symmetry

MDPI

Article

Generalized Neutrosophic Soft Expert Set for Multiple-Criteria Decision-Making

Vakkas Uluçay [1], Memet Şahin [1] and Nasruddin Hassan [2,*]

[1] Department of Mathematics, Gaziantep University, Gaziantep 27310, Turkey; vulucay27@gmail.com (V.U.); mesahin@gantep.edu.tr (M.Ş.)

[2] School of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, UKM Bangi, Selangor 43600, Malaysia

* Correspondence: nas@ukm.edu.my; Tel.: +60-19-214-5750

Received: 12 September 2018; Accepted: 22 September 2018; Published: 26 September 2018

Abstract: Smarandache defined a neutrosophic set to handle problems involving incompleteness, indeterminacy, and awareness of inconsistency knowledge, and have further developed it neutrosophic soft expert sets. In this paper, this concept is further expanded to generalized neutrosophic soft expert set (GNSES). We then define its basic operations of complement, union, intersection, AND, OR, and study some related properties, with supporting proofs. Subsequently, we define a GNSES-aggregation operator to construct an algorithm for a GNSES decision-making method, which allows for a more efficient decision process. Finally, we apply the algorithm to a decision-making problem, to illustrate the effectiveness and practicality of the proposed concept. A comparative analysis with existing methods is done and the result affirms the flexibility and precision of our proposed method.

Keywords: aggregation operator; complement; intersection; membership; neutrosophic soft set

1. Introduction

For a proper description of objects in an uncertain and ambiguous environment, indeterminate and incomplete information has to be properly handled. Intuitionistic fuzzy sets were introduced by Atanassov [1], followed by Molodtsov on soft sets [2] and neutrosophy logic [3] and neutrosophic sets [4] were introduced by Smarandache. The term neutro-sophy means knowledge of neutral thought and this neutral represents the main distinction between fuzzy and intuitionistic fuzzy logic and a set. At present, work on the soft set theory is progressing rapidly. Various operations and applications of soft sets have been developed rapidly, including the possibility of fuzzy soft set [5], soft multiset theory [6], multiparameterized soft set [7], soft intuitionistic fuzzy sets [8], Q-fuzzy soft sets [9–11], multi Q-fuzzy sets [12–14], N-soft set [15], Hesitant N-soft set [16], and Fuzzy N-soft set [17], thereby, opening avenues to genetic applications [18,19]. Later, Maji [20] have introduced a more generalized concept—which is a combination of neutrosophic sets and soft sets—and have studied its properties. Alhazaymeh and Hassan [21,22] have studied the concept of vague soft set, which were later extended to vague soft expert set theory [23,24], bipolar fuzzy soft expert set [25], and multi Q-fuzzy soft expert set [26]. Şahin et al. [27] introduced neutrosophic soft expert sets, while Al-Quran and Hassan [28,29] extended it further to neutrosophic vague soft expert set. Neutrosophic set theory has also been applied to multiple attribute decision-making [30–32]. Fuzzy modelling has long been widely applied to physical problems, which include intuitionistic hesitant fuzzy [33], t-concept lattices [34], fuzzy operators [35], medical image retrieval [36], and artificial bee colony [37] and multi criteria decision making [38,39]. Neutrosophic sets have also gained traction with recent publications on neutrosophic triplets [40,41], Q-neutrosophic soft relations [42], Q-neutrosophic soft sets [43], and Q-neutrosophic soft expert set [44].

This paper anticipates the neutrosophic set discussions to handle problems involving incompleteness, indeterminacy, and awareness of inconsistency of knowledge, which is further developed to neutrosophic soft expert sets. We intend to extend the discussion further, by proposing the concept of generalized neutrosophic soft expert set (GNSES) and its basic operations of complement, union, intersection, AND, and OR, along with a definition of GNSES-aggregation operator, to construct an algorithm of a GNSES decision method. Finally we provide an application of the constructed algorithm to solve a decision-making problem.

2. Preliminaries

In this section, we review the basic definitions of a neutrosophic set, neutrosophic soft set, soft expert sets, neutrosophic soft expert sets, and neutrosophic parametrized (NP)-aggregation operator, which are required as preliminaries.

Definition 1. *[4] Let U be a universe of discourse, with a generic element in U denoted by u, then a neutrosophic (NS) set A is an object having the form*

$$A = \{< u: T_A(u),\ I_A(u),\ F_A(u)>, u \in U\}$$

where the functions T, I, F: U→]⁻0, 1⁺[define, respectively, the degree of membership (or Truth) , the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $u \in U$ to the set A with the condition.

$$^-0 \le T_A(u) + I_A(u) + F_A(u) \le 3^+$$

Definition 2. *[20] Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $NS(U)$ denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be the neutrosophic soft set over U, where F is a mapping given by $F: A \to NS(U)$.*

Definition 3. *[23] U is an initial universe, E is a set of parameters, X is a set of experts (agents), and $O = \{agree = 1, disagree = 0\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$. A pair (F, A) is called a soft expert set over U, where F is a mapping given by $F: A \to P(U)$ where $P(U)$ denoted the power set of U.*

Definition 4. *[27] A pair (F, A) is called a neutrosophic soft expert set over U, where F is a mapping given by $F: A \to P(U)$ where $P(U)$ denotes the power neutrosophic set of U.*

Definition 5. *[27] The complement of a neutrosophic soft expert set (F, A) is denoted by $(F, A)^c$, and is defined as $(F, A)^c = (F^c, \neg A)$ where $F^c = \neg A \to P(U)$ is a mapping given by $F^c(x)$= neutrosophic soft expert complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$, $F_{F^c(x)} = T_{F(x)}$.*

Definition 6. *[27] The agree-neutrosophic soft expert set $(F, A)_1$ over U is a neutrosophic soft expert subset of (F, A) defined as*

$$(F, A)_1 = \{F_1(m): m \in E \times X \times \{1\}\}.$$

Definition 7. *[27] The disagree-neutrosophic soft expert set $(F, A)_0$ over U is a neutrosophic soft expert subset of (F, A), defined as*

$$(F, A)_0 = \{F_0(m): m \in E \times X \times \{0\}\}.$$

Definition 8. *[27] Let (H, A) and (G, B) be two neutrosophic soft expert sets (NSESs) over the common universe U. Then the union of (H, A) and (G, B) is denoted by "$(H, A) \tilde{\cup} (G, B)$", and is defined by $(H, A) \tilde{\cup} (G, B) = (K, C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership, and falsity-membership of (K, C) are as follows:*

$$T_{K(e)}(m) = \begin{cases} T_{H(e)}(m), & if\ e \in A - B \\ T_{G(e)}(m), & if\ e \in B - A \\ max\left(T_{H(e)}(m), T_{G(e)}(m)\right), & if\ e \in A \cap B \end{cases}$$

$$I_{K(e)}(m) = \begin{cases} I_{H(e)}(m), & if\ e \in A - B \\ I_{G(e)}(m), & if\ e \in B - A \\ \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}, & if\ e \in A \cap B \end{cases}$$

$$F_{K(e)}(m) = \begin{cases} F_{H(e)}(m), & if\ e \in A - B \\ F_{G(e)}(m), & if\ e \in B - A \\ min\left(F_{H(e)}(m), F_{G(e)}(m)\right), & if\ e \in A \cap B \end{cases}$$

Definition 9. *[27] Let (H, A) and (G, B) be two NSESs over the common universe U. Then the intersection of (H, A) and (G, B) is denoted by "$(H, A) \tilde{\cap} (G, B)$" and is defined by $(H, A) \tilde{\cap} (G, B) = (K, C)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership, and falsity-membership of (K, C) are as follows:*

$$T_{K(e)}(m) = \min\left(T_{H(e)}(m), T_{G(e)}(m)\right)$$

$$I_{K(e)}(m) = \frac{I_{H(e)}(m) + I_{G(e)}(m)}{2}$$

$$F_{K(e)}(m) = \max\left(F_{H(e)}(m), F_{G(e)}(m)\right), if\ e \in A \cap B.$$

Definition 10. *[45] Let $\Psi_K \in$ NP-soft set. Then an NP-aggregation operator of Ψ_K, denoted by Ψ_K^{agg}, is defined by*

$$\Psi_K^{agg} = \left\{\left(\langle u, \mu_K^{agg}, \vartheta_K^{agg}, \omega_K^{agg}\rangle\right): u \in U\right\}, \quad (1)$$

which is a neutrosophic set over U,

$$\mu_K^{agg}(u) = \frac{1}{|U|}\sum_{\substack{e\in E \\ u\in U}} \mu_K(u).\lambda f_{K(x)}(u),\ \mu_K^{agg}: U \to [0,1] \quad (2)$$

$$\vartheta_K^{agg}(u) = \frac{1}{|U|}\sum_{\substack{e\in E \\ u\in U}} \vartheta_K(u).\lambda f_{K(x)}(u),\ \vartheta_K^{agg}: U \to [0,1] \quad (3)$$

$$\omega_K^{agg} = \frac{1}{|U|}\sum_{\substack{e\in E \\ u\in U}} \omega_K(u).\lambda f_{K(x)}(u),\ \omega_K^{agg}: U \to [0,1] \quad (4)$$

and where,

$$\lambda f_{K(x)}(u) = \begin{cases} 1, x \in f_{K(x)}(u), \\ 0, otherwise, \end{cases}$$

such that $|U|$ is the cardinality of U.

3. Generalized Neutrosophic Soft Expert Set

In this section, we introduce the concept of generalized neutrosophic soft expert set (GNSES) and define some of its properties. Throughout this paper, U is an initial universe, E is a set of parameters, X is a set of experts (agents), and $O = \{\text{agree} = 1, \text{disagree} = 0\}$ a set of opinions. Let $Z = E \times X \times O$ and $A \subseteq Z$ and μ is a fuzzy set of A; that is, $\mu: A \to I = [0,1]$.

