believe that the proposed formulation will be more applicable than the traditional FLP and MOTP. In fact, it will be useful to the models of transportation systems, emergency services, and online-shopping systems.

Nowadays, the parameters of the MOT-*p*-FLP are conflicting and imprecise nature due to lack of proper information. In fact, this type of mathematical formulation is difficult to tackle by traditional approaches. To overcome this situation, Zadeh (1965) introduced the *fuzzy set* (FS). Thereafter, Zimmermann (1978) incorporated fuzzy programming to solve a multi-objective linear programming problem. But, there is a drawback of the FS, it could not manage the certain case of uncertainty. Because of that, the *intuitionistic fuzzy set* (IFS) was developed by Atanassov (1986) as a generalization of the FS. The IFS was applied in a multi-objective optimization problem like Roy, Ebrahimnejad, Verdegay, and Das (2018). Although the FS and IFS deal with all types of fuzzy uncertainty, still they cannot handle the indeterminate situation. For instance, a survey is done on a particular statement, then there are a few who said the possibility of the statement is true 0.7, the statement is false 0.4, and the statement is not sure 0.3. This issue is beyond the scope of the FS and IFS, and thus dealing with a kind of indeterminate situations of uncertain information indeed becomes a true challenge. Based on this instance, the neutrosophic set, an extended form of the FS and IFS was developed by Smarandache (1999). It provides a more general structure and suitable form to deal with the mentioned uncertainties. The neutrosophic set is formulated based on logic in which elements are represented by three degrees, explicitly, truth degree, indeterminacy degree, and falsity degree.

The main contributions of this study are as follows:

- An integrated nonlinear optimization model based on the FLP and MOTP is introduced.
- The model finds the decision regarding the assignment from multiple existing facilities to multiple potential facilities in the continuous planner surface with a hyperbolic approximation of Euclidean distance.
- The total transportation cost, total transportation time and total carbon emission cost are considered.
- The impact of variable carbon emission under TCTP due to transportation is also incorporated, a major contribution in the modern age.
- An improved hybrid approach is followed to find the optimal solution of the MOT-*p*-FLP.
- The nature of the obtained optimal solution is also studied.

The outline of this study is as follows: In the next section, the proposed problem is formulated. Section 3 presents the methodology of a

Table 1
Some remarkable research works related to FLP, TP and carbon emission.

References	FLP	Transportation cost	Transportation time	Carbon emission cost	Cap & trade policy	Solution methods
Klibi et al. (2010)	Yes	Yes	–	–	–	Hierarchical heuristic
Elhedhli and Merrick (2012)	Yes	Yes	–	Yes	–	Lagrangian relaxation
Mahapatra et al. (2013)	–	Yes		–	–	Generalized reduced gradient
Harris et al. (2014)	Yes	Yes	–	Yes	–	Proposed evolutionary algorithm
Gabrel et al. (2014)	Yes	Yes	–	–	–	Cutting plane algorithm
Sabbagh et al. (2015)	–	Yes	–	–	–	Proposed algorithm
Maity et al. (2016)	–	Yes	–	–	–	Fuzzy multi-choice goal programming
Saif and Elhedhli (2016)	Yes	Yes	–	–	–	Lagrangian heuristic
Dua et al. (2016)	–	–	–	Yes	Yes	Classical approach
Turken et al. (2017)	Yes	–	–	Yes	Yes	Exact algorithm
Roy et al. (2017)	–	Yes	–	–	–	Conic scalarization
Wu et al. (2017)	Yes	Yes	–	Yes	–	Classical approach
Cao et al. (2017)	–	–	–	Yes	Yes	Stackleberg game
Jaafari and Delage (2017)	Yes	Yes	–	–	–	Row generation algorithm
Mišković et al. (2017)	Yes	Yes	–	–	–	Proposed memetic algorithm
Carlo et al. (2017)	Yes	Yes	–	–	–	Decomposition heuristic, Simulated annealing
Roy et al. (2018)	–	Yes	Yes	–	–	Intuitionistic fuzzy programming
This investigation	Yes	Yes	Yes	Yes	Yes	Proposed hybrid algorithm

hybrid approach along with its pros and cons. Then, Section 4 explores the non-dominated nature of the compromise solution. Moreover, the effectiveness of the stated model and the approach are evaluated with an example in Section 5. In Section 6, the obtained results for two cases are discussed. The sensitivity of the stated model is investigated in Section 7. Thereafter, Section 8 depicts the important managerial insights. At last, conclusions and future research directions based on our study are provided.

2. Mathematical description

In this section, we first define the proposed problem, i.e., MOT-p-FLP. Thereafter, the mathematical formulation is introduced on the following premises and notations. Moreover, the connection between the MOT-p-FLP and a MOTP, and some basic definitions are presented.

2.1. Problem background

Here, a logistical problem is inspected from an economical and environmental point of view. Our proposed problem deals with a transportation network which consists of multiple existing sites or sources, potential facility sites or demand points, and products are transported from existing sites to potential facility sites. The main aim is to minimize the total transportation cost, time, and carbon emission cost under TCTP by locating the potential facility sites simultaneously. Besides the transportation cost and time, the following postures are also handled in our model: (i) variable carbon emissions under TCTP, (ii) weights of conveyances which affect the transportation cost and carbon emission cost, (iii) weights of obstacles in the path which are reflected in transportation time, (iv) selling cost as a reward to reduce carbon emission, and (v) penalty cost to avoid unnecessary carbon discharges. Fig. 1 illustrates the structure of the MOT-p-FLP network.

Assume that there are three existing sites S_1, S_2, and S_3 and four potential facility sites D_1, D_2, D_3, and D_4. In fact, the supply and demand of the corresponding sites are also known. Moreover, the locations of S_1, S_2, and S_3 are provided. But, the locations of D_1, D_2, D_3, and D_4 are not known in the Euclidean plane. Consequently, the dotted lines denote the product flow by conveyances (i.e., T_1, T_2, T_3, and T_4) from S_1, S_2, and S_3 to D_1, D_2, D_3, and D_4, respectively. Furthermore, the obstacle is designated by B_1. In this situation, the decision maker has to seek the optimal locations of the potential facility sites with mentioned objective functions.

2.2. Notations and assumptions

The following notations and assumptions are employed to formulate the model.

m: number of existing facility sites.
p: number of potential facility sites.
k: number of objective functions.
a_i: availability at ith existing facility site ($i = 1, 2, \ldots, m$).
b_j: demand at jth potential facility site ($j = 1, 2, \ldots, p$).
e_i: in a location problem, the decision maker may put more important of the existing facility site, expressed as weight. Therefore, with each ith existing site, we associate a weight e_i.
t_{ij}: there may be some obstacles (e.g., railway level crossing, bridge crossing, broken-down, etc.) of the path from ith site to jth site which are affected the transportation time. These will be designated as t_{ij}.
δ_{ij}:there may be used different type of conveyances to transport the goods from ith site to jth site. Depend on their machine performance, we assign the weight δ_{ij}.
α:tax for each unit product that emit carbon.
β:carbon trading (buying) cost per unit item.
γ:carbon trading (selling) cost per unit item.
C:emission cap (i.e., limited capacity of carbon emission permit).
P_c:penalty cost per unit emitted in excess of the cap.
(u_i, v_i):coordinates of the ith existing facility site ($i = 1, 2, \ldots, m$).
(x_j, y_j):coordinates of the jth potential facility site ($j = 1, 2, \ldots, p$).
w_{ij}:amount of flow to be transported from ith existing facility site to jth potential facility site.
w: $\{(w_{ij})$: subject to the constraints $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, p)\}$.
w^B: $(w_{ij}^B$: $i = 1, 2, \ldots m; j = 1, 2, \ldots, p)$, the optimal feasible solution.
F: $\mathbb{R}^{2p} \times W$, where $(x, y) \in \mathbb{R}^{2p}$ and $w \in W$, the feasible set.
$\mathbf{Z}$:objective function vector.
S:neutrosophic set.
T_r:truth membership.
I_n:indeterminacy membership.
F_a:falsity membership.
L_k:lower value of the kth objective function.
U_k:upper value of the kth objective function.
ϕ:transportation cost function per unit flow from an existing facility site to a potential facility site depends on weight of conveyances.
ψ:time function per unit item from an existing facility site to a potential facility site depends on obstacle of path.
φ:average carbon emission function per unit product from an existing facility site to a potential facility site depends on weight of conveyances.

- Type of transportation cost function is a hyperbolic approximation of Euclidean distance in two-dimensional space ($\phi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + \delta_{ij}}$).
- Transportation time function is a hyperbolic approximation of Euclidean distance in two-dimensional space ($\psi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + t_{ij}}$).
- The carbon emission function is a hyperbolic approximation of Euclidean distance in two-dimensional space ($\varphi(u_i, v_i; x_j, y_j) = \sqrt{(u_i - x_j)^2 + (v_i - y_j)^2 + \delta_{ij}}$).

