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Generalized Neutrosophic TOPSIS to Solve Multi-Criteria Decision-Making Problems

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Abstract: Multi-criteria decision making (MCDM) is the technique of selecting the best alternative from multiple alternatives and multiple conditions. The technique for order preference by similarity to an ideal solution (TOPSIS) is a crucial practical technique for ranking and selecting different options by using a distance measure. In this article, we protract the fuzzy TOPSIS technique to neutrosophic fuzzy TOPSIS, and prove the accuracy of the method by explaining the MCDM problem with single-value neutrosophic information, and use the method for supplier selection in the production industry. We hope that this article will promote future scientific research on numerous existence issues based on multi-criteria decision making.

Keywords: Neutrosophic set, Single valued Neutrosophic set, TOPSIS, MCDM

1. Introduction

We faced a lot of complications in different areas of life which contains vagueness such as engineering, economics, modeling, and medical diagnoses, etc. However, a general question is raised that in mathematical modeling how we can express and use the uncertainty. A lot of researchers in the world proposed and recommended different approaches to solve those problems that contain uncertainty. In decision-making problems, multiple attribute decision making (MADM) is the most essential part which provides us to find the most appropriate and extraordinary alternative. However, to choose the appropriate alternative is very difficult because of vague information in some cases. To overcome such situations, Zadeh developed the notion of fuzzy sets (FSs) [1] to solve those problems which contain uncertainty and vagueness. It is observed that in some cases circumstances cannot be handled by fuzzy sets, to overcome such types of situations Turksen [2] gave the idea of interval-valued fuzzy sets (IVFSs). In some cases, we must deliberate membership unbiassed as the non- membership values for the suitable representation of an object in uncertain and indeterminate conditions that could not be handled by FSs nor IVFSs. To overcome these difficulties Atanassov offered the concept of Intuitionistic fuzzy sets (IFSs) [3]. The theory which was presented by Atanassov only deals the insufficient data considering both the membership and non-membership values, but the intuitionistic fuzzy set theory cannot handle the incompatible and imprecise information. To deal with such incompatible and imprecise data Smarandache [4] extended the work of Atanassov IFSs and proposed a powerful tool comparative to FSs and IFSs to deal with indeterminate, incomplete, and inconsistent information's which faced in real-life problems. Since the direct use of Neutrosophic sets (NSs) for TOPSIS is somewhat difficult. To apply the NSs, Wang et al. introduced a subclass of NSs known as single-valued Neutrosophic sets (SVNSs) in [5]. In [6] the author proposed a geometric interpretation by using NSs. Gulfam et al. [7] introduced a new distance formula for SVNSs and developed some new techniques under the Neutrosophic environment. The concept of a single-valued Neutrosophic soft expert set proposed in [8] by combining the SVNSs and soft expert sets.

To solve MCDM problems with single-valued Neutrosophic numbers (SVNNs) presented by Deli and Subas in [9], they constructed the concept of cut sets of SVNNs. On the base of the correlation of IFSs, the term correlation coefficient of SVNSs [10] introduced and proposed a decision-making method by using a weighted correlation coefficient or the weighted cosine similarity measure of SVNSs. In [11] the idea of simplified Neutrosophic sets introduced with some operational laws and aggregation operators such as real-life Neutrosophic weighted arithmetic average operator and weighted geometric average operator. They constructed an MCDM method on the base of proposed aggregation operators and cosine similarity measure for simplified neutrosophic sets. Sahin and Yiğider [12] extended the TOPSIS method to MCDM with a single-valued neutrosophic technique.

The TOPSIS method is presented in [13] to solve multi-criteria decision problems with different choices. In [14], Chen & Hwang extended the idea of the TOPSIS method and proposed a new TOPSIS model. The author uses the newly proposed decision-making method to solve uncertain data [15]. In [16], the authors applied this method to the prediction of diabetic patients in medical diagnosis. In [17–19] the authors studied the soft set TOPSIS, fuzzy TOPSIS, and Intuitionistic Fuzzy TOPSIS respectively and used for decision making. In [20], for the solution of single-valued neutrosophic soft set expert based multi-attribute decision-making problems, the authors proposed the TOPSIS technique. Generalized fuzzy TOPSIS was given in [21,22] with accuracy function. Maji [23] proposed the concept of neutrosophic soft sets (NSSs) with some properties and operations. Authors studied NSSs and gave some new definitions on NSSs [24], they also gave the idea of neutrosophic soft matrices with some operations and proposed a decision-making method. Many researchers developed the decision-making models by using the NSSs reported in the literature [25–27]. Elhassouny and Smarandache [28] extended the work on a simplified TOPSIS method and by using single-valued Neutrosophic information they proposed Neutrosophic simplified TOPSIS method. Saqlain et.al [21] presented generalized neutrosophic TOPSIS using accuracy function for the neutrosophic hypersoft set environment. The concept of single-valued neutrosophic cross-entropy measure introduced by Jun [29], he also constructed an MCDM method and claimed that this proposed method is more appropriate than previous methods for decision making.

Saha and Broumi [31], studied the interval-valued neutrosophic sets (IVNSs) and developed some new set-theoretic operations on IVNSs with their properties. The idea of an Interval-valued generalized single valued neutrosophic trapezoidal number (IVGSVTrN) was presented by Deli [32] with some operations and discussed their properties based on neutrosophic numbers. Hashim et al [33], studied the vague set and interval neutrosophic set and established a new theory known as interval neutrosophic vague set (INVS), they also presented some operations for INVS with their properties and derived the properties by using numerical examples. In [34], Abdel basset et al. applied TODIM and TOPSIS methods based on the best-worst method to increase the accuracy of evaluation under uncertainty according to the NSs. They also used the plithogenic set theory to resolve the indeterminate information and evaluate the economic performance of manufacturing industries, they used the AHP method to find the weight vector of the financial ratios to achieve this goal after that they used the VIKOR and TOPSIS methods to utilize the companies ranking [35, 36].

In the following paragraph, we explain some positive impacts of this research. The concentration of this study is to evaluate the best supplier for the production industry. This research is a very suitable illustration of Neutrosophic TOPSIS. A group of decision-makers chooses the best supplier for the production industry. The Neutrosophic TOPSIS method increases alternative performances based on the best and worst solutions.

1.1 Motivation and Contribution

Classical TOPSIS uses clear techniques for language assessment, but due to the imprecision and ambiguity of language assessment, we propose neutrosophic TOPSIS. In this paper, we discuss the

NSs and SVNSs with some operations. We presented the generalization of TOPSIS for the SVNSs and use the proposed method for supplier selection.

1.2 Structure of Article

In Section 2, some basic definitions have been added, which will help the rest of this article. Section 3 consists of the main work of the article, which defines the neutrosophic TOPSIS algorithm. The application of the proposed method and calculations are presented in section 4 and finally, the conclusion draws in Section 5.

2. Preliminaries

In this section, we remind some basic definitions such as NSs and SVNSs with some operations that will be used in the following sequel.

Neutrosophic Set (NS) [30]: Let X be a space of points and x be an arbitrary element of X. A neutrosophic set A in X is defined by a Truth-membership function $T_A(x)$, an Indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$ i.e.; $T_A(x), I_A(x), F_A(x)$: $X \to]0^-, 1^+[$, and $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Single Valued Neutrosophic Sets [5]**:** Let E be a universe. An SVNS over E is an NS over E, but truthiness, indeterminacy, and falsity membership functions are defined

$T_A(x)$: X → [0, 1], $I_A(x)$: X → [0, 1], $F_A(x)$: X → [0, 1], and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Multiplication of SVNS [11]: Let A = $\{\alpha_1, \alpha_2, \alpha_3\}$ and B = $\{\beta_1, \beta_2, \beta_3\}$ are two SVN numbers, then their multiplication is defined as follows A ⊗ B = $(\alpha_1\beta_1, \alpha_2 + \beta_2 - \alpha_2\beta_2, \alpha_3 + \beta_3 - \alpha_3\beta_3)$.

