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Generalized Single-Valued Neutrosophic Hesitant Fuzzy Prioritized Aggregation Operators and Their Applications to Multiple Criteria Decision-Making

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Abstract: Single-valued neutrosophic hesitant fuzzy set (SVNHFS) is a combination of single-valued neutrosophic set and hesitant fuzzy set, and its aggregation tools play an important role in the multiple criteria decision-making (MCDM) process. This paper investigates the MCDM problems in which the criteria under SVNHF environment are in different priority levels. First, the generalized single-valued neutrosophic hesitant fuzzy prioritized weighted average operator and generalized single-valued neutrosophic hesitant fuzzy prioritized weighted geometric operator are developed based on the prioritized average operator. Second, some desirable properties and special cases of the proposed operators are discussed in detail. Third, an approach combined with the proposed operators and the score function of single-valued neutrosophic hesitant fuzzy element is constructed to solve MCDM problems. Finally, an example of investment selection is provided to illustrate the validity and rationality of the proposed method.

Keywords: multiple criteria decision-making (MCDM); single-valued neutrosophic hesitant fuzzy set (SVNHFS); generalized single-valued neutrosophic hesitant fuzzy prioritized weighted average operator; generalized single-valued neutrosophic hesitant fuzzy prioritized weighted geometric operator

1. Introduction

In daily life, MCDM problems happen in many fields; decision makers determine the best one from several alternatives through evaluating them with respect to the corresponding criteria. Due to the high complexity of the social environment, the evaluation information given by decision makers is often uncertain, incomplete, and inconsistent. With the demand for accuracy of decision-making results is getting higher and higher, much research in recent years has focused on the MCDM problems under fuzzy environment [1]. In 1965, Zadeh [2] developed the fuzzy set (FS) theory, which is a powerful tool to express the fuzzy information. However, there are several obvious limitations of FS theory in expressing uncertain information, which are attracting widespread interest in improving FS theory.

Atanassov [3] introduced the non-membership function to extend FS theory and proposed the intuitionistic fuzzy set (IFS) theory. IFS can express the membership and non-membership information simultaneously; the property can deal with some applications effectively, which FS cannot. For example, ten decision makers vote for an affair, four present agreement, three suggest different opinions, and the others choose to give up. The example above can be characterized by IFS, i.e., the value of membership is 0.4, and the value of non-membership is 0.3. However, expressing the voting information by FS is impossible. To describe the fuzziness of evaluation information more effective, Atanassov and Gargov [4] utilized the interval number to extend the membership and non-membership functions and put forward the interval-valued intuitionistic fuzzy set (IVIFS) theory. Nevertheless, in the real decision-making process, only considering the membership and non-membership information is not comprehensive sometimes. For instance, a decision maker gives her/his evaluation on a viewpoint, she/he may think the positive probability is 0.5, the false probability is 0.6, and the indeterminacy probability is 0.2 [5]. Obviously, IFS and IVIFS theory cannot deal with this situation. Therefore, Smarandache [6] defined the neutrosophic set (NS), which can be regarded as a generalization of FS and IFS [7]. NS consists of three independent membership functions, namely, truth-membership, indeterminacy-membership, and falsity-membership functions. Whereas, NS theory was originally proposed from a philosophical point of view, and it is difficult to apply NS theory in the field of science and engineering. To solve this problem, Wang [8,9] defined the concepts of interval neutrosophic set (INS) and single-valued neutrosophic set (SVNS), which are specific cases of NS.

Another drawback of FS is that its membership value is single; while determining the exact value of membership may be difficult for decision makers due to doubt. To deal with this situation, Torra and Narukawa [10] and Torra [11] extended the FS theory to hesitant fuzzy set (HFS) theory through allowing decision makers to give several different values of membership. Furthermore, Chen [12] defined the concept of interval-valued hesitant fuzzy set (IVHFS), in which the possible membership values can be expressed by interval numbers. Considering the complex information given by decision makers, Zhu [13] introduced the non-membership hesitancy function to propose the dual hesitant fuzzy set (DHFS) theory. According to the aforementioned analysis of improved FS theory from two directions, Ye [14] developed the single-valued neutrosophic hesitant fuzzy set (SVNHFS) combined with NS and HFS theory, in addition, Liu and Shi [7] extended the SVNHFS to interval neutrosophic hesitant fuzzy set (INHFS). Consequently, SVNHFS and INHFS not only can characterize the inconsistent and indeterminate information but also allow decision makers to give several possible values of truth-membership, indeterminacy-membership, and falsity-membership functions.

Besides the evaluation information, aggregation tools also are important parts of MCDM process. Ye [14] developed the operational laws and cosine measure of single-valued neutrosophic hesitant fuzzy elements (SVNHFEs), and proposed the single-valued neutrosophic hesitant fuzzy weighted average (SVNHFWA) operator and single-valued neutrosophic hesitant fuzzy weighted geometric (SVNHFWG) operator to aggregate SVNHFEs. Şahin and Liu [15] constructed the decision-making approach based on the correlation coefficient and weighted correlation coefficient of SVNHFEs. Biswas et al. [16] put forward several approaches for decision-making under SVNHF environment by using distance measures of SVNHFEs. Liu and Luo [17] proposed the single-valued neutrosophic hesitant fuzzy ordered weighted average (SVNHFOWA) operator and single-valued neutrosophic hesitant fuzzy hybrid weighted average (SVNHFHWA) operator, and applied them into MCDM process. Liu and Zhang [18] developed the single-valued neutrosophic hesitant fuzzy Heronian mean aggregation operators to deal with MCDM problems. Liu and Shi [7] defined the operational laws of INHFSs and proposed interval neutrosophic hesitant fuzzy generalized weighted average (INHFGWA) operator, interval neutrosophic hesitant fuzzy generalized ordered weighted average (INHFGOWA) operator, and interval neutrosophic hesitant fuzzy generalized hybrid weighted average (INHFGHWA) operator. Ye [19] determined the ranking of alternatives combined with the correlation coefficient of INHFSs.

The aforementioned decision-making methods are applied to the situation of the aggregated arguments and are in the same priority; whereas, in many real situations, criteria always have different priorities. For example, a mother chooses the dried milk for her baby, the criteria she considers are price and safety. Obviously, a prioritization ordering exists between the criteria, i.e., safety is much more important than price [20]. To deal with this situation, Yager [21] proposed the prioritized average (PA) operator to aggregate the evaluation information concerning the criteria of different priorities. Since the PA operator was presented, many scholars have focused on extending the PA operator into the fuzzy environment. For instance, Yu [20] proposed the intuitionistic fuzzy prioritized weighted average (IFPWA) operator and intuitionistic fuzzy prioritized weighted geometric (IFPWG) operator, and investigated their properties. Yu et al. [22] extended the PA operator into IVIF environment and developed the interval-valued intuitionistic fuzzy prioritized weighted average (IVIFPWA) operator and interval-valued intuitionistic fuzzy prioritized weighted geometric (IVIFPWG) operator. Liu and Wang [23] studied the aggregation operator under IN environment and put forward the interval neutrosophic prioritized ordered weighted average (INPOWA) operator. Furthermore, Wei [24] extended the PA operator into hesitant fuzzy MCDM problems. Jin et al. [25] developed interval-valued hesitant fuzzy Einstein prioritized weighted average (IVHFEPWA) operator and the interval-valued hesitant fuzzy Einstein prioritized weighted geometric (IVHFEPWG) operator through improving the operations of IVHFSs. However, to our best knowledge, little attention has been paid to the prioritized aggregation operators under SVNHF environment.

This paper proposes the aggregation operators for SVNHFEs, in which the aggregation arguments have different priority levels, and develops an approach for decision-making. To do this, the rest of this paper is organized as follows. Section 2 briefly introduces some basic concepts of SVNS, HFS, SVNHFS, and the PA operator. Section 3 develops the generalized single-valued neutrosophic hesitant fuzzy prioritized weighted average (GSVNHFPWA) operator and generalized single-valued neutrosophic hesitant fuzzy prioritized weighted geometric (GSVNHFPWG) operator, and investigates some desirable properties and special cases of the proposed operators. Section 4 constructs an approach for decision-making based on the proposed operators. Section 5 provides a numerical example to illustrate the applications and advantages of the proposed method. Section 6 summarizes the conclusions of this research.

2. Preliminaries

In this section, we briefly introduce some basic concepts, including the definitions of NS, SVNS, HFS, and SVNHFS. The operations of SVNHFEs and the PA operator are also presented, which are used in the subsequent discussion.

2.1. The Single-Valued Neutrosophic Set

Definition 1. *Ref. [6] Let X be a universe of discourse, with a generic element in X denoted by x. An NS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]^{-}0,1^{+}[$, i.e., $T_A(x) : X \to]^{-}0,1^{+}[$, $I_A(x) : X \to]^{-}0,1^{+}[$, and $F_A(x) : X \to]^{-}0,1^{+}[$. Thus, the sum of three aforementioned functions satisfies the condition of $^{-}0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.*

NS theory was originally proposed from the angle of philosophy and can be regarded as a generalization of FS, IFS, and IVIFS. However, the NS is not easily used for real scientific and engineering decision-making problems. To solve this limitation, Wang [8] defined the concept of SVNS, which is a special case of NS.

Definition 2. *Ref. [8] Let X be a universe of discourse, with a generic element in X denoted by x. An SVNS A is given by*

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}, \tag{1}$$

where $T_A(x)$ is the truth-membership function, $I_A(x)$ is the indeterminacy-membership function, and $F_A(x)$ is the falsity-membership function. For each point x in X, the functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ satisfy the conditions of $T_A(x), I_A(x), F_A(x) \in [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

2.2. The Hesitant Fuzzy Set

During the decision-making process, decision makers sometimes may be confused when determining the exact membership value of an element to the set because of the existing several possible membership values. Considering this situation, Torra and Narukawa [10] defined the concept of HFS.