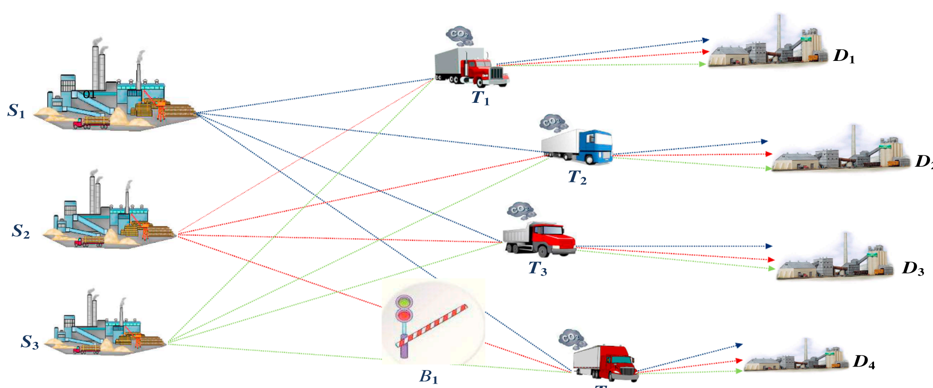

Fig. 1. Network for the multi-objective transportation-p-facility location problem.

- •Facilities are capacitated.
- •No relationship exists between potential facility sites.
- •The opening costs of new potential facility sites are ignored.
- •The solution space is continuous.
- •The parameters are deterministic.
- •The potential facility sites are located in the Euclidean plane.
- •The potential facility sites are assumed as points.
- •Transportation cost, transportation time, and average carbon emissions are directly proportional to the amount of transported goods.

2.3. Model identification

Herein, a mathematical formulation is incorporated in light of the FLP and MOTP. In fact, this model asks transportation amounts and optimal locations for the facilities simultaneously. The mathematical formulation of the MOT-*p*-FLP under TCTP can be stated as follows:

Model 1

$$\text{minimize } Z_{1(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\phi(u_i, v_i; x_j, y_j) \tag{2.1}$$

$$\text{minimize } Z_{2(x,y,w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\psi(u_i, v_i; x_j, y_j) \tag{2.2}$$

$$\begin{aligned}\text{minimize } Z_{3(x,y,w)} = {} & \alpha \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) \\ & + P_c\beta\Big(\textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) - C\Big)^{+} \\ & - \gamma\Big(C - \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j)\Big)^{+}\end{aligned} \tag{2.3}$$

$$\text{subject to } \sum_{j=1}^{p} w_{ij} \leqslant a_i \ (i = 1, 2, \ldots, m), \tag{2.4}$$

$$\sum_{i=1}^{m} w_{ij} \geqslant b_j (j = 1, 2, \ldots, p), \tag{2.5}$$

$$w_{ij} \geqslant 0 \ \forall\ i, j, \tag{2.6}$$

$$\sum_{i=1}^{m} a_i \geqslant \sum_{j=1}^{p} b_j. \tag{2.7}$$

Where

$$\begin{aligned}\left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j)\right)^{+} &= \max\left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j), 0\right) \\ &= \begin{cases} C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) & \text{if} C \geqslant \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j), \\ 0 & \text{otherwise.}\end{cases}; \left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) - C\right)^{+} \\ &= \max\left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) - C, 0\right) = \begin{cases} \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) - C & \text{if} \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) \geqslant C, \\ 0 & \text{otherwise.}\end{cases}\end{aligned}$$

The objective function (2.1) aims to seek optimal locations for *p*-facilities, which minimizes the total transportation cost. The objective function (2.2) indicates to minimize the total transportation time by determining the optimal locations for *p*-facilities. The objective function (2.3) intents to minimize the total carbon emission cost under TCTP by locating the optimal locations for the *p*-facilities. Constraints (2.4) enforce that the total amounts of each existing site which cannot surpass its availability. Constraints (2.5) ensure that the total items of each potential site fulfill its desired demand. Constraints (2.6) are non-negativity conditions. Finally, constraints (2.7) suggest the feasibility condition.

The objective function (2.3) demonstrates that, depending on the cap, there are two feasible regions. Case 1 occurs when $C \geqslant \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j)$. And the second one is occurred when $C \leqslant \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j)$.

Case 1: This case can be represented by the following model:

Model 1.1

$$\begin{aligned}\text{minimize} Z_{1(x,y,w)} &= \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\phi(u_i, v_i; x_j, y_j) \\ \text{minimize} Z_{2(x,y,w)} &= \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\psi(u_i, v_i; x_j, y_j) \\ \text{minimize} Z_{3(x,y,w)} &= \alpha\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) \\ &\quad - \gamma\left(C - \sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j)\right)\end{aligned} \tag{2.8}$$

$$\begin{aligned}\text{subject to}\quad & \text{the constraints (2.4) -- (2.7),} \\ & C \geqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j).\end{aligned} \tag{2.9}$$

Case 2: The following model is described for case 2:

Model 1.2

$$\begin{aligned}\text{minimize} Z_{1(x,y,w)} &= \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\phi(u_i, v_i; x_j, y_j) \\ \text{minimize} Z_{2(x,y,w)} &= \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\psi(u_i, v_i; x_j, y_j) \\ \text{minimize} Z_{3(x,y,w)} &= \alpha\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) \\ &\quad + P_c\beta\left(\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j) - C\right)\end{aligned} \tag{2.10}$$

$$\begin{aligned}\text{subject to}\quad & \text{the constraints (2.4) -- (2.7),} \\ & C \leqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j).\end{aligned} \tag{2.11}$$

2.4. Connection between the MOT-p-FLP and MOTP

The functions (i.e., ϕ, ψ and φ) are only dependent on the locations of the potential facility sites. In fact, if we fix the location of potential sites by finding optimal locations, then the functions should be converted into constant functions. Consequently, we designate (x_j^*, y_j^*) for the optimal locations, $e_i\phi(u_i, v_i; x_j^*, y_j^*) = c_{ij}'$ for the unit transportation cost from *i*th source to *j*th demand point, $e_i\psi(u_i, v_i; x_j^*, y_j^*) = t_{ij}'$ as the

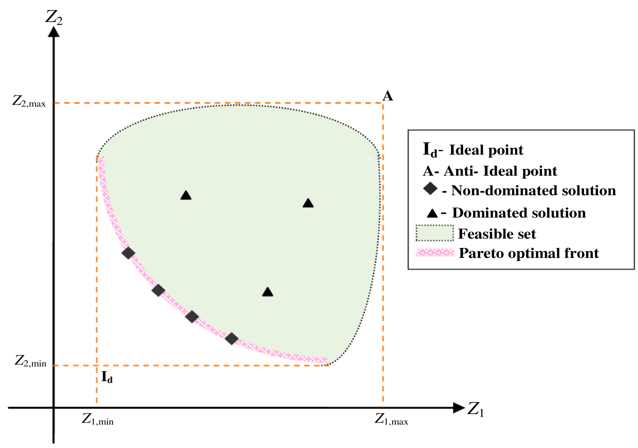

Fig. 2. The solution concept of a multi-objective optimization.

unit transportation time from ith source to jth demand point, and $\varphi(u_i, v_i;x_j^*, y_j^*) = d'_{ij}$ for the unit carbon emission for transportation of product flow from ith source to jth destination. Henceforth, Model 1 is read as follows:

Model 2

$$\text{minimize} Z_1(w) = \sum_{i=1}^{m}\sum_{j=1}^{p} c'_{ij}w_{ij} \tag{2.12}$$

$$\text{minimize} Z_2(w) = \sum_{i=1}^{m}\sum_{j=1}^{p} t'_{ij}w_{ij} \tag{2.13}$$

$$\text{minimize} Z_3(w) = (\alpha + \gamma + P_c\beta)\sum_{i=1}^{m}\sum_{j=1}^{p} d'_{ij}w_{ij} - (\gamma + P_c\beta)C \tag{2.14}$$

subject to the constraints (2.4) − −(2.7),

which is the well-known form of a MOTP.

2.5. Basic definitions

Here, some basic definitions related to the solution method of MOT-p-FLP are presented.

Definition 1. (Ideal solution)

An ideal solution of the MOT-p-FLP is the one which minimizes each of the objective function simultaneously, i.e., $Z_k(x^*, y^*, w^*) = \min_{(x,y,w)\in F} Z_k(x, y, w)$, $k = 1, 2, 3$.

Definition 2. (Anti-ideal solution)

The anti-ideal solution of the MOT-p-FLP is $Z_k(x^A, y^A, w^A) = \max_{(x,y,w)\in F} Z_k(x, y, w)$, $k = 1, 2, 3$.

Definition 3. (Non-dominated solution)

A solution $(x^N, y^N, w^N) \in F$ yields a non-dominated solution (otherwise called Pareto-optimal solution, efficient or non-inferior solution) of Model 1 iff there is no other solution $(x, y, w) \in F$ such that

$Z_k(x, y, w) \leqslant Z_k(x^N, y^N, w^N)$ for $k = 1, 2, 3$ and
$Z_k(x, y, w) < Z_k(x^N, y^N, w^N)$ for at least one k.

Definition 4. (Compromise solution)

A non-dominated solution $(x^N, y^N, w^N) \in F$ is said to be the compromise solution of the MOT-p-FLP iff $\mathbf{Z}(x^N, y^N, w^N) \leqslant \wedge_{(x,y,w)\in F} \mathbf{Z}(x, y, w)$, where $\wedge$ indicates the minimum.

The ideal, anti-ideal, non-dominated and compromise solutions are described graphically in Fig. 2.