3. Neutrosophic TOPSIS [11]

3. 1. Algorithm for Neutrosophic TOPSIS using SVNNs

To explain the procedure of Neutrosophic TOPSIS using SVNNs the following steps are followed. Let A = {A_1, A_2, A_3, …., A_m} be a set of alternatives and C = {C_1, C_2, C_3, …., C_n} be a set of evaluation criteria and *DM* be a set of "l" decision-makers as follows *DM* = {DM_1, DM_2, DM_3,…, DM_l}. In the form of linguistic variables, the importance of the evaluation criteria, DMs, and alternative ratings are given in Table 1.

Step 1: Computation of weights of the DMs

Let the SVN number for rating the k^{th} DM is denoted by

$D_k = (T_k^{dm}, I_k^{dm}, F_k^{dm})$

Weight of the k^{th} DM can be found by the following formula

$$\lambda_k = \frac{1-\left[\frac{1}{3}\left\{\left(1-T_k^{dm}(x)\right)^2+\left(I_k^{dm}(x)\right)^2+\left(F_k^{dm}(x)\right)^2\right\}\right]^{0.5}}{\sum_{k=1}^{l}\left(1-\left[\frac{1}{3}\left\{\left(1-T_k^{dm}(x)\right)^2+\left(I_k^{dm}(x)\right)^2+\left(F_k^{dm}(x)\right)^2\right\}\right]^{0.5}\right)} \text{ ; where } \lambda_k \geq 0 \text{ and } \sum_{k=1}^{l}\lambda_k = 1$$

Step 2: Computation of the Aggregated Neutrosophic Decision Matrix (ANDM)

The ANDM is given as follows

$$D = \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} = [r_{ij}]_{m\times n}$$

where r_{ij} can be defined as

$r_{ij} = (T_{ij},\ I_{ij},\ F_{ij}) = (T_{A_i}(x_j),\ I_{A_i}(x_j),\ F_{A_i}(x_j))$, where $i = 1, 2, 3, \ldots, m;\ j = 1, 2, 3, \ldots, n$

Therefore, ANDM written as follows

$$D = \begin{bmatrix} (T_{A_1}(x_1), I_{A_1}(x_1), F_{A_1}(x_1)) & (T_{A_1}(x_2), I_{A_1}(x_2), F_{A_1}(x_2)) & \cdots & (T_{A_1}(x_n), I_{A_1}(x_n), F_{A_1}(x_n)) \\ (T_{A_2}(x_1), I_{A_2}(x_1), F_{A_2}(x_1)) & (T_{A_2}(x_2), I_{A_2}(x_2), F_{A_2}(x_2)) & \cdots & (T_{A_2}(x_n), I_{A_2}(x_n), F_{A_2}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{A_m}(x_1), I_{A_m}(x_1), F_{A_m}(x_1)) & (T_{A_m}(x_2), I_{A_m}(x_2), F_{A_m}(x_2)) & \cdots & (T_{A_m}(x_n), I_{A_m}(x_n), F_{A_m}(x_n)) \end{bmatrix}$$

rating for the i^{th} alternative w.r.t. the j^{th} criterion by the k^{th} *DM*

$$r_{ij}^{(k)} = (T_{ij}^{(k)},\ I_{ij}^{(k)},\ F_{ij}^{(k)})$$

For *DM* weights and alternative ratings r_{ij} can be calculated by using a single-valued neutrosophic weighted averaging operator (SVNWAO)

$$r_{ij} = \left[1 - \prod_{k=1}^{l}(1 - T_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(I_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(F_{ij}^{(k)})^{\lambda_k}\right]$$

Step 3: Computation of the weights for the criteria

Let an SVNN allocated to the criterion by X_j the k^{th} *DM* is denoted as

$$w_j^{(k)} = \left(T_j^{(k)}, I_j^{(k)}, F_j^{(k)}\right)$$

SVNWAO to compute the weights of the criteria is given as follows

$$w_j = \left[1 - \prod_{k=1}^{l}(1 - T_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(I_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(F_j^{(k)})^{\lambda_k}\right]$$

The aggregated weight for the criterion X_j is represented as

$$w_j = (T_j,\ I_j,\ F_j) \qquad j = 1, 2, 3, \ldots, n$$

$$W = [w_1, w_2, w_3, \ldots, w_n]^{Transpose}$$

Step 4: Computation of Aggregated Weighted Neutrosophic Decision Matrix (AWNDM)

The AWNDM is calculated as follows

$$R' = \begin{bmatrix} r'_{11} & r'_{12} & \cdots & r'_{1n} \\ r'_{21} & r'_{22} & \cdots & r'_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r'_{m1} & r'_{m2} & \cdots & r'_{mn} \end{bmatrix} = \left[r'_{ij}\right]_{m\times n}$$

where $r'_{ij} = \left(T_{A_i.W}(x_j), I_{A_i.W}(x_j), F_{A_i.W}(x_j)\right)$ where $i = 1, 2, 3, \ldots, m;\ j = 1, 2, 3, \ldots, n.$

Therefore, R' can be written as

$$R' = \begin{bmatrix} (T_{A_1.W}(x_1), I_{A_1.W}(x_1), F_{A_1.W}(x_1)) & (T_{A_1.W}(x_2), I_{A_1.W}(x_2), F_{A_1.W}(x_2)) & \cdots & (T_{A_1.W}(x_n), I_{A_1.W}(x_n), F_{A_1.W}(x_n)) \\ (T_{A_2.W}(x_1), I_{A_2.W}(x_1), F_{A_2.W}(x_1)) & (T_{A_2.W}(x_2), I_{A_2.W}(x_2), F_{A_2.W}(x_2)) & \cdots & (T_{A_2.W}(x_n), I_{A_2.W}(x_n), F_{A_2.W}(x_n)) \\ \vdots & \vdots & \ddots & \vdots \\ (T_{A_m.W}(x_1), I_{A_m.W}(x_1), F_{A_m.W}(x_1)) & (T_{A_m.W}(x_2), I_{A_m.W}(x_2), F_{A_m.W}(x_2)) & \cdots & (T_{A_m.W}(x_n), I_{A_m.W}(x_n), F_{A_m.W}(x_n)) \end{bmatrix}$$

To find $T_{A_i.W}(x_j)$, $I_{A_i.W}(x_j)$ and $F_{A_i.W}(x_j)$ we used

$$R \otimes W = \left\{\langle x, T_{A_i.W}(x)\rangle, \langle x, I_{A_i.W}(x)\rangle, \langle x, F_{A_i.W}(x)\rangle \,\middle|\, x \in X\right\}$$

The components of the product given as

$T_{A_i.W}(x) = T_{A_i}(x).\ T_j$

$I_{A_i.W}(x) = I_{A_i}(x) + I_j(x) - I_{A_i}(x) \times I_j(x)$

$F_{A_i.W}(x) = F_{A_i}(x) + F_j(x) - F_{A_i}(x) \times F_j(x)$

Step 5: Computation of Single Valued Neutrosophic Positive Ideal Solution (SVN-PIS) and Single Valued Neutrosophic Positive Ideal Solution (SVN-NIS)

Let J_1 be the benefit criteria and J_2 be the cost criteria. A^* be an SVN-PIS and A' be an SVN-NIS as follows

$A^* = (T_{A^*W}(x_j),\ I_{A^*W}(x_j),\ F_{A^*W}(x_j))$ and

$A' = (T_{A'W}(x_j),\ I_{A'W}(x_j),\ F_{A'W}(x_j))$

The components of SVN-PIS and SVN-NIS are following

$$T_{A^*W}(x_j) = \left(\left(\max_i T_{A_i.W}(x_j) \mid j \in j_1\right), \left(\min_i T_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

$$I_{A^*W}(x_j) = \left(\left(\min_i I_{A_i.W}(x_j) \mid j \in j_1\right), \left(\max_i I_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

$$F_{A^*W}(x_j) = \left(\left(\min_i F_{A_i.W}(x_j) \mid j \in j_1\right), \left(\max_i F_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

$$T_{A'W}(x_j) = \left(\left(\min_i T_{A_i.W}(x_j) \mid j \in j_1\right), \left(\max_i T_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

$$I_{A'W}(x_j) = \left(\left(\max_i I_{A_i.W}(x_j) \mid j \in j_1\right), \left(\min_i I_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

$$F_{A'W}(x_j) = \left(\left(\max_i F_{A_i.W}(x_j) \mid j \in j_1\right), \left(\min_i F_{A_i.W}(x_j) \mid j \in j_2\right)\right)$$