Definition 11. *A pair (F^{μ}, A) is called a generalized neutrosophic soft expert set (GNSES) over U, where F^{μ} is a mapping given by*

$$F^{\mu}: A \to \mathcal{N}(U) \times I,$$

with $\mathcal{N}(U)$ being the set of all neutrosophic soft expert subsets of U. For any parameter $e \in A$, $F(e)$ is referred as the neutrosophic value set of parameter e, i.e.,

$$F(e) = \left\{\langle u/T_{F(e)}(u), I_{F(e)}(u), F_{F(e)}(u)\rangle\right\},$$

where $T, I, F: U \to]^-0, 1^+[$ are the membership function of truth, indeterminacy, and falsity, respectively, of the element $u \in U$. For any $u \in U$ and $e \in A$

$$^-0 \le T_{F(e)}(u) + I_{F(e)}(u) + F_{F(e)}(u) \le 3^+$$

In fact, F^{μ} is a parameterized family of neutrosophic soft expert sets on U, which has the degree of possibility of the approximate value set which is prepresented by $\mu(e)$ for each parameter e, which can be written as follows:

$$F^{\mu}(e) = \left\{\left(\frac{u_1}{F(e)(u_1)}, \frac{u_2}{F(e)(u_2)}, \frac{u_3}{F(e)(u_3)}, \cdots, \frac{u_n}{F(e)(u_n)}\right), \mu(e)\right\}.$$

Example 1. *Suppose that* $U = \{u_1, u_2, u_3\}$ *is a set of computers and* $E = \{e_1, e_2, e_3\}$ *is a set of decision parameters. Let* $X = \{p, q, r\}$ *be set of experts. Suppose that*

$$F^{\mu}(e_1, p, 1) = \left\{\left(\frac{u_1}{0.4,0.3,0.2}, \frac{u_2}{0.6,0.1,0.8}, \frac{u_3}{0.5,0.7,0.2}\right), 0.3\right\}$$
$$F^{\mu}(e_1, q, 1) = \left\{\left(\frac{u_1}{0.3,0.2,0.5}, \frac{u_2}{0.5,0.6,0.2}, \frac{u_3}{0.8,0.1,0.4}\right), 0.4\right\}$$
$$F^{\mu}(e_1, r, 1) = \left\{\left(\frac{u_1}{0.8,0.4,0.3}, \frac{u_2}{0.7,0.3,0.5}, \frac{u_3}{0.2,0.6,0.5}, \right), 0.8\right\}$$
$$F^{\mu}(e_2, p, 1) = \left\{\left(\frac{u_1}{0.7,0.3,0.6}, \frac{u_2}{0.5,0.1,0.4}, \frac{u_3}{0.8,0.6,0.3}\right), 0.2\right\}$$
$$F^{\mu}(e_2, q, 1) = \left\{\left(\frac{u_1}{0.6,0.7,0.1}, \frac{u_2}{0.8,0.4,0.7}, \frac{u_3}{0.5,0.1,0.7}\right), 0.6\right\}$$
$$F^{\mu}(e_2, r, 1) = \left\{\left(\frac{u_1}{0.5,0.1,0.8}, \frac{u_2}{0.9,0.3,0.6}, \frac{u_3}{0.4,0.1,0.7}\right), 0.5\right\}$$
$$F^{\mu}(e_3, p, 1) = \left\{\left(\frac{u_1}{0.6,0.3,0.2}, \frac{u_2}{0.5,0.6,0.7}, \frac{u_3}{0.8,0.1,0.4}\right), 0.7\right\}$$
$$F^{\mu}(e_3, q, 1) = \left\{\left(\frac{u_1}{0.7,0.3,0.4}, \frac{u_2}{0.6,0.2,0.5}, \frac{u_3}{0.7,0.4,0.6}\right), 0.4\right\}$$
$$F^{\mu}(e_3, r, 1) = \left\{\left(\frac{u_1}{0.8,0.4,0.3}, \frac{u_2}{0.5,0.3,0.6}, \frac{u_3}{0.1,0.4,0.2}\right), 0.5\right\}$$
$$F^{\mu}(e_1, p, 0) = \left\{\left(\frac{u_1}{0.4,0.1,0.2}, \frac{u_2}{0.7,0.3,0.5}, \frac{u_3}{0.4,0.1,0.6}\right), 0.1\right\}$$
$$F^{\mu}(e_1, q, 0) = \left\{\left(\frac{u_1}{0.7,0.3,0.5}, \frac{u_2}{0.6,0.2,0.4}, \frac{u_3}{0.4,0.5,0.1}\right), 0.3\right\}$$
$$F^{\mu}(e_1, r, 0) = \left\{\left(\frac{u_1}{0.6,0.4,0.3}, \frac{u_2}{0.7,0.2,0.6}, \frac{u_3}{0.4,0.1,0.3}\right), 0.2\right\}$$
$$F^{\mu}(e_2, p, 0) = \left\{\left(\frac{u_1}{0.5,0.1,0.7}, \frac{u_2}{0.4,0.5,0.1}, \frac{u_3}{0.7,0.1,0.4}\right), 0.2\right\}$$
$$F^{\mu}(e_2, q, 0) = \left\{\left(\frac{u_1}{0.4,0.3,0.6}, \frac{u_2}{0.7,0.2,0.5}, \frac{u_3}{0.8,0.1,0.4}\right), 0.6\right\}$$
$$F^{\mu}(e_2, r, 0) = \left\{\left(\frac{u_1}{0.3,0.2,0.6}, \frac{u_2}{0.4,0.3,0.5}, \frac{u_3}{0.5,0.1,0.4}\right), 0.4\right\}$$
$$F^{\mu}(e_3, p, 0) = \left\{\left(\frac{u_1}{0.4,0.3,0.6}, \frac{u_2}{0.5,0.1,0.6}, \frac{u_3}{0.6,0.2,0.5}\right), 0.5\right\}$$
$$F^{\mu}(e_3, q, 0) = \left\{\left(\frac{u_1}{0.6,0.2,0.7}, \frac{u_2}{0.8,0.1,0.4}, \frac{u_3}{0.5,0.3,0.4}\right), 0.7\right\}$$
$$F^{\mu}(e_3, r, 0) = \left\{\left(\frac{u_1}{0.5,0.4,0.6}, \frac{u_2}{0.6,0.4,0.3}, \frac{u_3}{0.7,0.2,0.1}\right), 0.2\right\}$$

The generalized neutrosophic soft expert set (GNSES) is a parameterized family $\{F(e_i), i = 1,2, \ldots\}$ *of all neutrosophic sets of* U *and describes a collection of approximation of an object.*

Definition 12. *Let* (F^{μ}, A) *and* (G^{η}, B) *be two generalized neutrosophic soft expert sets (GNSESs) over* U. *Then* (F^{μ}, A) *is said to be a generalized neutrosophic soft expert subset of* (G^{η}, B) *if*

i. $B \subseteq A$, *and*
ii. $G^{\eta}(\varepsilon)$ *is a generalized neutrosophic soft expert subset* $F^{\mu}(\varepsilon)$, *for all* $\varepsilon \in B$,

Example 2. *Consider Example 1. Suppose that A and B are as follows.*

$$A = \{(e_1, p, 1), (e_2, p, 1), (e_2, q, 0), (e_3, r, 1)\}$$
$$B = \{(e_1, p, 1), (e_2, p, 1), (e_3, r, 1)\}.$$

Since B *is a neutrosophic soft expert subset of* A, *clearly* $B \subset A$. *Let* (G^{η}, B) *and* (F^{μ}, A) *be defined as follows:*

$$(F^{\mu}, A) = \left\{ \left[(e_1, p, 1), \left(\frac{u_1}{0.4,0.3,0.2}, \frac{u_2}{0.6,0.1,0.8}, \frac{u_3}{0.5,0.7,0.2}\right), 0.3\right],\right.$$

$$\left[(e_2,p,1),\left(\frac{u_1}{0.7,0.3,0.6},\frac{u_2}{0.5,0.1,0.4},\frac{u_3}{0.8,0.6,0.3}\right),0.2\right],$$
$$\left[(e_2,q,0),\left(\frac{u_1}{0.4,0.3,0.6},\frac{u_2}{0.7,0.2,0.5},\frac{u_3}{0.8,0.1,0.4}\right),0.6\right],$$
$$\left[(e_3,r,1),\left(\frac{u_1}{0.8,0.4,0.3},\frac{u_2}{0.5,0.3,0.6},\frac{u_3}{0.1,0.4,0.2}\right),0.5\right]\Big\}.$$

$$(G^{\eta},B)=\Big\{\left[(e_1,p,1),\left(\frac{u_1}{0.4,0.3,0.2},\frac{u_2}{0.6,0.1,0.8},\frac{u_3}{0.5,0.7,0.2}\right),0.3\right],$$
$$\left[(e_2,p,1),\left(\frac{u_1}{0.7,0.3,0.6},\frac{u_2}{0.5,0.1,0.4},\frac{u_3}{0.8,0.6,0.3}\right),0.2\right],$$
$$\left[(e_3,r,1),\left(\frac{u_1}{0.8,0.4,0.3},\frac{u_2}{0.5,0.3,0.6},\frac{u_3}{0.1,0.4,0.2}\right),0.5\right]\Big\}.$$

Therefore $(G^{\eta},B)\subseteq(F^{\mu},A)$.

Definition 13. *Two GNSESs* (F^{μ},A) *and* (G^{η},B) *over* U *are said to be equal if* (F^{μ},A) *is a GNSES subset of* (G^{η},B) *and* (G^{η},B) *is a GNSES subset of* (F^{μ},A).

Definition 14. *An agree-GNSESs* $(F^{\mu},A)_1$ *over* U *is a GNSES subset of* (F^{μ},A) *defined as follows.*

$$(F^{\mu},A)_1=\{F_1(\propto):\propto\in E\times X\times\{1\}\}.$$

Example 3. *Consider Example 1. The agree-GNSES* $(F^{\mu},Z)_1$ *over* U *is*

$$(F^{\mu},Z)_1=\Big\{\left[(e_1,p,1),\left(\frac{u_1}{0.4,0.3,0.2},\frac{u_2}{0.6,0.1,0.8},\frac{u_3}{0.5,0.7,0.2},\right),0.3\right],$$
$$\left[(e_1,q,1),\left(\frac{u_1}{0.3,0.2,0.5},\frac{u_2}{0.5,0.6,0.2},\frac{u_3}{0.8,0.1,0.4},\right),0.4\right],$$
$$\left[(e_1,r,1),\left(\frac{u_1}{0.8,0.4,0.3},\frac{u_2}{0.7,0.3,0.5},\frac{u_3}{0.2,0.6,0.5},\right),0.8\right],$$
$$\left[(e_2,p,1),\left(\frac{u_1}{0.7,0.3,0.6},\frac{u_2}{0.5,0.1,0.4},\frac{u_3}{0.8,0.6,0.3},\right),0.2\right],$$
$$\left[(e_2,q,1),\left(\frac{u_1}{0.6,0.7,0.1},\frac{u_2}{0.8,0.4,0.7},\frac{u_3}{0.5,0.1,0.7},\right),0.6\right],$$
$$\left[(e_2,r,1),\left(\frac{u_1}{0.5,0.1,0.8},\frac{u_2}{0.9,0.3,0.6},\frac{u_3}{0.4,0.1,0.7},\right),0.5\right],$$
$$\left[(e_3,p,1),\left(\frac{u_1}{0.6,0.3,0.2},\frac{u_2}{0.5,0.6,0.7},\frac{u_3}{0.8,0.1,0.4},\right),0.7\right],$$
$$\left[(e_3,q,1),\left(\frac{u_1}{0.7,0.3,0.4},\frac{u_2}{0.6,0.2,0.5},\frac{u_3}{0.7,0.4,0.6},\right),0.4\right],$$
$$\left[(e_3,r,1),\left(\frac{u_1}{0.8,0.4,0.3},\frac{u_2}{0.5,0.3,0.6},\frac{u_3}{0.1,0.4,0.2},\right),0.5\right]\Big\}.$$

