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Fuzzy Parameterized Complex Neutrosophic Soft Expert Set for Decision under Uncertainty

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Abstract: In the definition of the complex neutrosophic soft expert set (CNSES), parameters set is a classical set, and the parameters have the same degree of importance, which is considered as 1. This poses a limitation in modeling of some problems. This paper introduces the concept of fuzzy parameterized complex neutrosophic soft expert set (FP-CNSES) to handle this issue by assigning a degree of importance to each of the problem parameters. We further develop FP-CNSES by establishing the concept of weighted fuzzy parameterized complex neutrosophic soft expert set (WFP-CNSES) based on the idea that each expert has a relative weight. These new mathematical frameworks reduce the chance of unfairness in the decision making process. Some essential operations with their properties and relevant laws related to the notion of FP-CNSES are defined and verified. The notation of mapping on fuzzy parameterized complex neutrosophic soft expert classes is defined and some properties of fuzzy parameterized complex neutrosophic soft expert images and inverse images was investigated. FP-CNSES is used to put forth an algorithm on decision-making by converting it from complex state to real state and subsequently provided the detailed decision steps. Then, we provide the comparison of FP-CNSES to the current methods to show the ascendancy of our proposed method.

Keywords: complex neutrosophic set; complex neutrosophic soft expert set; fuzzy parameterized single valued neutrosophic soft expert set; single valued neutrosophic set; soft expert set

1. Introduction

In a world where not everything is certain, the need to represent uncertain data was successfully fulfilled by Zadeh [1] by introducing the concept of fuzzy sets. Intuitionistic fuzzy sets were introduced by Atanassov [2] as an extension of Zadeh's notion of fuzzy set, which proved to be a better model of uncertainty. The words "neutrosophy" and "neutrosophic" were introduced for the first time by Smarandache [3]. Then, neutrosophic set [4] was defined as a more general platform, which extends the concepts of the fuzzy set and intuitionistic fuzzy set. To apply neutrosophic set to real- life problems, its operators need to be specified. Thus, single valued neutrosophic set and its basic operations were defined [5] as a special case of neutrosophic set. Molodtsov [6] proposed the concept of soft set, to bring a topological flavor to the models of uncertainty and associate family of subsets of universe to parameters. Soft set was then extended to the soft expert set [7], which was further developed to fuzzy soft expert set (FSES) [8], intuitionistic fuzzy soft expert sets (IFSES) [9] and single valued neutrosophic soft expert sets (SVNSES) [10]. At the same time, there have been some practical applications in soft expert set theory and its extensions that are used in decision making [11–13].

Among the significant milestones in the development of soft sets and soft expert sets and their generalizations is the introduction of the fuzzy parameterized aspect. This new aspect has further improved the theories of soft and soft expert sets and made them better suited to be used in solving decision making problems, especially when used with the more accurate generalizations of soft and soft expert sets such as FSESs, IFSESs and other hybrid models mentioned above. The fuzzy parameterized aspect was firstly established by Cagman et al. [14] who introduced the concept of fuzzy parameterized soft set by giving a degree of importance to each element in the set of parameters. Subsequently, this aspect was attached and/or added to the existing generalizations of the soft sets, soft expert sets and fuzzy sets. Bashir and Salleh [15] introduced the notion of fuzzy parameterized soft expert sets (FP-SES), while Hazaymeh et al. [16] the notion of fuzzy parameterized fuzzy soft expert sets (FP-FSES), followed by Selvachandran and Salleh [17] who introduced the notion of fuzzy parameterized intuitionistic fuzzy soft expert set (FP-IFSES) as a generalization of the work by Hazaymeh et al. [16]. However, these sets can only handle incomplete and uncertainty information but not indeterminate and inconsistent information, which usually exists in real situations. To treat this deficiency, Al-Quran and Hassan [18] defined the fuzzy parameterized single valued neutrosophic soft expert set (FP-SVNSES), which proves superior to these models with three independent membership functions. The FP-SVNSES model is also significantly more advantageous compared to SVNSES, as it has added advantages to SVNSES by virtue of the fuzzy parameterized feature, which provides more information enhancing the quality of the information presented by the SVNSES, which in turn, increases the accuracy of the final decision.

The development of the uncertainty sets that have been mentioned above are not limited to the real field but extended to the complex field. The introduction of fuzzy sets was followed by their extension to the complex fuzzy set [19,20]. Alkouri and Salleh [21] introduced the concept of complex intuitionistic fuzzy set (CIFS) to represent information that happens repeatedly over a period of time. To handle imprecise, indeterminate, inconsistent, and incomplete information that has periodic nature, Ali and Smarandache [22] introduced complex neutrosophic set, where each membership function associates with a phase term. This feature gives wave-like properties that could be used to describe constructive and destructive interference depending on the phase value of an element, as well as its ability to deal with indeterminacy. Inspired by this, Al-Quran and Hassan [23] generalized the CNSES from the definitions of the complex neutrosophic set and soft expert set on the basis of the SVNSES. In depth, the rationales of introducing the complex neutrosophic set and the soft expert set are considered as a potent motivation to the introduction of the concept of CNSES. CNSES is actually an extension of the SVNSES to the complex space, which makes it superior to all of the aforementioned uncertainty sets. Subsequently, Al-Quran and Hassan [24] studied the CNSESs further by establishing a novel structure of relation between two CNSESs, called complex neutrosophic soft expert relation, to evaluate the degree of interaction between the CNSESs, which is defined as a subset of the Cartesian product of the CNSESs.

Over the years, many techniques and methods have been proposed as tools to be used to find the solutions of problems that are nonlinear or vague in nature, with every method introduced superior to its predecessors. Following in this direction, we extend the studies on CNSESs [23] and FP-SVNSES [18] through the establishment of the notion of FP-CNSES to keep the advantages of CNSES while holding the FP-SVNSES features. On the one hand, the novelty of CNSES appears in its ability to provide a succinct, elegant and comprehensive representation of two-dimensional neutrosophic information as well as the adequate parameterization and the opinions of the experts, all in a single set. This two-dimensional information is presented by amplitude and phase terms simultaneously where the phase terms give neutrosophic information that may interfere, constructively or destructively, with the neutrosophic information presented by the associated amplitude terms, thus making this model highly suitable for use in decision making problems to select the best alternative. On the other hand, FP-SVNSES has the fuzzy parameterized feature, which gives a degree of importance to each parameter in the domain of the SVNSES. All of these features together are contained in the proposed FP-CNSES.

To facilitate our discussion, we first review some background on complex neutrosophic set and FP-SVNSES in Section 2. In Section 3, we introduce the concept of FP-CNSES and give its theoretic operations. In Section 4, we study a mapping on fuzzy parameterized complex neutrosophic soft expert classes and its properties. In Section 5 we discuss an application of the FP-CNSES in decision making. Section 6 provides a comprehensive comparison among FP-CNSES and other recent approaches to manifest the dominance of our proposed method. In Section 7, we define the concept of the WFP-CNSES where experts' relative weights are considered and applied to solve a decision making problem. Section 8 outlines the conclusion of this paper.

2. Preliminaries

In this section, a summary of the literature on complex neutrosophic set and FP-SVNSES relevant to this paper is presented.

We begin by recalling the definition of complex neutrosophic set and its basic operations in the following two definitions.

Definition 1. [22] *Let X be a universe of discourse; a complex neutrosophic set S in X is characterized by a truth membership function $T_S(x)$, an indeterminacy membership function $I_S(x)$, and a falsity membership function $F_S(x)$ that assign an element $x \in X$ a complex-valued grade of $T_S(x)$, $I_S(x)$, and $F_S(x)$ in S. By definition, the values $T_S(x)$, $I_S(x)$, and $F_S(x)$ and their sum may all be within the unit circle in the complex plane and are of the form, $T_S(x) = p_S(x).e^{j\mu_S(x)}$, $I_S(x) = q_S(x).e^{j\nu_S(x)}$ and $F_S(x) = r_S(x).e^{j\omega_S(x)}$; $p_S(x), q_S(x), r_S(x)$ and $\mu_S(x), \nu_S(x), \omega_S(x)$ are, respectively, real valued and $p_S(x), q_S(x), r_S(x) \in [0,1]$ such that $0^- \leq P_S(x) + q_S(x) + r_S(x) \leq 3^+$.*

Definition 2. [22] *Let A and B be two complex neutrosophic sets, where A is characterized by a truth membership function $T_A(x) = p_A(x).e^{j\mu_A(x)}$, an indeterminacy membership function $I_A(x) = q_A(x).e^{j\nu_A(x)}$ and a falsity membership function $F_A(x) = r_A(x).e^{j\omega_A(x)}$ and B is characterized by a truth membership function $T_B(x) = p_B(x).e^{j\mu_B(x)}$, an indeterminacy membership function $I_B(x) = q_B(x).e^{j\nu_B(x)}$ and a falsity membership function $F_B(x) = r_B(x).e^{j\omega_B(x)}$.*

We define the the complement, subset, union and intersection operations as follows.

1. *The complement of A, denoted as $\tilde{c}(A)$, is specified by functions:*
   $T_{\tilde{c}(A)}(u) = p_{\tilde{c}(A)}(u).e^{j\mu_{\tilde{c}(A)}(u)} = r_A(u).e^{j(2\pi - \mu_A(u))}$,
   $I_{\tilde{c}(A)}(u) = q_{\tilde{c}(A)}(u).e^{j\nu_{\tilde{c}(A)}(u)} = (1 - q_A(u)).e^{j(2\pi - \nu_A(u))}$, *and*
   $F_{\tilde{c}(A)}(u) = r_{\tilde{c}(A)}(u).e^{j\omega_{\tilde{c}(A)}(u)} = p_A(u).e^{j(2\pi - \omega_A(u))}$.

2. *A is said to be complex neutrosophic subset of B ($A \subseteq B$) if and only if the following conditions are satisfied:*
   (a) $T_A(u) \leq T_B(u)$ such that $p_A(u) \leq p_B(u)$ and $\mu_A(u) \leq \mu_B(u)$.
   (b) $I_A(u) \geq I_B(u)$ such that $q_A(u) \geq q_B(u)$ and $\nu_A(u) \geq \nu_B(u)$.
   (c) $F_A(u) \geq F_B(u)$ such that $r_A(u) \geq r_B(u)$ and $\omega_A(u) \geq \omega_B(u)$.

3. *The union(intersection) of A and B, denoted as $A \cup (\cap) B$ and the truth membership function $T_{A\cup(\cap)B}(u)$, the indeterminacy membership function $I_{A\cup(\cap)B}(u)$, and the falsity membership function $F_{A\cup(\cap)B}(u)$ are defined as:*
   $T_{A\cup(\cap)B}(u) = [(p_A(u) \vee (\wedge) p_B(u))].e^{j(\mu_A(u) \vee (\wedge) \mu_B(u))}$,
   $I_{A\cup(\cap)B}(u) = [(q_A(u) \wedge (\vee) q_B(u))].e^{j(\nu_A(u) \wedge (\vee) \nu_B(u))}$ *and*
   $F_{A\cup(\cap)B}(u) = [(r_A(u) \wedge (\vee) r_B(u))].e^{j(\omega_A(u) \wedge (\vee) \omega_B(u))}$,
   where $\vee$ = max and $\wedge$ = min .

Al-Quran and Hassan [18] defined the FP-SVNSES, agree FP-SVNSES and disagree FP-SVNSES as follows.