Step 6: Computation of Separation Measures

For the separation measures d^* and d', Normalized Euclidean Distance is used as given as

$$d_i^* = \left(\frac{1}{3n}\sum_{j=1}^{n}\left[\left(T_{A_i.W}(x_j) - T_{A^*W}(x_j)\right)^2 + \left(I_{A_i.W}(x_j) - I_{A^*W}(x_j)\right)^2 + \left(F_{A_i.W}(x_j) - F_{A^*W}(x_j)\right)^2\right]\right)^{0.5}$$

$$d_i' = \left(\frac{1}{3n}\sum_{j=1}^{n}\left[\left(T_{A_i.W}(x_j) - T_{A'W}(x_j)\right)^2 + \left(I_{A_i.W}(x_j) - I_{A'W}(x_j)\right)^2 + \left(F_{A_i.W}(x_j) - F_{A'W}(x_j)\right)^2\right]\right)^{0.5}$$

Step 7: Computation of Relative Closeness Coefficient (*RCC*)

The *RCC* of an alternative *Ai* w.r.t. the SVN-PIS A^* is computed as

$$RCCi = \frac{d_i'}{d_i' + d_i^*} \qquad \text{where } 0 \le RCCi \le 1$$

Step 8: Ranking alternatives

After computation of *RCCi* for each alternative A_i, the rank of the alternatives presented in descending orders of *RCCi*.

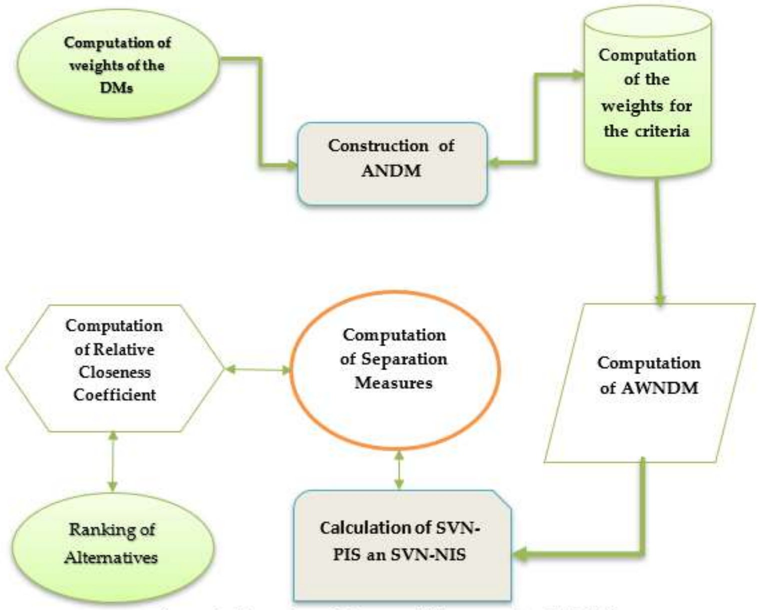

Figure 1 *Algorithm of Proposed Neutrosophic TOPSIS*

4. Application of Neutrosophic TOPSIS in decision making

A production industry wants to hire a supplier, for the selection of supplier managing director of the industry decides the criteria for supplier selection. The industry hires a team of decision-makers for the selection of the best supplier. Consider A = {A_i: i = 1, 2, 3, 4, 5} be a set of supplier and $DM = \{DM_1, DM_2, DM_3, DM_4\}$ be a team of decision-makers (l = 4). The evaluation criteria (n = 5) for the selection of supplier given as follows,

$$C = \begin{cases} Benifit\ Criteria \\ Cost\ Criteria \end{cases} \quad j_1 = \begin{cases} X_1: & Delivery \\ X_2: & Quality \\ X_3: & Flexibility \\ X_4: & Service \end{cases} \quad j_2 = \{X_5 : Price$$

Calculations of the problem using the proposed SVN-TOPSIS for the importance of criteria and DMs SVN rating scale is given in the following Table

Table 1. Linguistic variables LV's for rating the importance of criteria and decision-makers

LVs	SVNNs
VI	(.90, .10, .10)
I	(.75, .25, .20)
M	(.50, .50, .50)
UI	(.35, .75, .80)
VUI	(.10, .90, .90)

Where VI, I, M, UI, VUI stand for very important, important, medium, unimportant, very unimportant respectively. The alternative ratings are given in the following table

Table 2. Alternative Ratings for Linguistic Variables

LVs	SVNNs
EG	(1.0, 0.0,0.0)

VVG	(.90, .10, .10)
VG	(.80, .15, .20)
G	(.70, .25, .30)
MG	(.60, .35, .40)
M	(.50, .50, .50)
MB	(.40, .65, .60)
B	(.30, .75, .70)
VB	(.20, .85, .80)
VVB	(.10, .90, .90)
EB	(0.0,1.0,1.0)

Where EG, VVG, VG, G, MG, M, MB, B, VB, VVB, EB are representing extremely good, very very good, very good, good, medium good, medium, medium bad, bad, very bad, very very bad, extremely bad respectively.

Step 1: Determine the weights of the DMs

Weights for the *DMs* are calculated as follows

$$\lambda_k = \frac{1-\left[\frac{1}{3}\left\{\left(1-T_k^{dm}(x)\right)^2+\left(I_k^{dm}(x)\right)^2+\left(F_k^{dm}(x)\right)^2\right\}\right]^{0.5}}{\sum_{k=1}^{l}\left(1-\left[\frac{1}{3}\left\{\left(1-T_k^{dm}(x)\right)^2+\left(I_k^{dm}(x)\right)^2+\left(F_k^{dm}(x)\right)^2\right\}\right]^{0.5}\right)} \;;\; \lambda_k \geq 0 \text{ and } \sum_{k=1}^{l}\lambda_k = 1$$

$$\lambda_1 = \frac{1-\left[\frac{1}{3}\left\{\left(1-T_1^{dm}(x)\right)^2+\left(I_1^{dm}(x)\right)^2+\left(F_1^{dm}(x)\right)^2\right\}\right]^{0.5}}{\sum_{k=1}^{l}\left(1-\left[\frac{1}{3}\left\{\left(1-T_k^{dm}(x)\right)^2+\left(I_k^{dm}(x)\right)^2+\left(F_k^{dm}(x)\right)^2\right\}\right]^{0.5}\right)}$$

$$\lambda_1 = \frac{1-\left[\frac{1}{3}\left\{\left(1-T_1^{dm}(x)\right)^2+\left(I_1^{dm}(x)\right)^2+\left(F_1^{dm}(x)\right)^2\right\}\right]^{0.5}}{\begin{array}{c}1-\left[\frac{1}{3}\left\{\left(1-T_1^{dm}(x)\right)^2+\left(I_1^{dm}(x)\right)^2+\left(F_1^{dm}(x)\right)^2\right\}\right]^{0.5}+1-\left[\frac{1}{3}\left\{\left(1-T_2^{dm}(x)\right)^2+\left(I_2^{dm}(x)\right)^2+\left(F_2^{dm}(x)\right)^2\right\}\right]^{0.5}+\\ 1-\left[\frac{1}{3}\left\{\left(1-T_3^{dm}(x)\right)^2+\left(I_3^{dm}(x)\right)^2+\left(F_3^{dm}(x)\right)^2\right\}\right]^{0.5}+1-\left[\frac{1}{3}\left\{\left(1-T_4^{dm}(x)\right)^2+\left(I_4^{dm}(x)\right)^2+\left(F_4^{dm}(x)\right)^2\right\}\right]^{0.5}\end{array}}$$

$$\lambda_1 = \frac{1-\left[\frac{1}{3}\left\{(1-0.9)^2+(0.10)^2+(0.10)^2\right\}\right]^{0.5}}{\begin{array}{c}1-\left[\frac{1}{3}\left\{(1-0.9)^2+(0.10)^2+(0.10)^2\right\}\right]^{0.5}+1-\left[\frac{1}{3}\left\{(1-0.75)^2+(0.25)^2+(0.20)^2\right\}\right]^{0.5}+\\ 1-\left[\frac{1}{3}\left\{(1-0.50)^2+(0.50)^2+(0.50)^2\right\}\right]^{0.5}+1-\left[\frac{1}{3}\left\{(1-0.35)^2+(0.75)^2+(0.80)^2\right\}\right]^{0.5}\end{array}}$$