Definition 15. *A disagree-GNSESs* $(F^{\mu},A)_0$ *over* U *is a GNSES subset of* (F^{μ},A) *is defined as follows:*

$$(F^{\mu},A)_0=\{F_0(\propto):\propto\in E\times X\times\{0\}\}.$$

Example 4. *Consider Example 1. The disagree-GNSES* $(F^{\mu},Z)_0$ *over* U *is*

$$(F^{\mu},Z)_0=\Big\{\left[(e_1,p,0),\left(\frac{u_1}{0.4,0.1,0.2},\frac{u_2}{0.7,0.3,0.5},\frac{u_3}{0.4,0.1,0.6},\right),0.1\right],$$
$$\left[(e_1,q,0),\left(\frac{u_1}{0.7,0.3,0.5},\frac{u_2}{0.6,0.2,0.4},\frac{u_3}{0.4,0.5,0.1},\right),0.3\right],$$
$$\left[(e_1,r,0),\left(\frac{u_1}{0.6,0.4,0.3},\frac{u_2}{0.7,0.2,0.6},\frac{u_3}{0.4,0.1,0.3},\right),0.2\right],$$
$$\left[(e_2,p,0),\left(\frac{u_1}{0.5,0.1,0.7},\frac{u_2}{0.4,0.5,0.1},\frac{u_3}{0.7,0.1,0.4},\right),0.2\right],$$
$$\left[(e_2,q,0),\left(\frac{u_1}{0.4,0.3,0.6},\frac{u_2}{0.7,0.2,0.5},\frac{u_3}{0.8,0.1,0.4},\right),0.6\right],$$
$$\left[(e_2,r,0),\left(\frac{u_1}{0.3,0.2,0.6},\frac{u_2}{0.4,0.3,0.5},\frac{u_3}{0.5,0.1,0.4},\right),0.4\right],$$

$$\left[(e_3,p,0),\left(\frac{u_1}{0.4,0.3,0.6},\frac{u_2}{0.5,0.1,0.6},\frac{u_3}{0.6,0.2,0.5},\right),0.5\right],$$
$$\left[(e_3,q,0),\left(\frac{u_1}{0.6,0.2,0.7},\frac{u_2}{0.8,0.1,0.4},\frac{u_3}{0.5,0.3,0.4},\right),0.7\right],$$
$$\left[(e_3,r,0),\left(\frac{u_1}{0.5,0.4,0.6},\frac{u_2}{0.6,0.4,0.3},\frac{u_3}{0.7,0.2,0.1},\right),0.2\right]\Big\}.$$

Definition 16. *The complement of a GNSES* (F^{μ},A)*, denoted by* $(F^{\mu},A)^c$*, is defined as* $(F^{\mu},A)^c=\left(F^{\mu(c)},\neg A\right)$ *where* $F^{\mu(c)}\colon \neg A\to\mathcal{N}(U)\times I$ *is a mapping given by*

$$F^{\mu(c)}(\propto)=\left\{\begin{array}{c} T_{F(\propto)^{(c)}}=F_{F(\propto)},\\ I_{F(\propto)^{(c)}}=\overline{1}-I_{F(\propto)},\\ F_{F(\propto)^{(c)}}=T_{F(\propto)},\\ \mu^c(\propto)=\overline{1}-\mu(\propto)\end{array}\right\}\ \textit{for each}\ \propto\in E.$$

Example 5. *Consider Example 1. By using the definition of GNSES complement, the complement of* F^{μ} *denoted by* $F^{\mu(c)}$*, is as follows:*

$$(F^{\mu(c)},Z)=\left\{\left[(\neg e_1,p,1),\left(\frac{u_1}{0.2,0.7,0.4},\frac{u_2}{0.8,0.9,0.6},\frac{u_3}{0.2,0.3,0.5},\right),0.7\right],\right.$$
$$\left[(\neg e_1,q,1),\left(\frac{u_1}{0.5,0.8,0.3},\frac{u_2}{0.2,0.4,0.5},\frac{u_3}{0.4,0.9,0.8},\right),0.6\right],$$
$$\left[(\neg e_1,r,1),\left(\frac{u_1}{0.3,0.6,0.8},\frac{u_2}{0.5,0.7,0.7},\frac{u_3}{0.5,0.4,0.2},\right),0.2\right],$$
$$\left[(\neg e_2,p,1),\left(\frac{u_1}{0.6,0.7,0.7},\frac{u_2}{0.4,0.9,0.5},\frac{u_3}{0.3,0.4,0.8},\right),0.8\right],$$
$$\left[(\neg e_2,q,1),\left(\frac{u_1}{0.1,0.3,0.6},\frac{u_2}{0.7,0.6,0.8},\frac{u_3}{0.7,0.9,0.5},\right),0.4\right],$$
$$\left[(\neg e_2,r,1),\left(\frac{u_1}{0.8,0.9,0.5},\frac{u_2}{0.6,0.7,0.9},\frac{u_3}{0.7,0.9,0.4},\right),0.5\right],$$
$$\left[(\neg e_3,p,1),\left(\frac{u_1}{0.2,0.7,0.6},\frac{u_2}{0.7,0.4,0.5},\frac{u_3}{0.4,0.9,0.8},\right),0.3\right],$$
$$\left[(\neg e_3,q,1),\left(\frac{u_1}{0.4,0.7,0.7},\frac{u_2}{0.5,0.8,0.6},\frac{u_3}{0.6,0.6,0.7},\right),0.6\right],$$
$$\left[(\neg e_3,r,1),\left(\frac{u_1}{0.3,0.6,0.8},\frac{u_2}{0.6,0.7,0.5},\frac{u_3}{0.2,0.6,0.1},\right),0.5\right],$$
$$\left[(\neg e_1,p,0),\left(\frac{u_1}{0.2,0.9,0.4},\frac{u_2}{0.5,0.7,0.7},\frac{u_3}{0.6,0.9,0.4},\right),0.9\right],$$
$$\left[(\neg e_1,q,0),\left(\frac{u_1}{0.5,0.7,0.7},\frac{u_2}{0.4,0.8,0.6},\frac{u_3}{0.1,0.5,0.4},\right),0.7\right],$$
$$\left[(\neg e_1,r,0),\left(\frac{u_1}{0.3,0.6,0.6},\frac{u_2}{0.6,0.8,0.7},\frac{u_3}{0.3,0.9,0.4},\right),0.8\right],$$
$$\left[(\neg e_2,p,0),\left(\frac{u_1}{0.7,0.9,0.5},\frac{u_2}{0.1,0.5,0.4},\frac{u_3}{0.4,0.9,0.7},\right),0.8\right],$$
$$\left[(\neg e_2,q,0),\left(\frac{u_1}{0.6,0.7,0.4},\frac{u_2}{0.5,0.8,0.7},\frac{u_3}{0.4,0.9,0.8},\right),0.4\right],$$
$$\left[(\neg e_2,r,0),\left(\frac{u_1}{0.6,0.8,0.3},\frac{u_2}{0.5,0.7,0.4},\frac{u_3}{0.4,0.9,0.5},\right),0.6\right],$$
$$\left[(\neg e_3,p,0),\left(\frac{u_1}{0.6,0.7,0.4},\frac{u_2}{0.6,0.9,0.5},\frac{u_3}{0.5,0.8,0.6},\right),0.5\right],$$
$$\left[(\neg e_3,q,0),\left(\frac{u_1}{0.7,0.8,0.6},\frac{u_2}{0.4,0.9,0.8},\frac{u_3}{0.4,0.7,0.5},\right),0.3\right],$$
$$\left[(\neg e_3,r,0),\left(\frac{u_1}{0.6,0.6,0.5},\frac{u_2}{0.3,0.6,0.6},\frac{u_3}{0.1,0.8,0.7},\right),0.8\right]\Big\}.$$

Proposition 1. *If* (F^{μ},A) *is a generalized neutrosophic soft expert set over* U*, then*

1. $((F^{\mu},A)^c)^c=(F^{\mu},A)$
2. $((F^{\mu},A)_1)^c=(F^{\mu},A)_0$
3. $((F^{\mu},A)_0)^c=(F^{\mu},A)_1$

Proof. (1) From Definition 16, we have $(F^{\mu}, A)^c = (F^{\mu(c)}, \neg A)$, where $F^{\mu(c)}(\propto) = T_{F(\propto)^{(c)}} = F_{F(\propto)}, I_{F(\propto)^{(c)}} = \bar{1} - I_{F(\propto)}, F_{F(\propto)^{(c)}} = T_{F(\propto)}$ and $\mu^c(\propto) = \bar{1} - \mu(\propto)$ for each $\propto \in E$.

Now $((F^{\mu}, A)^c)^c = ((F^{\mu(c)})^c, A)$ where

$$\begin{aligned}(F^{\mu(c)})^c(\propto) &= \begin{bmatrix} T_{F(\propto)^{(c)}} = F_{F(\propto)}, I_{F(\propto)^{(c)}} = \bar{1} - I_{F(\propto)}, \\ F_{F(\propto)^{(c)}} = T_{F(\propto)}, \mu^c(\propto) = \bar{1} - \mu(\propto) \end{bmatrix}^c \\ &= \begin{bmatrix} T_{F(\propto)} = F_{F(\propto)^{(c)}}, I_{F(\propto)} = \bar{1} - I_{F(\propto)^{(c)}}, \\ F_{F(\propto)} = T_{F(\propto)^{(c)}}, \mu(\propto) = \bar{1} - \mu^c(\propto) \end{bmatrix} \\ &= \bar{1} - (\bar{1} - I_{F(\propto)}) = \bar{1} - (\bar{1} - \mu(\propto)) = I_{F(\propto)} \\ &= \mu(\propto).\end{aligned}$$

Thus $((F^{\mu}, A)^c)^c = ((F^{\mu(c)})^c, A) = (F^{\mu}, A)$, for all $\propto \in E$.

The proofs of assertions (2) and (3) are obvious. □

Definition 17. *The union of two GNSESs (F^{μ}, A) and (G^{η}, B) over U, denoted by $(F^{\mu}, A) \tilde{\cup} (G^{\eta}, B)$, is the GNSESs (H^{Ω}, C), where $C = A \cup B$ and the truth-membership, indeterminacy-membership, and falsity-membership of (H^{Ω}, C) are as follows:*

$$T_{H^{\Omega}(e)} = \begin{cases} T_{F^{\mu}(e)}(m) & if\ e \in A - B \\ T_{G^{\eta}(e)}(m) & if\ e \in B - A \\ max\left(T_{F^{\mu}(e)}(m), T_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

$$I_{H^{\Omega}(e)} = \begin{cases} I_{F^{\mu}(e)}(m) & if\ e \in A - B \\ I_{G^{\eta}(e)}(m) & if\ e \in B - A \\ min\left(I_{F^{\mu}(e)}(m), I_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

$$F_{H^{\Omega}(e)} = \begin{cases} F_{F^{\mu}(e)}(m) & if\ e \in A - B \\ F_{G^{\eta}(e)}(m) & if\ e \in B - A \\ min\left(F_{F^{\mu}(e)}(m), F_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

where $\Omega(m) = max\left(\mu_{(e)}(m), \eta_{(e)}(m)\right)$.