Definition 3. *Ref. [10] Let X be a non-empty and finite set, an HFS A on X is defined by a function $h_A(x)$ that when applied to X returns a finite subset of $[0,1]$, which can be expressed as*

$$A = \{\langle x, h_A(x)\rangle | x \in X\}, \tag{2}$$

where $h_A(x)$ is a set of some different values in $[0,1]$, indicating the possible membership degrees of the element $x \in X$ to A.

2.3. The Single-Valued Neutrosophic Hesitant Fuzzy Set

Based on the combination of SVNS and HFS, Ye [14] proposed the concept of SVNHFS.

Definition 4. *Ref. [14] Let X be a non-empty and finite set, an SVNHFS N on X is expressed as*

$$N = \left\{\left\langle x, \widetilde{t}(x), \widetilde{i}(x), \widetilde{f}(x)\right\rangle | x \in X\right\}, \tag{3}$$

where $\widetilde{t}(x) = \{\gamma | \gamma \in \widetilde{t}(x)\}$, $\widetilde{i}(x) = \left\{\delta \middle| \delta \in \widetilde{i}(x)\right\}$, and $\widetilde{f}(x) = \left\{\eta \middle| \eta \in \widetilde{f}(x)\right\}$ are three sets of some different values in $[0,1]$, denoting the possible truth-membership hesitant, possible indeterminacy-membership hesitant, and possible falsity-membership hesitant degrees of the element $x \in X$ to N. And they satisfy the conditions of γ, δ, $\eta \subseteq [0,1]$ and $0 \leq \sup\gamma^+ + \sup\delta^+ + \sup\eta^+ \leq 3$, where $\gamma^+ = \cup_{\gamma \in \widetilde{t}(x)} \max\{\gamma\}$, $\delta^+ = \cup_{\delta \in \widetilde{i}(x)} \max\{\delta\}$, and $\eta^+ = \cup_{\eta \in \widetilde{f}(x)} \max\{\eta\}$ for $x \in X$. For convenience, we call $\widetilde{n} = \left\{\widetilde{t}(x), \widetilde{i}(x), \widetilde{f}(x)\right\}$ is an SVNHFE, denoted by $\widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$.

Definition 5. *Ref. [14] Let $\widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$, $\widetilde{n}_1 = \left\{\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1\right\}$ and $\widetilde{n}_2 = \left\{\widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2\right\}$ be three SVNHFEs, $\lambda > 0$, then the basic operations of SVNHFEs are defined as*

$$\widetilde{n}_1 \oplus \widetilde{n}_2 = \left\{\widetilde{t}_1 \oplus \widetilde{t}_2, \widetilde{i}_1 \otimes \widetilde{i}_2, \widetilde{f}_1 \otimes \widetilde{f}_2\right\} = \underset{\gamma_1 \in \widetilde{t}_1, \delta_1 \in \widetilde{i}_1, \eta_1 \in \widetilde{f}_1, \gamma_2 \in \widetilde{t}_2, \delta_2 \in \widetilde{i}_2, \eta_2 \in \widetilde{f}_2}{\cup} \{\{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}, \{\delta_1\delta_2\}, \{\eta_1\eta_2\}\}; \tag{4}$$

$$\widetilde{n}_1 \otimes \widetilde{n}_2 = \left\{\widetilde{t}_1 \otimes \widetilde{t}_2, \widetilde{i}_1 \oplus \widetilde{i}_2, \widetilde{f}_1 \oplus \widetilde{f}_2\right\} = \underset{\gamma_1 \in \widetilde{t}_1, \delta_1 \in \widetilde{i}_1, \eta_1 \in \widetilde{f}_1, \gamma_2 \in \widetilde{t}_2, \delta_2 \in \widetilde{i}_2, \eta_2 \in \widetilde{f}_2}{\cup} \{\{\gamma_1\gamma_2\}, \{\delta_1 + \delta_2 - \delta_1\delta_2\}, \{\eta_1 + \eta_2 - \eta_1\eta_2\}\}; \tag{5}$$

$$\lambda\widetilde{n} = \underset{\gamma \in \widetilde{t}, \delta \in \widetilde{i}, \eta \in \widetilde{f}}{\cup} \left\{\left\{1 - (1-\gamma)^\lambda\right\}, \left\{\delta^\lambda\right\}, \left\{\eta^\lambda\right\}\right\}; \tag{6}$$

$$\widetilde{n}^\lambda = \underset{\gamma \in \widetilde{t}, \delta \in \widetilde{i}, \eta \in \widetilde{f}}{\cup} \left\{\left\{\gamma^\lambda\right\}, \left\{1 - (1-\delta)^\lambda\right\}, \left\{1 - (1-\eta)^\lambda\right\}\right\}. \tag{7}$$

Definition 6. *Ref. [18] Let $\widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$ be an SVNHFE, then the score function $s(\widetilde{n})$ of $\widetilde{n}$ is given by*

$$s(\widetilde{n}) = \left[\frac{1}{l}\sum_{i=1}^{l} \gamma_i + \frac{1}{p}\sum_{i=1}^{p} (1-\delta_i) + \frac{1}{q}\sum_{i=1}^{q} (1-\eta_i)\right] \Big/ 3, \tag{8}$$

where l, p, q are the numbers of values in $\widetilde{t}, \widetilde{i}, \widetilde{f}$, respectively. Obviously, the range of $s(\widetilde{n})$ is limited to $[0,1]$.

Definition 7. *Ref. [18] Let $\widetilde{n}_1 = \left\{\widetilde{t}_1, \widetilde{i}_1, \widetilde{f}_1\right\}$ and $\widetilde{n}_2 = \left\{\widetilde{t}_2, \widetilde{i}_2, \widetilde{f}_2\right\}$ be two SVNHFEs, then the comparison method of them is expressed by*

(1) If $s(\widetilde{n}_1) > s(\widetilde{n}_2)$, then $\widetilde{n}_1 > \widetilde{n}_2$;
(2) If $s(\widetilde{n}_1) < s(\widetilde{n}_2)$, then $\widetilde{n}_1 < \widetilde{n}_2$;
(3) If $s(\widetilde{n}_1) = s(\widetilde{n}_2)$, then $\widetilde{n}_1 = \widetilde{n}_2$.

2.4. The Prioritized Average Operator

Aggregation operators play an important role in group decision-making to fusion the evaluation information. In view of priority relations between the criteria, Yager [21] developed the PA operator to solve this problem.

Definition 8. *Ref. [21] Let* $C = \{C_1, C_2, \ldots, C_n\}$ *be a collection of criteria, and priority relations between the criteria exist which can be expressed by the ordering of* $C_1 \succ C_2 \succ C_3 \succ \cdots \succ C_n$. *That means criteria* C_j *has a higher priority level than criteria* C_k *if* $j < k$. *The value* $C_j(x)$ *is the evaluation information of alternative* x *with respect to criteria* C_j. *Thus, if*

$$\mathrm{PA}\big(C_j(x)\big) = \textstyle\sum_{j=1}^{n} w_j C_j(x), \tag{9}$$

then the function PA is called the prioritized average (PA) operator, where $w_j = T_j / \sum_{j=1}^{n} T_j, T_j = \prod_{k=1}^{j-1} C_k(x)$, $T_1 = 1$.

3. Generalized Single-Valued Neutrosophic Hesitant Fuzzy Prioritized Aggregation Operators

The PA operator can effectively solve the decision-making problems that the criteria have different priorities; however, it can only be used in the situation where the aggregated arguments are exact values. Combined with the PA operator and the generalized mean operators [26], we extend the PA operator to deal with the decision-making problems under SVNHF environment. In this section, the GSVNHFPWA operator and GSVNHFPWG operator are proposed, and their properties are presented simultaneously. Besides, several special cases of the GSVNHFPWA operator and GSVNHFPWG operator are also discussed through changing the values of the parameter λ.

3.1. Generalized Single-Valued Neutrosophic Hesitant Fuzzy Prioritized Average Operator

Definition 9. *Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\} (j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, and let* $\mathrm{GSVNHFPWA} : \Omega^n \to \Omega$, *if*

$$\mathrm{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^\lambda \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}_2^\lambda \oplus \cdots \oplus \frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_n^\lambda\right)^{1/\lambda}, \tag{10}$$

then the function GSVNHFPWA is called the GSVNHFPWA operator. Where $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k) (j = 2, \ldots, n)$, $T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$.

According to the operational laws of SVHFEs in Definition 5, we can obtain the theorem as follows.

Theorem 1. *Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\} (j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, then their aggregated value by using the GSVNHFPWA operator is also an SVNHFE, and*

$$\begin{aligned} &\mathrm{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^\lambda \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}_2^\lambda \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^{n} T_j}\widetilde{n}_n^\lambda\right)^{1/\lambda} \\ &= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_n \in \widetilde{t}_n, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_n \in \widetilde{i}_n, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_n \in \widetilde{f}_n} \left\{\left\{\left(1 - \prod_{j=1}^{n}\left(1 - \gamma_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \prod_{j=1}^{n}\left(1 - (1 - \delta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\},\right. \\ &\qquad \left.\left\{1 - \left(1 - \prod_{j=1}^{n}\left(1 - (1 - \eta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}\right\}. \end{aligned} \tag{11}$$

where $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k) (j = 2, \ldots, n)$, $T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$.

Proof. We can use mathematical induction to prove the Theorem 1:

(a) For $n = 1$, since

$$\text{GSVNHFPWA}_{\lambda}(\widetilde{n}_1) = \left(\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^{\lambda}\right)^{1/\lambda} = \left(\frac{T_1}{T_1}\widetilde{n}_1^{\lambda}\right)^{1/\lambda} = \widetilde{n}_1.$$

Obviously, Equation (11) holds for $n = 1$.