$$\lambda_1 = \frac{0.9}{0.9+0.76548+0.5+0.26402}$$

$$\lambda_1 = \frac{0.9}{2.42950} = 0.37045$$

$\lambda_1 = 0.37045$

Similarly, we get the weights for the other decision-makers as follows

$$\lambda_2 = \frac{0.76548}{2.42950} = 0.31508$$

$\lambda_2 = 0.31508$

$$\lambda_3 = \frac{0.5}{2.42950} = 0.20580$$

$\lambda_3 = 0.20580$

$$\lambda_4 = \frac{0.26402}{2.42950} = 0.10867$$

$\lambda_4 = 0.10867$

The weights for *DMs* are given in the following Table

Table 3. Weights of Decision Makers

Criteria	**Alternatives**	**Decision Makers**			
		DM1	**DM2**	**DM3**	**DM4**
X1	*A1*	VG (0.80,0.15,0.20)	MG (0.60,0.35,0.40)	VG (0.80,0.15,0.20)	G (0.70,0.25,0.30)
		$r_{11}^{(1)} = (T_{11}^{(1)}, I_{11}^{(1)}, F_{11}^{(1)})$	$r_{11}^{(2)} = (T_{11}^{(2)}, I_{11}^{(2)}, F_{11}^{(2)})$	$r_{11}^{(3)} = (T_{11}^{(3)}, I_{11}^{(3)}, F_{11}^{(3)})$	$r_{11}^{(4)} = (T_{11}^{(4)}, I_{11}^{(4)}, F_{11}^{(4)}$
	A2	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)
		$r_{21}^{(1)} = (T_{21}^{(1)}, I_{21}^{(1)}, F_{21}^{(1)})$	$r_{21}^{(2)} = (T_{21}^{(2)}, I_{21}^{(2)}, F_{21}^{(2)})$	$r_{21}^{(3)} = (T_{21}^{(3)}, I_{21}^{(3)}, F_{21}^{(3)})$	$r_{21}^{(4)} = (T_{21}^{(4)}, I_{21}^{(4)}, F_{21}^{(4)})$
	A3	M (0.50,0.50,0.50)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	M (0.50,0.50,0.50)
		$r_{31}^{(1)} = (T_{31}^{(1)}, I_{31}^{(1)}, F_{31}^{(1)})$	$r_{31}^{(2)} = (T_{31}^{(2)}, I_{31}^{(2)}, F_{31}^{(2)})$	$r_{31}^{(3)} = (T_{31}^{(3)}, I_{31}^{(3)}, F_{31}^{(3)})$	$r_{31}^{(4)} = (T_{31}^{(4)}, I_{31}^{(4)}, F_{31}^{(4)})$
	A4	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)
		$r_{41}^{(1)} = (T_{41}^{(1)}, I_{41}^{(1)}, F_{41}^{(1)})$	$r_{41}^{(2)} = (T_{41}^{(2)}, I_{41}^{(2)}, F_{41}^{(2)})$	$r_{41}^{(3)} = (T_{41}^{(3)}, I_{41}^{(3)}, F_{41}^{(3)})$	$r_{41}^{(4)} = (T_{41}^{(4)}, I_{41}^{(4)}, F_{41}^{(4)})$
	A5	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	VG (0.80,0.15,0.20)
		$r_{51}^{(1)} = (T_{51}^{(1)}, I_{51}^{(1)}, F_{51}^{(1)})$	$r_{51}^{(2)} = (T_{51}^{(2)}, I_{51}^{(2)}, F_{51}^{(2)})$	$r_{51}^{(3)} = (T_{51}^{(3)}, I_{51}^{(3)}, F_{51}^{(3)})$	$r_{51}^{(4)} = (T_{51}^{(4)}, I_{51}^{(4)}, F_{51}^{(4)})$
X2	*A1*	G (0.70,0.25,0.30)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)
		$r_{12}^{(1)} = (T_{12}^{(1)}, I_{12}^{(1)}, F_{12}^{(1)})$	$r_{12}^{(2)} = (T_{12}^{(2)}, I_{12}^{(2)}, F_{12}^{(2)})$	$r_{12}^{(3)} = (T_{12}^{(3)}, I_{12}^{(3)}, F_{12}^{(3)})$	$r_{12}^{(4)} = (T_{12}^{(4)}, I_{12}^{(4)}, F_{12}^{(4)})$
	A2	VG (0.80,0.15,0.20)	MG (0.60,0.35,0.40)	M (0.50,0.50,0.50)	MG (0.60,0.35,0.40)
		$r_{22}^{(1)} = (T_{22}^{(1)}, I_{22}^{(1)}, F_{22}^{(1)})$	$r_{22}^{(2)} = (T_{22}^{(2)}, I_{22}^{(2)}, F_{22}^{(2)})$	$r_{22}^{(3)} = (T_{22}^{(3)}, I_{22}^{(3)}, F_{22}^{(3)})$	$r_{22}^{(4)} = (T_{22}^{(4)}, I_{22}^{(4)}, F_{22}^{(4)})$
	A3	M (0.50,0.50,0.50)	VG (0.80,0.15,0.20)	G (0.70,0.25,0.30)	G (0.70,0.25,0.30)
		$r_{32}^{(1)} = (T_{32}^{(1)}, I_{32}^{(1)}, F_{32}^{(1)})$	$r_{32}^{(2)} = (T_{32}^{(2)}, I_{32}^{(2)}, F_{32}^{(2)})$	$r_{32}^{(3)} = (T_{32}^{(3)}, I_{32}^{(3)}, F_{32}^{(3)})$	$r_{32}^{(4)} = (T_{32}^{(4)}, I_{32}^{(4)}, F_{32}^{(4)})$
	A4	MG (0.60,0.35,0.40)	M (0.50,0.50,0.50)	VG (0.80,0.15,0.20)	M (0.50,0.50,0.50)
		$r_{42}^{(1)} = (T_{42}^{(1)}, I_{42}^{(1)}, F_{42}^{(1)})$	$r_{42}^{(2)} = (T_{42}^{(2)}, I_{42}^{(2)}, F_{42}^{(2)})$	$r_{42}^{(3)} = (T_{42}^{(3)}, I_{42}^{(3)}, F_{42}^{(3)})$	$r_{42}^{(4)} = (T_{42}^{(4)}, I_{42}^{(4)}, F_{42}^{(4)})$
	A5	G (0.70,0.25,0.30)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	VG (0.80,0.15,0.20)
		$r_{52}^{(1)} = (T_{52}^{(1)}, I_{52}^{(1)}, F_{52}^{(1)})$	$r_{52}^{(2)} = (T_{52}^{(2)}, I_{52}^{(2)}, F_{52}^{(2)})$	$r_{52}^{(3)} = (T_{52}^{(3)}, I_{52}^{(3)}, F_{52}^{(3)})$	$r_{52}^{(4)} = (T_{52}^{(4)}, I_{52}^{(4)}, F_{52}^{(4)})$
X3	*A1*	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)	M (0.50,0.50,0.50)	M (0.50,0.50,0.50)
		$r_{13}^{(1)} = (T_{13}^{(1)}, I_{13}^{(1)}, F_{13}^{(1)})$	$r_{13}^{(2)} = (T_{13}^{(2)}, I_{13}^{(2)}, F_{13}^{(2)})$	$r_{13}^{(3)} = (T_{13}^{(3)}, I_{13}^{(3)}, F_{13}^{(3)})$	$r_{13}^{(4)} = (T_{13}^{(4)}, I_{13}^{(4)}, F_{13}^{(4)})$
	A2	VG (0.80,0.15,0.20)	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	VG (0.80,0.15,0.20)
		$r_{23}^{(1)} = (T_{23}^{(1)}, I_{23}^{(1)}, F_{23}^{(1)})$	$r_{23}^{(2)} = (T_{23}^{(2)}, I_{23}^{(2)}, F_{23}^{(2)})$	$r_{23}^{(3)} = (T_{23}^{(3)}, I_{23}^{(3)}, F_{23}^{(3)})$	$r_{23}^{(4)} = (T_{23}^{(4)}, I_{23}^{(4)}, F_{23}^{(4)})$
	A3	M (0.50,0.50,0.50)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)
		$r_{33}^{(1)} = (T_{33}^{(1)}, I_{33}^{(1)}, F_{33}^{(1)})$	$r_{33}^{(2)} = (T_{33}^{(2)}, I_{33}^{(2)}, F_{33}^{(2)})$	$r_{33}^{(3)} = (T_{33}^{(3)}, I_{33}^{(3)}, F_{33}^{(3)})$	$r_{33}^{(4)} = (T_{33}^{(4)}, I_{33}^{(4)}, F_{33}^{(4)})$
	A4	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)	MG (0.60,0.35,0.40)
		$r_{43}^{(1)} = (T_{43}^{(1)}, I_{43}^{(1)}, F_{43}^{(1)})$	$r_{43}^{(2)} = (T_{43}^{(2)}, I_{43}^{(2)}, F_{43}^{(2)})$	$r_{43}^{(3)} = (T_{43}^{(3)}, I_{43}^{(3)}, F_{43}^{(3)})$	$r_{43}^{(4)} = (T_{43}^{(4)}, I_{43}^{(4)}, F_{43}^{(4)})$
	A5	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	G (0.70,0.25,0.30)
		$r_{53}^{(1)} = (T_{53}^{(1)}, I_{53}^{(1)}, F_{53}^{(1)})$	$r_{53}^{(2)} = (T_{53}^{(2)}, I_{53}^{(2)}, F_{53}^{(2)})$	$r_{53}^{(3)} = (T_{53}^{(3)}, I_{53}^{(3)}, F_{53}^{(3)})$	$r_{53}^{(4)} = (T_{53}^{(4)}, I_{53}^{(4)}, F_{53}^{(4)})$
X4	*A1*	G (0.70,0.25,0.30)	M (0.50,0.50,0.50)	MG (0.60,0.35,0.40)	M (0.50,0.50,0.50)
		$r_{14}^{(1)} = (T_{14}^{(1)}, I_{14}^{(1)}, F_{14}^{(1)})$	$r_{14}^{(2)} = (T_{14}^{(2)}, I_{14}^{(2)}, F_{14}^{(2)})$	$r_{14}^{(3)} = (T_{14}^{(3)}, I_{14}^{(3)}, F_{14}^{(3)})$	$r_{14}^{(4)} = (T_{14}^{(4)}, I_{14}^{(4)}, F_{14}^{(4)})$
	A2	VG (0.80,0.15,0.20)	VG (0.80,0.15,0.20)	M (0.50,0.50,0.50)	G (0.70,0.25,0.30)
		$r_{24}^{(1)} = (T_{24}^{(1)}, I_{24}^{(1)}, F_{24}^{(1)})$	$r_{24}^{(2)} = (T_{24}^{(2)}, I_{24}^{(2)}, F_{24}^{(2)})$	$r_{24}^{(3)} = (T_{24}^{(3)}, I_{24}^{(3)}, F_{24}^{(3)})$	$r_{24}^{(4)} = (T_{24}^{(4)}, I_{24}^{(4)}, F_{24}^{(4)})$