Definition 5. (Neutrosophic set)

Let D be a universal set and $s \in D$. A neutrosophic set S in D is defined by three membership functions respectively, truth $T_r(s)$, indeterminacy $I_n(s)$ and falsity $F_a(s)$, and denoted by $S = \{(s, T_r(s), I_n(s), F_a(s)): s \in D\}$, where.

(i) $T_r(s): D \to [0, 1]$, $I_n(s): D \to [0, 1]$ and $F_a(s): D \to [0, 1]$,
(ii) $0 \leqslant \sup(T_r(s)) + \sup(I_n(s)) + \sup(F_a(s)) \leqslant 3$.

3. Methodology

In this section, a hybrid approach is presented to solve the proposed MOT-p-FLP. Thereafter, the advantages and disadvantages of the stated approach are also discussed.

3.1. Hybrid approach

Herein, a hybrid approach is developed based on an *alternating locate-allocate* (Loc-Alloc) heuristic (Cooper, 1964), and the *neutrosophic compromise programming* (NCP) (Rizk-Allah, Hassanien, & Elhoseny, 2018). Our hybrid approach comprises two parts. In the first part, three single objective *transportation-p-facility location problems* (T-p-FLPs) are solved by an alternating Loc-Alloc heuristic, and in the second part, the compromise non-dominated solution for the MOT-p-FLP is received by the NCP.

Alternating Loc-Alloc heuristic: The proposed heuristic is again divided into two parts. In Part 1, the heuristic seeks the initial locations, and in Part 2, it finds the optimum locations. Here, at first, the locations are placed for p-facilities from m-existing sites. If $p < m$, we generate all possible combinations of the m-existing sites taken p at a time. For each combination, the existing sites are to be considered as potential facility sites, and other existing sites are designated depending on which potential facility sites have the smallest distance. Finally, all designated distances are summed up. In fact, this phenomenon is repeated for all combinations. Therefore, the final initial potential locations for three distance functions are the combinations with the minimum sum of distances. With these final allocations, the distances between p-facilities and m-existing locations for three distance functions are easily computed. When $p = m$, the case is trivial and we easily get the distances between them. However, if $p > m$, we introduce a new heuristic concept to resolve this issue. Initially, we choose the m facility allocations as m existing sites randomly and allocate the remaining $(p - m)$ facilities in some Euclidean points with large coordinates such that the distances of those coordinates become very large numbers from facilities. Then, we easily compute the distances between p-facilities and m-existing sites and a large positive number is assigned for such distances which cannot be calculated. Now, it is already assumed that the distances are connected with cost, time and carbon emission functions per unit commodity from the ith site to the jth site. We take these distances as the cost, time and carbon emission coefficients. Then the problem converts into three classical transportation problems. Utilizing the initial potential location (x_j^I, y_j^I), we solve these problems individually:

Model 3

minimize $Z_{1(w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\phi(u_i, v_i;x_j^I, y_j^I)$
subject to the constraints (2.4) − −(2.7).

Model 4

minimize $Z_{2(w)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}\psi(u_i, v_i;x_j^I, y_j^I)$
subject to the constraints (2.4) − −(2.7).

Model 5

$$\begin{aligned}\text{minimize}\quad & Z_{3(w)} = \alpha \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I)\\ & + P_c\beta\Big(\textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I) - C\Big)^+\\ & - \gamma\Big(C - \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I)\Big)^+\\ \text{subject to}\quad & \text{the constraints (2.4)} - -(2.7).\end{aligned}$$

From Model 3 and Model 4, we can easily find optimal feasible solutions (w^B). But, to find the optimal feasible solution of Model 5, we split the model into two parts as Model 5.1 and Model 5.2, respectively. They are given as follows:

Model 5.1

$$\begin{aligned}\text{minimize}\quad & Z_{3(w)} = (\alpha + \gamma)\textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I) - \gamma C\\ \text{subject to}\quad & \text{the constraints (2.4)} - -(2.7),\\ & C \geqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I).\end{aligned}$$

Model 5.2

$$\begin{aligned}\text{minimize}\quad & Z_{3(w)} = (\alpha + P_c\beta)\textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I) - P_c\beta C\\ \text{subject to}\quad & \text{the constraints (2.4)} - -(2.7),\\ & C \leqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j^I, y_j^I).\end{aligned}$$

Thereafter, we solve Model 5.1 & Model 5.2 to extract the feasible solutions which lead the optimal solutions of these models. Then, we compare the solutions to find the optimal solution for Model 5. However, if one of them (Model 5.1 and Model 5.2) has the feasible solution (and the other has no feasible solution) then the optimal solution of the corresponding model is the optimal solution of Model 5. Employing these optimal feasible solutions (w^B), we minimize objective functions as the optimal solutions are arisen (see Appendix A). The iterative formula (see Appendix B) are used to minimize objective functions. Hence, $(x, y, w)^{(l)}$ is the local optimal (ideal) solution for the lth single objective T-p-FLP, where $l = 1, 2, 3$.

NCP: Here, a payoff table with entries $Z_{lk} := Z_k((x, y, w)^{(l)}),\ l, k = 1, 2, 3$ are calculated for non-dominated solution of the MOT-p-FLP. Afterwards, the upper (U_k) and lower (L_k) bounds for each objective function are estimated as follows: $U_k = \max\{Z_{1k}, Z_{2k}, Z_{3k}\}$ and $L_k = Z_{kk},\ k = 1, 2, 3$. Consequently, the upper and lower values for the neutrosophic environment are computed as

$$\begin{aligned}& U_k^{Tr} = U_k,\ L_k^{Tr} = L_k, \quad \text{for truth membership},\\ & U_k^{In} = L_k^{Tr} + q_k'(U_k^{Tr} - L_k^{Tr}),\ L_k^{In} = L_k^{Tr},\\ & \text{for indeterminacy membership},\\ & U_k^{Fa} = U_k^{Tr},\ L_k^{Fa} = L_k^{Tr} + q_k(U_k^{Tr} - L_k^{Tr}), \quad \text{for falsity membership}.\end{aligned}$$

where q_k and q_k' are tolerance variables, choose by the decision maker for falsity and indeterminacy membership functions. The membership functions for neutrosophic environment can be constructed as follows:

$$T_{rk}(Z_k(x, y, w)) = \begin{cases} 1 & Z_k(x, y, w) \leqslant L_k^{Tr},\\ 1 - \frac{Z_k(x,y,w) - L_k^{Tr}}{U_k^{Tr} - L_k^{Tr}} & L_k^{Tr} \leqslant Z_k(x, y, w) \leqslant U_k^{Tr},\\ 0 & Z_k(x, y, w) \geqslant U_k^{Tr}.\end{cases}$$

$$I_{nk}(Z_k(x, y, w)) = \begin{cases} 1 & Z_k(x, y, w) \leqslant L_k^{In},\\ 1 - \frac{Z_k(x,y,w) - L_k^{In}}{U_k^{In} - L_k^{In}} & L_k^{In} \leqslant Z_k(x, y, w) \leqslant U_k^{In},\\ 0 & Z_k(x, y, w) \geqslant U_k^{In}.\end{cases}$$

$$F_{ak}(Z_k(x, y, w)) = \begin{cases} 1 & Z_k(x, y, w) \geqslant U_k^{Fa},\\ 1 - \frac{U_k^{Fa} - Z_k(x,y,w)}{U_k^{Fa} - L_k^{Fa}} & L_k^{Fa} \leqslant Z_k(x, y, w) \leqslant U_k^{Fa},\\ 0 & Z_k(x, y, w) \leqslant L_k^{Fa}.\end{cases}$$

As the objective functions are conflicting in nature, hence, $U_k^{Tr} = L_k^{Tr}$, $U_k^{In} = L_k^{In}$ and $U_k^{Fa} - L_k^{Fa}$ are not possible for any (x_k^*, y_k^*, w_k^*) $(k = 1, 2, 3)$.

The neutrosophic model for the MOT-p-FLP can be stated as follows:

Model 6 (For Case 1)

$$\begin{aligned}\text{maximize}\quad & \theta\\ \text{minimize}\quad & \mu\\ \text{maximize}\quad & \nu\\ \text{subject to}\quad & T_{rk}(Z_k(x, y, w)) \geqslant \theta,\ F_{ak}(Z_k(x, y, w)) \leqslant \mu,\ I_{nk}(Z_k(x, y, w)) \geqslant \nu,\\ & \text{the constraints (2.4)} - -(2.7),\\ & C \geqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j, y_j),\\ & \theta \geqslant \mu,\ \theta \geqslant \nu,\ \theta + \mu + \nu \leqslant 3,\\ & 0 \leqslant q_k \leqslant U_k - L_k,\\ & 0 \leqslant q_k' \leqslant U_k - L_k,\\ & \theta, \mu, \nu \in [0, 1], \quad k = 1, 2, 3.\end{aligned}$$

where θ, μ and ν represent the global degree of satisfaction, indeterminacy and dissatisfaction of a solution, respectively.