(b) For $n = 2$, since

$$\widetilde{n}_1^{\lambda} = \bigcup_{\gamma_1\in\widetilde{t}_1,\delta_1\in\widetilde{i}_1,\eta_1\in\widetilde{f}_1}\left\{\left\{\gamma_1^{\lambda}\right\},\left\{1-(1-\delta_1)^{\lambda}\right\},\left\{1-(1-\eta_1)^{\lambda}\right\}\right\},$$

$$\widetilde{n}_2^{\lambda} = \bigcup_{\gamma_2\in\widetilde{t}_2,\delta_2\in\widetilde{i}_2,\eta_2\in\widetilde{f}_2}\left\{\left\{\gamma_2^{\lambda}\right\},\left\{1-(1-\delta_2)^{\lambda}\right\},\left\{1-(1-\eta_2)^{\lambda}\right\}\right\},$$

Then

$$\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^{\lambda} = \bigcup_{\gamma_1\in\widetilde{t}_1,\delta_1\in\widetilde{i}_1,\eta_1\in\widetilde{f}_1}\left\{\left\{1-\left(1-\gamma_1^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right\},\left\{\left(1-(1-\delta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right\},\left\{\left(1-(1-\eta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right\}\right\},$$

$$\frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}_2^{\lambda} = \bigcup_{\gamma_2\in\widetilde{t}_2,\delta_2\in\widetilde{i}_2,\eta_2\in\widetilde{f}_2}\left\{\left\{1-\left(1-\gamma_2^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right\},\left\{\left(1-(1-\delta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right\},\left\{\left(1-(1-\eta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right\}\right\}.$$

We have

$$\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^{\lambda} \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}_2^{\lambda} =$$

$$\bigcup_{\gamma_1\in\widetilde{t}_1,\delta_1\in\widetilde{i}_1,\eta_1\in\widetilde{f}_1,\gamma_2\in\widetilde{t}_2,\delta_2\in\widetilde{i}_2,\eta_2\in\widetilde{f}_2}\left\{\left\{1-\left(1-\gamma_1^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}+1-\left(1-\gamma_2^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}-\left(1-\left(1-\gamma_1^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(1-\left(1-\gamma_2^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right\},\right.$$

$$\left.\left\{\left(\left(1-(1-\delta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\delta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right\},\left\{\left(\left(1-(1-\eta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\eta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right\}\right\}.$$

$$= \bigcup_{\gamma_1\in\widetilde{t}_1,\delta_1\in\widetilde{i}_1,\eta_1\in\widetilde{f}_1,\gamma_2\in\widetilde{t}_2,\delta_2\in\widetilde{i}_2,\eta_2\in\widetilde{f}_2}\left\{\left\{1-\left(1-\gamma_1^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\left(1-\gamma_2^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right\},\right.$$

$$\left.\left\{\left(\left(1-(1-\delta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\delta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right\},\left\{\left(\left(1-(1-\eta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\eta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right\}\right\}.$$

Thus

$$\text{GSVNHFPWA}_{\lambda}(\widetilde{n}_1,\widetilde{n}_1) = \left(\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}_1^{\lambda} \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}_2^{\lambda}\right)^{1/\lambda} =$$

$$\bigcup_{\gamma_1\in\widetilde{t}_1,\delta_1\in\widetilde{i}_1,\eta_1\in\widetilde{f}_1,\gamma_2\in\widetilde{t}_2,\delta_2\in\widetilde{i}_2,\eta_2\in\widetilde{f}_2}\left\{\left\{\left(1-\left(1-\gamma_1^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\left(1-\gamma_2^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\},\right.$$

$$\left.\left\{1-\left(1-\left(\left(1-(1-\delta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\delta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right)^{1/\lambda}\right\},\left\{1-\left(1-\left(\left(1-(1-\eta_1)^{\lambda}\right)^{\frac{T_1}{\sum_{j=1}^{n} T_j}}\right)\left(\left(1-(1-\eta_2)^{\lambda}\right)^{\frac{T_2}{\sum_{j=1}^{n} T_j}}\right)\right)^{1/\lambda}\right\}\right\}.$$

i.e., Equation (11) holds for $n = 2$.

(c) If Equation (11) holds for $n = k$, we have

$$\text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_k) = \left(\frac{T_1}{\sum_{j=1}^n T_j} \widetilde{n}_1^\lambda \oplus \frac{T_2}{\sum_{j=1}^n T_j} \widetilde{n}_2^\lambda \oplus \cdots \oplus \frac{T_k}{\sum_{j=1}^n T_j} \widetilde{n}_k^\lambda \right)^{1/\lambda}$$

$$= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_k \in \widetilde{t}_k, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_k \in \widetilde{i}_k, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_k \in \widetilde{f}_k} \left\{ \left\{ \left(1 - \prod_{j=1}^k \left(1 - \gamma_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^k \left(1 - (1-\delta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^k \left(1 - (1-\eta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\} \right\}.$$

When $n = k+1$, combined with the operations of SVNHFE in Definition 5, we have

$$\frac{T_1}{\sum_{j=1}^n T_j} \widetilde{n}_1^\lambda \oplus \frac{T_2}{\sum_{j=1}^n T_j} \widetilde{n}_2^\lambda \oplus \cdots \oplus \frac{T_k}{\sum_{j=1}^n T_j} \widetilde{n}_k^\lambda \oplus \frac{T_{k+1}}{\sum_{j=1}^n T_j} \widetilde{n}_{k+1}^\lambda =$$

$$= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_k \in \widetilde{t}_k, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_k \in \widetilde{i}_k, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_k \in \widetilde{f}_k} \left\{ \left\{ 1 - \prod_{j=1}^k \left(1 - \gamma_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\}, \left\{ \prod_{j=1}^k \left(1 - (1-\delta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\}, \left\{ \prod_{j=1}^k \left(1 - (1-\eta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\} \right\} \oplus$$

$$\bigcup_{\gamma_{k+1} \in \widetilde{t}_{k+1}, \delta_{k+1} \in \widetilde{i}_{k+1}, \eta_{k+1} \in \widetilde{f}_{k+1}} \left\{ \left\{ 1 - \left(1 - \gamma_{k+1}^\lambda\right)^{\frac{T_{k+1}}{\sum_{j=1}^n T_j}} \right\}, \left\{ \left(1 - (1-\delta_{k+1})^\lambda\right)^{\frac{T_{k+1}}{\sum_{j=1}^n T_j}} \right\}, \left\{ \left(1 - (1-\eta_{k+1})^\lambda\right)^{\frac{T_{k+1}}{\sum_{j=1}^n T_j}} \right\} \right\}$$

$$= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_{k+1} \in \widetilde{t}_{k+1}, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_{k+1} \in \widetilde{i}_{k+1}, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_{k+1} \in \widetilde{f}_{k+1}} \left\{ \left\{ 1 - \prod_{j=1}^{k+1} \left(1 - \gamma_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\}, \left\{ \prod_{j=1}^{k+1} \left(1 - (1-\delta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\}, \left\{ \prod_{j=1}^{k+1} \left(1 - (1-\eta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right\} \right\}.$$

Then

$$\text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_{k+1}) = \left(\frac{T_1}{\sum_{j=1}^n T_j} \widetilde{n}_1^\lambda \oplus \frac{T_2}{\sum_{j=1}^n T_j} \widetilde{n}_2^\lambda \oplus \cdots \oplus \frac{T_{k+1}}{\sum_{j=1}^n T_j} \widetilde{n}_{k+1}^\lambda \right)^{1/\lambda}$$

$$= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_{k+1} \in \widetilde{t}_{k+1}, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_{k+1} \in \widetilde{i}_{k+1}, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_{k+1} \in \widetilde{f}_{k+1}} \left\{ \left\{ \left(1 - \prod_{j=1}^{k+1} \left(1 - \gamma_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^{k+1} \left(1 - (1-\delta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^{k+1} \left(1 - (1-\eta_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}} \right)^{1/\lambda} \right\} \right\}.$$

i.e., Equation (11) holds for $n = k+1$, thus we can confirm Equation (11) holds for all n. The proof of Theorem 1 is completed. □

Some desirable properties of the GSVNHFPWA operator are presented as below.

Theorem 2. *(Idempotency) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\} (j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k) (j = 2, \ldots, n), T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$. *If all* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\} (j = 1, 2, \ldots, n)$ *are equal, i.e.,* $\widetilde{n}_j = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$, $\widetilde{t} = \gamma$, $\widetilde{i} = \delta$, *and* $\widetilde{f} = \eta$, *then*

$$\text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}. \tag{12}$$

Proof. Since $\widetilde{n}_j = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$, by Theorem 1, we have

$$\mathrm{GSVNHFPWA}_{\lambda}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^{n} T_j}\widetilde{n}^{\lambda} \oplus \frac{T_2}{\sum_{j=1}^{n} T_j}\widetilde{n}^{\lambda} \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^{n} T_j}\widetilde{n}^{\lambda}\right)^{1/\lambda}$$

$$= \bigcup_{\widetilde{\gamma}\in\widetilde{t},\widetilde{\delta}\in\widetilde{i},\widetilde{\eta}\in\widetilde{f}} \left\{\left\{\left(1 - \prod_{j=1}^{n}\left(1-\gamma^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \prod_{j=1}^{n}\left(1-(1-\delta)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \prod_{j=1}^{n}\left(1-(1-\eta)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}\right\}$$

$$= \bigcup_{\widetilde{\gamma}\in\widetilde{t},\widetilde{\delta}\in\widetilde{i},\widetilde{\eta}\in\widetilde{f}} \left\{\left\{\left(1 - \left(1-\gamma^{\lambda}\right)^{\frac{\sum_{j=1}^{n} T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \left(1-(1-\delta)^{\lambda}\right)^{\frac{\sum_{j=1}^{n} T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \left(1-(1-\eta)^{\lambda}\right)^{\frac{\sum_{j=1}^{n} T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda}\right\}\right\}$$

$$= \bigcup_{\widetilde{\gamma}\in\widetilde{t},\widetilde{\delta}\in\widetilde{i},\widetilde{\eta}\in\widetilde{f}} \left\{\left\{\left(\gamma^{\lambda}\right)^{1/\lambda}\right\}, \left\{1 - \left((1-\delta)^{\lambda}\right)^{1/\lambda}\right\}, \left\{1 - \left((1-\eta)^{\lambda}\right)^{1/\lambda}\right\}\right\} = \bigcup_{\widetilde{\gamma}\in\widetilde{t},\widetilde{\delta}\in\widetilde{i},\widetilde{\eta}\in\widetilde{f}} \{\{\gamma\}, \{\delta\}, \{\eta\}\} = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}.$$