Definition 3. *[18] Let U be a universe set, E be a set of parameters, I^E denote all fuzzy subsets of E, X be a set of experts, and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Z = \Psi \times X \times O$ and $A \subseteq Z$, where $\Psi \subset I^E$. A pair $(f, A)_\Psi$ is called a fuzzy parameterized single valued neutrosophic soft expert set $(FPSVNSES)$ over U, where F is a mapping given by*

$$f_\Psi : A \to SVN(U),$$

and $SVN(U)$ denotes the set of all single valued neutrosophic subsets of U.

Definition 4. *An agree FP-SVNSES $(f, A)_{\Psi_1}$ over U is a FP-SVNSE subset of $(f, A)_\Psi$ where the opinions of all experts are "agree" and is defined as follows:*

$$(f, A)_{\Psi_1} = \{F_\Psi(\epsilon) : \epsilon \in \Psi \times X \times \{1\}\}.$$

A disagree FP-SVNSES $(f, A)_{\Psi_0}$ over U is a FP-SVNSE subset of $(f, A)_\Psi$ where the opinions of all experts are "disagree" and is defined as follows:

$$(f, A)_{\Psi_0} = \{F_\Psi(\epsilon) : \epsilon \in \Psi \times X \times \{0\}\}.$$

3. Fuzzy Parameterized Complex Neutrosophic Soft Expert Set

In this section, we introduce the definition of FP-CNSES, which is a generalization of the concept of FP-SVNSES. We define some operations on this concept, namely subset, equality, complement, union and intersection. We also give some properties on these operations.

We begin by proposing the definition of FP-CNSES, and give an illustrative example of it.

Definition 5. *Let U be a universe set, E be a set of parameters, $FZ(E)$ denote all fuzzy subsets of E, X be a set of experts, and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Y = \Gamma \times X \times O$ and $A \subseteq Y$ where $\Gamma \subset FZ(E)$. A pair $(H, A)_\Gamma$ is called a fuzzy parameterized complex neutrosophic soft expert set (FP-CNSES) over U, where H is a mapping given by*

$$H_\Gamma : A \to CN^U,$$

and CN^U denotes the power complex neutrosophic set of U.

The FP-CNSES $(H, A)_\Gamma$ can be written as the following set of ordered pairs:

$$(H, A)_\Gamma = \Big\{\big(a, \{\frac{u}{H_\Gamma(a)(u)} : u \in U\}\big) : a \in A\Big\},$$

where $A \subseteq \Gamma \times X \times O = \Big\{\big(\frac{e}{\mu_\Gamma(e)}, x, o\big) : e \in E, x \in X \text{and } o \in O\Big\}$, such that $\mu_\Gamma(e)$ is the corresponding membership function of the fuzzy set Γ and $\forall u \in U, \forall a \in A$, $H_\Gamma(a)(u) = \langle T_{H_\Gamma(a)}(u), I_{H_\Gamma(a)}(u),\ F_{H_\Gamma(a)}(u)\rangle$, where $T_{H_\Gamma(a)}(u) = p_{H_\Gamma(a)}(u).e^{j\mu_{H_\Gamma(a)}(u)}, I_{H(a)}(u) = q_{H_\Gamma(a)}(u).e^{j\nu_{H_\Gamma(a)}(u)}$ and $F_{H_\Gamma(a)}(u) = r_{H_\Gamma(a)}(u).e^{j\omega_{H_\Gamma(a)}(u)}$ with $T_{H_\Gamma(a)}(u), I_{H_\Gamma(a)}(u)$ and $F_{H_\Gamma(a)}(u)$ representing the complex-valued truth membership function, complex-valued indeterminacy membership function and complex-valued falsity membership function, respectively, for the FP-CNSES $(H, A)_\Gamma$. The values $T_{H_\Gamma(a)}(u), I_{H_\Gamma(a)}(u), F_{H_\Gamma(a)}(u)$ are within the unit circle in the complex plane and both the amplitude terms $p_{H_\Gamma(a)}(u), q_{H_\Gamma(a)}(u), r_{H_\Gamma(a)}(u)$ and the phase terms $\mu_{H_\Gamma(a)}(u), \nu_{H_\Gamma(a)}(u), \omega_{H_\Gamma(a)}(u)$ are real valued such that $p_{H_\Gamma(a)}(u), q_{H_\Gamma(a)}(u)$, $r_{H_\Gamma(a)}(u) \in [0, 1]$ and $0 \le p_{H_\Gamma(a)}(u) + q_{H_\Gamma(a)}(u) + r_{H_\Gamma(a)}(u) \le 3$.

Example 1. *Suppose that a car company produces two different models of cars and wish to take the opinion of its team of experts concerning these two models of cars before and after testing these cars. These two models of cars form the universe of elements,* $U = \{u_1, u_2\}$. *The team of experts is represented by the set* $X = \{x_1, x_2\}$. *Suppose that the team of experts consider a set of parameters,* $E = \{e_1, e_2, e_3\}$, *where* $e_i (i = 1, 2, 3)$ *denotes the decisions "reliability", "comfortability" and "durability", respectively, and suppose* $\Gamma = \left\{ \frac{e_1}{0.4}, \frac{e_2}{0.6}, \frac{e_3}{0.5} \right\}$ *is a fuzzy subset of* $FZ(E)$. *It is to be noted that the parameters may be affected and altered after the cars are tested. By applying the FP-CNSES and considering the opinions of the experts in process one (before testing the car) as amplitude terms of membership, non-membership and indeterminate membership, and setting the opinions of the experts in the second process (after testing the car) as phase terms of membership, non-membership, and indeterminacy, the first and second processes form a FP-CNSES as a whole, which is shown below:*

$$
\begin{aligned}
&(H, A)_\Gamma = \\
&\Big\{ \Big\{ \Big(\tfrac{e_1}{0.4}, x_1, 1 \Big), \Big\{ \frac{u_1}{\langle 0.6e^{2\pi(0.5)}, 0.1e^{2\pi(0)}, 0.2e^{2\pi(0.4)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.8)}, 0.4e^{2\pi(0.7)}, 0.3e^{2\pi(1)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_1}{0.4}, x_2, 1 \Big), \Big\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.7e^{2\pi(0.5)}, 0.1e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.8e^{2\pi(0.7)}, 0.7e^{2\pi(0.5)}, 0.4e^{2\pi(0.3)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_2}{0.6}, x_1, 1 \Big), \Big\{ \frac{u_1}{\langle 0.3e^{2\pi(0.2)}, 0.5e^{2\pi(0.6)}, 0.4e^{2\pi(0.5)} \rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.5)}, 0.7e^{2\pi(0.6)}, 0.1e^{2\pi(0.2)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_2}{0.6}, x_2, 1 \Big), \Big\{ \frac{u_1}{\langle 0.9e^{2\pi(0.8)}, 0.5e^{2\pi(0.3)}, 0.7e^{2\pi(0.4)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.4)}, 0.8e^{2\pi(0.9)}, 0.1e^{2\pi(0.2)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_3}{0.5}, x_1, 1 \Big), \Big\{ \frac{u_1}{\langle 0.4e^{2\pi(0.2)}, 0.5e^{2\pi(0.3)}, 0.7e^{2\pi(0.6)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_3}{0.5}, x_2, 1 \Big), \Big\{ \frac{u_1}{\langle 0.8e^{2\pi(0.1)}, 0.3e^{2\pi(0.2)}, 0.7e^{9\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.6)}, 0.2e^{2\pi(0.4)}, 0.6e^{2\pi(0.8)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_1}{0.4}, x_1, 0 \Big), \Big\{ \frac{u_1}{\langle 0.6e^{2\pi(0.5)}, 0.7e^{2\pi(0.4)}, 0.9e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.9e^{2\pi(0.8)}, 0.8e^{2\pi(0.8)}, 0.7e^{2\pi(0.7)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_1}{0.4}, x_2, 0 \Big), \Big\{ \frac{u_1}{\langle 0.4e^{2\pi(0.2)}, 0.6e^{2\pi(0.4)}, 0.7e^{2\pi(0.8)} \rangle}, \frac{u_2}{\langle 0.2e^{2\pi(0.4)}, 0.5e^{2\pi(0.4)}, 0.6e^{2\pi(0.5)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_2}{0.6}, x_1, 0 \Big), \Big\{ \frac{u_1}{\langle 0.3e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.4)}, 0.5e^{2\pi(0.6)}, 0.7e^{2\pi(0.8)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_2}{0.6}, x_2, 0 \Big), \Big\{ \frac{u_1}{\langle 0.5e^{2\pi(0.8)}, 0.3e^{2\pi(0.1)}, 0.8e^{2\pi(0.9)} \rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.7)}, 0.8e^{2\pi(0.7)}, 0.7e^{2\pi(0.6)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_3}{0.5}, x_1, 0 \Big), \Big\{ \frac{u_1}{\langle 0.4e^{2\pi(0.7)}, 0.9e^{2\pi(0.8)}, 0.1e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.8e^{2\pi(0.2)}, 0.3e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)} \rangle} \Big\} \Big\}, \\
&\Big\{ \Big(\tfrac{e_3}{0.5}, x_2, 0 \Big), \Big\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.6e^{2\pi(0.7)}, 0.1e^{2\pi(0.9)} \rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0)}, 0.4e^{2\pi(0.3)}, 0.1e^{2\pi(0.1)} \rangle} \Big\} \Big\} \Big\}.
\end{aligned}
$$

In the FP-CNSES $(H, A)_\Gamma$ *above, the amplitude term of the membership in the first process and the phase term in the second process form a complex-valued truth membership function. Similarly, the amplitude term of non-membership in process one and the phase term of non-membership in the second process form a complex-valued falsity membership function. In addition, the amplitude term of undecidedness in the first process and the phase term of indeterminacy in the second process form the complex-valued indeterminate membership function.*

Now, we put forward the definition of an agree FP-CNSES and the definition of a disagree FP-CNSES.

Definition 6. *An agree FP-CNSES* $(H, A)_{\Gamma_1}$ *over* U *is a FP-CNSE subset of* $(H, A)_\Gamma$ *where the opinions of all experts are "agree" and is defined as follows:*

$$(H, A)_{\Gamma_1} = \Big\{ \big(a, \{ \frac{u}{H_\Gamma(a)(u)} : u \in U \} \big) : a \in A \subseteq \Gamma \times X \times \{1\} \Big\}$$

Definition 7. *A disagree FP-CNSES* $(H, A)_{\Gamma_0}$ *over* U *is a FP-CNSE subset of* $(H, A)_\Gamma$ *where the opinions of all experts are "disagree" and is defined as follows:*

$$(H, A)_{\Gamma_0} = \Big\{ \big(a, \{ \frac{u}{H_\Gamma(a)(u)} : u \in U \} \big) : a \in A \subseteq \Gamma \times X \times \{0\} \Big\}$$

In the following, we give some basic definitions and set theoretic operations of FP-CNSESs.

We begin by proposing the definition of the subset of two FP-CNSESs and the equality of two FP-CNSESs.

Definition 8. *Let $(H,A)_\Gamma$ and $(G,B)_\Delta$ be two FP-CNSESs over a universe U, we say that $(H,A)_\Gamma$ is a fuzzy parameterized complex neutrosophic soft expert subset of $(G,B)_\Delta$ denoted by $(H,A)_\Gamma \subseteq (G,B)_\Delta$ if and only if*
1. $A \subseteq B$, and 2. $\forall a \in A, H_\Gamma(a)$ is complex neutrosophic subset of $G_\Delta(a)$.