Model 7 (For Case 2)

$$\begin{aligned}\text{maximize}\quad & \theta\\ \text{minimize}\quad & \mu\\ \text{maximize}\quad & \nu\\ \text{subject to}\quad & T_{rk}(Z_k(x, y, w)) \geqslant \theta,\ F_{ak}(Z_k(x, y, w)) \leqslant \mu,\ I_{nk}(Z_k(x, y, w)) \geqslant \nu,\\ & \text{the constraints (2.4)} - -(2.7),\\ & C \leqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j, y_j),\\ & \theta \geqslant \mu,\ \theta \geqslant \nu,\ \theta + \mu + \nu \leqslant 3,\\ & 0 \leqslant q_k \leqslant U_k - L_k,\\ & 0 \leqslant q_k' \leqslant U_k - L_k,\\ & \theta, \mu, \nu \in [0, 1], \quad k = 1, 2, 3.\end{aligned}$$

Thereafter, the simplified neutrosophic model of the MOT-p-FLP can be represented to derive the compromise non-dominated solution as follows:

Model 8 (For Case 1)

$$\begin{aligned}\text{maximize}\quad & \theta - \mu + \nu\\ \text{subject to}\quad & Z_k(x, y, w) + (U_k^{Tr} - L_k^{Tr})\theta \leqslant U_k^{Tr}\\ & Z_k(x, y, w) + (U_k^{In} - L_k^{In})\nu \leqslant U_k^{In}\\ & Z_k(x, y, w) - (U_k^{Fa} - L_k^{Fa})\mu \leqslant L_k^{Fa}\\ & \text{the constraints (2.4)} - -(2.7),\\ & C \geqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i;x_j, y_j),\\ & \theta \geqslant \mu,\ \theta \geqslant \nu,\ \theta + \mu + \nu \leqslant 3,\\ & 0 \leqslant q_k \leqslant U_k - L_k,\\ & 0 \leqslant q_k' \leqslant U_k - L_k,\\ & \theta, \mu, \nu \in [0, 1], \quad k = 1, 2, 3.\end{aligned}$$

Model 9 (For Case 2)

$$\begin{aligned}
\text{maximize} \quad & \theta - \mu + \nu \\
\text{subject to} \quad & Z_k(x, y, w) + (U_k^{Tr} - L_k^{Tr})\theta \leqslant U_k^{Tr} \\
& Z_k(x, y, w) + (U_k^{In} - L_k^{In})\nu \leqslant U_k^{In} \\
& Z_k(x, y, w) - (U_k^{Fa} - L_k^{Fa})\mu \leqslant L_k^{Fa} \\
& \text{the constraints } (2.4) - -(2.7), \\
& C \leqslant \textstyle\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}\varphi(u_i, v_i; x_j, y_j), \\
& \theta \geqslant \mu, \theta \geqslant \nu, \theta + \mu + \nu \leqslant 3, \\
& 0 \leqslant q_k \leqslant U_k - L_k, \\
& 0 \leqslant q_k' \leqslant U_k - L_k, \\
& \theta, \mu, \nu \in [0, 1], \quad k = 1, 2, 3.
\end{aligned}$$

3.2. Advantages of the proposed approach

In this subsection, we explore the main advantages of our hybrid approach.

- The main advantage of the hybrid approach is to give a general structure for dealing with the indeterminacy uncertainties in available data. Moreover, it does not require trade-offs or complicated parameters or any other reference directions from the decision maker. In fact, the employ of the approach guarantees a solution that maximizes the global degree of satisfaction and dissatisfaction, and minimizes indeterminacy level, and truly, it is a non-dominated optimal solution.
- The information about the data of the MOT-*p*-FLP is not precisely defined, the mathematical formulation of our approach has the capability to manipulate vague ideas like the number of objective functions and constraints.
- The stated approach provides a simple mathematical structure which makes easier for understanding and using. In fact, it always gives a compromise solution within a relatively short computational time for small scale entries.

3.3. Disadvantages of our approach

The main limitation of our approach is that it cannot deal with the fixed-charge cost for route selection or vehicle. If the fixed-charge cost is incurred, then the continuous structure of the problem will be lost. Furthermore, we have used the C++ programming language for the iterations and optimization solver for the NCP. Therefore, if an algorithm is specially designed for this complex structure might yields result faster, and is certainly necessary to solve large scale instances.

4. Analysis of non-dominated solution

Here, we first demonstrate that if (x^*, y^*, w^*) is a non-dominated solution of the MOT-*p*-FLP, then (x^*, y^*) is a non-dominated solution of the unconstrained multi-objective FLPs of Eqs. (2.1), (2.2) and (2.8) or Eqs. (2.1), (2.2) and (2.10), where $w = w^*$.

Lemma 1. Let (x^*, y^*, w^*) is a non-dominated solution of the MOT-*p*-FLP of Eqs. (2.1), (2.2) and (2.8). Then (x^*, y^*) is a non-dominated solution of the multi-objective FLP:

$$\text{minimize} Z_{1(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^* \phi(u_i, v_i; x_j, y_j)$$

$$\text{minimize} Z_{2(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^* \psi(u_i, v_i; x_j, y_j)$$

$$\text{minimize} Z_{3(x,y)} = (\alpha + \gamma)\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}^* \varphi(u_i, v_i; x_j, y_j) - \gamma C$$

Proof. This lemma can be proved by the method of indirect proof (i.e., contradiction). Let $(\bar{x}, \bar{y})$ be a solution such that $Z_k(\bar{x}, \bar{y}, w^*) \leqslant Z_k(x^*, y^*, w^*)$ for $k(=1, 2, 3)$, and $Z_k(\bar{x}, \bar{y}, w^*) < Z_k(x^*, y^*, w^*)$ for at least one k. Again $(\bar{x}, \bar{y}, w^*)$ is a feasible solution of the problem, then there is a contradiction to a non-dominated solution of (x^*, y^*, w^*). This ends the lemma. □

Lemma 2. Let (x^*, y^*, w^*) is a non-dominated solution of the MOT-*p*-FLP of Eqs. (2.1), (2.2) and (2.10). Then (x^*, y^*) is a non-dominated solution of the multi-objective FLP:

$$\text{minimize} Z_{1(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^* \phi(u_i, v_i; x_j, y_j)$$

$$\text{minimize} Z_{2(x,y)} = \sum_{i=1}^{m}\sum_{j=1}^{p} e_i w_{ij}^* \psi(u_i, v_i; x_j, y_j)$$

$$\text{minimize} Z_{3(x,y)} = (\alpha + P_c\beta)\sum_{i=1}^{m}\sum_{j=1}^{p} w_{ij}^* \varphi(u_i, v_i; x_j, y_j) - P_c\beta C$$

Proof. It can be easily proved by similar way. □

Lemma 3. Let $(x^*, y^*, w^*, \theta^*, \mu^*, \nu^*)$ be an optimal solution of Model 8, then it should be also a non-dominated solution (x^*, y^*, w^*) of Model 1.1.

Proof. Let the contrary be true. Then there is a solution $(\bar{x}, \bar{y}, \bar{w}) \in F$ such that $Z_k(\bar{x}, \bar{y}, \bar{w}) < Z_k(x^*, y^*, w^*)\ \forall\ k,\ k = 1, 2, 3$. Now θ^*, μ^* and ν^* are the optimal values of Model 8, then

$$\begin{aligned}
& Z_k(\bar{x}, \bar{y}, \bar{w}) + (U_k^{Tr} - L_k^{Tr})\theta^* < Z_k(x^*, y^*, w^*) + (U_k^{Tr} - L_k^{Tr})\theta^* \leqslant U_k^{Tr}, \quad k = 1, 2, 3, \\
& Z_k(\bar{x}, \bar{y}, \bar{w}) + (U_k^{In} - L_k^{In})\nu^* < Z_k(x^*, y^*, w^*) + (U_k^{In} - L_k^{In})\nu^* \leqslant U_k^{In}, \quad k = 1, 2, 3, \\
& Z_k(\bar{x}, \bar{y}, \bar{w}) - (U_k^{Fa} - L_k^{Fa})\mu^* < Z_k(x^*, y^*, w^*) - (U_k^{Fa} - L_k^{Fa})\mu^* \leqslant L_k^{Fa}, \quad k = 1, 2, 3.
\end{aligned}$$

Henceforth, there exist $\theta > \theta^*$, $\mu > \mu^*$, $\nu > \nu^*$ and an $l \in \{1, 2, 3\}$ such that

$$\begin{aligned}
& Z_l(\bar{x}, \bar{y}, \bar{w}) + (U_l^{Tr} - L_l^{Tr})\theta = U_l^{T}, \\
& Z_k(\bar{x}, \bar{y}, \bar{w}) + (U_k^{Tr} - L_k^{Tr})\theta \leqslant U_k^{T}, \quad k \neq l, \\
& Z_l(\bar{x}, \bar{y}, \bar{w}) + (U_l^{In} - L_l^{In})\nu = U_l^{In}, \\
& Z_k(\bar{x}, \bar{y}, \bar{w}) + (U_k^{In} - L_k^{In})\nu \leqslant U_k^{In}, \quad k \neq l, \\
& Z_l(\bar{x}, \bar{y}, \bar{w}) - (U_l^{Fa} - L_l^{Fa})\mu = L_l^{Fa}, \\
& Z_k(\bar{x}, \bar{y}, \bar{w}) - (U_k^{Fa} - L_k^{Fa})\mu \leqslant L_k^{Fa}, \quad k \neq l,
\end{aligned}$$

which contradict that $(x^*, y^*, w^*, \theta^*, \mu^*, \nu^*)$ is an optimal solution of Model 8. This completes the proof of lemma. □

Lemma 4. Let $(x^*, y^*, w^*, \theta^*, \mu^*, \nu^*)$ be an optimal solution of Model 9, then it should be also a non-dominated solution (x^*, y^*, w^*) of Model 1.2.