Then, the proof of Theorem 2 is completed. □

Theorem 3. *(Boundedness) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\} (j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k) (j = 2, \ldots, n), T_1 = 1,$ *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$*. And let* $\widetilde{n}^- = \{\{\gamma^-\}, \{\delta^+\}, \{\eta^+\}\}$ *and* $\widetilde{n}^+ = \{\{\gamma^+\}, \{\delta^-\}, \{\eta^-\}\}$*, where* $\gamma^+ = \cup_{\gamma_j\in\widetilde{t}_j}\max\{\gamma_j\}$, $\delta^+ = \cup_{\delta_j\in\widetilde{i}_j}\max\{\delta_j\}$, $\eta^+ = \cup_{\eta_j\in\widetilde{f}_j}\max\{\eta_j\}$, $\gamma^- = \cup_{\gamma_j\in\widetilde{t}_j}\min\{\gamma_j\}$, $\delta^- = \cup_{\delta_j\in\widetilde{i}_j}\min\{\delta_j\}$, *and* $\eta^- = \cup_{\eta_j\in\widetilde{f}_j}\min\{\eta_j\}$*. Then*

$$\widetilde{n}^- \leq \mathrm{GSVNHFPWA}_{\lambda}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \widetilde{n}^+. \quad (13)$$

Proof. Since $\gamma^- \leq \gamma_j \leq \gamma^+$, $\delta^- \leq \delta_j \leq \delta^+$, and $\eta^- \leq \eta_j \leq \eta^+$. First, when $\lambda \in (0, \infty)$, then

$$\gamma_j^{\lambda} \geq \left(\gamma^-\right)^{\lambda}, 1-\gamma_j^{\lambda} \leq 1-\left(\gamma^-\right)^{\lambda}, \left(1-\gamma_j^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \leq \left(1-\left(\gamma^-\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}},$$

$$\prod_{j=1}^{n}\left(1-\gamma_j^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \leq \prod_{j=1}^{n}\left(1-\left(\gamma^-\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}},$$

$$1-\prod_{j=1}^{n}\left(1-\gamma_j^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \geq 1-\prod_{j=1}^{n}\left(1-\left(\gamma^-\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}},$$

$$\left(1-\prod_{j=1}^{n}\left(1-\gamma_j^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda} \geq \left(1-\prod_{j=1}^{n}\left(1-\left(\gamma^-\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda} = \gamma^-.$$

Similarly, we have

$$\left(1-\prod_{j=1}^{n}\left(1-\gamma_j^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda} \leq \left(1-\prod_{j=1}^{n}\left(1-\left(\gamma^+\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}}\right)^{1/\lambda} = \gamma^+.$$

And as $\delta^- \leq \delta_j \leq \delta^+$, then

$$1-\delta_j \leq 1-\delta^-, \left(1-\delta_j\right)^{\lambda} \leq \left(1-\delta^-\right)^{\lambda}, 1-\left(1-\delta_j\right)^{\lambda} \geq 1-\left(1-\delta^-\right)^{\lambda},$$

$$\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}} \geq \left(1-\left(1-\delta^-\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n} T_j}},$$

$$\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}} \geq \prod_{j=1}^{n}\left(1-\left(1-\delta^{-}\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}},$$

$$1-\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}} \leq 1-\prod_{j=1}^{n}\left(1-\left(1-\delta^{-}\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}},$$

$$\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} \leq \left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta^{-}\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda},$$

$$1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} \geq 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta^{-}\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} = \delta^{-}.$$

Similarly, we have

$$1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} \leq 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta^{+}\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} = \delta^{+}.$$

On the other hand,

$$\eta^{-} \leq 1-\left(1-\prod_{j=1}^{n}\left(1-\left(1-\delta_j\right)^{\lambda}\right)^{\frac{T_j}{\sum_{j=1}^{n}T_j}}\right)^{1/\lambda} \leq \eta^{+}.$$

Let $\mathrm{GSVNHFPWA}_{\lambda}(\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_n)=\widetilde{n}=\{\{\gamma\},\{\delta\},\{\eta\}\}$, then

$$s(\widetilde{n})=\frac{\frac{1}{l}\sum_{i=1}^{l}\gamma_i+\frac{1}{p}\sum_{i=1}^{p}(1-\delta_i)+\frac{1}{q}\sum_{i=1}^{q}(1-\eta_i)}{3} \geq \frac{\frac{1}{l^{-}}\sum_{i=1}^{l^{-}}\gamma_i^{-}+\frac{1}{p^{-}}\sum_{i=1}^{p^{-}}(1-\delta_i^{+})+\frac{1}{q^{-}}\sum_{i=1}^{q^{-}}(1-\eta_i^{+})}{3}=s(\widetilde{n}^{-}),$$

And

$$s(\widetilde{n})=\frac{\frac{1}{l}\sum_{i=1}^{l}\gamma_i+\frac{1}{p}\sum_{i=1}^{p}(1-\delta_i)+\frac{1}{q}\sum_{i=1}^{q}(1-\eta_i)}{3} \leq \frac{\frac{1}{l^{+}}\sum_{i=1}^{l^{+}}\gamma_i^{+}+\frac{1}{p^{+}}\sum_{i=1}^{p^{+}}(1-\delta_i^{-})+\frac{1}{q^{+}}\sum_{i=1}^{q^{+}}(1-\eta_i^{-})}{3}=s(\widetilde{n}^{+}).$$

If $s(\widetilde{n}^{-})<s(\widetilde{n})<s(\widetilde{n}^{+})$, we have

$$\widetilde{n}^{-}<\mathrm{GSVNHFPWA}_{\lambda}(\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_n)<\widetilde{n}^{+}.$$

If $s(\widetilde{n})=s(\widetilde{n}^{-})$, i.e.,

$$\frac{\frac{1}{l}\sum_{i=1}^{l}\gamma_i+\frac{1}{p}\sum_{i=1}^{p}(1-\delta_i)+\frac{1}{q}\sum_{i=1}^{q}(1-\eta_i)}{3} = \frac{\frac{1}{l^{-}}\sum_{i=1}^{l^{-}}\gamma_i^{-}+\frac{1}{p^{-}}\sum_{i=1}^{p^{-}}(1-\delta_i^{+})+\frac{1}{q^{-}}\sum_{i=1}^{q^{-}}(1-\eta_i^{+})}{3},$$

Then

$$\mathrm{GSVNHFPWA}_{\lambda}(\widetilde{n}_1,\widetilde{n}_2,\ldots,\widetilde{n}_n)=\widetilde{n}^{-}.$$

If $s(\widetilde{n})=s(\widetilde{n}^{-})$, i.e.,

$$\frac{\frac{1}{l}\sum_{i=1}^{l}\gamma_i+\frac{1}{p}\sum_{i=1}^{p}(1-\delta_i)+\frac{1}{q}\sum_{i=1}^{q}(1-\eta_i)}{3} \leq \frac{\frac{1}{l^{+}}\sum_{i=1}^{l^{+}}\gamma_i^{+}+\frac{1}{p^{+}}\sum_{i=1}^{p^{+}}(1-\delta_i^{-})+\frac{1}{q^{+}}\sum_{i=1}^{q^{+}}(1-\eta_i^{-})}{3},$$

Then

$$\text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \widetilde{n}^+.$$

Based on analysis above, we have

$$\widetilde{n}^- \leq \text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \widetilde{n}^+ \lambda \in (0, \infty).$$

Similarly, we can obtain

$$\widetilde{n}^- \leq \text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \widetilde{n}^+ \lambda \in (-\infty, 0).$$

The proof of Theorem 3 is completed. □

Theorem 4. *(Monotonicity) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *and* $\widetilde{n}_j^* = \left\{\widetilde{t}_j^*, \widetilde{i}_j^*, \widetilde{f}_j^*\right\}(j = 1, 2, \ldots, n)$ *be two collections of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n), T_j^* = \prod_{k=1}^{j-1} s(\widetilde{n}_k^*)(j = 2, \ldots, n)$, $T_1 = T_1^* = 1$, $s(\widetilde{n}_k)$ *and* $s(\widetilde{n}_k^*)$ *are the score values of SVNHFE* $\widetilde{n}_k$ *and* $\widetilde{n}_k^*$*, respectively. If* $\widetilde{n}_j \leq \widetilde{n}_j^*$ $(j = 1, 2, \ldots, n)$*, then*

$$\text{GSVNHFPWA}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \text{GSVNHFPWA}_\lambda(\widetilde{n}_1^*, \widetilde{n}_2^*, \ldots, \widetilde{n}_n^*). \quad (14)$$

Proof. It directly follows from Theorem 3. □

Special cases of the GSVNHFPWA operator are shown as follows.