Definition 9. *For two FP-CNSESs $(H,A)_\Gamma$ and $(G,B)_\Gamma$ over a universe U, we say that $(H,A)_\Gamma$ is equal to $(G,B)_\Gamma$ and we write $(H,A)_\Gamma = (G,B)_\Gamma$ if $(H,A)_\Gamma \subseteq (G,B)_\Gamma$ and $(G,B)_\Gamma \subseteq (H,A)_\Gamma$.*

In the following, we propose the definition of the complement of a FP-CNSES along with an illustrative example and give a proposition on the complement of a FP-CNSES.

Let U be a universe of discourse and $(H,A)_\Gamma$ be a FP-CNSES on U, which is as defined below:

$$(H,A)_\Gamma = \Big\{\big(a, \{\frac{u}{H_\Gamma(a)(u)} : u \in U\}\big) : a \in A\Big\}.$$

Definition 10. *The complement of $(H,A)_\Gamma$ is denoted by $(H,A)^c_\Gamma$ and is defined by*

$$(H,A)^c_\Gamma = \Big\{\big(a, \{\frac{u}{H^c_\Gamma(a)(u)} : u \in U\}\big) : a \in \neg A\Big\},$$

where

$$H^c_\Gamma(a)(u) = \langle T_{H^c_\Gamma(a)}(u), I_{H^c_\Gamma(a)}(u), F_{H^C_\Gamma(a)}(u)\rangle,$$

such that:

$$T_{H^c_\Gamma(a)}(u) = p_{H^c_\Gamma(a)}(u).e^{j\mu_{H^c_\Gamma(a)}(u)} = r_{H_\Gamma(a)}(u).e^{j(2\pi-\mu_{H_\Gamma(a)}(u))},$$
$$I_{H^c_\Gamma(a)}(u) = q_{H^c_\Gamma(a)}(u).e^{j\nu_{H^c_\Gamma(a)}(u)} = (1-q_{H_\Gamma(a)}(u)).e^{j(2\pi-\nu_{H_\Gamma(a)}(u))},$$
$$F_{H^c_\Gamma(a)}(u) = r_{H^c_\Gamma(a)}(u).e^{j\omega_{H^c_\Gamma(a)}(u)} = p_{H_\Gamma(a)}(u).e^{j(2\pi-\omega_{H_\Gamma(a)}(u))},$$

and $\neg A \subseteq \Gamma^{\tilde{c}} \times X \times O$, where $\tilde{c}$ is the fuzzy complement.

Example 2. *Consider the approximation given in Example 1, where*

$$H_\Gamma(\frac{e_1}{0.4}, x_1, 1) = \{\frac{u_1}{\langle 0.6e^{j2\Pi(0.5)},\ 0.1e^{j2\Pi(0)},\ 0.2e^{j2\Pi(0.4)}\rangle}, \frac{u_2}{\langle 0.5e^{j2\Pi(0.8)},\ 0.4e^{j2\Pi(0.7)},\ 0.3e^{j2\Pi(1)}\rangle}\}.$$

By using the complex neutrosophic complement and the fuzzy complement, we obtain the complement of the approximation given by

$$H_\Gamma(\frac{e_1}{0.6}, x_1, 1) = \{\frac{u_1}{\langle 0.2e^{j2\Pi(0.5)},\ 0.9e^{j2\Pi(1)},\ 0.6e^{j2\Pi(0.6)}\rangle}, \frac{u_2}{\langle 0.3e^{j2\Pi(0.2)},\ 0.6e^{j2\Pi(0.3)},\ 0.5e^{j2\Pi(0)}\rangle}\}.$$

Proposition 1. *If $(H,A)_\Gamma$ is a FP-CNSES over U, then $\big((H,A)^c_\Gamma\big)^c = (H,A)_\Gamma$.*

Proof. From Definition 10, we have $(H,A)^c_\Gamma = \Big\{\big(a, \{\frac{u}{H^c_\Gamma(a)(u)} : u \in U\}\big) : a \in \neg A\Big\}$, where, $H^c_\Gamma(a)(u)$ $= \langle T_{H^c_\Gamma(a)}(u), I_{H^c_\Gamma(a)}(u), F_{H^c_\Gamma(a)}(u)\rangle = \langle p_{H^c_\Gamma(a)}(u).e^{j\mu_{H^c_\Gamma(a)}(u)}, q_{H^c_\Gamma(a)}(u).e^{j\nu_{H^c_\Gamma(a)}(u)}, r_{H^c_\Gamma(a)}(u).e^{j\omega_{H^c_\Gamma(a)}(u)}\rangle,$ $= \langle r_{H_\Gamma(a)}(u).e^{j(2\pi-\mu_{H_\Gamma(a)}(u))}, (1-q_{H_\Gamma(a)}(u)).e^{j(2\pi-\nu_{H_\Gamma(a)}(u))}, p_{H_\Gamma(a)}(u).e^{j(2\pi-\omega_{H_\Gamma(a)}(u))}\rangle$, and $\neg A \subseteq \Gamma^{\tilde{c}} \times X \times O$ Thus, $((H,A)^c_\Gamma)^c = \Big\{\big(a, \{\frac{u}{(H^c_\Gamma)^c(a)(u)} : u \in U\}\big) : a \in \neg(\neg A)\Big\}$, where $(H^c_\Gamma)^c(a)(u) = \langle r_{H^c_\Gamma(a)}(u).e^{j(2\pi-\mu_{H^c_\Gamma(a)}(u))}, (1-q_{H^c_\Gamma(a)}(u)).e^{j(2\pi-\nu_{H^c_\Gamma(a)}(u))}, p_{H^c_\Gamma(a)}(u)\ .e^{j(2\pi-\omega_{H^c_\Gamma(a)}(u))}\rangle,$

$= \langle p_{H_{\Gamma(a)}}(u).e^{j\left(2\pi-(2\pi-\mu_{H_{\Gamma(a)}(u)})\right)}, \left(1-(1-q_{H_{\Gamma(a)}}(u))\right).e^{j\left(2\pi-(2\pi-\nu_{H_{\Gamma(a)}(u)})\right)},$
$r_{H_{\Gamma(a)}}(u).e^{j\left(2\pi-(2\pi-\omega_{H_{\Gamma(a)}(u)})\right)}\rangle = \langle p_{H_{\Gamma(a)}}(u).e^{j\mu_{H_{\Gamma(a)}(u)}}, q_{H_{\Gamma(a)}}(u).e^{j\nu_{H_{\Gamma(a)}(u)}}, r_{H_{\Gamma(a)}}(u).e^{j\omega_{H_{\Gamma(a)}(u)}}\rangle,$
$= \langle T_{H_{\Gamma(a)}}(u), I_{H_{\Gamma(a)}}(u), F_{H_{\Gamma(a)}}(u)\rangle, = H_{\Gamma}(a)(u)$, and $\neg(\neg A) \subseteq (\Gamma^{\tilde{c}})^{\tilde{c}} \times X \times O$ and since $(\Gamma^{\tilde{c}})^{\tilde{c}} = \Gamma$, this completes the proof. □

We introduce the definition of union and intersection operations of two FP-CNSESs along with an illustrative example and some propositions on these two operations.

Let $(H,A)_\Gamma$ and $(G,B)_\Delta$ be two FP-CNSESs over a universe U, where $(H,A)_\Gamma = \left\{\left(a, \left\{\frac{u}{H_\Gamma(a)(u)} : u \in U\right\}\right) : a \in A\right\}$ and $(G,B)_\Delta = \left\{\left(b, \left\{\frac{u}{G_\Delta(b)(u)} : u \in U\right\}\right) : b \in B\right\}$.

Definition 11. *The union of $(H,A)_\Gamma$ and $(G,B)_\Delta$, denoted by $(H,A)_\Gamma \,\tilde{\cup}\, (G,B)_\Delta$, is the FP-CNSES $(K,C)_\Theta$ such that $C_\Theta = A_\Gamma \cup B_\Delta$ and $\Theta = \Gamma \hat{\cup} \Delta$, $\hat{\cup}$ is the fuzzy union, and $\forall \epsilon \in C_\Theta, \forall\, u \in U$,*

$$T_{K_{\Theta(\epsilon)}}(u) = \begin{cases} p_{H_{\Gamma(\epsilon)}}(u).e^{j\mu_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ p_{G_{\Delta(\epsilon)}}(u).e^{j\mu_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (p_{H_{\Gamma(\epsilon)}}(u) \vee p_{G_{\Delta(\epsilon)}}(u)).e^{j(\mu_{H_{\Gamma(\epsilon)}(u)} \vee \mu_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

$$I_{K_{\Theta(\epsilon)}}(u) = \begin{cases} q_{H_{\Gamma(\epsilon)}}(u).e^{j\nu_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ q_{G_{\Delta(\epsilon)}}(u).e^{j\nu_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (q_{H_{\Gamma(\epsilon)}}(u) \wedge q_{G_{\Delta(\epsilon)}}(u)).e^{j(\nu_{H_{\Gamma(\epsilon)}(u)} \wedge \nu_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

$$F_{K_{\Theta(\epsilon)}}(u) = \begin{cases} r_{H_{\Gamma(\epsilon)}}(u).e^{j\omega_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ r_{G_{\Delta(\epsilon)}}(u).e^{j\omega_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (r_{H_{\Gamma(\epsilon)}}(u) \wedge r_{G_{\Delta(\epsilon)}}(u)).e^{j(\omega_{H_{\Gamma(\epsilon)}(u)} \wedge \omega_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

where $\vee = max$, and $\wedge = min$.

Definition 12. *The intersection of $(H,A)_\Gamma$ and $(G,B)_\Delta$, denoted by $(H,A)_\Gamma \,\tilde{\cap}\, (G,B)_\Delta$, is the FP-CNSES $(K,C)_\Theta$ such that $C_\Theta = A_\Gamma \cup B_\Delta$ and $\Theta = \Gamma \hat{\cap} \Delta$, $\hat{\cap}$ is the fuzzy intersection, and $\forall \epsilon \in C_\Theta, \forall\, u \in U$,*

$$T_{K_{\Theta(\epsilon)}}(u) = \begin{cases} p_{H_{\Gamma(\epsilon)}}(u).e^{j\mu_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ p_{G_{\Delta(\epsilon)}}(u).e^{j\mu_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (p_{H_{\Gamma(\epsilon)}}(u) \wedge p_{G_{\Delta(\epsilon)}}(u)).e^{j(\mu_{H_{\Gamma(\epsilon)}(u)} \wedge \mu_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

$$I_{K_{\Theta(\epsilon)}}(u) = \begin{cases} q_{H_{\Gamma(\epsilon)}}(u).e^{j\nu_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ q_{G_{\Delta(\epsilon)}}(u).e^{j\nu_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (q_{H_{\Gamma(\epsilon)}}(u) \vee q_{G_{\Delta(\epsilon)}}(u)).e^{j(\nu_{H_{\Gamma(\epsilon)}(u)} \vee \nu_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

$$F_{K_{\Theta(\epsilon)}}(u) = \begin{cases} r_{H_{\Gamma(\epsilon)}}(u).e^{j\omega_{H_{\Gamma(\epsilon)}(u)}} & ,if\ \epsilon \in A_\Gamma - B_\Delta \\ r_{G_{\Delta(\epsilon)}}(u).e^{j\omega_{G_{\Delta(\epsilon)}(u)}} & ,if\ \epsilon \in B_\Delta - A_\Gamma \\ (r_{H_{\Gamma(\epsilon)}}(u) \vee r_{G_{\Delta(\epsilon)}}(u)).e^{j(\omega_{H_{\Gamma(\epsilon)}(u)} \vee \omega_{G_{\Delta(\epsilon)}(u)})} & ,if\ \epsilon \in A_\Gamma \cap B_\Delta, \end{cases}$$

where $\vee = max$, and $\wedge = min$.