Proof. The proof is left to the reader. □

5. Experimental example

Herein, a real-life based example is presented to validate our model and methodology. In the example, an industrial association wishes to start a few new firms with the aim of minimizing the total transportation cost, time and carbon emission cost under tax, cap and trade policy. The association has 4 existing firms: S_1, S_2, S_3 and S_4, and they want to establish 3 new firms: D_1, D_2 and D_3. They transport goods by conveyances. In fact, we consider the weights of the conveyances depend on the machine performance which is reflected in transportation and carbon emission cost. Thereafter, the obstacles of the paths are also appraised in transportation time to become the problem more realistic. Under TCTP, the industrial association is allocated a carbon emission cap C. When they emit less (more) than the cap C, then they can sell (buy) the extra permit in (from) the carbon trading market. Moreover, if the firms emit more than the cap, then they have to pay extra cost as a penalty to reduce carbon emission. Hypothetical data of real-life scenarios are created. Here, we take carbon tax $\alpha = 0.3$, carbon buying cost from the trade market $\beta = 0.5$, carbon selling cost $\gamma = 0.7$ and penalty cost $P_c = 0.9$. The availability of S_1, S_2, S_3 and S_4 and the demand of the firms D_1, D_2, and D_3 are known. Moreover, the locations and the weights of the plants S_1, S_2, S_3 and S_4 are also provided. Table 2 represents the locations and weights of the existing firms. The supply, demand and non-negative weights are given in Table 3.

Two special cases are considered for TCTP.

Case 5.1: First, when the carbon cap is $C = 800$.

5.1. Performance of the hybrid approach

The steps for solving the proposed MOT-p-FLP are as follows:

Step 1. At first, three initial potential locations are picked up from Table 2 for three plants. Thereafter, four *cases* are appeared which are shown in Tables 4–7.

Step 2. Now, the distances (i.e., for each individual distance function) are estimated among assigned initial potential locations and the rest site for each case. Thereafter, the smallest distance is chosen for each individual distance function from the said four cases. The final initial potential locations are Case 5.1.4 for the first distance function, Case 5.1.3 for the 2nd distance function, and Case 5.1.4 for the 3rd distance function.

Step 3. The distances between existing and initial locations of plants are calculated, and the distances are considered as a cost, time and average carbon emission coefficients. The coefficients are as follows:

Cost coefficients (c'_{ij}):

$c'_{11} = 4.745,\ c'_{12} = 0,\ c'_{13} = 4.595,\ c'_{21} = 2.691,\ c'_{22} = 1.531,\ c'_{23} = 0,\ c'_{31},$
$= 8.094,\ c'_{32} = 7.688,\ c'_{33} = 6.001,\ c'_{41} = 0$
$c'_{42} = 3.166,\ c'_{43} = 5.382.$

Time coefficients (t'_{ij}):

$t'_{11} = 5.763,\ t'_{12} = 4.749,\ t'_{13} = 0.164,\ t'_{21} = 1.500,\ t'_{22} = 2.691,\ t'_{23},$
$= 1.532,\ t'_{31} = 0.218,\ t'_{32} = 8.094,\ t'_{33} = 7.685,\ t'_{41} = 4.047$
$t'_{42} = 0.155,\ t'_{43} = 3.165.$

Carbon emission coefficients (d'_{ij}):

$d'_{11} = 15.817,\ d'_{12} = 0,\ d'_{13} = 15.313,\ d'_{21} = 26.912,\ d'_{22} = 15.310,\ d'_{23}.$
$= 0,\ d'_{31} = 20.236,\ d'_{32} = 19.219,\ d'_{33} = 15.003,\ d'_{41} = 0,\ d'_{42}$
$= 15.830,\ d'_{43} = 26.909$

Step 4. Thereafter, the LINGO 17.0 iterative scheme is employed to obtain the individual optimal feasible solution as follows:

For **Model 3**: $w_{12} = 60,\ w_{22} = 10,\ w_{23} = 30,\ w_{31} = 25,\ w_{33} = 5,\ w_{41} = 25$ with all other $w_{ij} = 0$ and $Z_1 = 247.676$.

For **Model 4**: $w_{12} = 25,\ w_{13} = 35,\ w_{21} = 20,\ w_{22} = 20,\ w_{31} = 30,\ w_{42} = 25$ with all other $w_{ij} = 0$ and $Z_2 = 218.751$.

For **Model 5**:

For Model 5.1: $w_{12} = 60,\ w_{22} = 5,\ w_{23} = 35,\ w_{31} = 25,\ w_{32} = 5,\ w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 118.545$.

For Model 5.2: $w_{11} = 2.548,\ w_{12} = 57.451,\ w_{22} = 12.548,\ w_{23} = 27.452,\ w_{31} = 22.452,\ w_{33} = 7.548,\ w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 240$. Therefore, the optimal solution of Model 5 is the optimal solution corresponding to Model 5.1.

Table 2
Locations and weights of the existing firms.

	Position (u_i, v_i)	Weight (e_i)
S_1	(5, 10)	0.3
S_2	(2, 25)	0.1
S_3	(17, 25)	0.4
S_4	(20, 5)	0.2

Table 3
Pay-off table (t_{ij}, δ_{ij}).

	D_1	D_2	D_3	Supply (a_i)
S_1	(0.0, 0.2)	(0.7, 0.0)	(0.3, 0.5)	60
S_2	(0.1, 0.3)	(0.2, 0.4)	(0.7, 0.0)	40
S_3	(0.3, 0.5)	(0.5, 0.4)	(0.2, 0.1)	30
S_4	(0.0, 0.0)	(0.6, 0.6)	(0.4, 0.1)	25
Demand (b_j)	50	70	35	

Table 4
Case 5.1.1.

	Position	Weight
D_1	(5, 10)	0.3
D_2	(2, 25)	0.1
D_3	(17, 25)	0.4

Table 5
Case 5.1.2.

	Position	Weight
D_1	(2, 25)	0.1
D_2	(17, 25)	0.4
D_3	(20, 5)	0.2

Table 6
Case 5.1.3.

	Position	Weight
D_1	(17, 25)	0.4
D_2	(20, 5)	0.2
D_3	(5, 10)	0.3

Table 7
Case 5.1.4.

	Position	Weight
D_1	(20, 5)	0.2
D_2	(5, 10)	0.3
D_3	(2, 25)	0.1

Step 5. The C+ + programming language is explored for executing our model as a single objective function to get the individual optimal potential locations for the plants. The respective optimal potential locations are as follows:

$(x_1, y_1)^{(1)} = (17.065, 24.568),\ (x_2, y_2)^{(1)} = (5.000, 10.000),\ (x_3, y_3)^{(1)},$
$= (2.458, 25.000)$
$(x_1, y_1)^{(2)} = (16.907, 25.000),\ (x_2, y_2)^{(2)} = (5.613, 10.040),\ (x_3, y_3)^{(2)},$
$= (5.000, 10.000)$
$(x_1, y_1)^{(3)} = (18.512, 14.918),\ (x_2, y_2)^{(3)} = (5.000, 10.000)$ and $(x_3, y_3)^{(3)} = (2.000, 25.000)$.

Step 6. Using the obtained solutions, we compute the upper and lower values for each objective function and they are as follows:

$U_1 = \max\{153.021, 157.550, 268.432\},\ L_1 = 153.021;$
$U_2 = \max\{168.263, 157.136, 283.457\},\ L_2 = 157.136;$
$U_3 = \max\{195.02, 489.503, 118.856\},\ L_3 = 118.856.$

Step 7. Upper and lower bounds based on the NCP are calculated for each objective function:

For $Z_1(x, y, w)$:
$U_1^{Tr} = U_1 = 268.432,\ L_1^{Tr} = L_1 = 153.021,$

$$U_1^{In} = L_1^{Tr} + q_1'(U_1^{Tr} - L_1^{Tr}) = 153.021 + 115.411q_1',\ L_1^{In} = L_1^{Tr} = 153.021,$$

$U_1^{F_a} = U_1^{T_r} = 268.432$, $L_1^{F_a} = L_1^{T_r} + q_1(U_1^{T_r} - L_1^{T_r}) = 153.021 + 115.411q_1$.

For $Z_2(x, y, w)$:

$U_2^{T_r} = U_2 = 283.457$, $L_2^{T_r} = L_2 = 157.136$,

$U_2^{I_n} = L_2^{T_r} + q_2'(U_2^{T_r} - L_2^{T_r}) = 157.361 + 126.321q_2'$, $L_2^{I_n} = L_2^{T_r} = 157.361$,

$U_2^{F_a} = U_2^{T_r} = 283.457$, $L_2^{F_a} = L_2^{T_r} + q_2(U_2^{T_r} - L_2^{T_r}) = 157.361 + 126.321q_2$.