(1) If $\lambda = 1$, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy prioritized weighted average (SVNHFPWA) operator:

$$\text{SVNHFPWA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^n T_j}\widetilde{n}_1 \oplus \frac{T_2}{\sum_{j=1}^n T_j}\widetilde{n}_2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^n T_j}\widetilde{n}_n\right). \quad (15)$$

(2) If $\lambda \to 0$, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy prioritized weighted geometric (SVNHFPWG) operator:

$$\text{SVNHFPWG}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left((\widetilde{n}_1)^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes (\widetilde{n}_2)^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes (\widetilde{n}_n)^{\frac{T_n}{\sum_{j=1}^n T_j}}\right). \quad (16)$$

(3) If $\lambda = 2$, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy prioritized weighted quadratic average (SVNHFPWQA) operator:

$$\text{SVNHFPWQA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^n T_j}\widetilde{n}_1^2 \oplus \frac{T_2}{\sum_{j=1}^n T_j}\widetilde{n}_2^2 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^n T_j}\widetilde{n}_n^2\right)^{1/2}. \quad (17)$$

(4) If $\lambda = 3$, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy prioritized weighted cubic average (SVNHFPWCA) operator:

$$\text{SVNHFPWCA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left(\frac{T_1}{\sum_{j=1}^n T_j}\widetilde{n}_1^3 \oplus \frac{T_2}{\sum_{j=1}^n T_j}\widetilde{n}_2^3 \oplus \cdots \oplus \frac{T_n}{\sum_{j=1}^n T_j}\widetilde{n}_n^3\right)^{1/3}. \quad (18)$$

(5) If $\lambda = 1$ and the aggregated arguments are in the same priority level, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy weighted average (SVNHFWA) operator [14]:

$$\text{SVNHFWA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = (w_1\widetilde{n}_1 \oplus w_2\widetilde{n}_2 \oplus \cdots \oplus w_n\widetilde{n}_n). \tag{19}$$

(6) If $\lambda \to 0$ and the aggregated arguments are in the same priority level, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy weighted geometric (SVNHFWG) operator [14]:

$$\text{SVNHFWG}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = (\widetilde{n}_1^{w_1} \otimes \widetilde{n}_2^{w_2} \otimes \cdots \otimes \widetilde{n}_n^{w_n}). \tag{20}$$

(7) If $w = (1/n, 1/n, \ldots, 1/n)^T$, $\lambda = 1$, and the aggregated arguments are in the same priority level, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy arithmetic average (SVNHFAA) operator:

$$\text{SVNHFAA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \frac{1}{n}(\widetilde{n}_1 \oplus \widetilde{n}_2 \oplus \cdots \oplus \widetilde{n}_n). \tag{21}$$

(8) If $w = (1/n, 1/n, \ldots, 1/n)^T$, $\lambda \to 0$, and the aggregated arguments are in the same priority level, then the GSVNHFPWA operator is reduced to the single-valued neutrosophic hesitant fuzzy geometric average (SVNHFGA) operator:

$$\text{SVNHFGA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = (\widetilde{n}_1 \otimes \widetilde{n}_2 \otimes \cdots \otimes \widetilde{n}_n)^{1/n}. \tag{22}$$

3.2. Generalized Single-Valued Neutrosophic Hesitant Fuzzy Prioritized Geometric Operator

Based on the GSVNHFPWA operator investigated above, we develop the GSVNHFPWG operator as the following.

Definition 10. *Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, and let* GSVNHFPWG : $\Omega^n \to \Omega$, *if*

$$\text{GSVNHFPWG}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \frac{1}{\lambda}\left((\lambda\widetilde{n}_1)^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes (\lambda\widetilde{n}_2)^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes (\lambda\widetilde{n}_n)^{\frac{T_n}{\sum_{j=1}^n T_j}}\right), \tag{23}$$

then the function GSVNHFPWG is called the GSVNHFPWG operator. Where $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n)$, $T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$.

Similarly, according to the operations of SVHFEs in Definition 5, the theorem is obtained as below.

Theorem 5. *Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, then their aggregated value by using the GSVNHFPWG operator is also an SVNHFE, and*

$$\begin{aligned} &\text{GSVNHFPWG}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \frac{1}{\lambda}\left((\lambda\widetilde{n}_1)^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes (\lambda\widetilde{n}_2)^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes (\lambda\widetilde{n}_n)^{\frac{T_n}{\sum_{j=1}^n T_j}}\right) \\ &= \bigcup_{\widetilde{\gamma}_1 \in \widetilde{t}_1, \widetilde{\gamma}_2 \in \widetilde{t}_2, \ldots, \widetilde{\gamma}_n \in \widetilde{t}_n, \widetilde{\delta}_1 \in \widetilde{i}_1, \widetilde{\delta}_2 \in \widetilde{i}_2, \ldots, \widetilde{\delta}_n \in \widetilde{i}_n, \widetilde{\eta}_1 \in \widetilde{f}_1, \widetilde{\eta}_2 \in \widetilde{f}_2, \ldots, \widetilde{\eta}_n \in \widetilde{f}_n} \left\{\left\{1 - \left(1 - \prod_{j=1}^n \left(1 - (1-\gamma_j)^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}}\right)^{1/\lambda}\right\}, \left\{\left(1 - \prod_{j=1}^n \left(1 - \delta_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}}\right)^{1/\lambda}\right\}, \right. \\ &\left. \left\{\left(1 - \prod_{j=1}^n \left(1 - \eta_j^\lambda\right)^{\frac{T_j}{\sum_{j=1}^n T_j}}\right)^{1/\lambda}\right\}\right\}. \end{aligned} \tag{24}$$

where $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n)$, $T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$.

Proof. The proof procedure of Theorem 5 is similar to Theorem 1. □

Some desirable properties of the GSVNHFPWG operator are presented as below.

Theorem 6. *(Idempotency) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n), T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score function value of SVNHFE* $\widetilde{n}_k$. *If all* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *are equal, i.e.,* $\widetilde{n}_j = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}$, $\widetilde{t} = \gamma$, $\widetilde{i} = \delta$, *and* $\widetilde{f} = \eta$, *then*

$$\mathrm{GSVNHFPWG}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \widetilde{n} = \left\{\widetilde{t}, \widetilde{i}, \widetilde{f}\right\}. \tag{25}$$

Proof. The proof procedure of Theorem 6 is similar to Theorem 2. □

Theorem 7. *(Boundedness) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *be a collection of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n), T_1 = 1$, *and* $s(\widetilde{n}_k)$ *is the score value of SVNHFE* $\widetilde{n}_k$. *And let* $\widetilde{n}^- = \{\{\gamma^-\}, \{\delta^+\}, \{\eta^+\}\}$ *and* $\widetilde{n}^+ = \{\{\gamma^+\}, \{\delta^-\}, \{\eta^-\}\}$, *where* $\gamma^+ = \cup_{\gamma_j \in \widetilde{t}_j} \max\{\gamma_j\}$, $\delta^+ = \cup_{\delta_j \in \widetilde{i}_j} \max\{\delta_j\}$, $\eta^+ = \cup_{\eta_j \in \widetilde{f}_j} \max\{\eta_j\}$, $\gamma^- = \cup_{\gamma_j \in \widetilde{t}_j} \min\{\gamma_j\}$, $\delta^- = \cup_{\delta_j \in \widetilde{i}_j} \min\{\delta_j\}$, *and* $\eta^- = \cup_{\eta_j \in \widetilde{f}_j} \min\{\eta_j\}$. *Then*

$$\widetilde{n}^- \leq \mathrm{GSVNHFPWG}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \widetilde{n}^+. \tag{26}$$

Proof. The proof procedure of Theorem 7 is similar to Theorem 3. □

Theorem 8. *(Monotonicity) Let* $\widetilde{n}_j = \left\{\widetilde{t}_j, \widetilde{i}_j, \widetilde{f}_j\right\}(j = 1, 2, \ldots, n)$ *and* $\widetilde{n}_j^* = \left\{\widetilde{t}_j^*, \widetilde{i}_j^*, \widetilde{f}_j^*\right\}(j = 1, 2, \ldots, n)$ *be two collections of SVNHFEs, where* $T_j = \prod_{k=1}^{j-1} s(\widetilde{n}_k)(j = 2, \ldots, n), T_j^* = \prod_{k=1}^{j-1} s\left(\widetilde{n}_k^*\right)(j = 2, \ldots, n)$, $T_1 = T_1^* = 1$, $s(\widetilde{n}_k)$ *and* $s\left(\widetilde{n}_k^*\right)$ *are the score function values of SVNHFE* $\widetilde{n}_k$ *and* $\widetilde{n}_k^*$, *respectively. If* $\widetilde{n}_j \leq \widetilde{n}_j^*(j = 1, 2, \ldots, n)$, *then*

$$\mathrm{GSVNHFPWG}_\lambda(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) \leq \mathrm{GSVNHFPWG}_\lambda(\widetilde{n}_1^*, \widetilde{n}_2^*, \ldots, \widetilde{n}_n^*). \tag{27}$$

Proof. It directly follows from Theorem 7. □

Special cases of the GSVNHFPWG operator are shown as follows:

(1) If $\lambda = 1$, then the GSVNHFPWG operator is reduced to the single-valued neutrosophic hesitant fuzzy prioritized weighted geometric (SVNHFPWG) operator:

$$\mathrm{SVNHFPWG}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left((\widetilde{n}_1)^{\frac{T_1}{\sum_{j=1}^n T_j}} \otimes (\widetilde{n}_2)^{\frac{T_2}{\sum_{j=1}^n T_j}} \otimes \cdots \otimes (\widetilde{n}_n)^{\frac{T_n}{\sum_{j=1}^n T_j}} \right). \tag{28}$$

(2) If $\lambda = 1$ and the aggregated arguments are in the same priority level, then the GSVNHFPWG operator is reduced to the SVNHFWG operator [14]:

$$\mathrm{SVNHFWG}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = \left((\widetilde{n}_1)^{w_1} \otimes (\widetilde{n}_2)^{w_2} \otimes \cdots \otimes (\widetilde{n}_n)^{w_n} \right). \tag{29}$$

(3) If $w = (1/n, 1/n, \ldots, 1/n)^T$, $\lambda = 1$, and the aggregated arguments are in the same priority level, then the GSVNHFPWG operator is reduced to the SVNHFGA operator:

$$\mathrm{SVNHFGA}(\widetilde{n}_1, \widetilde{n}_2, \ldots, \widetilde{n}_n) = (\widetilde{n}_1 \otimes \widetilde{n}_2 \otimes \cdots \otimes \widetilde{n}_n)^{1/n}. \tag{30}$$