Example 3. *Consider Example 1. Let* $\Gamma = \left\{\frac{e_1}{0.3}, \frac{e_2}{0.1}, \frac{e_3}{0.9}\right\}$ *be a fuzzy subset of E, and* $\Delta = \left\{\frac{e_1}{0.4}, \frac{e_2}{0.8}, \frac{e_3}{0.2}\right\}$ *be another fuzzy subset over E.*

$$A_{\Gamma} = \left\{\left(\frac{e_1}{0.3}, x_1, 1\right), \left(\frac{e_2}{0.1}, x_2, 1\right), \left(\frac{e_3}{0.9}, x_2, 0\right)\right\},$$
$$B_{\Upsilon} = \left\{\left(\frac{e_1}{0.4}, x_2, 0\right), \left(\frac{e_2}{0.8}, x_2, 1\right), \left(\frac{e_3}{0.2}, x_1, 0\right)\right\}.$$

Suppose $(H, A)_{\Gamma}$ *and* $(G, B)_{\Delta}$ *are two FP-CNSESs over the same U given by*

$$\begin{aligned}
&(H, A)_{\Gamma} = \\
&\left\{\left\{\left(\frac{e_1}{0.3}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.5)}, 0.5e^{2\pi(0.6)}, 0.8e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.8)}, 0.7e^{2\pi(0.1)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\},\right. \\
&\left\{\left(\frac{e_2}{0.1}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.4)}, 0.9e^{2\pi(0.3)}, 0.4e^{2\pi(0.1)}\rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.3)}, 0.2e^{2\pi(0.6)}, 0.4e^{2\pi(0.7)}\rangle}\right\}\right\}, \\
&\left.\left\{\left(\frac{e_3}{0.9}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.1)}, 0.3e^{2\pi(0.4)}, 0.5e^{2\pi(0.9)}\rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.8)}, 0.1e^{2\pi(0.3)}\rangle}\right\}\right\}\right\},
\end{aligned}$$

and

$$\begin{aligned}
&(G, B)_{\Delta} = \\
&\left\{\left\{\left(\frac{e_1}{0.4}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.9e^{2\pi(0.5)}, 0.5e^{2\pi(0.8)}, 0.4e^{2\pi(0.4)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.9)}, 0.7e^{2\pi(0.2)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\},\right. \\
&\left\{\left(\frac{e_2}{0.8}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.6)}, 0.4e^{2\pi(0.3)}, 0.6e^{2\pi(0.4)}\rangle}, \frac{u_2}{\langle 0.7e^{2\pi(0.9)}, 0.2e^{2\pi(0.4)}, 0.1e^{2\pi(0.6)}\rangle}\right\}\right\}, \\
&\left.\left\{\left(\frac{e_3}{0.2}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.4)}, 0.2e^{2\pi(0.5)}, 0.9e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.5)}, 0.3e^{2\pi(0.7)}, 0.9e^{2\pi(0.8)}\rangle}\right\}\right\}\right\}.
\end{aligned}$$

By using the complex neutrosophic union and the fuzzy union (maximum), we have

$$\begin{aligned}
&(H, A)_{\Gamma} \mathbin{\tilde{\cup}} (G, B)_{\Delta} = \\
&\left\{\left\{\left(\frac{e_1}{0.3}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.5)}, 0.5e^{2\pi(0.6)}, 0.8e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.8)}, 0.7e^{2\pi(0.1)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\},\right. \\
&\left\{\left(\frac{e_2}{0.8}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.6)}, 0.4e^{2\pi(0.3)}, 0.4e^{2\pi(0.1)}\rangle}, \frac{u_2}{\langle 0.7e^{2\pi(0.9)}, 0.2e^{2\pi(0.4)}, 0.1e^{2\pi(0.6)}\rangle}\right\}\right\}, \\
&\left\{\left(\frac{e_3}{0.9}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.1)}, 0.3e^{2\pi(0.4)}, 0.5e^{2\pi(0.9)}\rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.8)}, 0.1e^{2\pi(0.3)}\rangle}\right\}\right\}, \\
&\left\{\left(\frac{e_1}{0.4}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.9e^{2\pi(0.5)}, 0.5e^{2\pi(0.8)}, 0.4e^{2\pi(0.4)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.9)}, 0.7e^{2\pi(0.2)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\}, \\
&\left.\left\{\left(\frac{e_3}{0.2}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.4)}, 0.2e^{2\pi(0.5)}, 0.9e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.5)}, 0.3e^{2\pi(0.7)}, 0.9e^{2\pi(0.8)}\rangle}\right\}\right\}\right\}.
\end{aligned}$$

By using the complex neutrosophic intersection and the fuzzy intersection (minimum), we have

$$\begin{aligned}
&(H, A)_{\Gamma} \mathbin{\tilde{\cap}} (G, B)_{\Delta} = \\
&\left\{\left\{\left(\frac{e_1}{0.3}, x_1, 1\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.5)}, 0.5e^{2\pi(0.6)}, 0.8e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.8)}, 0.7e^{2\pi(0.1)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\},\right. \\
&\left\{\left(\frac{e_2}{0.8}, x_2, 1\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.4)}, 0.9e^{2\pi(0.3)}, 0.6e^{2\pi(0.4)}\rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.3)}, 0.2e^{2\pi(0.6)}, 0.4e^{2\pi(0.7)}\rangle}\right\}\right\}, \\
&\left\{\left(\frac{e_3}{0.9}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.4e^{2\pi(0.1)}, 0.3e^{2\pi(0.4)}, 0.5e^{2\pi(0.9)}\rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.8)}, 0.1e^{2\pi(0.3)}\rangle}\right\}\right\}, \\
&\left\{\left(\frac{e_1}{0.4}, x_2, 0\right), \left\{\frac{u_1}{\langle 0.9e^{2\pi(0.5)}, 0.5e^{2\pi(0.8)}, 0.4e^{2\pi(0.4)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.9)}, 0.7e^{2\pi(0.2)}, 0.6e^{2\pi(0.7)}\rangle}\right\}\right\}, \\
&\left.\left\{\left(\frac{e_3}{0.2}, x_1, 0\right), \left\{\frac{u_1}{\langle 0.7e^{2\pi(0.4)}, 0.2e^{2\pi(0.5)}, 0.9e^{2\pi(0.3)}\rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.5)}, 0.3e^{2\pi(0.7)}, 0.9e^{2\pi(0.8)}\rangle}\right\}\right\}\right\}.
\end{aligned}$$

Proposition 2. *Let* $(H, A)_{\Gamma}$, $(G, B)_{\Delta}$ *and* $(S, W)_{\Theta}$ *be any three FP-CNSESs over a universe U. Then,*

1. $((H, A)_{\Gamma} \mathbin{\tilde{\cup}} (G, B)_{\Delta}) \mathbin{\tilde{\cap}} (S, W)_{\Theta} = ((H, A)_{\Gamma} \mathbin{\tilde{\cap}} (S, W)_{\Theta}) \mathbin{\tilde{\cup}} ((G, B)_{\Delta} \mathbin{\tilde{\cap}} (S, W)_{\Theta})$;
2. $((H, A)_{\Gamma} \mathbin{\tilde{\cap}} (G, B)_{\Delta}) \mathbin{\tilde{\cup}} (S, W)_{\Theta} = ((H, A)_{\Gamma} \mathbin{\tilde{\cup}} (S, W)_{\Theta}) \mathbin{\tilde{\cap}} ((G, B)_{\Delta} \mathbin{\tilde{\cup}} (S, W)_{\Theta})$.

Proof.
(1) Assume that $((H, A)_{\Gamma} \mathbin{\tilde{\cup}} (G, B)_{\Delta}) = (Q, R)_{\Pi}$, where $R_{\Pi} = A_{\Gamma} \cup B_{\Delta}$ and $\Pi = \Gamma \hat{\cup} \Delta$, $(L, D)_{\Phi} = (Q, R)_{\Pi} \mathbin{\tilde{\cap}} (S, W)_{\Theta}$ where $D_{\Phi} = R_{\Pi} \cup W_{\Theta}$ and $\Phi = \Pi \hat{\cap} \Theta$. Thus, $\Phi = (\Gamma \hat{\cup} \Delta) \hat{\cap} \Theta = (\Gamma \hat{\cap} \Theta) \hat{\cup} (\Delta \hat{\cap} \Theta)$, since the distributive property is valid for fuzzy sets.

Let $(L, D)_\Phi = \left\{(\epsilon, \{\frac{u}{L_\Phi(\epsilon)(u)} : u \in U\}) : \epsilon \in D\right\}$, where $L_\Phi(\epsilon)(u) = \langle T_{L_\Phi(\epsilon)}(u), I_{L_\Phi(\epsilon)}(u), F_{L_\Phi(\epsilon)}(u)\rangle$. We consider the case when $\epsilon \in A_\Gamma \cap B_\Delta \cap W_\Theta$ as the other cases are trivial. Then, we have,

$$\begin{aligned}
&T_{L_\Phi(\epsilon)}(u) = T_{(Q_\Pi(\epsilon)\tilde{\cap}S_\Theta(\epsilon))}(u) = T_{(H_\Gamma(\epsilon)\tilde{\cup}G_\Delta(\epsilon))\tilde{\cap}S_\Theta(\epsilon)}(u),\\
&= min(max(p_{H_\Gamma(\epsilon)}(u), p_{G_\Delta(\epsilon)}(u)), p_{S_\Theta(\epsilon)}(u)).e^{jmin(max(\mu_{H_\Gamma(\epsilon)}(u),\mu_{G_\Delta(\epsilon)}(u)),\mu_{S_\Theta(\epsilon)}(u))},\\
&= max(min(p_{H_\Gamma(\epsilon)}(u), p_{S_\Theta(\epsilon)}(u)), min(p_{G_\Delta(\epsilon)}(u), p_{S_\Theta(\epsilon)}(u))).e^{jmax(min(\mu_{H_\Gamma(\epsilon)}(u),\mu_{S_\Theta(\epsilon)}(u)),min(\mu_{G_\Delta(\epsilon)}(u),\mu_{S_\Theta(\epsilon)}(u)))},\\
&= T_{(H_\Gamma(\epsilon)\tilde{\cap}S_\Theta(\epsilon))\tilde{\cup}(G_\Delta(\epsilon)\tilde{\cap}S_\Theta(\epsilon))}(u),
\end{aligned}$$

which implies that $T_{(H_\Gamma(\epsilon)\tilde{\cup}G_\Delta(\epsilon))\tilde{\cap}S_\Theta(\epsilon)}(u) = T_{(H_\Gamma(\epsilon)\tilde{\cap}S_\Theta(\epsilon))\tilde{\cup}(G_\Delta(\epsilon)\tilde{\cap}S_\Theta(\epsilon))}(u)$.