	A_3	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)	MG (0.60,0.35,0.40)
		$r_{34}^{(1)} = (T_{34}^{(1)}, I_{34}^{(1)}, F_{34}^{(1)})$	$r_{34}^{(2)} = (T_{34}^{(2)}, I_{34}^{(2)}, F_{34}^{(2)})$	$r_{34}^{(3)} = (T_{34}^{(3)}, I_{34}^{(3)}, F_{34}^{(3)})$	$r_{34}^{(4)} = (T_{34}^{(4)}, I_{34}^{(4)}, F_{34}^{(4)})$
	A_4	M (0.50,0.50,0.50)	MB (0.40,0.65,0.60)	MG (0.60,0.35,0.40)	VG (0.80,0.15,0.20)
		$r_{44}^{(1)} = (T_{44}^{(1)}, I_{44}^{(1)}, F_{44}^{(1)})$	$r_{44}^{(2)} = (T_{44}^{(2)}, I_{44}^{(2)}, F_{44}^{(2)})$	$r_{44}^{(3)} = (T_{44}^{(3)}, I_{44}^{(3)}, F_{44}^{(3)})$	$r_{44}^{(4)} = (T_{44}^{(4)}, I_{44}^{(4)}, F_{44}^{(4)})$
	A_5	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	G (0.70,0.25,0.30)
		$r_{54}^{(1)} = (T_{54}^{(1)}, I_{54}^{(1)}, F_{54}^{(1)})$	$r_{54}^{(2)} = (T_{54}^{(2)}, I_{54}^{(2)}, F_{54}^{(2)})$	$r_{54}^{(3)} = (T_{54}^{(3)}, I_{54}^{(3)}, F_{54}^{(3)})$	$r_{54}^{(4)} = (T_{54}^{(4)}, I_{54}^{(4)}, F_{54}^{(4)})$
X5	A_1	M (0.50,0.50,0.50)	MG (0.60,0.35,0.40)	VG (0.80,0.15,0.20)	M (0.50,0.50,0.50)
		$r_{15}^{(1)} = (T_{15}^{(1)}, I_{15}^{(1)}, F_{15}^{(1)})$	$r_{15}^{(2)} = (T_{15}^{(2)}, I_{15}^{(2)}, F_{15}^{(2)})$	$r_{15}^{(3)} = (T_{15}^{(3)}, I_{15}^{(3)}, F_{15}^{(3)})$	$r_{15}^{(4)} = (T_{15}^{(4)}, I_{15}^{(4)}, F_{15}^{(4)})$
	A_2	VG (0.80,0.15,0.20)	M (0.50,0.50,0.50)	G (0.70,0.25,0.30)	G (0.70,0.25,0.30)
		$r_{25}^{(1)} = (T_{25}^{(1)}, I_{25}^{(1)}, F_{25}^{(1)})$	$r_{25}^{(2)} = (T_{25}^{(2)}, I_{25}^{(2)}, F_{25}^{(2)})$	$r_{25}^{(3)} = (T_{25}^{(3)}, I_{25}^{(3)}, F_{25}^{(3)})$	$r_{25}^{(4)} = (T_{25}^{(4)}, I_{25}^{(4)}, F_{25}^{(4)})$
	A_3	G (0.70,0.25,0.30)	G (0.70,0.25,0.30)	M (0.50,0.50,0.50)	MG (0.60,0.35,0.40)
		$r_{35}^{(1)} = (T_{35}^{(1)}, I_{35}^{(1)}, F_{35}^{(1)})$	$r_{35}^{(2)} = (T_{35}^{(2)}, I_{35}^{(2)}, F_{35}^{(2)})$	$r_{35}^{(3)} = (T_{35}^{(3)}, I_{35}^{(3)}, F_{35}^{(3)})$	$r_{35}^{(4)} = (T_{35}^{(4)}, I_{35}^{(4)}, F_{35}^{(4)})$
	A_4	M (0.50,0.50,0.50)	M (0.50,0.50,0.50)	MG (0.60,0.35,0.40)	G (0.70,0.25,0.30)
		$r_{45}^{(1)} = (T_{45}^{(1)}, I_{45}^{(1)}, F_{45}^{(1)})$	$r_{45}^{(2)} = (T_{45}^{(2)}, I_{45}^{(2)}, F_{45}^{(2)})$	$r_{45}^{(3)} = (T_{45}^{(3)}, I_{45}^{(3)}, F_{45}^{(3)})$	$r_{45}^{(4)} = (T_{45}^{(4)}, I_{45}^{(4)}, F_{45}^{(4)})$
	A_5	G (0.70,0.25,0.30)	VG (0.80,0.15,0.20)	VG (0.80,0.15,0.20)	VG (0.80,0.15,0.20)
		$r_{55}^{(1)} = (T_{55}^{(1)}, I_{55}^{(1)}, F_{55}^{(1)})$	$r_{55}^{(2)} = (T_{55}^{(2)}, I_{55}^{(2)}, F_{55}^{(2)})$	$r_{55}^{(3)} = (T_{55}^{(3)}, I_{55}^{(3)}, F_{55}^{(3)})$	$r_{55}^{(4)} = (T_{55}^{(4)}, I_{55}^{(4)}, F_{55}^{(4)})$

Table 4. Importance and Weights of Decision-Makers

	DM1	*DM2*	*DM3*	*DM4*
Linguistic	VI(0.90,0.10,0.10)	I (0.75,0.25,0.20)	M (0.50,0.50,0.50)	UI (0.35,0.75,0.80)
Variables	$(T_1^{dm}, I_1^{dm}, F_1^{dm})$	$(T_2^{dm}, I_2^{dm}, F_2^{dm})$	$(T_3^{dm}, I_3^{dm}, F_3^{dm})$	$(T_4^{dm}, I_4^{dm}, F_4^{dm})$
Weights	λ_{DM_1}= 0.37045	λ_{DM_2}= 0.31508	λ_{DM_3}= 0.20580	λ_{DM_4}= 0.10867

Step 2: Computation of Aggregated Single Valued Neutrosophic Decision Matrix (ASVNDM)

To find the ASVNDM not only the weights of the DMs, but the alternative ratings are also required. The alternative ratings, according to the *DMs* given in the following table.