4. An Approach for Decision-Making under Single-Valued Neutrosophic Hesitant Fuzzy Environment

In this section, we utilize the GSVNHFPWA operator and GSVNHFPWG operator to solve the MCDM problems under SVNHF environment, respectively. For a MCDM problem, let $A = \{A_1, A_2, \ldots A_m\}$ be a set of m alternatives to be evaluated, $C = \{C_1, C_2, \ldots, C_n\}$ be a collection of criteria that prioritizations between the criteria expressed by the linear ordering $C_1 \succ C_2 \succ \cdots \succ C_n$ exist, i.e., criteria C_j has a higher priority level than the criteria C_k if $j < k$. Decision makers evaluates the alternatives over the criteria by using SVNHFEs, let $N = (\widetilde{n}_{ij})_{m\times n} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ be an SVNHF decision matrix, and $\widetilde{n}_{ij} = \left\{\widetilde{t}_{ij}, \widetilde{i}_{ij}, \widetilde{f}_{ij}\right\}$ is the evaluation information given by decision maker. Where $\widetilde{t}_{ij} = \{\gamma_{ij} | \gamma_{ij} \in \widetilde{t}_{ij}\}$ represents the possible degrees that the alternative A_i satisfies the criteria C_j provided by decision maker, $\widetilde{i}_{ij} = \left\{\delta_{ij} \middle| \delta_{ij} \in \widetilde{i}_{ij}\right\}$ represents the possible indeterminacy degrees that decision maker judges whether the alternative A_i satisfies the criteria C_j, and $\widetilde{f}_{ij} = \left\{\eta_{ij} \middle| \eta_{ij} \in \widetilde{f}_{ij}\right\}$ represents the possible degrees that the alternative A_i does not satisfy the criteria C_j provided by decision maker.

Based on the assumptions above, we use the GSVNHFPWA operator or GSVNHFPWG operator to construct an approach for decision-making under SVNHF environment. The main steps are presented below.

Step 1. Calculate the values of $T_{ij} (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ by the equations as follows.

$$T_{ij} = \prod_{k=1}^{j-1} s(\widetilde{n}_{ik}) (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n), T_{i1} = 1. \quad (31)$$

Step 2. Utilize the GSVNHFPWA operator:

$$\widetilde{n}_i = \mathrm{GSVNHFPWA}_\lambda(\widetilde{n}_{i1}, \widetilde{n}_{i2}, \ldots, \widetilde{n}_{in}) = \left(\frac{T_{i1}}{\sum_{j=1}^n T_{ij}}(\widetilde{n}_{i1})^\lambda \oplus \frac{T_{i2}}{\sum_{j=1}^n T_{ij}}(\widetilde{n}_{i2})^\lambda \oplus \cdots \oplus \frac{T_{in}}{\sum_{j=1}^n T_{ij}}(\widetilde{n}_{in})^\lambda\right)^{1/\lambda}$$
$$= \bigcup_{\widetilde{\gamma}_{i1}\in\widetilde{t}_{i1}, \widetilde{\gamma}_{i2}\in\widetilde{t}_{i2}, \ldots, \widetilde{\gamma}_{in}\in\widetilde{t}_{in}, \widetilde{\delta}_{i1}\in\widetilde{i}_{i1}, \widetilde{\delta}_{i2}\in\widetilde{i}_{i2}, \ldots, \widetilde{\delta}_{in}\in\widetilde{i}_{in}, \widetilde{\eta}_{i1}\in\widetilde{f}_{i1}, \widetilde{\eta}_{i2}\in\widetilde{f}_{i2}, \ldots, \widetilde{\eta}_{in}\in\widetilde{f}_{in}} \left\{\left\{\left(1 - \prod_{j=1}^n \left(1 - (\gamma_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}, \quad (32)$$
$$\left\{1 - \left(1 - \prod_{j=1}^n \left(1 - (1 - \delta_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}, \left\{1 - \left(1 - \prod_{j=1}^n \left(1 - (1 - \eta_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}\right\}.$$

or the GSVNHFPWG operator:

$$\widetilde{n}_i = \mathrm{GSVNHFPWG}_\lambda(\widetilde{n}_{i1}, \widetilde{n}_{i2}, \ldots, \widetilde{n}_{in}) = \frac{1}{\lambda}\left((\lambda\widetilde{n}_{i1})^{\frac{T_{i1}}{\sum_{j=1}^n T_{ij}}} \otimes (\lambda\widetilde{n}_{i2})^{\frac{T_{i2}}{\sum_{j=1}^n T_{ij}}} \otimes \cdots \otimes (\lambda\widetilde{n}_{in})^{\frac{T_{in}}{\sum_{j=1}^n T_{ij}}}\right)$$
$$= \bigcup_{\widetilde{\gamma}_{i1}\in\widetilde{t}_{i1}, \widetilde{\gamma}_{i2}\in\widetilde{t}_{i2}, \ldots, \widetilde{\gamma}_{in}\in\widetilde{t}_{in}, \widetilde{\delta}_{i1}\in\widetilde{i}_{i1}, \widetilde{\delta}_{i2}\in\widetilde{i}_{i2}, \ldots, \widetilde{\delta}_{in}\in\widetilde{i}_{in}, \widetilde{\eta}_{i1}\in\widetilde{f}_{i1}, \widetilde{\eta}_{i2}\in\widetilde{f}_{i2}, \ldots, \widetilde{\eta}_{in}\in\widetilde{f}_{in}} \left\{\left\{1 - \left(1 - \prod_{j=1}^n \left(1 - (1 - \gamma_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}, \quad (33)$$
$$\left\{\left(1 - \prod_{j=1}^n \left(1 - (\delta_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}, \left\{\left(1 - \prod_{j=1}^n \left(1 - (\eta_{ij})^\lambda\right)^{\frac{T_{ij}}{\sum_{j=1}^n T_{ij}}}\right)^{1/\lambda}\right\}\right\}.$$

to aggregate the SVNHF decision matrix $N = (\widetilde{n}_{ij})_{m\times n}$ into the SVNHFE $\widetilde{n}_i = \left\{\widetilde{t}_i, \widetilde{i}_i, \widetilde{f}_i\right\}$ of each alternative.

Step 3. Rank all the alternatives by calculating the score function value of the SVNHFE $\widetilde{n}_i = \left\{\widetilde{t}_i, \widetilde{i}_i, \widetilde{f}_i\right\}$ combined with Definition 6.

$$s(\widetilde{n}_i) = \left[\frac{1}{l_i}\sum_{\gamma_i \in \widetilde{t}_i} \gamma_i + \frac{1}{p_i}\sum_{\delta_i \in \widetilde{i}_i} (1-\delta_i) + \frac{1}{q_i}\sum_{\eta_i \in \widetilde{f}_i} (1-\eta_i)\right] \Big/ 3. \quad (34)$$

Then the bigger the score function value $s(\widetilde{n}_i)$, the higher the ranking of alternative x_i will be.

5. Numerical Example

In this section, we apply a numerical example of MCDM problem under SVNHF environment to illustrate the applications and advantages of the proposed method [14].

5.1. Implementation

Suppose that an investment company wants to invest a sum of money in a target company. After a market survey, four alternative companies are identified to be chosen from, namely, a car company (A_1), a food company (A_2), a computer company (A_3), and an arms company (A_4). To evaluate the investment potential of a company needs to consider many aspects, such as the growth prospects of the company, risk degree of the investment, and the impact of the company on the environment. Therefore, the investment company shall evaluate the four alternative companies above with respect to three criteria, namely, the environmental impact (C_1), the risk (C_2), and the growth (C_3). In the real decision-making process, compared with determining the weights of criteria, identifying the priority level of criteria is more feasible and accurate. Then, according to the weight vector of three criteria $\boldsymbol{w} = (0.40, 0.35, 0.25)^T$ [14], we set up the criteria C_1 with the first priority level, followed by criteria C_2 and C_3. Decision makers from the investment company express the evaluation information combined with SVNHFEs, and the SVNHF decision matrix $N = \left(\widetilde{n}_{ij}\right)_{m\times n}$ is obtained shown in Table 1 [14].

Table 1. SVNHF decision matrix.

Alternatives	C_1	C_2	C_3
A_1	{{0.2, 0.3}, {0.1, 0.2}, {0.5, 0.6}}	{{0.3, 0.4, 0.5}, {0.1}, {0.3, 0.4}}	{{0.5, 0.6}, {0.2, 0.3}, {0.3, 0.4}}
A_2	{{0.6, 0.7}, {0.1, 0.2}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1, 0.2}, {0.2, 0.3}}	{{0.6, 0.7}, {0.1}, {0.3}}
A_3	{{0.5, 0.6}, {0.1}, {0.3}}	{{0.5, 0.6}, {0.4}, {0.2, 0.3}}	{{0.6}, {0.3}, {0.4}}
A_4	{{0.3, 0.5}, {0.2}, {0.1, 0.2, 0.3}}	{{0.7, 0.8}, {0.1}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1}, {0.2}}

Then, we use the proposed method to determine the ranking result of the four alternative companies, which are presented as follows.