The proofs for the identity and falsity terms follow similarly. Therefore,

$$((H, A)_\Gamma \tilde{\cup} (G, B)_\Delta)\tilde{\cap}(S, W)_\Theta = ((H, A)_\Gamma \tilde{\cap} (S, W)_\Theta)\tilde{\cup}((G, B)_\Delta\tilde{\cap}(S, W)_\Theta).$$

(2) The proof is similar to that in Part (1) and therefore is omitted. □

4. MAPPING ON FP-CNSESs

In this section, we introduce the notion of a mapping on fuzzy parameterized complex neutrosophic soft expert classes. fuzzy parameterized complex neutrosophic soft expert classes are collections of FP-CNSESs. We define the fuzzy parameterized complex neutrosophic soft expert images and fuzzy parameterized complex neutrosophic soft expert inverse images of FP-CNSESs. We give some operations and properties related with this concept.

Definition 13. *Let U be a universe set, E be a set of parameters, $FZ(E)$ denote all fuzzy subsets of E, where $\Gamma \subset FZ(E)$, X be a set of experts, and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Y = \Gamma \times X \times O$. Then, the collection of all FP-CNSESs over U with parameters from Y is called a fuzzy parameterized complex neutrosophic soft expert class and is denoted as $(\widetilde{U, Y})$.*

Definition 14. *Let $(\widetilde{U, Y})$ and $(\widetilde{V, Y'})$ be fuzzy parameterized complex neutrosophic soft expert classes. Let $r : U \to V$ and $s : Y \to Y'$ be mappings. Then, a mapping $F : (\widetilde{U, Y}) \to (\widetilde{V, Y'})$ is defined as follows:*

For a FP-CNSES $(H, A)_\Gamma$ in $(\widetilde{U, Y})$, $(F(H, A)_\Gamma, M)$, where $M = s(Y) \subseteq Y'$ is a FP-CNSES in $(\widetilde{V, Y'})$ obtained as follows:

$$F(H, A)_\Gamma(\beta)(v) = \begin{cases} \bigcup_{u\in r^{-1}(v)} \Big(\bigcup_{\alpha\in s^{-1}(\beta)\cap A} H_\Gamma(\alpha)\Big)(u) & \text{if } r^{-1}(v) \text{ and } s^{-1}(\beta) \cap A \neq \emptyset,\\ (0, 1, 1) & \text{otherwise.}\end{cases}$$

For $u \in r^{-1}(v)$, $\beta \in M \subseteq Y'$, $v \in V$ and $\forall \alpha \in s^{-1}(\beta) \cap A$, $(F(H, A)_\Gamma, M)$ is called a fuzzy parameterized complex neutrosophic soft expert image of the FP-CNSES $(H, A)_\Gamma$. If $M = Y'$, then we shall write $(F(H, A)_\Gamma, M)$ as $F(H, A)_\Gamma$.

Definition 15. *Let $(\widetilde{U, Y})$ and $(\widetilde{V, Y'})$ be two fuzzy parameterized complex neutrosophic soft expert classes. Let $r : U \to V$ and $s : Y \to Y'$ be mappings. Then, a mapping $F^{-1} : (\widetilde{V, Y'}) \to (\widetilde{U, Y})$ is defined as follows:*

For a FP-CNSES $(G, B)_\Delta$ in $(\widetilde{V, Y'})$, $(F^{-1}(G, B)_\Delta, N)$, where $N = s^{-1}(B)$, is a FP-CNSES in $(\widetilde{U, Y})$ obtained as follows:

$$F^{-1}(G, B)_\Delta(\alpha)(u) = \begin{cases} G_\Delta(s(\alpha))(r(u)) & \text{if } s(\alpha) \in B,\\ (0, 1, 1) & \text{otherwise,}\end{cases}$$

For $\alpha \in N \subseteq Y$ and $u \in U$. $(F^{-1}(G, B)_\Delta, N)$ is called a fuzzy parameterized complex neutrosophic soft expert inverse image of the FP-CNSES $(G, B)_\Delta$. If $N = Y$, we write $(F^{-1}(G, B)_\Delta, N)$ as $F^{-1}(G, B)_\Delta$.

Definition 16. *Let $F : (\widetilde{U,Y}) \to (\widetilde{V,Y'})$ be a mapping and $(H,A)_\Gamma$, $(H',A')_\Delta$ be FP-CNSESs in $(\widetilde{U,Y})$. Then, for $\beta \in Y'$, $\nu \in V$, the union and intersection of the fuzzy parameterized complex neutrosophic soft expert images $F(H,A)_\Gamma$ and $F(G,B)_\Delta$ are defined as follows.*

$$\left(F\left(H,A\right)_\Gamma \tilde{\cup} F\left(H',A'\right)_\Delta\right)\left(\beta\right)\left(\nu\right) = F\left(H,A\right)_\Gamma\left(\beta\right)\left(\nu\right) \cup F\left(H',A'\right)_\Delta\left(\beta\right)\left(\nu\right),$$

$$\left(F\left(H,A\right)_\Gamma \tilde{\cap} F\left(H',A'\right)_\Delta\right)\left(\beta\right)\left(\nu\right) = F\left(H,A\right)_\Gamma\left(\beta\right)\left(\nu\right) \cap F\left(H',A'\right)_\Delta\left(\beta\right)\left(\nu\right),$$

where $\tilde{\cup}$ and $\tilde{\cap}$ denote fuzzy parameterized complex neutrosophic soft expert union and intersection of fuzzy parameterized complex neutrosophic soft expert images in $(\widetilde{V,Y'})$.

Definition 17. *Let $F : (\widetilde{U,Y}) \to (\widetilde{V,Y'})$ be a mapping and $(G,B)_\Sigma$, $(G',B')_\Omega$ FP-CNSESs in $(\widetilde{V,Y'})$. Then, for $\alpha \in Y$, $u \in U$, the union and intersection of the fuzzy parameterized complex neutrosophic soft expert inverse images $F^{-1}(G,B)_\Sigma$ and $F^{-1}(G',B')_\Omega$ are defined as follows.*

$$\left(F^{-1}\left(G,B\right)_\Sigma \tilde{\cup} F^{-1}\left(G',B'\right)_\Omega\right)\left(\alpha\right)\left(u\right) = F^{-1}\left(G,B\right)_\Sigma\left(\alpha\right)\left(u\right) \cup F^{-1}\left(G',B'\right)_\Omega\left(\alpha\right)\left(u\right),$$

$$(F^{-1}(G,B)_\Sigma \tilde{\cap} F^{-1}(G',B')_\Omega)(\alpha)(u) = F^{-1}(G,B)_\Sigma(\alpha)(u) \cap F^{-1}(G',B')_\Omega(\alpha)(u),$$

where $\tilde{\cup}$ and $\tilde{\cap}$ denote fuzzy parameterized complex neutrosophic soft expert union and intersection of fuzzy parameterized complex neutrosophic soft expert inverse images in $(\widetilde{U,Y})$.

Proposition 3. *Let $F : (\widetilde{U,Y}) \to (\widetilde{V,Y'})$ be a mapping. Then, for FP-CNSESs $(H,A)_\Gamma$ and $(H',A')_\Delta$ in the fuzzy parameterized complex neutrosophic soft expert class $(\widetilde{U,Y})$, we have:*

1. $F(\tilde{\phi}) = \tilde{\phi}$.
2. $F(\tilde{\psi}) = \tilde{\psi}$.
3. $F\left((H,A)_\Gamma \tilde{\cup} (H',A')_\Delta\right) = F\left(H,A\right)_\Gamma \tilde{\cup} F\left(H',A'\right)_\Delta$.
4. $F\left((H,A)_\Gamma \tilde{\cap} (H',A')_\Delta\right) \subseteq F\left(H,A\right)_\Gamma \tilde{\cap} F\left(H',A'\right)_\Delta$.
5. *If* $(H,A)_\Gamma \subseteq (H',A')_\Delta$, *then* $F\left(H,A\right)_\Gamma \subseteq F\left(H',A'\right)_\Delta$.

Proof. The proof is straightforward by Definitions 14 and 16. □

Proposition 4. *Let $F : (\widetilde{U,Y}) \to (\widetilde{V,Y'})$ be a mapping. Then, for FP-CNSESs $(G,B)_\Sigma$ and $(G',B')_\Omega$ in the fuzzy parameterized complex neutrosophic soft expert class $(\widetilde{V,Y'})$, we have:*

1. $F^{-1}(\tilde{\phi}) = \tilde{\phi}$.
2. $F^{-1}(\tilde{\psi}) = \tilde{\psi}$.
3. $F^{-1}\left((G,B)_\Sigma \tilde{\cup} (G',B')_\Omega\right) = F^{-1}\left(G,B\right)_\Sigma \tilde{\cup} F^{-1}\left(G',B'\right)_\Omega$.
4. $F^{-1}\left((G,B)_\Sigma \tilde{\cap} (G',B')_\Omega\right) = F^{-1}\left(G,B\right)_\Sigma \tilde{\cap} F^{-1}\left(G',B'\right)_\Omega$.
5. *If* $(G,B)_\Sigma \subseteq (G',B')_\Omega$, *then* $F^{-1}\left(G,B\right)_\Sigma \subseteq F^{-1}\left(G',B'\right)_\Omega$.

Proof. The proof is straightforward by Definitions 15 and 17. □

5. An Application of Fuzzy Parameterized Complex Neutrosophic Soft Expert Set

In this section, we present an application of FP-CNSES in a decision making problem by considering the following example.

Example 4. *Suppose that an engineering company wishes to evaluate two kinds of a certain product from a manufacturer and choose the most suitable one. Suppose that the company takes the opinion of its experts concerning these two kinds of product on two phases: once before using the products and again after trying a sample of each of the two kinds of the product. Suppose that $U = \{u_1, u_2\}$ is the universe consisting of the two alternatives (the two kinds of the product) and $E = \{e_1, e_2, e_3\}$ is the attributes set, where e_1 stands for "easy to use", e_2 stands for " functional " and e_3 stands for "durable". The attributes e_1, e_2 and e_3 are important* *with degrees 0.3, 0.6 and 0.8, respectively. That is, the fuzzy subset of parameters is* $\Gamma = \left\{ \frac{e_1}{0.3}, \frac{e_2}{0.6}, \frac{e_3}{0.8} \right\}$. *Let* $X = \{x_1, x_2\}$ *be a set of experts. Now, the team of experts are requested to make a decision about the most desirable alternative based on the goals and the constraints according to a chosen subset* Γ *of* $FZ(E)$ *to construct a FP-CNSES.*