Now by using the alternative ratings $r_{ij}^{(k)}$ and the DM weights λ_k we get

$r_{ij} = \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \lambda_3 r_{ij}^{(3)} \oplus \cdots \oplus \lambda_l r_{ij}^{(l)}$

$r_{ij} = \left(1 - \prod_{k=1}^{l}(1 - T_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(I_{ij}^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(F_{ij}^{(k)})^{\lambda_k}\right)$

where *i* = 1, 2, 3, 4, 5; *j* = 1, 2, 3, 4, 5 and (*l* = 4).

For *i* = *j* = 1 and *l* = 4

$r_{11} = \lambda_1 r_{11}^{(1)} \oplus \lambda_2 r_{11}^{(2)} \oplus \lambda_3 r_{11}^{(3)} \oplus \cdots \oplus \lambda_l r_{11}^{(l)}$

$r_{11} = \left(1 - \prod_{k=1}^{4}(1 - T_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^{4}(I_{11}^{(k)})^{\lambda_k}, \prod_{k=1}^{4}(F_{11}^{(k)})^{\lambda_k}\right)$

r_{11} = (1- $(1 - T_{11}^{(1)})^{\lambda_1}(1 - T_{11}^{(2)})^{\lambda_2}(1 - T_{11}^{(3)})^{\lambda_3}(1 - T_{11}^{(4)})^{\lambda_4}$, $(I_{11}^{(1)})^{\lambda_1}(I_{11}^{(2)})^{\lambda_2}(I_{11}^{(3)})^{\lambda_3}(I_{11}^{(4)})^{\lambda_4}$, $(F_{11}^{(1)})^{\lambda_1}(F_{11}^{(2)})^{\lambda_2}(F_{11}^{(3)})^{\lambda_3}(F_{11}^{(4)})^{\lambda_4}$)

r_{11} = (1-($(1 - 0.8)^{0.37045}(1 - 0.6)^{0.31508}(1 - 0.8)^{0.20580}(1 - 0.7)^{0.10867}$),

($(0.15)^{0.37045}(0.35)^{0.31508}(0.15)^{0.20580}(0.25)^{0.10867}$) ,

($(0.20)^{0.37045}(0.40)^{0.31508}(0.20)^{0.20580}(0.30)^{0.10867}$))

r_{11} = (0.740, 0.207, 0.260)

Similarly, we can find other values

r_{21} = (0.711, 0.237, 0.289)

r_{31} = (0.593, 0.373, 0.407)
r_{41} = (0.661, 0.288, 0.339)
r_{51} = (0.706, 0.241, 0.294)
r_{12} = (0.682, 0.268, 0.318)
r_{22} = (0.676, 0.275, 0.324)
r_{32} = (0.681, 0.275, 0.324)
r_{42} = (0.619, 0.342, 0.381)
r_{52} = (0.695, 0.253, 0.305)
r_{13} = (0.505, 0.392, 0.429)
r_{23} = (0.773, 0.176, 0.227)
r_{33} = (0.603, 0.359, 0.397)
r_{43} = (0.661, 0.288, 0.339)
r_{53} = (0.693, 0.255, 0.307)
r_{14} = (0.605, 0.359, 0.395)
r_{24} = (0.748, 0.203, 0.252)
r_{34} = (0.600, 0.350, 0.400)
r_{44} = (0.542, 0.443, 0.458)
r_{54} = (0.693, 0.339, 0.307)
r_{15} = (0.614, 0.349, 0.386)
r_{25} = (0.697, 0.257, 0.303)
r_{35} = (0.656, 0.299, 0.344)
r_{45} = (0.548, 0.431, 0.452)
r_{55} = (0.768, 0.181, 0.232)

Table 5. Aggregated Single Valued Neutrosophic Decision Matrix D = $[r_{ij}]_{5\times4}$

	X_1	X_2	X_3	X_4	X_5
A_1	r_{11} = (0.740, 0.207, 0.260)	r_{12} = (0.682, 0.268, 0.318)	r_{13} = (0.505, 0.392, 0.429)	r_{14} = (0.605, 0.359, 0.395)	r_{15} = (0.614, 0.349, 0.386)
A_2	r_{21} = (0.711, 0.237, 0.289)	r_{22} = (0.676, 0.275, 0.324)	r_{23} = (0.773, 0.176, 0.227)	r_{24} = (0.748, 0.203, 0.252)	r_{25} = (0.697, 0.257, 0.303)
A_3	r_{31} = (0.593, 0.373, 0.407)	r_{32} = (0.681, 0.275, 0.324)	r_{33} = (0.603, 0.359, 0.397)	r_{34} = (0.600, 0.350, 0.400)	r_{35} = (0.656, 0.299, 0.344)
A_4	r_{41} = (0.661, 0.288, 0.339)	r_{42} = (0.619, 0.342, 0.381)	r_{43} = (0.661, 0.288, 0.339)	r_{43} = (0.661, 0.288, 0.339)	r_{45} = (0.548, 0.431, 0.452)
A_5	r_{51} = (0.706, 0.241, 0.294)	r_{52} = (0.695, 0.253, 0.305)	r_{53} = (0.693, 0.255, 0.307)	r_{54}= (0.693, 0.339, 0.307)	r_{55} = (0.768, 0.181, 0.232)

Step 3: Computation of the weights of the criteria

The individual weights given by each DM is given in Table 6.

Table 6. Weights of alternatives determined by the DMs $w_j^{(k)}= (T_j^{(k)}, I_j^{(k)}, F_j^{(k)})$

Criteria	***DM*₁**	***DM*₂**	***DM*₃**	***DM*₄**
X_1	VI (0.90,0.10,0.10)	VI (0.90,0.10,0.10)	VI (0.90,0.10,0.10)	I (0.75,0.25,0.20)
(DELIVERY)	$w_1^{(1)} =(T_1^{(1)}, I_1^{(1)}, F_1^{(1)})$	$w_1^{(2)} = (T_1^{(2)}, I_1^{(2)}, F_1^{(2)})$	$w_1^{(3)} = (T_1^{(3)}, I_1^{(3)}, F_1^{(3)})$	$w_1^{(4)} = (T_1^{(4)}, I_1^{(4)}, F_1^{(4)})$