Example 6. *Suppose that (F^{μ}, A) and (G^{η}, B) are two GNSESs over U, such that*

$$\begin{aligned}(F^{\mu}, A) = \Big\{&\Big[(e_1, p, 1), \Big(\frac{u_1}{0.4,0.3,0.2}, \frac{u_2}{0.6,0.1,0.8}, \frac{u_3}{0.5,0.7,0.2}\Big), 0.3\Big], \\ &\Big[(e_2, q, 1), \Big(\frac{u_1}{0.7,0.3,0.6}, \frac{u_2}{0.5,0.1,0.4}, \frac{u_3}{0.7,0.6,0.3}\Big), 0.2\Big], \\ &\Big[(e_2, q, 0), \Big(\frac{u_1}{0.4,0.3,0.6}, \frac{u_2}{0.7,0.2,0.5}, \frac{u_3}{0.8,0.1,0.4}\Big), 0.6\Big], \\ &\Big[(e_3, r, 1), \Big(\frac{u_1}{0.8,0.4,0.3}, \frac{u_2}{0.5,0.3,0.6}, \frac{u_3}{0.1,0.4,0.2}\Big), 0.5\Big]\Big\}.\end{aligned}$$

$$\begin{aligned}(G^{\eta}, B) = \Big\{&\Big[(e_1, p, 1), \Big(\frac{u_1}{0.6,0.5,0.1}, \frac{u_2}{0.8,0.2,0.3}, \frac{u_3}{0.9,0.2,0.3}\Big), 0.1\Big], \\ &\Big[(e_2, q, 1), \Big(\frac{u_1}{0.6,0.7,0.1}, \frac{u_2}{0.8,0.4,0.7}, \frac{u_3}{0.5,0.1,0.7}\Big), 0.4\Big], \\ &\Big[(e_3, r, 1), \Big(\frac{u_1}{0.4,0.1,0.2}, \frac{u_2}{0.5,0.4,0.2}, \frac{u_3}{0.3,0.6,0.4}\Big), 0.8\Big]\Big\}.\end{aligned}$$

Then $(F^{\mu}, A) \tilde{\cup} (G^{\eta}, B) = (H^{\Omega}, C)$ where

$$\begin{aligned}(H^{\Omega}, C) = \Big\{&\Big[(e_1, p, 1), \Big(\frac{u_1}{0.6,0.3,0.1}, \frac{u_2}{0.8,0.1,0.3}, \frac{u_3}{0.9,0.2,0.2}\Big), 0.3\Big], \\ &\Big[(e_2, q, 1), \Big(\frac{u_1}{0.6,0.3,0.1}, \frac{u_2}{0.8,0.2,0.5}, \frac{u_3}{0.7,0.1,0.4}\Big), 0.4\Big], \\ &\Big[(e_2, q, 0), \Big(\frac{u_1}{0.4,0.3,0.6}, \frac{u_2}{0.7,0.2,0.5}, \frac{u_3}{0.8,0.1,0.4}\Big), 0.6\Big], \\ &\Big[(e_3, r, 1), \Big(\frac{u_1}{0.8,0.1,0.2}, \frac{u_2}{0.5,0.3,0.2}, \frac{u_3}{0.3,0.4,0.2}\Big), 0.8\Big]\Big\}.\end{aligned}$$

Proposition 2. *If* (F^{μ}, A), (G^{η}, B) *and* (H^{Ω}, C) *are three GNSESs over* U, *then*

1. $\left((F^{\mu}, A) \tilde{\cup} (G^{\eta}, B)\right) \tilde{\cup} (H^{\Omega}, C) = (F^{\mu}, A) \tilde{\cup} \left((G^{\eta}, B) \tilde{\cup} (H^{\Omega}, C)\right)$.
2. $(F^{\mu}, A) \tilde{\cup} (F^{\mu}, A) \subseteq (F^{\mu}, A)$.

Proof. (1) We want to prove that

$$\left((F^{\mu}, A) \tilde{\cup} (G^{\eta}, B)\right) \tilde{\cup} (H^{\Omega}, C) = (F^{\mu}, A) \tilde{\cup} \left((G^{\eta}, B) \tilde{\cup} (H^{\Omega}, C)\right)$$

By using Definition 17, we consider the case when $e \in A \cap B$, as other cases are trivial. We will have

$$(F^{\mu}, A) \tilde{\cup} (G^{\eta}, B) = \left\{ \left(u \Big/ \begin{pmatrix} max \begin{pmatrix} T_{F^{\mu}(e)}(m), \\ T_{G^{\eta}(e)}(m) \end{pmatrix}, \\ min \begin{pmatrix} \mathrm{I}_{F^{\mu}(e)}(m), \\ \mathrm{I}_{G^{\eta}(e)}(m) \end{pmatrix}, \\ min \begin{pmatrix} \mathrm{F}_{F^{\mu}(e)}(m), \\ \mathrm{F}_{G^{\eta}(e)}(m) \end{pmatrix} \end{pmatrix} \right), max \begin{pmatrix} \mu_{(e)}(m), \\ \eta_{(e)}(m) \end{pmatrix}, u \in U \right\}.$$

Also consider the case when $e \in H$, as the other cases are trivial. We will have

$$\left((F^{\mu}, A) \tilde{\cup} (G^{\eta}, B)\right) \tilde{\cup} (H^{\Omega}, C) = \left\{ \left(u \Big/ \begin{pmatrix} max\left(T_{F^{\mu}(e)}(m), T_{G^{\eta}(e)}(m)\right), \\ min\left(\mathrm{I}_{F^{\mu}(e)}(m), \mathrm{I}_{G^{\eta}(e)}(m)\right), \\ min\left(\mathrm{F}_{F^{\mu}(e)}(m), \mathrm{F}_{G^{\eta}(e)}(m)\right) \end{pmatrix} \right), \begin{matrix} \left(u/T_{H^{\Omega}(e)}(m), \mathrm{I}_{H^{\Omega}(e)}(m), F_{H^{\Omega}(e)}(m)\right), \\ max\left(\mu_{(e)}(m), \eta_{(e)}(m), \Omega(m)\right), u \in U \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \left(u/T_{F^{\Omega}(e)}(m), \mathrm{I}_{F^{\Omega}(e)}(m), F_{F^{\Omega}(e)}(m)\right), \\ \left(u \Big/ \begin{pmatrix} max\left(T_{G^{\mu}(e)}(m), T_{H^{\eta}(e)}(m)\right), \\ min\left(\mathrm{I}_{G^{\mu}(e)}(m), \mathrm{I}_{H^{\eta}(e)}(m)\right), \\ min\left(\mathrm{F}_{G^{\mu}(e)}(m), \mathrm{F}_{H^{\eta}(e)}(m)\right) \end{pmatrix} \right) max\left(\mu_{(e)}(m), \eta_{(e)}(m), \Omega(m)\right), u \in U \end{matrix} \right\}$$

$$= (F^{\mu}, A) \tilde{\cup} \left((G^{\eta}, B) \tilde{\cup} (H^{\Omega}, C)\right).$$

(2) The proof is straightforward. □

Definition 18. *Let* (F^{μ}, A) *and* (G^{η}, B) *be two GNSESs over a common universe* U. *Then the intersection of* (F^{μ}, A) *and* (G^{η}, B) *is denoted by* $(F^{\mu}, A) \tilde{\cap} (G^{\eta}, B) = (K^{\delta}, C)$, *where* $C = A \cap B$ *and the truth-membership, indeterminacy-membership, and falsity-membership of* (K^{δ}, C) *are as follows:*

$$T_{K^{\delta}(e)} = \begin{cases} T_{F^{\mu}(e)}(m) & if\ e \in A - B \\ T_{G^{\eta}(e)}(m) & if\ e \in B - A \\ min\left(T_{F^{\mu}(e)}(m), T_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

$$I_{K^{\delta}(e)} = \begin{cases} \mathrm{I}_{F^{\mu}(e)}(m) & if\ e \in A - B \\ \mathrm{I}_{G^{\eta}(e)}(m) & if\ e \in B - A \\ min\left(\mathrm{I}_{F^{\mu}(e)}(m), \mathrm{I}_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

$$F_{K^{\delta}(e)} = \begin{cases} \mathrm{F}_{F^{\mu}(e)}(m) & if\ e \in A - B \\ \mathrm{F}_{G^{\eta}(e)}(m) & if\ e \in B - A \\ max\left(\mathrm{F}_{F^{\mu}(e)}(m), \mathrm{F}_{G^{\eta}(e)}(m)\right) & if\ e \in A \cap B \end{cases}$$

where $\delta(m) = min\left(\mu_{(e)}(m), \eta_{(e)}(m)\right)$.

Example 7. *Suppose that* (F^{μ}, A) *and* (G^{η}, B) *are two GNSESs over* U, *such that*

$$(F^{\mu}, A) = \left\{ \left[(e_1, p, 1), \left(\frac{u_1}{0.4,0.3,0.2}, \frac{u_2}{0.6,0.1,0.8}, \frac{u_3}{0.5,0.7,0.2}, \right), 0.3\right], \right.$$
$$\left[(e_2, q, 1), \left(\frac{u_1}{0.7,0.3,0.6}, \frac{u_2}{0.5,0.1,0.4}, \frac{u_3}{0.7,0.6,0.3}, \right), 0.2\right],$$
$$\left. \left[(e_2, q, 0), \left(\frac{u_1}{0.4,0.3,0.6}, \frac{u_2}{0.7,0.2,0.5}, \frac{u_3}{0.8,0.1,0.4}, \right), 0.6\right] \right\}.$$

$$(G^{\eta}, B) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.6,0.5,0.1}, \frac{u_2}{0.8,0.2,0.3}, \frac{u_3}{0.9,0.2,0.3}, \right), 0.1\right], \left[(e_3, r, 1), \left(\frac{u_1}{0.4,0.1,0.2}, \frac{u_2}{0.5,0.4,0.2}, \frac{u_3}{0.3,0.6,0.4}, \right), 0.8\right]\right\}.$$

Then $(F^{\mu}, A) \tilde{\cap} (G^{\eta}, B) = (K^{\delta}, C)$ *where*

$$(K^{\delta}, C) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.4,0.3,0.2}, \frac{u_2}{0.6,0.1,0.8}, \frac{u_3}{0.5,0.2,0.3}, \right), 0.1\right]\right\}.$$

Proposition 3. *If* (F^{μ}, A), (G^{η}, B) *and* (H^{Ω}, C) *are three GNSESs over* U, *then*

1. $\left((F^{\mu}, A) \tilde{\cap} (G^{\eta}, B)\right) \tilde{\cap} (K^{\delta}, C) = (F^{\mu}, A) \tilde{\cap} \left((G^{\eta}, B) \tilde{\cap} (K^{\delta}, C)\right)$
2. $(F^{\mu}, A) \tilde{\cap} (F^{\mu}, A) \subseteq (F^{\mu}, A)$.