$$
\begin{aligned}
&(H, A)_\Gamma = \\
&\left\{ \left\{ \left(\tfrac{e_1}{0.3}, x_1, 1 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.2e^{2\pi(0.4)}, 0.1e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.7)}, 0.5e^{2\pi(0.4)}, 0.1e^{2\pi(1)} \rangle} \right\} \right\}, \right. \\
&\left\{ \left(\tfrac{e_1}{0.3}, x_2, 1 \right), \left\{ \frac{u_1}{\langle 0.6e^{2\pi(0.5)}, 0.6e^{2\pi(0.4)}, 0.1e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.9e^{2\pi(0.8)}, 0.5e^{2\pi(0.6)}, 0.5e^{2\pi(0.4)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_2}{0.6}, x_1, 1 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.3)}, 0.7e^{2\pi(0.8)}, 0.8e^{2\pi(0.7)} \rangle}, \frac{u_2}{\langle 0.4e^{2\pi(0.9)}, 0.2e^{2\pi(0.5)}, 0.1e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_2}{0.6}, x_2, 1 \right), \left\{ \frac{u_1}{\langle 0.8e^{2\pi(0.9)}, 0.4e^{2\pi(0.3)}, 0.7e^{2\pi(0.4)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.4)}, 0.8e^{2\pi(0.6)}, 0.6e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_3}{0.8}, x_1, 1 \right), \left\{ \frac{u_1}{\langle 0.2e^{2\pi(0.2)}, 0.3e^{2\pi(0.3)}, 0.7e^{2\pi(0.6)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_3}{0.8}, x_2, 1 \right), \left\{ \frac{u_1}{\langle 0.8e^{2\pi(0.1)}, 0.3e^{2\pi(0.2)}, 0.7e^{9\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.6)}, 0.2e^{2\pi(0.4)}, 0.6e^{2\pi(0.8)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_1}{0.3}, x_1, 0 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.5)}, 0.5e^{2\pi(0.4)}, 0.9e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.9e^{2\pi(0.8)}, 0.7e^{2\pi(0.8)}, 0.5e^{2\pi(0.7)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_1}{0.3}, x_2, 0 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.6)}, 0.7e^{2\pi(0.4)}, 0.7e^{2\pi(0.8)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.4)}, 0.5e^{2\pi(0.4)}, 0.3e^{2\pi(0.5)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_2}{0.6}, x_1, 0 \right), \left\{ \frac{u_1}{\langle 0.3e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.7e^{2\pi(0.4)}, 0.5e^{2\pi(0.8)}, 0.6e^{2\pi(0.8)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_2}{0.6}, x_2, 0 \right), \left\{ \frac{u_1}{\langle 0.3e^{2\pi(0.5)}, 0.3e^{2\pi(0.1)}, 0.9e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.7)}, 0.8e^{2\pi(0.7)}, 0.7e^{2\pi(0.6)} \rangle} \right\} \right\}, \\
&\left\{ \left(\tfrac{e_3}{0.8}, x_1, 0 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.4e^{2\pi(0.5)}, 0.4e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.8e^{2\pi(0.2)}, 0.3e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left. \left\{ \left(\tfrac{e_3}{0.8}, x_2, 0 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.6)}, 0.2e^{2\pi(0.3)}, 0.1e^{2\pi(0.6)} \rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0)}, 0.6e^{2\pi(0.4)}, 0.3e^{2\pi(0.1)} \rangle} \right\} \right\} \right\}.
\end{aligned}
$$

In the FP-CNSES $(H, A)_\Gamma$ above, the amplitude terms of membership, non-membership and indeterminate membership represent the opinions of the experts in phase one (before using the products), while the phase terms of membership, non-membership, and indeterminacy represent the opinions of the experts in the second phase (after trying a sample of each of the two kinds of the product). Thus, the amplitude term of the membership in the first phase and the phase term of the membership in the second phase form a complex-valued truth membership function of the FP-CNSES $(H, A)_\Gamma$. Similarly, the amplitude term of non-membership in phase one and the phase term of non-membership in the second phase form a complex-valued falsity membership function. In addition, the amplitude term of undecidedness in the first phase and the phase term of indeterminacy in the second phase form the complex-valued indeterminate membership function. Now, our problem is to select the most desirable kind of the product to the engineering company.

To solve this decision making problem, we use the FP-CNSES $(H, A)_\Gamma$ together with a generalized algorithm. This algorithm converts the fuzzy parameterized complex neutrosophic soft expert values (FP-CNSEVs) to fuzzy parameterized single-valued neutrosophic soft expert values (FP-SVNSEVs) using a practical formula which give a decision-making with a simple computational process without the need to carry out directed operations on complex numbers. In this formula, we give a weight to the amplitude terms (a weight to the experts' opinions before using the products) by multiplying the weight vector to each amplitude term. Similarly we give a weight to the phase terms (a weight to the experts' opinions after using the products) by multiplying the weight vector to each phase term. Then, we combine the values of the weighted amplitude terms and phase terms to obtain the FP-SVNSEVs which represent the experts' opinions on both phases together. Thus, after doing these simple calculations to all membership functions of the FP-CNSES $(H, A)_\Gamma$, we proceed to the final decision using the fuzzy parameterized single valued neutrosophic soft expert method (FPSVNSEM) [18].

The generalized algorithm is as follows.

Algorithm 1: Fuzzy parameterized complex neutrosophic soft expert method (FP-CNSEM).

1. Input the FP-CNSES $(H, A)_\Gamma$
2. Convert the FP-CNSES $(H, A)_\Gamma$ to the FPSVNSES $(\widehat{H}, A)_\Gamma$ by obtaining the weighted aggregation values of $T_{\widehat{H}_\Gamma(a_i)}(u_j)$, $I_{\widehat{H}_\Gamma(a_i)}(u_j)$ and $F_{\widehat{H}_\Gamma(a_i)}(u_j)$,$\forall a_i \in A$ and $\forall u_j \in U$ as the following formulas:

$$T_{\widehat{H}_\Gamma(a_i)}(u_j) = w_1 p_{H_\Gamma(a_i)}(u_j) + w_2(1/2\pi)\mu_{H_\Gamma(a_i)}(u_j),$$

$$I_{\widehat{H}_\Gamma(a_i)}(u_j) = w_1 q_{H_\Gamma(a_i)}(u_j) + w_2(1/2\pi)\nu_{H_\Gamma(a_i)}(u_j),$$

$$F_{\widehat{H}_\Gamma(a_i)}(u_j) = w_1 r_{H_\Gamma(a_i)}(u_j) + w_2(1/2\pi)\omega_{H_\Gamma(a_i)}(u_j),$$

where $p_{H_\Gamma(a_i)}(u_j), q_{H_\Gamma(a_i)}(u_j), r_{H_\Gamma(a_i)}(u_j)$ and $\mu_{H_\Gamma(a_i)}(u_j), \nu_{H_\Gamma(a_i)}(u_j), \omega_{H_\Gamma(a_i)}(u_j)$ are the amplitude and phase terms in the FP-CNSES $(H, A)_\Gamma$, respectively. $T_{\widehat{H}_\Gamma(a_i)}(u_j)$, $I_{\widehat{H}_\Gamma(a_i)}(u_j)$ and $F_{\widehat{H}_\Gamma(a_i)}(u_j)$ are the truth-membership function, indeterminacy-membership function and falsity-membership function in the FP-SVNSES $(\widehat{H}, A)_\Gamma$, respectively, and w_1, w_2 are the weights for the amplitude terms (the first decision process) and the phase terms (the second decision process), respectively, where w_1 and $w_2 \in [0, 1]$ and $w_1 + w_2 = 1$.
3. Find the values of $c_{ij} = T_{\widehat{H}_\Gamma(a_i)}(u_j) - I_{\widehat{H}_\Gamma(a_i)}(u_j) - F_{\widehat{H}_\Gamma(a_i)}(u_j), \forall u_j \in U$ and $\forall a_i \in A$.
4. Compute the score of each element $u_j \in U$ by the following formulas :

$$K_j = \sum_{x\in X}\sum_{i=1}^{n} c_{ij}(\mu_\Gamma(e_i)),\ S_j = \sum_{x\in X}\sum_{i=1}^{n} c_{ij}(\mu_\Gamma(e_i))$$

for the agree FP-SVNSES and disagree FP-SVNSES, where $\mu_\Gamma(e_i)$ is the corresponding membership function of the fuzzy set Γ, X is the set of the experts and n is the number of the parameters (attributes).
5. Find the values of the score $r_j = K_j - S_j$ for each element $u_j \in U$.
6. Determine the value of the highest score $m = max_{u_j \in U}\{r_j\}$. Then, the decision is to choose element u_j as the optimal solution to the problem. If there are more than one elements with the highest r_j score, then any one of those elements can be chosen as the optimal solution.

Then, we can conclude that the optimal choice for the team of the experts is to select the kind u_j as the most desirable kind of the product to the company.

Now, to convert the FP-CNSES $(H, A)_\Gamma$ to the FP-SVNSES $(\widehat{H}, A)_\Gamma$, suppose that the weight vector for the amplitude terms is $w_1 = 0.3$ and the weight vector for the phase terms is $w_2 = 0.7$ and obtain the weighted aggregation values of $T_{\widehat{H}_\Gamma(a_i)}(u_j)$, $I_{\widehat{H}_\Gamma(a_i)}(u_j)$ and $F_{\widehat{H}_\Gamma(a_i)}(u_j)$,$\forall a_i \in A$ and $\forall u_j \in U$. To illustrate this step, we calculate $T_{\widehat{H}_\Gamma(a_1)}(u_1)$, $I_{\widehat{H}_\Gamma(a_1)}(u_1)$ and $F_{\widehat{H}_\Gamma(a_1)}(u_1)$, such that $a_1 = (\frac{e_1}{0.3}, x_1, 1)$ as shown below:

$$\begin{aligned} T_{\widehat{H}_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) &= w_1 p_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) + w_2(1/2\pi)\mu_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) \\ &= 0.3(0.5) + 0.7(1/2\pi)(2\pi)(0.4) \\ &= 0.43 \end{aligned}$$

$$\begin{aligned} I_{\widehat{H}_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) &= w_1 q_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) + w_2(1/2\pi)\nu_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) \\ &= 0.3(0.2) + 0.7(1/2\pi)(2\pi)(0.4) \\ &= 0.34 \end{aligned}$$

$$F_{\widehat{H}_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) = w_1 r_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1) + w_2(1/2\pi)\omega_{H_\Gamma(\frac{e_1}{0.3},x_1,1)}(u_1)$$
$$= 0.3(0.1) + 0.7(1/2\pi)(2\pi)(0.2)$$
$$= 0.17.$$

Then, the FP-SVNSEV

$$(T_{\widehat{H}_\Gamma(a_1)}(u_1), I_{\widehat{H}_\Gamma(a_1)}(u_1), F_{\widehat{H}_\Gamma(a_1)}(u_1)) = (0.43, 0.34, 0.17).$$

In the same manner, we calculate the others FP-SVNSEVs, $\forall a \in A$ and $\forall u \in U$ as in Table 1.

Table 1. Values of $(\widehat{H}, A)_\Gamma$.

	u_1	u_2
$\left(\frac{e_1}{0.3}, x_1, 1\right)$	$\langle 0.43, 0.34, 0.17\rangle$	$\langle 0.67, 0.43, 0.73\rangle$
$\left(\frac{e_1}{0.3}, x_2, 1\right)$	$\langle 0.53, 0.46, 0.24\rangle$	$\langle 0.83, 0.57, 0.53\rangle$
$\left(\frac{e_2}{0.6}, x_1, 1\right)$	$\langle 0.33, 0.77, 0.73\rangle$	$\langle 0.75, 0.41, 0.71\rangle$
$\left(\frac{e_2}{0.6}, x_2, 1\right)$	$\langle 0.87, 0.33, 0.49\rangle$	$\langle 0.43, 0.66, 0.32\rangle$
$\left(\frac{e_3}{0.8}, x_1, 1\right)$	$\langle 0.20, 0.30, 0.63\rangle$	$\langle 0.29, 0.35, 0.38\rangle$
$\left(\frac{e_3}{0.8}, x_2, 1\right)$	$\langle 0.31, 0.23, 0.35\rangle$	$\langle 0.45, 0.34, 0.74\rangle$
$\left(\frac{e_1}{0.3}, x_1, 0\right)$	$\langle 0.50, 0.43, 0.48\rangle$	$\langle 0.83, 0.77, 0.64\rangle$
$\left(\frac{e_1}{0.3}, x_2, 0\right)$	$\langle 0.54, 0.49, 0.77\rangle$	$\langle 0.43, 0.43, 0.44\rangle$
$\left(\frac{e_2}{0.6}, x_1, 0\right)$	$\langle 0.23, 0.32, 0.41\rangle$	$\langle 0.49, 0.71, 0.74\rangle$
$\left(\frac{e_2}{0.6}, x_2, 0\right)$	$\langle 0.44, 0.16, 0.48\rangle$	$\langle 0.58, 0.73, 0.63\rangle$
$\left(\frac{e_3}{0.8}, x_1, 0\right)$	$\langle 0.43, 0.47, 0.33\rangle$	$\langle 0.38, 0.23, 0.29\rangle$
$\left(\frac{e_3}{0.8}, x_2, 0\right)$	$\langle 0.54, 0.27, 0.45\rangle$	$\langle 0.09, 0.46, 0.16\rangle$

Tables 2 and 3 give the values of $c_{ij} = T_{\widehat{H}_\Gamma(a_i)}(u_j) - I_{\widehat{H}_\Gamma(a_i)}(u_j) - F_{\widehat{H}_\Gamma(a_i)}(u_j)$ and the score of each element $u_j \in U$ for agree FP-SVNSES and disagree FP-SVNSES, respectively.