X_2	I (0.75,0.25,0.20)	M (0.50,0.50,0.50)	M (0.50,0.50,0.50)	I (0.75,0.25,0.20)
(QUALITY)	$w_2^{(1)} = (T_2^{(1)}, I_2^{(1)}, F_2^{(1)})$	$w_2^{(2)} = (T_2^{(2)}, I_2^{(2)}, F_2^{(2)})$	$w_2^{(3)} = (T_2^{(3)}, I_2^{(3)}, F_2^{(3)})$	$w_2^{(4)} = (T_2^{(4)}, I_2^{(4)}, F_2^{(4)})$
X_3	VI (0.90,0.10,0.10)	VI (0.90,0.10,0.10)	I (0.75,0.25,0.20)	VI (0.90,0.10,0.10)
(FLEXIBILITY)	$w_3^{(1)} = (T_3^{(1)}, I_3^{(1)}, F_3^{(1)})$	$w_3^{(2)} = (T_3^{(2)}, I_3^{(2)}, F_3^{(2)})$	$w_3^{(3)} = (T_3^{(3)}, I_3^{(3)}, F_3^{(3)})$	$w_3^{(4)} = (T_3^{(4)}, I_3^{(4)}, F_3^{(4)})$
X_4	I (0.75,0.25,0.20)	I (0.75,0.25,0.20)	M (0.50,0.50,0.50)	UI (0.35,0.75,0.80)
(SERVICE)	$w_4^{(1)} = (T_4^{(1)}, I_4^{(1)}, F_4^{(1)})$	$w_4^{(2)} = (T_4^{(2)}, I_4^{(2)}, F_4^{(2)})$	$w_4^{(3)} = (T_4^{(3)}, I_4^{(3)}, F_4^{(3)})$	$w_4^{(4)} = (T_4^{(4)}, I_4^{(4)}, F_4^{(4)})$
X_5	M (0.50,0.50,0.50)	M (0.50,0.50,0.50)	VI (0.90,0.10,0.10)	VI (0.90,0.10,0.10)
(PRICE)	$w_5^{(1)} = (T_5^{(1)}, I_5^{(1)}, F_5^{(1)})$	$w_5^{(2)} = (T_5^{(2)}, I_5^{(2)}, F_5^{(2)})$	$w_5^{(3)} = (T_5^{(3)}, I_5^{(3)}, F_5^{(3)})$	$w_5^{(4)} = (T_5^{(4)}, I_5^{(4)}, F_5^{(4)})$

By using the values from Table 6, the aggregated criteria weights are calculated as follows

$w_j = (T_j, I_j, F_j) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \lambda_3 w_j^{(3)} \oplus \cdots \oplus \lambda_l w_j^{(l)}$

$w_j = (1\text{-}\prod_{k=1}^{l}(1-T_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(I_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(F_j^{(k)})^{\lambda_k})$ where j = 1, 2, 3, 4, 5 and (l = 4).

For j = 1 and l = 4

$w_1 = \lambda_1 w_1^{(1)} \oplus \lambda_2 w_1^{(2)} \oplus \lambda_3 w_1^{(3)} \oplus \lambda_4 w_1^{(4)}$

$w_1 = (1\text{-}\prod_{k=1}^{4}(1-T_1^{(k)})^{\lambda_k}, \prod_{k=1}^{4}(I_1^{(k)})^{\lambda_k}, \prod_{k=1}^{4}(F_1^{(k)})^{\lambda_k})$

$w_1 = (1\text{-} (1-T_1^{(1)})^{\lambda_1}(1-T_1^{(2)})^{\lambda_2}(1-T_1^{(3)})^{\lambda_3}(1-T_1^{(4)})^{\lambda_4}, (I_1^{(1)})^{\lambda_1}(I_1^{(2)})^{\lambda_2}(I_1^{(3)})^{\lambda_3}(I_1^{(4)})^{\lambda_4},$
$(F_1^{(1)})^{\lambda_1}(F_1^{(2)})^{\lambda_2}(F_1^{(3)})^{\lambda_3}(F_1^{(4)})^{\lambda_4})$

$w_1 = (1 - ((1-0.9)^{0.37045}(1-0.9)^{0.31508}(1-0.9)^{0.20580}(1-0.75)^{0.10867}),$
$((0.10)^{0.37045}(0.10)^{0.31508}(0.10)^{0.20580}(0.25)^{0.10867})$,
$((0.10)^{0.37045}(0.10)^{0.31508}(0.10)^{0.20580}(0.20)^{0.10867}))$

r_{11} = (0.740, 0.207, 0.260)

$w_1 = (T_1, I_1, F_1)$ = (0.890, 0.110, 0.108)

Similarly, we can get other values

Therefore

$$W_{\{X_1,X_2,X_3,X_4\}} = \begin{bmatrix}(0.890, 0.110, 0.108)\\(0.641, 0.359, 0.322)\\(0.879, 0.121, 0.115)\\(0.680, 0.325, 0.281)\\(0.699, 0.301, 0.301)\end{bmatrix}^T$$

Step 4: Construction of Aggregated Weighted Single Valued Neutrosophic Decision Matrix (AWSVNDM)

After finding the weights of the criteria and the alternative ratings, the aggregated weighted single-valued neutrosophic ratings are calculated as follows

$r'_{ij} = (T'_{ij}, I'_{ij}, rF'_{ij}) = (T_{A_i}(x).T_j, I_{A_i}(x) + I_j - I_{A_i}(x).I_j, F_{A_i}(x) + F_j - F_{A_i}(x).F_j)$

By using the above equation, we can get an aggregated weighted single-valued neutrosophic decision matrix.

Table 7. Aggregated Weighted Single Valued Neutrosophic Decision Matrix $R' = [r'_{ij}]_{5\times5}$

	X1	**X2**	**X3**	**X4**	**X5**
A1	$r'_{11} =$ (0.659,0.294,0.340)	$r'_{12} =$ (0.437,0.531,0.538)	$r'_{13} =$ (0.444,0.466,0.495)	$r'_{14} =$ (0.411,0.567,0.565)	$r'_{15} =$ (0.429,0.545,0.571)

A2	$r'_{21}=$ (0.633,0.321,0.366)	$r'_{22}=$ (0.433,0.535,0.542)	$r'_{23}=$ (0.679,0.276,0.316)	$r'_{24}=$ (0.509,0.462,0.462)	$r'_{25}=$ (0.487,0.481,0.513)
A3	$r'_{31}=$ (0.528,0.442,0.471)	$r'_{32}=$ (0.437,0.535,0.542)	$r'_{33}=$ (0.530,0.437,0.466)	$r'_{34}=$ (0.408,0.561,0.569)	$r'_{35}=$ (0.459,0.510,0.541)
A4	$r'_{41}=$ (0.588,0.366,0.410)	$r'_{42}=$ (0.397,0.578,0.580)	$r'_{43}=$ (0.581,0.374,0.415)	$r'_{44}=$ (0.037,0.624,0.610)	$r'_{45}=$ (0.383,0.602,0.617)
A5	$r'_{51}=$ (0.628,0.324,0.3700	$r'_{52}=$ (0.445,0.521,0.529)	$r'_{53}=$ (0.609,0.345,0.387)	$r'_{54}=$ (0.471,0.554,0.502)	$r'_{55}=$ (0.537,0.428,0.463)

Step 5: Computation of SVN-PIS and SVN-NIS

Since Delivery, Quality, Flexibility, and Services are benefit criteria that is why they are in the set

$$J_1 = \{X_1, X_2, X_3, X_4\}$$

whereas Price being the cost criteria, so it is in the set $J_2 = \{X_2\}$ SVN-PIS and SVN-NIS are calculated as,

Table 8. SVN-PIS and SVN-NIS

SVN-PIS	SVN-NIS
$\boldsymbol{T_1^+}$ = max {0.659,0.633,0.528,0.588,0.628} = 0.659	T_1^- = min {0.659,0.633,0.528,0.588,0.628} = 0.528
$\boldsymbol{I_1^+}$ = min {0.294,0.321,0.442,0.366,0.324} = 0.294	I_1^- = max {0.294,0.321,0.442,0.366,0.324} = 0.442
$\boldsymbol{F_1^+}$ = min {0.340,0.366,0.471,0.410,0.370} = 0.340	F_1^- = max {0.340,0.366,0.471,0.410,0.370} = 0.471
$\boldsymbol{T_2^+}$ = max {0.437,0.433,0.437,0.397,0.445} = 0.445	T_2^- = min {0.437,0.433,0.437,0.397,0.445} = 0.397
$\boldsymbol{I_2^+}$ = min {0.531,0.535,0.535,0.578,0.521} = 0.521	I_2^- = max {0.531,0.535,0.535,0.578,0.521} = 0.578
$\boldsymbol{F_2^+}$ = min {0.538,0.542,0.542,0.580,0.529} = 0.529	F_2^- = max {0.538,0.542,0.542,0.580,0.529} = 0.580
$\boldsymbol{T_3^+}$= max {0.444,0.679,0.530,0.581,0.609} = 0.679	T_3^- = min {0.444,0.679,0.530,0.581,0.609} = 0.444
$\boldsymbol{I_3^+}$ = min {0.466,0.276,0.437,0.374,0.345} = 0.276	I_3^- = max {0.466,0.276,0.437,0.374,0.345} = 0.466
$\boldsymbol{F_3^+}$ = min {0.495,0.316,0.466,0.415,0.387} = 0.316	F_3^- = max {0.495,0.316,0.466,0.415,0.387} = 0.495
$\boldsymbol{T_4^+}$ = max {0.411,0.509,0.408,0.037,0.471} = 0.509	T_4^- = min {0.411,0.509,0.408,0.037,0.471} = 0.037
$\boldsymbol{I_4^+}$ = min {0.567,0.462,0.561,0.624,0.554} = 0.462	I_4^- = max {0.567,0.462,0.561,0.624,0.554} = 0.624
$\boldsymbol{F_4^+}$ = min {0.565,0.462,0.569,0.610,0.502} = 0.462	F_4^- = max {0.565,0.462,0.569,0.610,0.502} = 0.610
$\boldsymbol{T_5^+}$ = min {0.429,0.487,0.459,0.383,0.537} = 0.383	T_5^- = max {0.429,0.487,0.459,0.383,0.537} = 0.537
$\boldsymbol{I_5^+}$ = max {0.545,0.481,0.510,0.602,0.428} = 0.602	I_5^- = min {0.545,0.481,0.510,0.602,0.428} = 0.428
$\boldsymbol{F_5^+}$ = max {0.571,0.513,0.541,0.617,0.463} = 0.617	F_5^- = min {0.571,0.513,0.541,0.617,0.463} = 0.463