For $Z_3(x, y, w)$:

$U_3^{T_r} = U_3 = 489.503$, $L_3^{T_r} = L_3 = 118.856$,

$U_3^{I_n} = L_3^{T_r} + q_3'(U_3^{T_r} - L_3^{T_r}) = 118.856 + 370.647q_3'$, $L_3^{I_n} = L_3^{T_r} = 118.856$,

$U_3^{F_a} = U_3^{T_r} = 489.503$, $L_3^{F_a} = L_3^{T_r} + q_3(U_3^{T_r} - L_3^{T_r}) = 118.856 + 370.647q_3$.

Step 8. Using the LINGO 17.0 iterative scheme, we solve the simplified neutrosophic model (Model 8). The optimal compromise solution of the above MOT-*p*-FLP is as follows: $w_{11} = 50$, $w_{12} = 10$, $w_{13} = 0$, $w_{21} = 0$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 0$, $w_{32} = 30$, $w_{33} = 0$, $w_{41} = 0$, $w_{42} = 25$, $w_{43} = 0$, $q_1 = q_3 = 0.3$, $q_2 = 0.21$, $q_1' = q_3' = 1$, $q_2' = 0.80$, $\theta = 0.699$, $\mu = 0$, $\nu = 0.699$, $(x_1, y_1) = (5, 10)$, $(x_2, y_2) = (16.380, 22.108)$, $(x_3, y_3) = (2, 25)$, $Z_1 = 187.713$, $Z_2 = 184.191$, $Z_3 = 230.269$.

Case 5.2: Furthermore, consider the carbon cap as $C = 675$. Here, Steps 1 to 4 are exactly same with Case 5.1. Since, the feasible solution of Model 5.1 does not exist, therefore, we take the solution of Model 5.2 as Model 5. The optimal solution of Model 5 is as follows:

$w_{12} = 60$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 25$, $w_{32} = 5$, $w_{41} = 25$ with all other $w_{ij} = 0$, and $Z_3 = 205.159$.

Step 5. Using C++ programming, we obtain the individual optimal potential locations for the plants. The respective optimal potential locations are as follows:

$(x_1, y_1)^{(1)} = (17.065, 24.568)$, $(x_2, y_2)^{(1)} = (5.000, 10.000)$, $(x_3, y_3)^{(1)}$, $= (2.458, 25.000)$

$(x_1, y_1)^{(2)} = (16.907, 25.000)$, $(x_2, y_2)^{(2)} = (5.613, 10.040)$, $(x_3, y_3)^{(2)}$, $= (5.000, 10.000)$

$(x_1, y_1)^{(3)} = (18.512, 14.918)$, $(x_2, y_2)^{(3)} = (5.000, 10.000)$ and $(x_3, y_3)^{(3)} = (2.000, 25.000)$.

Step 6. Thereafter, we calculate upper and lower bounds and they are as follows:

$U_1 = \max\{153.021, 157.550, 268.432\}$, $L_1 = 153.021$;
$U_2 = \max\{168.263, 157.136, 283.457\}$, $L_2 = 157.136$;
$U_3 = \max\{262.515, 483.378, 205.392\}$, $L_3 = 205.392$.

Then, using Step 7, we obtain the optimal compromise solution of the proposed problem. And the solution is as follows: $w_{11} = 50$, $w_{12} = 10$, $w_{13} = 0$, $w_{21} = 0$, $w_{22} = 5$, $w_{23} = 35$, $w_{31} = 0$, w_{32} . $= 30$, $w_{33} = 0$, $w_{41} = 0$, $w_{42} = 25$, $w_{43} = 0$, $q_1 = 1.239$, $q_2 = q_3 = q_1' = q_2' = q_3' = 1.234$, $\theta = 0.699$, $\mu = 0$, $\nu = 0.699$, $(x_1, y_1) = (5, 10)$, $(x_2, y_2) = (16.380, 22.108)$, $(x_3, y_3) = (2, 25)$, $Z_1 = 187.713$, $Z_2 = 184.191$, $Z_3 = 288.952$

6. Computational results and discussion

An application example is provided to analyze the proposed model with the help of the hybrid approach. The approach first finds the initial locations, optimal feasible solutions, optimal locations, ideal solutions (individual minimum), and anti-ideal solutions (individual maximum), and then we determine the upper and lower bounds for truth, indeterminacy, and falsity. Thereafter, the neutrosophic models for two cases of the MOT-*p*-FLP are formulated to derive optimal compromise solutions. The obtained results of the example show that the total transportation cost, delivery time and the optimal locations of the firms are same in both cases. But under TCTP, the total carbon emission cost is varied. In fact, we analyze that when the cap is larger than the total emission, the carbon emission cost decreases as they sell the extra permit in the trade market. For that reason, the industrial organizations will make more profit. Again when the cap is less than a threshold, the carbon emission cost increases as they have to buy carbon emission permits from others, as well as they have to pay also the penalty cost which minimized the profit of firms. Thereafter, TCTP can affect to adjust the carbon emissions due to transportation at the same time to the green environment.The steps of the objective functions and optimal facilities locations for the given example are depicted in Figs. 3 and 4. The alternating Loc-Alloc heuristic approach is coded in C++ and conducted using a code-block compiler, and the NCP is coded in the LINGO 17.0 iterative scheme on a Lenovo z580 computer with 2.50 GHz Intel (R) Core (TM) i5-3210M CPU with 4 GB RAM. The computational results are compared with Linux terminal on a computer with Intel(R) Core (TM) i3-4130 CPU @3.40 GHz with 4 GB RAM.

7. Sensitivity analysis

In this section, we investigate the resiliency of optimal compromise solutions in the MOT-*p*-FLP by varying the parameters. For the MOT-*p*-FLP, the difficulty arises when the range of parameters are chosen after small changes for which the optimal solution remains optimal. In fact, the complexity increases when the number of variables and constraints are in large size. Due to this reason, a simple procedure is adopted to evaluate the sensitivity of the proposed problem with the fact that the basic variables stay the same (as basic variables) but their values will be replaced. The effective range of the parameters in the MOT-*p*-FLP are computed by the following steps:

Step 1. Fix the basic variables for a given optimal solution of the MOT-*p*-FLP, received from the hybrid approach.

Step 2. Replace the value of each parameter one at a time in the successive trial, and keeping the others unaltered. Thereafter, solve the corresponding the MOT-*p*-FLP.

Step 3. Proceed Step 2 until the solution appears for the MOT-*p*-FLP: either "no feasible solution" or "change the basic variable in optimal solution".

Step 4. Compute the range of each parameter by Step 3.

Sensitivity analysis for supply and demand parameters:

Let a_i be changed to $a_i^*(i = 1, 2, 3, 4)$ and b_j be altered to $b_j^*(j = 1, 2, 3)$. Utilizing the stepwise procedure from above, the values of a_i^* and b_j^* are easily calculated, which shown in Tables 8 and 9. Note that the range of the other parameters in the MOT-*p*-FLP are resolved in a similar way.

8. Managerial insights

The fact that MOT-*p*-FLP is an especially application-based region, makes it essential to receive deep insights into the characteristics of optimal solutions. Herein, we gather information about the optimal solutions derived when employing Model 1 into two sub-problems. Observing the outcomes, the management's discretion can easily pick the optimal solution between two sub-problems. A brief discussion of the effect of carbon emission under TCTP is depicted. From that discussion, the managements can easily decide the optimal potential facility sites so that they can easily reach the sites with minimum transportation cost, transportation time and carbon emission. There is an analysis of carbon tax, cap and trade policy in emission, from that the managements can decide which case emits the least amount of carbon emission. As a result, they can balance between their profits and green environment, which may gain reputation in the global market. On the other hand, the machine performance of the conveyances is displayed in transportation cost and emission. In case the machine performance is good, then the total transportation cost along with carbon emission will be reduced. For that reason, the managements can easily choose which type of conveyances are better for transporting goods. Again, the time for obstacles of the paths is also considered in transportation time, so that the managements can calculate the more accurate transportation