Table 2. Numerical degree of agree FP-SVNSES.

U	u_1	u_2
$\left(\frac{e_1}{0.3}, x_1\right)$	−0.08	−0.49
$\left(\frac{e_1}{0.3}, x_2\right)$	−0.17	−0.27
$\left(\frac{e_2}{0.6}, x_1\right)$	−1.17	−0.37
$\left(\frac{e_2}{0.6}, x_2\right)$	0.05	−0.55
$\left(\frac{e_3}{0.8}, x_1\right)$	−0.73	−0.44
$\left(\frac{e_3}{0.8}, x_2\right)$	−0.27	−0.63
$K_j = \sum_{x \in X} \sum_{i=1}^{3} c_{ij}(\mu_\Gamma(e_i))$	$K_1 = -1.547$	$K_2 = -1.636$

Table 3. Numerical degree for disagree FP-SVNSES.

U	u_1	u_2
$\left(\frac{e_1}{0.3}, x_1\right)$	−0.41	−0.58
$\left(\frac{e_1}{0.3}, x_2\right)$	−0.72	−0.44
$\left(\frac{e_2}{0.6}, x_1\right)$	−0.50	−0.96
$\left(\frac{e_2}{0.6}, x_2\right)$	−0.20	−0.78
$\left(\frac{e_3}{0.8}, x_1\right)$	−0.37	−0.14
$\left(\frac{e_3}{0.8}, x_2\right)$	−0.18	−0.53
$S_j = \sum_{x \in X} \sum_{i=1}^{3} c_{ij}(\mu_\Gamma(e_i))$	$S_1 = -1.199$	$S_2 = -1.886$

Let K_j and S_j represent the score of each numerical degree for the agree FP-SVNSES and disagree FP-SVNSES, respectively. These values are given in Table 4.

Table 4. The score $r_j = K_j - S_j$.

$K_j = \sum_{x \in X} \sum_{i=1}^{3} c_{ij}(\mu_\Gamma(e_i))$	$S_j = \sum_{x \in X} \sum_{i=1}^{3} c_{ij}(\mu_\Gamma(e_i))$	$r_j = K_j - S_j$
$K_1 = -1.547$	$S_1 = -1.199$	-0.348
$K_2 = -1.636$	$S_2 = -1.886$	0.25

Thus, $m = max_{u_j \in U}\{r_j\} = r_2$. Therefore, the team of experts advise selecting the kind u_2 of this product as a desirable alternative.

6. Comparison between FP-CNSES and Other Existing Methods

In this section, we compare our proposed FP-CNSES model to FP-SVNSES [18] and FP-IFSES [17], which are generalizations of FP-SES [15] and FP-FSES [16].

To reveal the significance of our proposed FP-CNSES compared to FP-SVNSES and FP-IFSES, let us consider Example 4 above. In this example, we apply the FP-CNSES to handle a decision-making problem, which consists of two decision processes. Our proposed model is applied to both the decision processes by considering the opinions in process one as amplitude terms of truth membership, falsity membership and indeterminacy membership, and setting the second decision process as phase terms of truth membership, falsity membership, and indeterminacy. Thus, both decision processes form a FP-CNSES as whole.

On the other hand, when we apply the FP-IFSES to both processes, it tells us only about the truth membership and falsity membership in the first decision process, but cannot tell anything about the undecidedness. The situation is similar in the second decision process. Thus, FP-IFSES fails to handle this situation. Now, when we apply the FP-SVNSES, it tells about the truth membership, falsity membership and indeterminacy membership in the first round, and, similarly, it tells about the second round but it cannot describe both decision processes simultaneously.

Thus, both FP-SVNSES and FP-IFSES cannot directly solve such a decision-making problem with fuzzy parameterized complex neutrosophic soft expert information. In contrast, the FP-CNSES can directly address the fuzzy parameterized single valued neutrosophic soft expert problem, since the FP-SVNSES is a special case of FP-CNSES and can be easily represented in the form of FP-CNSES. In other words, the FP-SVNSES is a FP-CNSES with phase terms equal zeros. For example, the FP-SVNSEV (0.2, 0.4, 0.6) can be represented as $(0.2e^{j2\Pi(0)}, 0.4e^{j2\Pi(0)}, 0.6e^{j2\Pi(0)})$ by means of FP-CNSES. Furthermore, our method is applicable for fuzzy parameterized intuitionistic fuzzy soft expert problem, since FP-IFSES is a special case of FP-SVNSES and consequently of FP-CNSES. For example, the fuzzy parameterized intuitionistic fuzzy soft expert value (0.1, 0.6) can be (0.1, 0.3, 0.6) by means of FP-SVNSES and hence can be $(0.1e^{j2\Pi(0)}, 0.3e^{j2\Pi(0)}, 0.6e^{j2\Pi(0)})$ by means of FP-CNSES, since the sum of the degrees of membership, nonmembership and indeterminacy of an intuitionistic fuzzy value equal to 1. Note that the indeterminacy degree in intuitionistic fuzzy set is provided by default and cannot be defined alone unlike the SVNS where the indeterminacy is defined independently and quantified explicitly.

The advantage of using FP-CNSES manifests in representing information of two dimensions for one object in the same time (i.e., the expert's time can be saved by constructing/building one set (FP-CNSES) instead of two sets (FP-SVNSESs), not to mention reducing the amount of both mathematical calculations and investigation of several operators in incorporating two FP-SVNSESs to find the optimal solution). A practical formula is employed to convert the FP-CNSES to the FP-SVNSES, which gives a decision-making with a simple computational process without the need to carry out directed operations on complex numbers.

7. Weighted Fuzzy Parameterized Complex Neutrosophic Soft Expert Set

Experts opinions are vital for any decision making process as most real-life situations deal with elements and parameters that are subjective, biased and have the potential to be distorted and interpreted differently by different parties. Each expert might have his/her own thought, which differs from others in various aspects but all of the experts should have a common goal to reach the final destination. Moreover, as the domains of expertise of different experts are different, one expert may be more confident on his/her opinion than the other on the same set of attributes. For this type of environment, equal weights assignment to different experts may lead to improper and biased solution. Feeling the need of prioritizing different experts motivates us to develop a new idea for assigning relative weight to each expert by establishing a novel notion called weighted fuzzy parameterized complex neutrosophic soft expert set (WFP-CNSES). The relative weight is assigned to each of the experts where the choice of the experts may not be of equal importance.

This new mathematical framework reduces the chance of unfairness in the decision making process and brings more credibility to the final decision.

We begin this section by first proposing the concept of the weighted fuzzy parameterized single-valued neutrosophic soft expert set (WFP-SVNSES).

Definition 18. *Let U be a universe set, E be a set of parameters, I^E denote all fuzzy subsets of E, X be a set of experts, I^X denotes all fuzzy subsets of X and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Z = \Psi \times W \times O$ and $A \subseteq Z$ where $\Psi \subset I^E$ and $W \subset I^X$. A pair $(f, A,)_{\Psi,W}$ is called a weighted fuzzy parameterized single-valued neutrosophic soft expert set over U, where f is a mapping given by*

$$f_{\Psi,W} : A \to SVN(U),$$

and $SVN(U)$ denotes the set of all single-valued neutrosophic subsets of U.

We then generalize the WFP-SVNSES by establishing the concept of the weighted fuzzy parameterized complex neutrosophic soft expert set (WFP-CNSES), which is actually an extended version of WFP-SVNSES, on the complex space.

Definition 19. *Let U be a universe set, E be a set of parameters, $FZ(E)$ denote all fuzzy subsets of E, X be a set of experts, $FZ(X)$ denotes all fuzzy subsets of X and $O = \{1 = agree,\ 0 = disagree\}$ a set of opinions. Let $Y = \Gamma \times W \times O$ and $A \subseteq Y$ where $\Gamma \subset FZ(E)$ and $W \subset FZ(X)$. A pair $(H, A,)_{\Gamma,W}$ is called a weighted fuzzy parameterized complex neutrosophic soft expert set $(WFP - CNSES)$ over U, where H is a mapping given by*

$$H_{\Gamma,W} : A \to CN^U,$$

and CN^U denotes the power complex neutrosophic set of U.

The WFP-CNSES $(H, A)_{\Gamma,W}$ can be written as the following set of ordered pairs: $(H, A)_{\Gamma,W} = \Big\{\big(a, \{\frac{u}{H_{\Gamma,W}(a)(u)} : u \in U\}\big) : a \in A\Big\}$, where $A \subseteq \Gamma \times W \times O =$ $\Big\{\big(\frac{e}{\mu_\Gamma(e)}, \frac{x}{\mu_W(x)}, o\big) : e \in E, x \in X \text{and } o \in O\Big\}$, such that $\mu_\Gamma(e)$ and $\mu_W(x)$ are the corresponding membership functions of the fuzzy sets Γ and W, respectively.

From Definition 19, it is clear that the expert set in the WFP-CNSES $(H, A)_{\Gamma,W}$ is a fuzzy set, where the membership function $\mu_W(x)$ of the fuzzy set W represents weight of the expert x, and $\mu_W(x) \in [0, 1], \forall x \in X$.

For more illustration, we consider the following example.