Proof. (1) We want to prove that

$$\left((F^{\mu}, A) \tilde{\cap} (G^{\eta}, B)\right) \tilde{\cap} (K^{\delta}, C) = (F^{\mu}, A) \tilde{\cap} \left((G^{\eta}, B) \tilde{\cap} (K^{\delta}, C)\right)$$

By using Definition 18, consider the case when $e \in A \cap B$, since other cases are trivial. We have

$$(F^{\mu}, A) \tilde{\cap} (G^{\eta}, B) = \left\{\left(u \middle/ \begin{matrix} min\left(T_{F^{\mu}(e)}(m), T_{G^{\eta}(e)}(m)\right), \\ min\left(I_{F^{\mu}(e)}(m), I_{G^{\eta}(e)}(m)\right), \\ max\left(F_{F^{\mu}(e)}(m), F_{G^{\eta}(e)}(m)\right) \end{matrix}\right), min\left(\mu_{(e)}(m), \eta_{(e)}(m)\right), u \in U\right\}.$$

Also consider the case when $e \in K$, as the other cases are trivial. Then we have

$$\begin{aligned} \left((F^{\mu}, A) \tilde{\cap} (G^{\eta}, B)\right) \tilde{\cap} (K^{\delta}, C) &= \left\{\left(u \middle/ \begin{matrix} min\left(T_{F^{\mu}(e)}(m), T_{G^{\eta}(e)}(m)\right), \\ min\left(I_{F^{\mu}(e)}(m), I_{G^{\eta}(e)}(m)\right), \\ max\left(F_{F^{\mu}(e)}(m), F_{G^{\eta}(e)}(m)\right) \end{matrix}\right), \begin{matrix} \left(u/T_{K^{\delta}(e)}(m), I_{K^{\delta}(e)}(m), F_{K^{\delta}(e)}(m)\right), \\ min\left(\mu_{(e)}(m), \eta_{(e)}(m), \delta(m)\right), u \in U \end{matrix}\right\}. \\ &= \left\{\begin{matrix} \left(u/T_{F^{\Omega}(e)}(m), I_{F^{\Omega}(e)}(m), F_{F^{\Omega}(e)}(m)\right), \\ \left(u \middle/ \begin{matrix} min\left(T_{G^{\mu}(e)}(m), T_{K^{\delta}(e)}(m)\right), \\ min\left(I_{G^{\mu}(e)}(m), I_{K^{\delta}(e)}(m)\right), \\ max\left(F_{G^{\mu}(e)}(m), F_{K^{\delta}(e)}(m)\right) \end{matrix}\right) \end{matrix} min\left(\mu_{(e)}(m), \eta_{(e)}(m), \delta(m)\right), u \in U\right\} \\ &= (F^{\mu}, A) \tilde{\cap} \left((G^{\eta}, B) \tilde{\cap} (K^{\delta}, C)\right). \end{aligned}$$

(2) The proof is straightforward. □

Proposition 4. *If* (F^{μ}, A), (G^{η}, B) *and* (K^{δ}, C) *are three GNSESs over* U. *Then*

1. $\left((F^{\mu}, A) \tilde{\cup} (G^{\eta}, B)\right) \tilde{\cap} (K^{\delta}, C) = \left((F^{\mu}, A) \tilde{\cap} (K^{\delta}, C)\right) \tilde{\cup} \left((G^{\eta}, B) \tilde{\cap} (K^{\delta}, C)\right)$.
2. $\left((F^{\mu}, A) \tilde{\cap} (G^{\eta}, B)\right) \tilde{\cup} (K^{\delta}, C) = \left((F^{\mu}, A) \tilde{\cup} (K^{\delta}, C)\right) \tilde{\cap} \left((G^{\eta}, B) \tilde{\cup} (K^{\delta}, C)\right)$.

Proof. The proofs can be easily obtained from Definitions 17 and 18. □

Definition 19. *If* (F^{μ}, A) *and* (G^{η}, B) *are two GNSESs over* U, *then "*(F^{μ}, A) *AND* (G^{η}, B)*" denoted by* $(F^{\mu}, A) \wedge (G^{\eta}, B)$, *is defined by*

$$(F^{\mu}, A) \wedge (G^{\eta}, B) = (H^{\Omega}, A \times B)$$

such that, $H^{\Omega}(\alpha, \beta) = F^{\mu}(\alpha) \cap G^{\eta}(\beta)$ *and the truth-membership, indeterminacy-membership, and falsity-membership of* $(H^{\Omega}, A \times B)$ *are as follows.*

$$T_{H^{\Omega}(\alpha,\beta)}(m) = min\left(T_{F^{\mu}(\alpha)}(m), T_{G^{\eta}(\beta)}(m)\right),$$

$$I_{H^{\Omega}(\alpha,\beta)}(m) = min\left(I_{F^{\mu}(\alpha)}(m), I_{G^{\eta}(\beta)}(m)\right),$$

$$F_{H^{\Omega}(\alpha,\beta)}(m) = max\left(F_{F^{\mu}(\alpha)}(m), F_{G^{\eta}(\beta)}(m)\right)$$

and $\Omega(m) = min\left(\mu_{(e)}(m), \eta_{(e)}(m)\right)$, $\forall \alpha \in A, \forall \beta \in B$.

Example 8. *Suppose that* (F^{μ}, A) *and* (G^{η}, B) *are two GNSESs over* U*, such that*

$$(F^{\mu}, A) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.2,0.3,0.5}, \frac{u_2}{0.4,0.1,0.2}, \frac{u_3}{0.6,0.3,0.7},\right), 0.4\right], \left[(e_3, r, 0), \left(\frac{u_1}{0.5,0.2,0.1}, \frac{u_2}{0.6,0.3,0.7}, \frac{u_3}{0.2,0.1,0.8},\right), 0.3\right]\right\}$$

$$(G^{\eta}, B) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.3,0.2,0.6}, \frac{u_2}{0.6,0.3,0.2}, \frac{u_3}{0.8,0.1,0.2},\right), 0.5\right], \left[(e_2, q, 0), \left(\frac{u_1}{0.1,0.3,0.5}, \frac{u_2}{0.7,0.1,0.6}, \frac{u_3}{0.4,0.3,0.6},\right), 0.6\right]\right\}.$$

Then $(F^{\mu}, A) \wedge (G^{\eta}, B) = (H^{\Omega}, A \times B)$ *where*

$$(H^{\Omega}, A \times B) = \left\{\left[(e_1, p, 1), (e_1, p, 1) \left(\frac{u_1}{0.2,0.2,0.6}, \frac{u_2}{0.4,0.1,0.2}, \frac{u_3}{0.6,0.1,0.7},\right), 0.4\right], \left[(e_1, p, 1), (e_2, q, 0), \left(\frac{u_1}{0.1,0.3,0.5}, \frac{u_2}{0.4,0.1,0.6}, \frac{u_3}{0.4,0.3,0.7},\right), 0.4\right], \left[(e_3, r, 0), (e_1, p, 1), \left(\frac{u_1}{0.3,0.2,0.6}, \frac{u_2}{0.6,0.3,0.7}, \frac{u_3}{0.2,0.1,0.8},\right), 0.3\right], \left[(e_3, r, 0), (e_2, q, 0), \left(\frac{u_1}{0.1,0.2,0.5}, \frac{u_2}{0.6,0.1,0.7}, \frac{u_3}{0.2,0.1,0.8},\right), 0.3\right]\right\}.$$

Definition 20. *If* (F^{μ}, A) *and* (G^{η}, B) *are two GNSESs over* U*, then "*$(F^{\mu}, A)\ OR\ (G^{\eta}, B)$*" denoted by* $(F^{\mu}, A) \vee (G^{\eta}, B)$*, is defined by*

$$(F^{\mu}, A) \vee (G^{\eta}, B) = \left(K^{\delta}, A \times B\right)$$

such that $K^{\delta}(\alpha, \beta) = F^{\mu}(\alpha) \cup G^{\eta}(\beta)$ *and the truth-membership, indeterminacy-membership, and falsity-membership of* $\left(K^{\delta}, A \times B\right)$ *are as follows.*

$$T_{K^{\delta}(\alpha,\beta)}(m) = max\left(T_{F^{\mu}(\alpha)}(m), T_{G^{\eta}(\beta)}(m)\right),$$

$$I_{K^{\delta}(\alpha,\beta)}(m) = min\left(I_{F^{\mu}(\alpha)}(m), I_{G^{\eta}(\beta)}(m)\right),$$

$$F_{K^{\delta}(\alpha,\beta)}(m) = min\left(F_{F^{\mu}(\alpha)}(m), F_{G^{\eta}(\beta)}(m)\right)$$

and $\delta(m) = max\left(\mu_{(e)}(m), \eta_{(e)}(m)\right)$, $\forall \alpha \in A, \forall \beta \in B$.

Example 9. *Suppose that* (F^{μ}, A) *and* (G^{η}, B) *are two GNSESs over* U*, such that*

$$(F^{\mu}, A) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.2,0.3,0.5}, \frac{u_2}{0.4,0.1,0.2}, \frac{u_3}{0.6,0.3,0.7},\right), 0.4\right], \left[(e_3, r, 0), \left(\frac{u_1}{0.5,0.2,0.1}, \frac{u_2}{0.6,0.3,0.7}, \frac{u_3}{0.2,0.1,0.8},\right), 0.3\right]\right\}$$

$$(G^{\eta}, B) = \left\{\left[(e_1, p, 1), \left(\frac{u_1}{0.3,0.2,0.6}, \frac{u_2}{0.6,0.3,0.2}, \frac{u_3}{0.8,0.1,0.2},\right), 0.5\right], \left[(e_2, q, 0), \left(\frac{u_1}{0.1,0.3,0.5}, \frac{u_2}{0.7,0.1,0.6}, \frac{u_3}{0.4,0.3,0.6},\right), 0.6\right]\right\}.$$

Then $(F^{\mu}, A) \vee (G^{\eta}, B) = \left(K^{\delta}, A \times B\right)$ *where*

$$\left(K^{\delta}, A \times B\right) = \left\{\left[(e_1, p, 1), (e_1, p, 1) \left(\frac{u_1}{0.3,0.2,0.5}, \frac{u_2}{0.6,0.1,0.2}, \frac{u_3}{0.8,0.1,0.2},\right), 0.5\right], \left[(e_1, p, 1), (e_2, q, 0), \left(\frac{u_1}{0.2,0.3,0.5}, \frac{u_2}{0.7,0.1,0.2}, \frac{u_3}{0.6,0.3,0.6},\right), 0.6\right], \left[(e_3, r, 0), (e_1, p, 1), \left(\frac{u_1}{0.5,0.2,0.1}, \frac{u_2}{0.7,0.3,0.6}, \frac{u_3}{0.8,0.1,0.2},\right), 0.5\right], \left[(e_3, r, 0), (e_2, q, 0), \left(\frac{u_1}{0.5,0.2,0.1}, \frac{u_2}{0.7,0.1,0.6}, \frac{u_3}{0.4,0.1,0.6},\right), 0.6\right]\right\}.$$

Proposition 5. *Let* (F^{μ}, A) *and* (G^{η}, B) *be GNSESs over* U*. Then*

1. $((F^{\mu}, A) \wedge (G^{\eta}, B)\ )^c = (F^{\mu}, A)^c \vee (G^{\eta}, B)^c$
2. $((F^{\mu}, A) \vee (G^{\eta}, B))^c = (F^{\mu}, A)^c \wedge (G^{\eta}, B)^c$

Proof. The proofs can be easily obtained from Definitions 16, 19 and 20. □

4. GNSES-Aggregation Operator

In this section, we define a GNSES-aggregation operator of a GNSES to construct a decision method by which approximate functions of a soft expert set are combined to produce a neutrosophic set that can be used to evaluate each alternative.