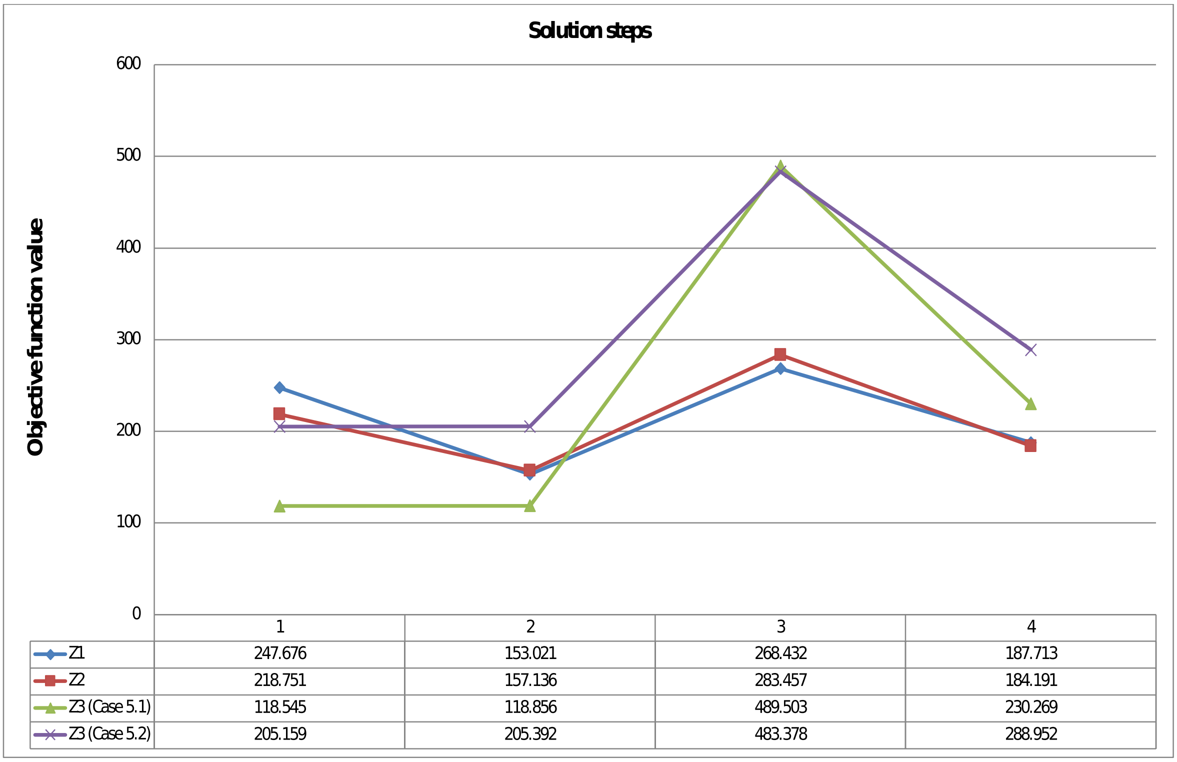

Fig. 3. Graphical representation of the proposed hybrid approach for example.

Fig. 4. The existing and optimal potential facilities in example.

Table 8
The range of supply and demand parameters for Case 5.1.

Real values of a_i and b_j	Changing values of a_i and b_j
$a_1 = 60$	$60 \leqslant a_1^* \leqslant 118.5$
$a_2 = 40$	$40 \leqslant a_2^* \leqslant 65.3$
$a_3 = 30$	$30 \leqslant a_3^* < \infty$
$a_4 = 25$	$25 \leqslant a_4^* < \infty$
$b_1 = 50$	$10.2 \leqslant b_1^* \leqslant 50$
$b_2 = 70$	$39.1 \leqslant b_2^* \leqslant 70$
$b_3 = 35$	$0.1 \leqslant b_3^* \leqslant 35$

time which improves their services to the customers. A sensitivity analysis is provided to show which range of the parameter is more appropriate for the managements. Finally, we can say that the mentioned formulation will be effective for the managements to seek optimal potential sites to transport with minimum cost, time and carbon emission.

Table 9
The range of supply and demand parameters for Case 5.2.

Real values of a_i and b_j	Changing values of a_i and b_j
$a_1 = 60$	$60 \leqslant a_1^* \leqslant 94.9$
$a_2 = 40$	$40 \leqslant a_2^* \leqslant 64.9$
$a_3 = 30$	$30 \leqslant a_3^* \leqslant 50$
$a_4 = 25$	$25 \leqslant a_4^* < \infty$
$b_1 = 50$	$22.5 \leqslant b_1^* \leqslant 50$
$b_2 = 70$	$54 \leqslant b_2^* \leqslant 70$
$b_3 = 35$	$-\infty < b_3^* \leqslant 35$

9. Conclusions and future research directions

This study has been presented a practical formulation for planning and transportation system with the objectives of minimizing the total transportation cost, total transportation time, and total carbon emission cost under TCTP on the entire transportation chain, and at the same time it also asks the potential facility sites along with the amounts of transported goods simultaneously. To the best of authors' knowledge, the problem of designing an MOT-*p*-FLP, considering variable carbon emission under TCTP, has not been studied before. Additionally, we have improved a hybrid approach to solve the proposed problem in an effective way. The stated formulation and improved hybrid approach have been tested by a real-life based example. Thereafter, the effect of variable carbon emissions under TCTP is investigated by two special cases. In fact, we explore the optimal decision to reduce carbon emission for companies under TCTP. Therefore, the nature of the obtained compromise solution is analyzed by four lemmas. Lastly, the sensitivity analysis has been given to check the resiliency of the parameters in the MOT-*p*-FLP. Moreover, our formulation can be utilized in other industrial applications like the manufacturing of plants, green supply chain model, production-inventory system, financial and further applications. We must underscore that in association with this investigation, there are other lines of research work of absolute significance and importance that we have not raised because they are outside the objectives initially set; however, in future investigation, one can analyze the MOT-p-FLP with neutrosophic parameters, and discuss the effect of variation in solution of the MOT-p-FLP. Another scope is to consider our model in the stochastic environment, then one may perform the statistical inference analysis using nonparametric hypothesis testing. Similarly, the possibilities of using Genetic Programming, Monte Carlo Method (Klibi et al., 2010) Simboloc Regression, techno-economic (cost) analysis and payback period are the interesting lines to be investigated in a forthcoming paper(s). In fact, researchers can then compare the study with our proposed study. Furthermore, interested scientists can analyze our model in different uncertain environments, e.g., type-2 fuzzy sets, intuitionistic fuzzy sets, rough sets or grey numbers. Besides, one may consider the membership functions as hyperbolic, exponential, etc., instead of linear membership functions for solving the MOT-p-FLP. In addition, researchers may employ the different types of distance functions such as rectangular distance, signed distance, Hausdorff distance, etc. The incorporation of fixed-charge costs in the MOT-p-FLP can be more realistic research modification. In this regard, a line of study that we design to explore in the future is the application of meta-heuristic algorithms to solve such problems. Nature-motivated metaheuristic algorithms, such as Particle Swarm Optimization, Genetic Algorithm, Simulated Annealing, etc. seem faster to successfully solve these problems with large scale entries and will be the fields for future research works.

Appendix A

Theorem 1.

(i) The objective function $Z_{1(x,y)}=\sum_{i=1}^{m}\sum_{j=1}^{p}e_i w_{ij}^B\phi(u_i,v_i;x_j,y_j)$ is convex on $\mathbb{R}^{2p}$.
(ii) The objective function $Z_{2(x,y)}=\sum_{i=1}^{m}\sum_{j=1}^{p}e_i w_{ij}^B\psi(u_i,v_i;x_j,y_j)$ is convex on $\mathbb{R}^{2p}$.
(iii) The objective function $Z_{3(x,y)}=(\alpha+\gamma)\sum_{i=1}^{m}\sum_{j=1}^{p}w_{ij}\varphi(u_i,v_i;x_j,y_j)-\gamma C$ or $(\alpha+P_c\beta)\sum_{i=1}^{m}\sum_{j=1}^{p}w_{ij}\varphi(u_i,v_i;x_j,y_j)-P_c\beta C$ is convex on $\mathbb{R}^{2p}$.

Proof.

(i) We know that a function Z_1 is convex on an area iff the related Hessian matrix of Z_1 is positive semidefinite on that area (Rockafellar, 1970). Now we designate Z_1 as $Z_1=\sum_{j=1}^{p}Z_{1j}$, where $Z_{1j}=\sum_{i=1}^{m}e_i w_{ij}^B\phi(u_i,v_i;x_j,y_j)$, and w_{ij}^B are given. Furthermore, Z_{1j} only depend on the variables x_j and y_j. The Hessian matrix for Z_{1j} at (x_j,y_j) is

$$H_{1j}=\begin{pmatrix}\frac{\partial^2 Z_{1j}}{\partial x_j^2} & \frac{\partial^2 Z_{1j}}{\partial x_j\partial y_j}\\ \frac{\partial^2 Z_{1j}}{\partial y_j\partial x_j} & \frac{\partial^2 Z_{1j}}{\partial y_j^2}\end{pmatrix}.$$

The principal minors of H_{1j} are $\frac{\partial^2 Z_{1j}}{\partial x_j^2}$ and $\det H_{1j}$ (determinant of H_{1j}).Now,

$$\frac{\partial^2 Z_{1j}}{\partial x_j^2}=\sum_{i=1}^{m}\frac{e_i w_{ij}^B(v_i-y_j)^2}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/2}},$$

$$\text{and}\det H_{1j}=\frac{\partial^2 Z_{1j}}{\partial x_j^2}\frac{\partial^2 Z_{1j}}{\partial y_j^2}-\left(\frac{\partial^2 Z_{1j}}{\partial x_j\partial y_j}\right)^2\left(\text{since}\frac{\partial^2 Z_{1j}}{\partial x_j\partial y_j}=\frac{\partial^2 Z_{1j}}{\partial y_j\partial x_j}\right)$$

$$=\left(\sum_{i=1}^{m}\frac{e_i w_{ij}^B(v_i-y_j)^2}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/2}}\right)\left(\sum_{i=1}^{m}\frac{e_i w_{ij}^B(u_i-x_j)^2}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/2}}\right)$$

$$-\left(\sum_{i=1}^{m}\frac{e_i w_{ij}^B(u_i-x_j)(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/2}}\right)^2$$

$$=\left(\sum_{i=1}^{m}\left(\frac{\sqrt{e_i w_{ij}^B}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\right)^2\right)\left(\sum_{i=1}^{m}\left(\frac{\sqrt{e_i w_{ij}^B}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\right)^2\right)$$

$$-\left(\sum_{i=1}^{m}\frac{\sqrt{e_i w_{ij}^B}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\frac{\sqrt{e_i w_{ij}^B}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\right)^2.$$