Example 5. *Consider Example 4. Suppose that the weights for the experts x_1 and x_2 are 0.5 and 0.8, respectively. Then, the fuzzy subset of experts is $W = \left\{ \frac{x_1}{0.5}, \frac{x_2}{0.8} \right\}$ and the FP-CNSES $(H, A)_\Gamma$ in Example 4 is converted to the WFP-CNSES $(H, A)_{\Gamma,W}$ where,*

$$
\begin{aligned}
&(H, A)_{\Gamma,W} = \\
&\Bigg\{ \left\{ \left(\frac{e_1}{0.3}, \frac{x_1}{0.5}, 1 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.2e^{2\pi(0.4)}, 0.1e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.6e^{2\pi(0.7)}, 0.5e^{2\pi(0.4)}, 0.1e^{2\pi(1)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_1}{0.3}, \frac{x_2}{0.8}, 1 \right), \left\{ \frac{u_1}{\langle 0.6e^{2\pi(0.5)}, 0.6e^{2\pi(0.4)}, 0.1e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.9e^{2\pi(0.8)}, 0.5e^{2\pi(0.6)}, 0.5e^{2\pi(0.4)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_2}{0.6}, \frac{x_1}{0.5}, 1 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.3)}, 0.7e^{2\pi(0.8)}, 0.8e^{2\pi(0.7)} \rangle}, \frac{u_2}{\langle 0.4e^{2\pi(0.9)}, 0.2e^{2\pi(0.5)}, 0.1e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_2}{0.6}, \frac{x_2}{0.8}, 1 \right), \left\{ \frac{u_1}{\langle 0.8e^{2\pi(0.9)}, 0.4e^{2\pi(0.3)}, 0.7e^{2\pi(0.4)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.4)}, 0.8e^{2\pi(0.6)}, 0.6e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_3}{0.8}, \frac{x_1}{0.5}, 1 \right), \left\{ \frac{u_1}{\langle 0.2e^{2\pi(0.2)}, 0.3e^{2\pi(0.3)}, 0.7e^{2\pi(0.6)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_3}{0.8}, \frac{x_2}{0.8}, 1 \right), \left\{ \frac{u_1}{\langle 0.8e^{2\pi(0.1)}, 0.3e^{2\pi(0.2)}, 0.7e^{9\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.1e^{2\pi(0.6)}, 0.2e^{2\pi(0.4)}, 0.6e^{2\pi(0.8)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_1}{0.3}, \frac{x_1}{0.5}, 0 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.5)}, 0.5e^{2\pi(0.4)}, 0.9e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.9e^{2\pi(0.8)}, 0.7e^{2\pi(0.8)}, 0.5e^{2\pi(0.7)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_1}{0.3}, \frac{x_2}{0.8}, 0 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.6)}, 0.7e^{2\pi(0.4)}, 0.7e^{2\pi(0.8)} \rangle}, \frac{u_2}{\langle 0.5e^{2\pi(0.4)}, 0.5e^{2\pi(0.4)}, 0.3e^{2\pi(0.5)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_2}{0.6}, \frac{x_1}{0.5}, 0 \right), \left\{ \frac{u_1}{\langle 0.3e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.2)} \rangle}, \frac{u_2}{\langle 0.7e^{2\pi(0.4)}, 0.5e^{2\pi(0.8)}, 0.6e^{2\pi(0.8)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_2}{0.6}, \frac{x_2}{0.8}, 0 \right), \left\{ \frac{u_1}{\langle 0.3e^{2\pi(0.5)}, 0.3e^{2\pi(0.1)}, 0.9e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0.7)}, 0.8e^{2\pi(0.7)}, 0.7e^{2\pi(0.6)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_3}{0.8}, \frac{x_1}{0.5}, 0 \right), \left\{ \frac{u_1}{\langle 0.5e^{2\pi(0.4)}, 0.4e^{2\pi(0.5)}, 0.4e^{2\pi(0.3)} \rangle}, \frac{u_2}{\langle 0.8e^{2\pi(0.2)}, 0.3e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)} \rangle} \right\} \right\}, \\
&\left\{ \left(\frac{e_3}{0.8}, \frac{x_2}{0.8}, 0 \right), \left\{ \frac{u_1}{\langle 0.4e^{2\pi(0.6)}, 0.2e^{2\pi(0.3)}, 0.1e^{2\pi(0.6)} \rangle}, \frac{u_2}{\langle 0.3e^{2\pi(0)}, 0.6e^{2\pi(0.4)}, 0.3e^{2\pi(0.1)} \rangle} \right\} \right\} \Bigg\}.
\end{aligned}
$$

Suppose we are interested in solving the decision making problem in Example 4, where the data are represented by the means of the WFP-CNSES $(H, A)_{\Gamma,W}$ above, where each expert has his/her own weight. Then, we may use the following algorithm, which is a generalization of Algorithm 1.

Now, we use Algorithm 2 to select the most desirable kind of the product to the engineering company.

Algorithm 2: Weighted fuzzy parameterized complex neutrosophic soft expert method (WFP-CNSEM).

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1. Input the WFP-CNSES $(H, A)_{\Gamma,W}$.
2. Convert the WFP-CNSES $(H, A)_{\Gamma,W}$ to the WFP-SVNSES $(\widehat{H}, A)_{\Gamma,W}$ as it was illustrated in step 2 of Algorithm 1. Note that the WFP-CNSES $(H, A)_{\Gamma,W}$ and the FP-CNSES $(H, A)_\Gamma$ has the same evaluation information and the difference between them lies in the structure of the expert set which does not affect the conversion process.
3. Find the values of c_{ij} for agree WFP-SVNSES and disagree WFP-SVNSES respectively, where $c_{ij} = T_{\widehat{H}_{\Gamma,W}(a_i)}(u_j) - I_{\widehat{H}_{\Gamma,W}(a_i)}(u_j) - F_{\widehat{H}_{\Gamma,W}(a_i)}(u_j), \forall u_j \in U$ and $\forall a_i \in A$.
4. Compute the score of each element $u_j \in U$ by the following formulas:
   $K_j = \sum_{x \in X} \sum_i c_{ij}(\mu_\Gamma(e))(\mu_W(x))$, $S_j = \sum_{x \in X} \sum_i c_{ij}(\mu_\Gamma(e))(\mu_W(x))$, for the agree WFP-SVNSES and disagree WFP-SVNSES, where $\mu_\Gamma(e)$ and $\mu_W(x)$ are the corresponding membership functions of the fuzzy sets Γ and W, respectively.
5. Find the values of the score $r_j = K_j - S_j$ for each element $u_j \in U$.
6. Determine the value of the highest score $m = \max_{u_j \in U} \{r_j\}$. Then, the decision is to choose element u_j as the optimal solution to the problem.

Tables 5 and 6 give the numerical degree of agree WFP-SVNSES and disagree WFP-SVNSES, respectively.

Table 5. Numerical degree of agree WFP-SVNSES.

U	u_1	u_2
$\left(\frac{e_1}{0.3}, \frac{x_1}{0.5}\right)$	-0.08	-0.49
$\left(\frac{e_1}{0.3}, \frac{x_2}{0.8}\right)$	-0.17	-0.27
$\left(\frac{e_2}{0.6}, \frac{x_1}{0.5}\right)$	-1.17	-0.37
$\left(\frac{e_2}{0.6}, \frac{x_2}{0.8}\right)$	0.05	-0.55
$\left(\frac{e_3}{0.8}, \frac{x_1}{0.5}\right)$	-0.73	-0.44
$\left(\frac{e_3}{0.8}, \frac{x_2}{0.8}\right)$	-0.27	-0.63
$K_j = \sum_{x\in X}\sum_{i} c_{ij}(\mu_\Gamma(e))(\mu_W(x))$	$K_1 = -0.845$	$K_2 = -1.093$

Table 6. Numerical degree of disagree WFP-SVNSES.

U	u_1	u_2
$\left(\frac{e_1}{0.3}, \frac{x_1}{0.5}\right)$	-0.41	-0.58
$\left(\frac{e_1}{0.3}, \frac{x_2}{0.8}\right)$	-0.72	-0.44
$\left(\frac{e_2}{0.6}, \frac{x_1}{0.5}\right)$	-0.50	-0.96
$\left(\frac{e_2}{0.6}, \frac{x_2}{0.8}\right)$	-0.20	-0.78
$\left(\frac{e_3}{0.8}, \frac{x_1}{0.5}\right)$	-0.37	-0.14
$\left(\frac{e_3}{0.8}, \frac{x_2}{0.8}\right)$	-0.18	-0.53
$S_j = \sum_{x\in X}\sum_{i} c_{ij}(\mu_\Gamma(e))(\mu_W(x))$	$S_1 = -0.744$	$S_2 = -1.250$

Let K_j and S_j represent the score of each numerical degree of agree WFP-SVNSES and disagree WFP-SVNSES, respectively. These values are given in Table 7.

Table 7. The score $r_j = K_j - S_j$.

$K_j = \sum_{x\in X}\sum_{i} c_{ij}(\mu_\Gamma(e))(\mu_W(x))$	$S_j = \sum_{x\in X}\sum_{i} c_{ij}(\mu_\Gamma(e))(\mu_W(x))$	$r_j = K_j - S_j$
$K_1 = -0.845$	$S_1 = -0.744$	-0.101
$K_2 = -1.093$	$S_2 = -1.250$	0.157

Thus, $m = \max_{u_j \in U}\{r_j\} = r_2$. Therefore, the team of experts advise selecting the kind u_2 of this product as a desirable alternative.

Using Algorithms 1 and 2, we obtained the same results. However, it is clear that giving more consideration to the expert (weight) reduces the chance of unfairness in the decision making process and as a result adds more credibility to the final decision.

To illustrate the significance of the adjustable approach using WFP-CNSES as compared to that of FP-CNSES, let us consider Example 5 above.

It can be seen that WFP-CNSES is basically a FP-CNSES with weighted experts and every FP-CNSES can be considered as a WFP-CNSES with a relative weight equal to 1 assigned to all of the experts' opinions.

For example, the approximation

$$\left\{\left(\frac{e_1}{0.3},\frac{x_1}{0.5},1\right),\left\{\frac{u_1}{\langle 0.5e^{2\pi(0.4)},0.2e^{2\pi(0.4)},0.1e^{2\pi(0.2)}\rangle},\frac{u_2}{\langle 0.6e^{2\pi(0.7)},0.5e^{2\pi(0.4)},0.1e^{2\pi(1)}\rangle}\right\}\right\}$$

can be represented as

$$\left\{\left(\frac{e_1}{0.3},\frac{x_1}{1},1\right),\left\{\frac{u_1}{\langle 0.5e^{2\pi(0.4)},0.2e^{2\pi(0.4)},0.1e^{2\pi(0.2)}\rangle},\frac{u_2}{\langle 0.6e^{2\pi(0.7)},0.5e^{2\pi(0.4)},0.1e^{2\pi(1)}\rangle}\right\}\right\}$$

by means of FP-CNSES.

In other words, the expert set in the FP-CNSES is a classical set, where the opinions of the experts have the same degree of importance, which is considered as 1. This limitation may lead to improper and biased solution. In contrast, the expert set in the WFP-CNSES is a fuzzy set, i.e., a membership degree that represents an importance degree (a relative weight) between zero and one is assigned to each of the experts. This makes the decision process more realistic and reduces the biased information given by the experts.

8. Conclusions

We introduced the concept of FP-CNSES, which is a FP-SVNSES defined in a complex setting. We defined FP-CNSES operations and their properties. We then studied a mapping on fuzzy parameterized complex neutrosophic soft expert classes and its properties. An adjustable approach to decision making problems based on FP-CNSES was also introduced. A comparison of the FP-IFSES and FP-SVNSES to FP-CNSES was made and the preferability of FP-CNSES was revealed. Finally, we defined the notion of WFP-CNSES where experts' relative weights were considered and applied it to solve a decision making problem. This adjustable approach to decision making can be helpful to deal with more room of uncertainty compared to using FP-CNSES on the same problem. Both the newly proposed approaches efficiently capture the incomplete, indeterminate, and inconsistent information and extend existing decision-making methods to provide a more comprehensive outlook for decision-makers. WFP-CNSES seems to be a promising new concept, paving the way toward numerous possibilities for future research. We intend to investigate this concept further by studying its operations and their properties to develop some real applications.

Author Contributions: A.A.-Q. proposed the concept of the fuzzy parameterized complex neutrosophic soft expert set; N.H. provided a concrete example on the decision making problem; A.A.-Q. and S.A. proposed the FP-CNSES decision making method; and N.H. edited the manuscript. All authors wrote the manuscript.

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