Definition 21. *Let $\Upsilon_A \in$ GNSESs. Then a GNSES-aggregation operator of Υ_A, denoted by Υ_A^{agg}, is defined by*

$$\Upsilon_A^{agg} = \left\{\left(\langle u, T_A^{agg}(u), I_A^{agg}(u), F_A^{agg}(u)\rangle\right) : u \in U\right\}, \tag{5}$$

which is a GNSES over U,

$$T_A^{agg} : U \to [0,1],\ \ T_A^{agg}(u) = \frac{1}{|U|}\sum_{\substack{e\in E\\ u\in U}} T_A(u).\mu, \tag{6}$$

$$F_A^{agg} : U \to [0,1], F_A^{agg}(u) = \frac{1}{|U|}\sum_{\substack{e\in E\\ u\in U}} F_A(u).\mu, \tag{7}$$

$$I_A^{agg} : U \to [0,1], I_A^{agg}(u) = \frac{1}{|U|}\sum_{\substack{e\in E\\ u\in U}} I_A(u)\,.\mu, \tag{8}$$

where $|U|$ is the cardinality of U and μ is defined below

$$\mu = \frac{1}{n}.\sum_{i=1}^{n}\mu(e_i). \qquad (e_i, i = 1,2,3,\ldots,n). \tag{9}$$

Definition 22. *Let $\Upsilon_A \in$ GNSESs, Υ_A^{agg} be the corresponding GNSES aggregation operator. Then a reduced fuzzy set of Υ_A^{agg} is a fuzzy set over U, denoted by*

$$\Upsilon_A^{agg} = \left\{\frac{\tau\Upsilon_A^{agg}(u)}{u} : u \in U\right\}, \tag{10}$$

where $\tau\Upsilon_A^{agg}(u) : U \to [0,1]$ and $u_i = \left|T_{A_i}^{agg} - F_{A_i}^{agg} - I_{A_i}^{agg}\right|$.

5. An Application of Generalized Neutrosophic Soft Expert Set

In this section, we present an application of generalized neutrosophic soft expert set theory in a decision-making problem. Based on Definitions 21 and 22, we constructed an algorithm for the GNSES decision-making method as follows.

Step 1—Choose a feasible subset of the set of parameters.
Step 2—Construct the GNSES tables for each opinion (agree, disagree) of experts.
Step 3—Compute the aggregation operator GNSES Υ_A^{agg} of Υ_A and the reduced fuzzy set $T_{A_i}^{agg}, F_{A_i}^{agg}, I_{A_i}^{agg}$ of Υ_A^{agg}.
Step 4—Score(u_i) = $\max agree\,(u_i) - \min disagree\,(u_i)$.
Step 5—Choose the element of u_i that has maximum score. This will be the optimal solution.

Example 10. *Suppose a company needs to employ a worker, which is to be decided by a few experts. The employee has to be chosen from five potential workers, $U = \{u_1, u_2, u_3, u_4, u_5\}$. Suppose there are four parameters $E = \{e_1, e_2, e_3, e_4\}$ where the parameters $e_i\ (i = 1,2,3,4)$ stand for "education," "age," "capability" and "experience", respectively. Let $X = \{p, q, r\}$ be a set of experts. After a serious discussion, the experts construct the following generalized neutrosophic soft expert set.*

Step 1—Choose a feasible subset of the set of parameters

$$(F^{\mu}, Z) = \left\{ \left[(e_1, p, 1), \left(\frac{u_1}{0.2,0.3,0.4}, \frac{u_2}{0.8,0.2,0.6}, \frac{u_3}{0.6,0.3,0.5}, \frac{u_4}{0.4,0.2,0.3}, \frac{u_5}{0.6,0.3,0.1},\right), 0.7\right],\right.$$
$$\left[(e_1, q, 1), \left(\frac{u_1}{0.3,0.1,0.4}, \frac{u_2}{0.2,0.1,0.5}, \frac{u_3}{0.4,0.2,0.3}, \frac{u_4}{0.4,0.2,0.3}, \frac{u_5}{0.7,0.2,0.5},\right), 0.6\right],$$
$$\left[(e_1, r, 1), \left(\frac{u_1}{0.3,0.5,0.1}, \frac{u_2}{0.6,0.2,0.5}, \frac{u_3}{0.1,0.4,0.2}, \frac{u_4}{0.5,0.2,0.3}, \frac{u_5}{0.4,0.3,0.2},\right), 0.2\right],$$

$$\left[(e_2,p,1),\left(\frac{u_1}{0.6,0.2,0.3},\frac{u_2}{0.4,0.2,0.5},\frac{u_3}{0.3,0.4,0.1},\frac{u_4}{0.7,0.3,0.6},\frac{u_5}{0.5,0.2,0.4},\right),0.8\right],$$
$$\left[(e_2,q,1),\left(\frac{u_1}{0.1,0.3,0.6},\frac{u_2}{0.7,0.3,0.1},\frac{u_3}{0.6,0.2,0.5},\frac{u_4}{0.3,0.1,0.6},\frac{u_5}{0.4,0.3,0.2},\right),0.4\right],$$
$$\left[(e_2,r,1),\left(\frac{u_1}{0.6,0.3,0.5},\frac{u_2}{0.7,0.3,0.6},\frac{u_3}{0.5,0.3,0.4},\frac{u_4}{0.2,0.1,0.3},\frac{u_5}{0.6,0.2,0.5},\right),0.5\right],$$
$$\left[(e_3,p,1),\left(\frac{u_1}{0.2,0.4,0.6},\frac{u_2}{0.7,0.4,0.2},\frac{u_3}{0.4,0.1,0.2},\frac{u_4}{0.8,0.4,0.3},\frac{u_5}{0.7,0.3,0.4},\right),0.3\right],$$
$$\left[(e_3,q,1),\left(\frac{u_1}{0.4,0.2,0.6},\frac{u_2}{0.5,0.3,0.6},\frac{u_3}{0.6,0.2,0.7},\frac{u_4}{0.8,0.2,0.4},\frac{u_5}{0.6,0.2,0.3},\right),0.4\right],$$
$$\left[(e_3,r,1),\left(\frac{u_1}{0.3,0.6,0.5},\frac{u_2}{0.6,0.2,0.5},\frac{u_3}{0.2,0.1,0.4},\frac{u_4}{0.5,0.3,0.2},\frac{u_5}{0.4,0.1,0.5},\right),0.5\right],$$
$$\left[(e_4,p,1),\left(\frac{u_1}{0.2,0.3,0.6},\frac{u_2}{0.7,0.1,0.5},\frac{u_3}{0.4,0.2,0.8},\frac{u_4}{0.9,0.2,0.4},\frac{u_5}{0.3,0.4,0.6},\right),0.6\right],$$
$$\left[(e_4,q,1),\left(\frac{u_1}{0.5,0.2,0.1},\frac{u_2}{0.2,0.3,0.4},\frac{u_3}{0.4,0.1,0.5},\frac{u_4}{0.6,0.3,0.2},\frac{u_5}{0.7,0.3,0.4},\right),0.6\right],$$
$$\left[(e_4,r,1),\left(\frac{u_1}{0.5,0.2,0.1},\frac{u_2}{0.6,0.3,0.5},\frac{u_3}{0.2,0.5,0.3},\frac{u_4}{0.5,0.1,0.4},\frac{u_5}{0.3,0.2,0.5},\right),0.3\right],$$
$$\left[(e_1,p,0),\left(\frac{u_1}{0.2,0.3,0.4},\frac{u_2}{0.5,0.3,0.1},\frac{u_3}{0.6,0.3,0.4},\frac{u_4}{0.6,0.2,0.4},\frac{u_5}{0.7,0.5,0.6},\right),0.9\right],$$
$$\left[(e_1,q,0),\left(\frac{u_1}{0.5,0.1,0.7},\frac{u_2}{0.4,0.2,0.3},\frac{u_3}{0.8,0.5,0.4},\frac{u_4}{0.7,0.3,0.6},\frac{u_5}{0.5,0.3,0.4},\right),0.7\right],$$
$$\left[(e_1,r,0),\left(\frac{u_1}{0.3,0.1,0.6},\frac{u_2}{0.6,0.3,0.7},\frac{u_3}{0.3,0.2,0.4},\frac{u_4}{0.8,0.1,0.4},\frac{u_5}{0.6,0.4,0.5},\right),0.6\right],$$
$$\left[(e_2,p,0),\left(\frac{u_1}{0.7,0.3,0.5},\frac{u_2}{0.6,0.2,0.4},\frac{u_3}{0.4,0.3,0.5},\frac{u_4}{0.3,0.2,0.5},\frac{u_5}{0.4,0.3,0.5},\right),0.8\right],$$
$$\left[(e_2,q,0),\left(\frac{u_1}{0.6,0.2,0.4},\frac{u_2}{0.5,0.3,0.7},\frac{u_3}{0.8,0.1,0.3},\frac{u_4}{0.2,0.3,0.6},\frac{u_5}{0.6,0.2,0.4},\right),0.4\right],$$
$$\left[(e_2,r,0),\left(\frac{u_1}{0.6,0.3,0.4},\frac{u_2}{0.5,0.2,0.4},\frac{u_3}{0.7,0.4,0.5},\frac{u_4}{0.5,0.2,0.4},\frac{u_5}{0.4,0.3,0.5},\right),0.2\right],$$
$$\left[(e_3,p,0),\left(\frac{u_1}{0.6,02,0.4},\frac{u_2}{0.6,0.1,0.5},\frac{u_3}{0.5,0.4,0.6},\frac{u_4}{0.8,0.3,0.6},\frac{u_5}{0.7,0.2,0.4},\right),0.5\right],$$
$$\left[(e_3,q,0),\left(\frac{u_1}{0.7,0.1,0.6},\frac{u_2}{0.4,0.5,0.8},\frac{u_3}{0.4,0.3,0.5},\frac{u_4}{0.6,0.2,0.5},\frac{u_5}{0.4,0.3,0.5},\right),0.3\right],$$
$$\left[(e_3,r,0),\left(\frac{u_1}{0.2,0.3,0.6},\frac{u_2}{0.7,0.4,0.5},\frac{u_3}{0.4,0.2,0.8},\frac{u_4}{0.9,0.1,0.4},\frac{u_5}{0.6,0.3,0.2},\right),0.3\right],$$
$$\left[(e_4,p,0),\left(\frac{u_1}{0.4,0.2,0.6},\frac{u_2}{0.5,0.2,0.6},\frac{u_3}{0.9,0.5,0.1},\frac{u_4}{0.3,0.2,0.6},\frac{u_5}{0.4,0.3,0.5},\right),0.6\right],$$
$$\left[(e_4,q,0),\left(\frac{u_1}{0.3,0.2,0.1},\frac{u_2}{0.6,0.1,0.5},\frac{u_3}{0.6,0.2,0.5},\frac{u_4}{0.8,0.3,0.2},\frac{u_5}{0.2,0.3,0.4},\right),0.5\right],$$
$$\left[(e_4,r,0),\left(\frac{u_1}{0.6,0.2,0.5},\frac{u_2}{0.7,0.1,0.6},\frac{u_3}{0.5,0.3,0.1},\frac{u_4}{0.3,0.2,0.6},\frac{u_5}{0.4,0.2,0.5},\right),0.1\right]\Big\}.$$

Step 2—Construct the GNSES tables for each opinion (agree, disagree) of experts, as shown in Tables 1 and 2.