$$A^+ = \begin{Bmatrix} (0.659, 0.294, 0.340), \\ (0.445, 0.521, 0.529), \\ (0.679, 0.276, 0.316), \\ (0.509, 0.462, 0.462), \\ (0.383, 0.602, 0.617) \end{Bmatrix} \qquad A^- = \begin{Bmatrix} (0.528, 0.442, 0.471), \\ (0.397, 0.578, 0.580), \\ (0.444, 0.466, 0.495), \\ (0.037, 0.624, 0.610), \\ (0.537, 0.428, 0.463) \end{Bmatrix}$$

Step 6: Computation of Separation Measures

Normalized Euclidean Distance Measure is used to find the negative and positive separation measures $\boldsymbol{d}^{+}$ and $\boldsymbol{d}^{-}$respectively. Now for the SVN-PIS, we use

$$d_i^{+}= \left(\frac{1}{3n}\sum_{j=1}^{n}\left[\left(T_{A_i.W}(x_j)-T_{A^*W}\,(x_j)\right)^2+\left(I_{A_i.W}(x_j)-I_{A^*W}\,(x_j)\right)^2+\left(F_{A_i.W}(x_j)-F_{A^*W}\,(x_j)\right)^2\right]\right)^{0.5}$$

For i = 1and n = 5

$$d_1^{+}= \left(\frac{1}{3(5)}\sum_{j=1}^{5}\left[\left(T_{A_1.W}(x_j)-T_{A^*W}\,(x_j)\right)^2+\left(I_{A_1.W}(x_j)-I_{A^*W}\,(x_j)\right)^2+\left(F_{A_1.W}(x_j)-F_{A^*W}\,(x_j)\right)^2\right]\right)^{0.5}$$

$$d_1^{+}= \left(\frac{1}{15}\begin{bmatrix}\left(T_{A_1.W}(X_1)-T_{A^*W}\,(X_1)\right)^2+\left(I_{A_1.W}(X_1)-I_{A^*W}\,(X_1)\right)^2+\left(F_{A_1.W}(X_1)-F_{A^*W}\,(X_1)\right)^2+\\ \left(T_{A_1.W}(X_2)-T_{A^*W}\,(X_2)\right)^2+\left(I_{A_1.W}(X_2)-I_{A^*W}\,(X_2)\right)^2+\left(F_{A_1.W}(X_2)-F_{A^*W}\,(X_2)\right)^2+\\ \left(T_{A_1.W}(X_3)-T_{A^*W}\,(X_3)\right)^2+\left(I_{A_1.W}(X_3)-I_{A^*W}\,(X_3)\right)^2+\left(F_{A_1.W}(X_3)-F_{A^*W}\,(X_3)\right)^2+\\ \left(T_{A_1.W}(X_4)-T_{A^*W}\,(X_4)\right)^2+\left(I_{A_1.W}(X_4)-I_{A^*W}\,(X_4)\right)^2+\left(F_{A_1.W}(X_4)-F_{A^*W}\,(X_4)\right)^2+\\ \left(T_{A_1.W}(X_5)-T_{A^*W}\,(X_5)\right)^2+\left(I_{A_1.W}(X_5)-I_{A^*W}\,(X_5)\right)^2+\left(F_{A_1.W}(X_5)-F_{A^*W}\,(X_5)\right)^2\end{bmatrix}\right)^{0.5}$$

$$d_1^{+}= \left(\frac{1}{15}\begin{bmatrix}(0659-0.659)^2+(0.294-0.294)^2+(0.340-0.340)^2+\\ (0.437-0.445)^2+(0.531-0.521)^2+(0.538-0.529)^2+\\ (0.444-0.679)^2+(0.466-0.276)^2+(0.495-0.316)^2+\\ (0.411-0.509)^2+(0.567-0.462)^2+(0.565-0.462)^2+\\ (0.429-0.383)^2+(0.545-0.602)^2+(0.571-0.617)^2\end{bmatrix}\right)^{0.5}$$

$$d_1^{+}= \left[\frac{1}{15}\,(0.000245+0.123366+0.031238+0.007481)\right]^{0.5}$$

d_1^{+}= 0.1040

Similarly, we can find other separation measures.

Step 7: Computation of Relative Closeness Coefficient (RCC)

The RCC is calculated by using

RCCi = $\frac{d_i'}{d_i'+d_i^*}$; i = 1, 2, 3, 4, 5

RCC1 = $\frac{d_1'}{d_1'+d_1^*}$ = $\frac{0.127532}{0.127532+0.104029}$ = 0.551

RCC2 = 0.896

RCC3 = 0.505

RCC4 = 0.363

RCC5 = 0.757

The separation measure and the value of relative closeness coefficient (RCC) expressed in the following figure.

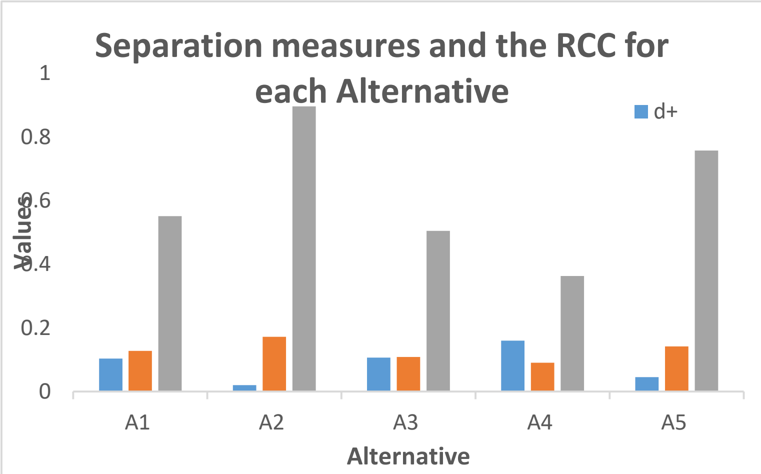

Figure 1. Separation measure and the RCC for each Alternative

Step 8: Ranking alternatives

From the above figure, we can see the RCC are ranked as follows

$RCC_2 > RCC_5 > RCC_1 > RCC_3 > RCC_4 \Rightarrow A_2 > A_5 > A_1 > A_3 > A_4$

By using the presented technique, we choose the best supplier for the production industry and observe that A_2 is the best alternative.

5. Conclusion

In this paper, we studied neutrosophic set and SVNSs with some basic operations and developed the generalized neutrosophic TOPSIS by using single-valued neutrosophic numbers. By using crisp data, it is more difficult to solve decision-making problems under uncertain environments, to overcome such uncertainties single-valued neutrosophic sets are more appropriate. We also developed the graphical model for generalized neutrosophic TOPSIS. Finally, to show the validity of the proposed technique an illustrated example of the best supplier in the production industry is presented and observed that A_2 is the best supplier for the production industry. We consider this technique will be helpful in problem-solving and will expand the area of investigations for more accuracy in real-life issues.

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