Step 1. Calculate the values of $T_{ij} (i = 1, 2, 3, 4; j = 1, 2, 3)$ according to Equation (31) as follows:

$$T_{ij} = \begin{bmatrix} 1.000 & 0.5167 & 0.3358 \\ 1.000 & 0.7833 & 0.5875 \\ 1.000 & 0.7167 & 0.4539 \\ 1.000 & 0.6667 & 0.5556 \end{bmatrix}.$$

Step 2. Utilize the GSVNHFPWA operator (which the parameter $\lambda = 1$) to aggregate the SVNHF decision matrix $N = \left(\widetilde{n}_{ij}\right)_{m\times n} (i = 1, 2, 3, 4; j = 1, 2, 3)$ into the SVNHFE $\widetilde{n}_i = \left\{\widetilde{t}_i, \widetilde{i}_i, \widetilde{f}_i\right\} (i = 1, 2, 3, 4)$ of each alternative company. Take the alternative company A_1 for instance, we have

$$\widetilde{n}_1 = \text{GSVNHFPWA}_1(\widetilde{n}_{11}, \widetilde{n}_{12}, \widetilde{n}_{13}) = \left(\frac{T_{11}}{\sum_{j=1}^{3} T_{1j}} (\widetilde{n}_{11})^1 \oplus \frac{T_{12}}{\sum_{j=1}^{3} T_{1j}} (\widetilde{n}_{12})^1 \oplus \frac{T_{13}}{\sum_{j=1}^{3} T_{1j}} (\widetilde{n}_{13})^1 \right)^{1/1}$$

$$= \bigcup_{\widetilde{\gamma}_{11}\in\widetilde{t}_{11},\widetilde{\gamma}_{12}\in\widetilde{t}_{12},\widetilde{\gamma}_{13}\in\widetilde{t}_{13},\widetilde{\delta}_{11}\in\widetilde{i}_{11},\widetilde{\delta}_{12}\in\widetilde{i}_{12},\widetilde{\delta}_{13}\in\widetilde{i}_{13},\widetilde{\eta}_{11}\in\widetilde{f}_{11},\widetilde{\eta}_{12}\in\widetilde{f}_{12},\widetilde{\eta}_{13}\in\widetilde{f}_{13}} \left\{ \left\{ \left(1 - \prod_{j=1}^{3} \left(1 - (\gamma_{1j})^1\right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^{3} \left(1 - (1-\delta_{1j})^1\right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\}, \left\{ 1 - \left(1 - \prod_{j=1}^{3} \left(1 - (1-\eta_{1j})^1\right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\} \right\}$$

$$\begin{aligned}
= \Big\{\Big\{ & 1-(1-0.2)^{0.54}(1-0.3)^{0.28}(1-0.5)^{0.18}, 1-(1-0.2)^{0.54}(1-0.3)^{0.28}(1-0.6)^{0.18}, 1-(1-0.2)^{0.54}(1-0.4)^{0.28}(1-0.5)^{0.18}, \\
& 1-(1-0.2)^{0.54}(1-0.4)^{0.28}(1-0.6)^{0.18}, 1-(1-0.2)^{0.54}(1-0.5)^{0.28}(1-0.5)^{0.18}, 1-(1-0.2)^{0.54}(1-0.5)^{0.28}(1-0.6)^{0.18}, \\
& 1-(1-0.3)^{0.54}(1-0.3)^{0.28}(1-0.5)^{0.18}, 1-(1-0.2)^{0.54}(1-0.3)^{0.28}(1-0.6)^{0.18}, 1-(1-0.2)^{0.54}(1-0.4)^{0.28}(1-0.5)^{0.18}, \\
& 1-(1-0.3)^{0.54}(1-0.4)^{0.28}(1-0.6)^{0.18}, 1-(1-0.2)^{0.54}(1-0.5)^{0.28}(1-0.5)^{0.18}, 1-(1-0.2)^{0.54}(1-0.5)^{0.28}(1-0.6)^{0.18}\Big\}, \\
& \Big\{1-\left(1-(1-(1-0.1))^{0.54}(1-(1-0.1))^{0.28}(1-(1-0.2))^{0.18}\right), 1-\left(1-(1-(1-0.1))^{0.54}(1-(1-0.1))^{0.28}(1-(1-0.3))^{0.18}\right), \\
& 1-\left(1-(1-(1-0.2))^{0.54}(1-(1-0.1))^{0.28}(1-(1-0.2))^{0.18}\right), 1-\left(1-(1-(1-0.2))^{0.54}(1-(1-0.1))^{0.28}(1-(1-0.3))^{0.18}\right)\Big\}, \\
& \Big\{1-\left(1-(1-(1-0.5))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.3))^{0.18}\right), 1-\left(1-(1-(1-0.5))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.4))^{0.18}\right), \\
& 1-\left(1-(1-(1-0.5))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.3))^{0.18}\right), 1-\left(1-(1-(1-0.5))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.4))^{0.18}\right), \\
& 1-\left(1-(1-(1-0.6))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.3))^{0.18}\right), 1-\left(1-(1-(1-0.6))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.4))^{0.18}\right), \\
& 1-\left(1-(1-(1-0.6))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.3))^{0.18}\right), 1-\left(1-(1-(1-0.6))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.4))^{0.18}\right)\Big\}.
\end{aligned}$$

and obtain the SVNHFE $\widetilde{n}_1$ as the following.

$$\widetilde{n}_1 = \{\{0.2922, 0.3203, 0.3220, 0.3414, 0.3489, 0.3556, 0.3675, 0.3691, 0.3811, 0.3941, 0.4004, 0.4242\}, \{0.1134, 0.1220, 0.1648, 0.1774\}, \{0.3953, 0.4164, 0.4283, 0.4512, 0.4361, 0.4595, 0.4726, 0.4979\}\}.$$

Similarly, the SVNHFEs of other alternative companies can be computed as follows:

$$\widetilde{n}_2 = \{\{0.6000, 0.6275, 0.6363, 0.6613, 0.6457, 0.6701, 0.6778, 0.7000\}, \{0.1000, 0.1257, 0.1340, 0.1684\}, \{0.1651, 0.1887, 0.2211, 0.2528\}\};$$

$$\widetilde{n}_3 = \{\{0.5228, 0.5567, 0.5694, 0.6000\}, \{0.1989\}, \{0.2787, 0.3186\}\};$$

$$\widetilde{n}_4 = \{\{0.5280, 0.5608, 0.5821, 0.6111, 0.5943, 0.6225, 0.6408, 0.6657\}, \{0.1366\}, \{0.1189, 0.1464, 0.1625, 0.2000, 0.1950, 0.2400\}\}.$$

Step 3. Calculate the score function value of the SVNHFE $\widetilde{n}_i$ by using Equation (34):

$$s(\widetilde{n}_1) = 0.5902, s(\widetilde{n}_2) = 0.7711, s(\widetilde{n}_3) = 0.6882, s(\widetilde{n}_4) = 0.7623.$$

Then, we can obtain the ranking order of four alternative companies is $A_2 \succ A_4 \succ A_3 \succ A_1$, the food company A_2 is the best alternative.

If we replace the GSVNHFPWA operator in the aforementioned procedures with the GSVNHFPWG operator, the decision-making steps of the proposed method can be described as follows.

Step 1′. See **Step 1.**

Step 2′. Utilize the GSVNHFPWG operator (which the parameter $\lambda = 1$) to aggregate the SVNHF decision matrix $N = (\widetilde{n}_{ij})_{m\times n}(i = 1,2,3,4; j = 1,2,3)$ into the SVNHFE $\widetilde{n}_i = \left\{\widetilde{t}_i, \widetilde{i}_i, \widetilde{f}_i\right\}(i = 1,2,3,4)$ of each alternative company. Take an alternative company A_1 for example, we have

$$\widetilde{n}_1 = \text{GSVNHFPWG}_1(\widetilde{n}_{11}, \widetilde{n}_{12}, \widetilde{n}_{13}) = \left((\widetilde{n}_{11})^{\frac{T_{11}}{\sum_{j=1}^{3} T_{1j}}} \otimes (\widetilde{n}_{12})^{\frac{T_{12}}{\sum_{j=1}^{3} T_{1j}}} \otimes (\widetilde{n}_{13})^{\frac{T_{13}}{\sum_{j=1}^{3} T_{1j}}} \right)$$

$$= \bigcup_{\widetilde{\gamma}_{11}\in\widetilde{t}_{11},\widetilde{\gamma}_{12}\in\widetilde{t}_{12},\widetilde{\gamma}_{13}\in\widetilde{t}_{13},\widetilde{\delta}_{11}\in\widetilde{i}_{11},\widetilde{\delta}_{12}\in\widetilde{i}_{12},\widetilde{\delta}_{13}\in\widetilde{i}_{13},\widetilde{\eta}_{11}\in\widetilde{f}_{11},\widetilde{\eta}_{12}\in\widetilde{f}_{12},\widetilde{\eta}_{13}\in\widetilde{f}_{13}} \left\{ \left\{ 1 - \left(1 - \prod_{j=1}^{3} \left(1 - (1-\gamma_{1j})^1 \right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\}, \left\{ \left(1 - \prod_{j=1}^{3} \left(1 - (\delta_{1j})^1 \right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\}, \left\{ \left(1 - \prod_{j=1}^{3} \left(1 - (\eta_{1j})^1 \right)^{\frac{T_{1j}}{\sum_{j=1}^{3} T_{1j}}} \right)^{1/1} \right\} \right\}$$