Table 1. Agree-GNSES.

U	u_1	u_2	u_3	u_4	u_5	μ
(e_1, p)	0.2,0.3,0.4	0.8,0.2,0.6	0.6,0.3,0.5	0.4,0.2,0.3	0.6,0.3,0.1	0.7
(e_2, p)	0.6,0.2,0.3	0.4,0.2,0.5	0.3,0.4,0.1	0.7,0.3,0.6	0.5,0.2,0.4	0.8
(e_3, p)	0.2,0.4,0.6	0.7,0.4,0.2	0.4,0.1,0.2	0.8,0.4,0.3	0.7,0.3,0.4	0.3
(e_4, p)	0.2,0.3,0.6	0.7,0.1,0.5	0.4,0.2,0.8	0.9,0.2,0.4	0.3,0.4,0.6	0.6
(e_1, q)	0.3,0.1,0.4	0.2,0.1,0.5	0.4,0.2,0.3	0.4,0.2,0.3	0.7,0.2,0.5	0.6
(e_2, q)	0.1,0.3,0.6	0.7,0.3,0.1	0.6,0.2,0.5	0.3,0.1,0.6	0.4,0.3,0.2	0.4
(e_3, q)	0.4,0.2,0.6	0.5,0.3,0.6	0.6,0.2,0.7	0.8,0.2,0.4	0.6,0.2,0.3	0.4
(e_4, q)	0.5,0.2,0.1	0.2,0.3,0.4	0.4,0.1,0.5	0.6,0.3,0.2	0.7,0.3,0.4	0.6
(e_1, r)	0.3,0.5,0.1	0.6,0.2,0.5	0.1,0.4,0.2	0.5,0.2,0.3	0.4,0.3,0.2	0.2
(e_2, r)	0.6,0.3,0.5	0.7,0.3,0.6	0.5,0.3,0.4	0.2,0.1,0.3	0.6,0.2,0.5	0.5
(e_3, r)	0.3,0.6,0.5	0.6,0.2,0.5	0.2,0.1,0.4	0.5,0.3,0.2	0.4,0.1,0.5	0.5
(e_4, r)	0.5,0.2,0.1	0.6,0.3,0.5	0.2,0.5,0.3	0.5,0.1,0.4	0.3,0.2,0.5	0.3

Table 2. Disagree-GNSES.

U	u_1	u_2	u_3	u_4	u_5	μ
(e_1, p)	0.2,0.3,0.4	0.5,0.3,0.1	0.6,0.3,0.4	0.6,0.2,0.4	0.7,0.5,0.6	0.9
(e_2, p)	0.7,0.3,0.5	0.6,0.2,0.4	0.4,0.3,0.5	0.3,0.2,0.5	0.4,0.3,0.5	0.8
(e_3, p)	0.6,02,0.4	0.6,0.1,0.5	0.5,0.4,0.6	0.8,0.3,0.6	0.7,0.2,0.4	0.5
(e_4, p)	0.4,0.2,0.6	0.5,0.2,0.6	0.9,0.5,0.1	0.3,0.2,0.6	0.4,0.3,0.5	0.6
(e_1, q)	0.5,0.1,0.7	0.4,0.2,0.3	0.8,0.5,0.4	0.7,0.3,0.6	0.5,0.3,0.4	0.7
(e_2, q)	0.6,0.2,0.4	0.5,0.3,0.7	0.8,0.1,0.3	0.2,0.3,0.6	0.6,0.2,0.4	0.4
(e_3, q)	0.7,0.1,0.6	0.4,0.5,0.8	0.4,0.3,0.5	0.6,0.2,0.5	0.4,0.3,0.5	0.3
(e_4, q)	0.3,0.2,0.1	0.6,0.1,0.5	0.6,0.2,0.5	0.8,0.3,0.2	0.2,0.3,0.4	0.5
(e_1, r)	0.3,0.1,0.6	0.6,0.3,0.7	0.3,0.2,0.4	0.8,0.1,0.4	0.6,0.4,0.5	0.6
(e_2, r)	0.6,0.3,0.4	0.5,0.2,0.4	0.7,0.4,0.5	0.5,0.2,0.4	0.4,0.3,0.5	0.2
(e_3, r)	0.2,0.3,0.6	0.7,0.4,0.5	0.4,0.2,0.8	0.9,0.1,0.4	0.6,0.3,0.2	0.3
(e_4, r)	0.6,0.2,0.5	0.7,0.1,0.6	0.5,0.3,0.1	0.3,0.2,0.6	0.4,0.2,0.5	0.1

Step 3—Now calculate the scores of agree (u_i) by using the data in Table 1, to obtain values in Table 3.

$$T_A^{agg}(p, u_1) = \left(\frac{T_{A_1} + T_{A_2} + T_{A_3} + T_{A_4}}{4}\right).\left(\frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}\right).$$
$$= \left(\frac{0.2 + 0.6 + 0.2 + 0.2}{4}\right).\left(\frac{0.7 + 0.8 + 0.3 + 0.6}{4}\right)$$
$$= 0.18$$

$$I_A^{agg}(q, u_1) = \left(\frac{I_{A_1} + I_{A_2} + I_{A_3} + I_{A_4}}{4}\right).\left(\frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}\right).$$
$$= \left(\frac{0.3 + 0.2 + 0.4 + 0.3}{4}\right).\left(\frac{0.7 + 0.8 + 0.3 + 0.6}{4}\right)$$
$$= 0.18$$

$$F_A^{agg}(r, u_1) = \left(\frac{F_{A_1} + F_{A_2} + F_{A_3} + F_{A_4}}{4}\right).\left(\frac{\mu_1 + \mu_2 + \mu_3 + \mu_4}{4}\right).$$
$$= \left(\frac{0.4 + 0.3 + 0.6 + 0.6}{4}\right).\left(\frac{0.7 + 0.8 + 0.3 + 0.6}{4}\right)$$
$$= 0.285$$

$$u_1 = \left|T_{A_i}^{agg} - F_{A_i}^{agg} - I_{A_i}^{agg}\right| = |0.18 - 0.18 - 0.285| = 0.285.$$

Table 3. Degree table of agree-GNSES.

U	u_1	u_2	u_3	u_4	u_5
p	0.285	0.015	0.135	0.015	0.09
q	0.18	0.15	0.105	0.12	0.015
r	0.165	0.09	0.24	0.06	0.045

Now calculate the score of disagree (u_i) by using the data in Table 2, to obtain values in Table 4.

$$T_A^{agg}(p,u_1) = \left(\frac{T_{A_1}+T_{A_2}+T_{A_3}+T_{A_4}}{4}\right).\left(\frac{\mu_1+\mu_2+\mu_3+\mu_4}{4}\right)$$
$$= \left(\frac{0.2+0.7+0.6+0.4}{4}\right).\left(\frac{0.9+0.8+0.5+0.6}{4}\right)$$
$$= 0.3325$$

$$I_A^{agg}(q,u_1) = \left(\frac{I_{A_1}+I_{A_2}+I_{A_3}+I_{A_4}}{4}\right).\left(\frac{\mu_1+\mu_2+\mu_3+\mu_4}{4}\right).$$
$$= \left(\frac{0.3+0.3+0.2+0.2}{4}\right).\left(\frac{0.9+0.8+0.5+0.6}{4}\right)$$
$$= 0.175$$

$$F_A^{agg}(r,u_1) = \left(\frac{F_{A_1}+F_{A_2}+F_{A_3}+F_{A_4}}{4}\right).\left(\frac{\mu_1+\mu_2+\mu_3+\mu_4}{4}\right).$$
$$= \left(\frac{0.4+0.5+0.4+0.6}{4}\right).\left(\frac{0.9+0.8+0.5+0.6}{4}\right)$$
$$= 0.3325$$

$$u_1 = \left|T_{A_i}^{agg} - F_{A_i}^{agg} - I_{A_i}^{agg}\right| = |0.3325 - 0.175 - 0.3325| = 0.175.$$

Table 4. Degree table of disagree-GNSES.

U	u_1	u_2	u_3	u_4	u_5
p	0.175	0.035	0.1225	0.175	0.1925
q	0.0525	0.2625	0.035	0.1225	0.0875
r	0.2275	0.1225	0.175	0.0175	0.1575

Step 4—The final score of u_i is computed as follows.

$$\text{Score}(u_1) = 0.285 - 0.0525 = 0.2325,$$
$$\text{Score}(u_2) = 0.15 - 0.035 = 0.115,$$
$$\text{Score}(u_3) = 0.24 - 0.035 = 0.205,$$
$$\text{Score}(u_4) = 0.12 - 0.0175 = 0.1025,$$
$$\text{Score}(u_5) = 0.09 - 0.0875 = 0.0025.$$

Step 5—$\text{Score}(u_1) = 0.2325$ is the maximum. Hence, the best decision for the experts is to select worker u_1 as the company's employee.

6. Comparison Analysis

A generalized neutrosophic soft expert model gives more precision, flexibility, and compatibility than the existing neutrosophic models. These are verified by a comparison analysis, using neutrosophic soft expert decision method, with those methods used by Sahin et al. [27], Hassan [44], and Maji [20], as given in Table 5. The comparison is done based on the same example as in Section 5. The ranking order results obtained are consistent with those in [20,27,44].

Table 5. Comparison of neutrosophic soft set to other variants.