Now, $\left(\sum_{i=1}^{m}(\frac{\sqrt{e_i w_{ij}^B}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}})^2\right)\left(\sum_{i=1}^{m}(\frac{\sqrt{e_i w_{ij}^B}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}})^2\right)\geqslant\left(\sum_{i=1}^{m}\frac{\sqrt{e_i w_{ij}^B}(v_i-y_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\frac{\sqrt{e_i w_{ij}^B}(u_i-x_j)}{[(u_i-x_j)^2+(v_i-y_j)^2+\delta_{ij}]^{3/4}}\right)^2$ (byCauchy.As

$-$ Schwarzinequality)

$e_i>0$, $w_{ij}^B\geqslant 0$, $(u_i-x_j)^2\geqslant 0$ and $(v_i-y_j)^2\geqslant 0$, then it is easily concluded that $\frac{\partial^2 Z_{1j}}{\partial x_j^2}\geqslant 0$ and $\det H_{1j}\geqslant 0$ for all values of x_j, y_j. Hence, Z_{1j} is convex with respect to x_j and y_j. Let us assume that $((x_1,y_1),(x_2,y_2),\ldots,(x_p,y_p))$ and $((x_1',y_1'),(x_2',y_2'),\ldots,(x_p',y_p'))$ be two arbitrary points of $\mathbb{R}^{2p}$, and $l'\in[0,1]$.Herewith,

$$Z_1(l'((x_1, y_1), (x_2, y_2), ..., (x_p, y_p)) + (1 - l')((x_1', y_1'), (x_2', y_2'), ..., (x_p', y_p')))$$
$$= \sum_{j=1}^{p} Z_{1j}(l'((x_1, y_1), (x_2, y_2), ..., (x_p, y_p)) + (1 - l')((x_1', y_1'), (x_2', y_2'), ..., (x_p', y_p')))$$
$$\leqslant l' \sum_{j=1}^{p} Z_{1j}((x_1, y_1), (x_2, y_2), ..., (x_p, y_p)) + (1 - l') \sum_{j=1}^{p} Z_{1j}((x_1', y_1'), (x_2', y_2'), ..., (x_p', y_p'))$$
$$= l'Z_1((x_1, y_1), (x_2, y_2), ..., (x_p, y_p)) + (1 - l')Z_1((x_1', y_1'), (x_2', y_2'), ..., (x_p', y_p')).$$

Henceforth, $Z_{1(x,y)}$ is convex on $\mathbb{R}^{2p}$. □

(ii) The proof is left to the reader. □

(iii) The proof is left to the reader. □

Appendix B

Iterative formula:

(i) Here, the iterative formula of $Z_{1(x,y)} = \sum_{i=1}^{m} \sum_{j=1}^{p} e_i w_{ij}^B \phi_(u_i, v_i;x_j, y_j)$ are presented. Differentiating Z_1 with respect to x_j and y_j, and then equating to zero.

$$\sum_{i=1}^{m} \frac{e_i w_{ij}^B (u_i - x_j)}{\phi(u_i, v_i;x_j, y_j)} = 0 \quad (j = 1, 2, ..., p), \tag{B.15}$$

$$\sum_{i=1}^{m} \frac{e_i w_{ij}^B (u_i - y_j)}{\phi(u_i, v_i;x_j, y_j)} = 0 \quad (j = 1, 2, ..., p). \tag{B.16}$$

Now, from Eqs. (B.15) and (B.16), we obtain as follows:

$$\sum_{i=1}^{m} \frac{e_i w_{ij}^B u_i}{\phi(u_i, v_i;x_j, y_j)} - x_j \sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j, y_j)} = 0 \quad (j = 1, 2, ..., p),$$
$$\sum_{i=1}^{m} \frac{e_i w_{ij}^B v_i}{\phi(u_i, v_i;x_j, y_j)} - y_j \sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j, y_j)} = 0 \quad (j = 1, 2, ..., p).$$

Then,

$$x_j = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B u_i}{\phi(u_i, v_i;x_j, y_j)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j, y_j)}} \quad (j = 1, 2, ..., p),$$
$$y_j = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B v_i}{\phi(u_i, v_i;x_j, y_j)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j, y_j)}} \quad (j = 1, 2, ..., p).$$

The equations are derived iteratively. The iteration equations for (x_j, y_j) are as follows (motivated by the concept Cooper (1964)):

$$x_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B u_i}{\phi(u_i, u_i;x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p;r \in \mathbb{N}),$$
$$y_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B v_i}{\phi(u_i, v_i;x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\phi(u_i, v_i;x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p;r \in \mathbb{N}),$$

where $\phi(u_i, v_i;x_j^r, y_j^r) = [(u_i - x_j^r)^2 + (v_i - y_j^r)^2 + \delta_{ij}]^{1/2}$. The initial iterations of (x_j, y_j) are the weighted mean coordinates:

$$x_j^0 = \frac{\sum_{i=1}^{m} e_i w_{ij}^B u_i}{\sum_{i=1}^{m} e_i w_{ij}^B} \quad (j = 1, 2, ..., p),$$
$$y_j^0 = \frac{\sum_{i=1}^{m} e_i w_{ij}^B v_i}{\sum_{i=1}^{m} e_i w_{ij}^B} \quad (j = 1, 2, ..., p).$$

(ii) As similar the iterations for $Z_{2(x,y)} = \sum_{i=1}^{m} \sum_{j=1}^{p} e_i w_{ij}^B \psi_(u_i, v_i;x_j, y_j)$ are

$$x_j^0 = \frac{\sum_{i=1}^{m} e_i w_{ij}^B u_i}{\sum_{i=1}^{m} e_i w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$y_j^0 = \frac{\sum_{i=1}^{m} e_i w_{ij}^B v_i}{\sum_{i=1}^{m} e_i w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$x_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B u_i}{\psi(u_i, u_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\psi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; \quad r \in \mathbb{N}),$$

$$y_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{e_i w_{ij}^B v_i}{\psi(u_i, v_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{e_i w_{ij}^B}{\psi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; \quad r \in \mathbb{N}),$$

where $\psi(u_i, v_i; x_j^r, y_j^r) = [(u_i - x_j^r)^2 + (v_i - y_j^r)^2 + t_{ij}]^{1/2}$.

(iii) Again the iterations for Z_3 are as follows:

Case 3.1: The iterations for $Z_{3(x,y)} = \sum_{i=1}^{m} \sum_{j=1}^{p} (\alpha + \gamma) w_{ij}^B \varphi(u_i, v_i; x_j, y_j) - \gamma C$ are as:

$$x_j^0 = \frac{\sum_{i=1}^{m} (\alpha + \gamma) w_{ij}^B u_i}{\sum_{i=1}^{m} (\alpha + \gamma) w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$y_j^0 = \frac{\sum_{i=1}^{m} (\alpha + \gamma) w_{ij}^B v_i}{\sum_{i=1}^{m} (\alpha + \gamma) w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$x_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{(\alpha + \gamma) w_{ij}^B u_i}{\varphi(u_i, u_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{(\alpha + \gamma) w_{ij}^B}{\varphi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; r \in \mathbb{N}),$$

$$y_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{(\alpha + \gamma) w_{ij}^B v_i}{\varphi(u_i, v_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{(\alpha + \gamma) w_{ij}^B}{\varphi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; r \in \mathbb{N}),$$

where $\varphi(u_i, v_i; x_j^r, y_j^r) = [(u_i - x_j^r)^2 + (v_i - y_j^r)^2 + \delta_{ij}]^{1/2}$.

Case 3.2: The iterations for $Z_{3(x,y)} = \sum_{i=1}^{m} \sum_{j=1}^{p} (\alpha + P_c \beta) w_{ij}^B \varphi(u_i, v_i; x_j, y_j) - P_c \beta C$ are as:

$$x_j^0 = \frac{\sum_{i=1}^{m} (\alpha + P_c \beta) w_{ij}^B u_i}{\sum_{i=1}^{m} (\alpha + P_c \beta) w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$y_j^0 = \frac{\sum_{i=1}^{m} (\alpha + P_c \beta) w_{ij}^B v_i}{\sum_{i=1}^{m} (\alpha + P_c \beta) w_{ij}^B} \quad (j = 1, 2, ..., p),$$

$$x_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{(\alpha + P_c \beta) w_{ij}^B u_i}{\varphi(u_i, u_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{(\alpha + P_c \beta) w_{ij}^B}{\varphi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; r \in \mathbb{N}),$$

$$y_j^{r+1} = \frac{\sum_{i=1}^{m} \frac{(\alpha + P_c \beta) w_{ij}^B v_i}{\varphi(u_i, v_i; x_j^r, y_j^r)}}{\sum_{i=1}^{m} \frac{(\alpha + P_c \beta) w_{ij}^B}{\varphi(u_i, v_i; x_j^r, y_j^r)}} \quad (j = 1, 2, ..., p; r \in \mathbb{N}),$$

where $\varphi(u_i, v_i; x_j^r, y_j^r) = [(u_i - x_j^r)^2 + (v_i - y_j^r)^2 + \delta_{ij}]^{1/2}$.

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