$$\begin{aligned}
= \Big\{ \Big\{ & 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.6))^{0.18}\right), \\
& 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.6))^{0.18}\right), \\
& 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.5))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.5))^{0.28}(1-(1-0.6))^{0.18}\right), \\
& 1 - \left(1-(1-(1-0.3))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.3))^{0.54}(1-(1-0.3))^{0.28}(1-(1-0.6))^{0.18}\right), \\
& 1 - \left(1-(1-(1-0.3))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.3))^{0.54}(1-(1-0.4))^{0.28}(1-(1-0.6))^{0.18}\right), \\
& 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.5))^{0.28}(1-(1-0.5))^{0.18}\right), 1 - \left(1-(1-(1-0.2))^{0.54}(1-(1-0.5))^{0.28}(1-(1-0.6))^{0.18}\right) \Big\}, \\
& \Big\{ 1-(1-0.1)^{0.54}(1-0.1)^{0.28}(1-0.2)^{0.18}, 1-(1-0.1)^{0.54}(1-0.1)^{0.28}(1-0.3)^{0.18}, 1-(1-0.2)^{0.54}(1-0.1)^{0.28}(1-0.2)^{0.18}, \\
& 1-(1-0.2)^{0.54}(1-0.1)^{0.28}(1-0.3)^{0.18} \Big\}, \Big\{ 1-(1-0.5)^{0.54}(1-0.3)^{0.28}(1-0.3)^{0.18}, 1-(1-0.5)^{0.54}(1-0.3)^{0.28}(1-0.4)^{0.18}, \\
& 1-(1-0.5)^{0.54}(1-0.4)^{0.28}(1-0.3)^{0.18}, 1-(1-0.5)^{0.54}(1-0.4)^{0.28}(1-0.4)^{0.18}, 1-(1-0.6)^{0.54}(1-0.3)^{0.28}(1-0.3)^{0.18}, \\
& 1-(1-0.6)^{0.54}(1-0.3)^{0.28}(1-0.4)^{0.18}, 1-(1-0.6)^{0.54}(1-0.4)^{0.28}(1-0.3)^{0.18}, 1-(1-0.6)^{0.54}(1-0.4)^{0.28}(1-0.4)^{0.18} \Big\}.
\end{aligned}$$

and obtain the SVNHFE $\widetilde{n}_1$ as the following:

$$\widetilde{n}_1 = \{\{0.2644, 0.2733, 0.2865, 0.2961, 0.3049, 0.3151, 0.3291, 0.3402, 0.3566, 0.3686, 0.3795, 0.3923\}, \{0.1190, 0.1401, 0.1733, 0.1931\}, \{0.4163, 0.4324, 0.4408, 0.4562, 0.4825, 0.4968, 0.5043, 0.5179\}\}.$$

Similarly, the SVNHFEs of other alternative companies can be computed as follows:

$$\widetilde{n}_2 = \{\{0.6000, 0.6234, 0.6314, 0.6559, 0.6403, 0.6652, 0.6738, 0.7000\}, \{0.1000, 0.1344, 0.1436, 0.1763\}, \{0.1866, 0.2217, 0.2260, 0.2594\}\};$$

$$\widetilde{n}_3 = \{\{0.5194, 0.5517, 0.5649, 0.6000\}, \{0.2531\}, \{0.2917, 0.3222\}\};$$

$$\widetilde{n}_4 = \{\{0.4600, 0.4781, 0.4788, 0.4976, 0.5789, 0.6016, 0.6026, 0.6262\}, \{0.1465\}, \{0.1261, 0.1565, 0.1712, 0.2000, 0.2196, 0.2467\}\}.$$

Step 3′. Calculate the score function value of the SVNHFE $\widetilde{n}_i$ by using Equation (34):

$$s(\widetilde{n}_1) = 0.5669, s(\widetilde{n}_2) = 0.7622, s(\widetilde{n}_3) = 0.6663, s(\widetilde{n}_4) = 0.7358.$$

Then, we can obtain the ranking order of four alternative companies is $A_2 \succ A_4 \succ A_3 \succ A_1$, and the food company A_2 is also the best alternative.

In real life, decision makers may determine the value of the parameter λ according to the decision-making problem itself or their preference. To analyze the influence of the parameter λ on the final ranking result, we change the parameter λ of the GSVNHFPWA operator and GSVNHFPWG operator in the numerical example above. Different values of the parameter λ are provided, such as 0.001, 0.5, 1, 2, 3, 5, 10, 20, and 50, which is determined by decision makers in decision-making process. Combined with the proposed method, we can obtain the score function values of four alternative companies, then the ranking results are determined as shown in Tables 2 and 3. Tables 2 and 3 show that when the GSVNHFPWA operator is used to aggregate arguments, the best alternative is the food company A_2 for $0 < \lambda \leq 3$, but the best alternative is the arms company A_4 for $5 \leq \lambda \leq 50$. Besides, when the GSVNHFPWG operator is used to aggregate arguments, the best alternative is always the food company A_2 for $0 < \lambda \leq 50$, however, there are some differences in specific ranking for $\lambda = 50$. Thus, the different ranking results indicate that the parameter λ plays a very important role in the aggregation process; decision makers should be cautious to determine the value of λ in real decision-making process.

Table 2. Score function values obtained by the GSVNHFPWA operator and the rankings of alternatives for different values of λ.

The Value of λ	$s(\tilde{n}_1)$	$s(\tilde{n}_2)$	$s(\tilde{n}_3)$	$s(\tilde{n}_4)$	Ranking
$\lambda = 0.001$	0.5834	0.7702	0.6856	0.7571	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 0.5$	0.5866	0.7706	0.6869	0.7596	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 1$	0.5902	0.7711	0.6882	0.7623	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 2$	0.5984	0.7721	0.6910	0.7676	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 3$	0.6071	0.7732	0.6937	0.7727	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 5$	0.6232	0.7753	0.6991	0.7811	$A_4 \succ A_2 \succ A_3 \succ A_1$
$\lambda = 10$	0.6500	0.7810	0.7109	0.7954	$A_4 \succ A_2 \succ A_3 \succ A_1$
$\lambda = 20$	0.6734	0.7902	0.7253	0.8104	$A_4 \succ A_2 \succ A_3 \succ A_1$
$\lambda = 50$	0.6927	0.8023	0.7394	0.8261	$A_4 \succ A_2 \succ A_3 \succ A_1$

Table 3. Score function values obtained by the GSVNHFPWG operator and the rankings of alternatives for different values of λ.

The Value of λ	$s(\tilde{n}_1)$	$s(\tilde{n}_2)$	$s(\tilde{n}_3)$	$s(\tilde{n}_4)$	Ranking
$\lambda = 0.01$	0.5735	0.7667	0.6766	0.7454	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 0.5$	0.5704	0.7647	0.6718	0.7408	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 1$	0.5669	0.7622	0.6663	0.7358	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 2$	0.5592	0.7569	0.6553	0.7251	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 3$	0.5512	0.7518	0.6459	0.7152	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 5$	0.5372	0.7435	0.6324	0.6998	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 10$	0.5166	0.7317	0.6132	0.6806	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 20$	0.5013	0.7311	0.5964	0.6686	$A_2 \succ A_4 \succ A_3 \succ A_1$
$\lambda = 50$	0.5718	1.0000	0.8765	0.8030	$A_2 \succ A_3 \succ A_4 \succ A_1$

5.2. Comparison and Discussion

To further verify the effectiveness of the proposed method, we compare the aforementioned ranking order with the results of other decision-making methods for analyzing the same numerical example as shown in Table 4; these methods include the SVNHFWA operator and SVNHFWG operator [14], correlation coefficient of DHFSs [27], correlation coefficient of SVNEs [28], and correlation coefficient of SVNHFEs [15]. From Table 4, we can see that the ranking order of four alternatives obtained by the SVNHFWA operator is $A_4 \succ A_2 \succ A_3 \succ A_1$ due to the feature of emphasizing group major points; besides, the ranking order of four alternatives in other methods are always $A_2 \succ A_4 \succ A_3 \succ A_1$, which is consistent with our proposed method.

Table 4. Comparison result of different decision-making methods.

Decision-Making Method	Ranking
The GSVNHFPWA operator ($\lambda = 1$)	$A_2 \succ A_4 \succ A_3 \succ A_1$
The GSVNHFPWG operator ($\lambda = 1$)	$A_2 \succ A_4 \succ A_3 \succ A_1$
The SVNHFWA operator	$A_4 \succ A_2 \succ A_3 \succ A_1$
The SVNHFWG operator	$A_2 \succ A_4 \succ A_3 \succ A_1$
Correlation coefficient of DHFSs	$A_2 \succ A_4 \succ A_3 \succ A_1$
Correlation coefficient of SVNEs	$A_2 \succ A_4 \succ A_3 \succ A_1$
Correlation coefficient of SVNHFEs	$A_2 \succ A_4 \succ A_3 \succ A_1$

With regard to the existing five decision-making methods above, the methods based on the correlation coefficient of DHFSs and correlation coefficient of SVNEs are only applicable to the DHF and SVN environment, respectively, while DHFS and SVNS are the specific cases of SVNHFS. On the other hand, the other three methods can only solve the decision-making problems that the criteria are in the same priority level. Therefore, the comparison result indicates that the proposed method, not only can deal with the decision-making problems effectively but, also has several advantages as follows: (1) decision makers evaluate the alternatives by using SVNHFEs, which contains truth-membership, indeterminacy-membership, and falsity-membership degrees, and SVNHFS is also a generalization of HFS, DHFS, and SVNS; thus, SVNHFEs can express more reliable evaluation information of decision makers; (2) the GSVNHFPWA operator and GSVNHFPWG operator can solve the decision-making problems that the criteria are in different priority levels, which is not considered in other decision-making methods under SVNHF environment; and (3) the GSVNHFPWA operator and GSVNHFPWG operator can be reduced to several aggregation operators through adjusting the value of the parameter λ, including the SVNHFWA operator and SVNHFWG operator [14]. Decision makers can determine the exact value of the parameter λ to respond to the possible situations in real life.

6. Conclusions

This paper studies the MCDM problems under SVNHF environment, while the criteria are in different priority levels. Motivated by the idea of the PA operator, we develop the GSVNHFPWA operator and GSVNHFPWG operator for aggregating SVNHFEs based on the related researches of SVNS and HFS theory. Some desirable properties of the proposed operators are investigated in detail, such as idempotency, boundedness, and monotonicity. Furthermore, we obtained several special cases that reduced from the proposed operators by changing the value of the parameter λ. Then, an approach for MCDM in which the criteria have different priorities is constructed combined with these operators. Finally, a numerical example is provided to illustrate the applications of the proposed method, and several advantages are reflected by the comparison between the proposed method and several existing decision-making methods. In the future, we shall investigate the SVNHF prioritized aggregation operators according to the different t-norm and t-conorm operational laws, and develop more aggregation operators for SVNHFSs.

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