Methods	Neutrosophic Soft Set	Neutrosophic Soft Expert Set	*Q*-Neutrosophic Soft Expert Set	Generalized Neutrosophic Soft Expert Set
Authors	Maji [20]	Sahin et al. [27]	Hassan et al. [44]	Proposed Method
Domain	Universe of discourse	Universe of discourse	Universe of discourse	Universe of discourse
Co-domain	$[0,1]^3$	$[0,1]^3$	$[0,1]^3$	$[0,1]^3$
True	Yes	Yes	Yes	Yes
Falsity	Yes	Yes	Yes	Yes
Indeterminacy	Yes	Yes	Yes	Yes
Expert	No	Yes	Yes	Yes
Q	No	No	Yes	No
Ranking	$u_2 > u_3 > u_1 > u_4 > u_5$	$u_2 > u_2 > u_1 > u_4 > u_5$	$u_3 > u_1 > u_2 > u_4 > u_5$	$u_1 > u_3 > u_2 > u_4 > u_5$

7. Conclusions

We have established the concept of generalized neutrosophic soft expert set (GNSES) as a generalization of NSES. The basic operations of GNSES of complement, union, intersection AND, and OR were defined. Subsequently, a definition of GNSES-aggregation operator was proposed to construct an algorithm of a GNSES decision method. Finally, an application of the constructed algorithm, to solve a decision-making , was provided. This new extension provides a significant contribution to current theories for handling indeterminacy, and it spurs the development of further research and pertinent applications.

Author Contributions: All authors contributed equally.

Funding: This research was funded by Universiti Kebangsaan Malaysia grant number GUP-2017-105.

Acknowledgments: We are indebted to Universiti Kebangsaan Malaysia for providing financial support and facilities for this research under the grant GUP-2017-105.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Set Syst.* **1986**, *20*, 87–96, doi:10.1016/S0165- 0114(86)80034-3.
2. Molodtsov, D. Soft set theory-first results. *Comput. Math. Appl.* **1999**, *37*, 19–31, doi:10.1016/S0898-1221(99)00056-5.
3. Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic*; American Research Press: Rehoboth, IL, USA, 1998.
4. Smarandache, F. Neutrosophic set—A generalization of the intuitionistic fuzzy set. *J. Def. Resour. Manag.* **2010**, *1*, 107–116.
5. Alkhazaleh, S.; Salleh, A.R.; Hassan, N. Possibility fuzzy soft set. *Adv. Decis. Sci.* **2011**, 479756, doi:10.1155/2011/479756.
6. Alkhazaleh, S.; Salleh, A.R.; Hassan, N. Soft multisets theory. *Appl. Math. Sci.* **2011**, *5*, 3561–3573**.**
7. Salleh, A.R.; Alkhazaleh, S.; Hassan, N.; Ahmad, A.G. Multiparameterized soft set. *J. Math. Stat.* **2012**, *8*, 92–97, doi:10.3844/jmssp.2012.92.97.
8. Alhazaymeh, K.; Halim, S.A.; Salleh, A.R.; Hassan, N. Soft intuitionistic fuzzy sets. *Appl. Math. Sci.* **2012**, *6*, 2669–2680.
9. Adam, F.; Hassan, N. Q-fuzzy soft matrix and its application. *AIP Conf. Proc.* **2014**, *1602*, 772–778, doi:10.1063/1.4882573.
10. Adam, F.; Hassan, N. Q-fuzzy soft set. *Appl. Math. Sci.* **2014**, *8*, 8689–8695, doi:10.12988/ams.2014.410865.
11. Adam, F.; Hassan, N. Operations on Q-fuzzy soft set. *Appl. Math. Sci.* **2014**, *8*, 8697–8701, doi:10.12988/ams.2014.410866.

12. Adam, F.; Hassan, N. Multi Q-fuzzy parameterized soft set and its application. *J. Intell. Fuzzy Syst*. **2014**, *27*, 419–424, doi:10.3233/IFS-131009.
13. Adam, F.; Hassan, N. Properties on the multi Q-fuzzy soft matrix. *AIP Conf. Proc.* **2014**, *1614*, 834–839, doi:10.1063/1.4895310.
14. Adam, F.; Hassan, N. Multi Q-fuzzy soft set and its application. *Far East J. Math. Sci.* **2015**, *97*, 871–881, doi:10.17654/FJMSAug2015_871_881.
15. Fatimah, F.; Rosadi, D.; Hakim, R.F.; Alcantud, J.C.R. N-soft sets and their decision making algorithms. *Soft Comput.* **2018**, *22*, 3829–3842.
16. Akram, M.; Adeel, A.; Alcantud, J.C.R. Group decision-making methods based on hesitant N-soft sets. *Expert Syst. Appl.* **2019**, *115*, 95–105.
17. Akram, M.; Adeel, A.; Alcantud, J.C.R. Fuzzy N-soft sets: A novel model with applications. *J. Intell. Fuzzy Syst.* **2018**, 1–15, doi:10.3233/JIFS-18244.
18. Varnamkhasti, M.J.; Hassan, N. A hybrid of adaptive neuro-fuzzy inference system and genetic algorithm. *J. Intell. Fuzzy Syst*. **2013**, *25*, 793–796, doi:10.3233/IFS-120685.
19. Varnamkhasti, M.J.; Hassan, N. Neurogenetic algorithm for solving combinatorial engineering problems. *J. Appl. Math.* **2012**, *2012*, 253714, doi:10.1155/2012/253714.
20. Maji, P.K. Neutrosophic soft set. *Comput. Math. Appl.* **2013**, *45*, 555–562, doi:10.1016/S0898-1221(03)00016-6.
21. Alhazaymeh, K.; Hassan, N. Generalized interval-valued vague soft set. *Appl. Math. Sci.* **2013**, *7*, 6983–6988, doi:10.12988/ams.2013.310575.
22. Alhazaymeh, K.; Hassan, N. Vague soft multiset theory. *Int. J. Pure Appl. Math.* **2014**, *93*, 511–523, doi:10.12732/ijpam.v93i4.3.
23. Hassan, N.; Alhazaymeh, K. Vague soft expert set theory. *AIP Conf. Proc.* **2013**, *1522*, 953–958, doi:10.1063/1.4801233.
24. Alhazaymeh, K.; Hassan, N. Mapping on generalized vague soft expert set. *Int. J. Pure Appl. Math.* **2014**, *93*, 369–376, doi:10.12732/ijpam.v93i3.7.
25. Adam, F.; Hassan, N. Multi Q-Fuzzy soft expert set and its applications. *J. Intell. Fuzzy Syst.* **2016**, *30*, 943–950, doi:10.3233/IFS-151816.
26. Al-Qudah, Y.; Hassan, N. Bipolar fuzzy soft expert set and its application in decision making. *Int. J. Decis. Sci.* **2017**, *10*, 175–191, doi:10.1504/IJADS.2017.084310.
27. Sahin, M.; Alkhazaleh, S.; Ulucay, V. Neutrosophic soft expert sets. *Appl. Math.* **2015**, *6*, 116–127, doi:10.4236/am.2015.61012.
28. Al-Quran, A.; Hassan, N. Neutrosophic vague soft expert set theory. *J. Intell. Fuzzy Syst.* **2016**, *30*, 3691–3702, doi:10.3233/IFS-162118.
29. Al-Quran, A.; Hassan, N. Fuzzy parameterised single valued neutrosophic soft expert set theory and its application in decision making. *Int. J. Appl. Decis. Sci.* **2016**, *9*, 212–227, doi:10.1504/IJADS.2016.080121.
30. Lu, Z.; Ye, J. Cosine measures of neutrosophic cubic sets for multiple attribute decision making. *Symmetry* **2017**, *9*, 121, doi:10.3390/sym9070121.
31. Tu, A.; Ye, J.; Wang, B. Multiple attribute decision-making method using similarity measures of neutrosophic cubic sets. *Symmetry* **2018**, *10*, 215, doi:10.3390/sym10060215.
32. Cui, W.; Ye, J. Multiple-attribute decision-making method using similarity measures of hesitant linguistic neutrosophic numbers regarding least common multiple cardinality. *Symmetry* **2018**, *10*, 330, doi:10.3390/sym10080330.
33. Beg, I.; Rashid, T. Group decision making using intuitonistic hesitant fuzzy sets. *Int. J. Fuzzy Logic Intell. Syst.* **2014**, *14*, 181–187, doi:10.5391/IJFIS.2014.14.3.181.
34. Medina, J.; Ojeda-Aciego, M. Multi-adjoint t-concept lattices. *Inf. Sci.* **2010**, *180*, 712–725, doi:10.1016/j.ins.2009.11.018.
35. Pozna, C.; Minculete, N.; Precup, R.E.; Kóczy, L.T.; Ballagi, Á. Signatures: Definitions, operators and applications to fuzzy modelling. *Fuzzy Sets Syst.* **2012**, *201*, 86-104, doi:10.1016/j.fss.2011.12.016.
36. Nowaková, J.; Prílepok, M.; Snášel, V. Medical image retrieval using vector quantization and fuzzy S-tree. *J. Med. Syst.* **2017**, *41*, 18, doi:10.1007/s10916-018-0957-y.
37. Kumar, A.; Kumar, D.; Jarial, S.K. A hybrid clustering method based on improved artificial bee colony and fuzzy C-Means algorithm. *Int. J. Artif. Intell*, **2017**, *15*, 24–44.

38. Liu, F.; Aiwu, G.; Lukovac, V.; Vukic, M. A multicriteria model for the selection of the transport service provider: A single valued neutrosophic DEMATEL multicriteria model. *Decis. Mak. Appl. Manag. Eng.* **2018**, *1*, doi:10.31181/dmame1802128l.
39. Mukhametzyanov, I.; Pamucar, D. A sensitivity analysis in MCDM problems: A statistical approach. *Decis. Mak. Appl. Manag. Eng.* **2018**, *1*, doi:10.31181/dmame1802050m.
40. Şahin, M.; Kargın, A. Neutrosophic triplet normed space. *Open Phys.* **2017**, *15*, 697–704, doi:10.1515/phys-2017-0082.
41. Şahin, M.; Kargın, A.; Çoban, M. Fixed point theorem for neutrosophic triplet partial metric space. *Symmetry* **2018**, *10*, 240, doi:10.3390/sym10070240.
42. Abu Qamar, M.; Hassan, N. Q-neutrosophic soft relations and its application in decision making. *Entropy* **2018**, *20*, 172, doi:10.3390/e20030172.
43. Abu Qamar, M.; Hassan, N. Entropy, measures of distance and similarity of Q-neutrosophic soft sets and some applications. *Entropy* **2018**, *20*, 672, doi:10.3390/e20090672.
44. Hassan, N.; Ulucay, V.; Sahin, M. Q-neutrosophic soft expert set and its application in decision making. *Int. J. Fuzzy Syst. Appl.* **2018**, *7*, 37–61, doi:10.4018/IJFSA.2018100103.
45. Broumi, S.; Deli, I.; Smarandache, F. Neutrosophic parametrized soft set theory and its decision making. *Int. Front. Sci. Lett.* **2014**, *1*, 1–11.