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GENERALIZED TOPOLOGICAL SPACES VIA NEUTROSOPHIC SETS

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Abstract. In this disquisition we have scrutinize about the traits of generalized topological spaces using neutrosophic sets. Depending on the nature of neutrosophic sets over the generalized topological spaces, some of the features has been contemplated.

Keywords: μ_N-openset; μ_N-int$A$; μ_N-cl$A$; μ_N-exterior; μ_N-frontier.

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1. INTRODUCTION

Some theories played a vital role towards the development of the neutrosophical topological space they are fuzzy set theory [1], Intuitionistic fuzzy set theory [2] and the Neutrosophic set theory [3]. Fuzzy set theory is a mathematical aid which deals with uncertainties in which element has a degree of membership, developed by L. Zadeh [10] in 1965. The concept of intutionistic fuzzy sets were developed by K.T. Atanassov [1]. Later on, Dogan Coker [2] laid down the foundations to intuitionistic fuzzy topological space. Followed by this, the new

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postulations of neutrosophic sets from the intutionistic fuzzy sets were put forth by Smarandache [3]. A.A. Salama and S.A. Albowi raised [6] [7] their thoughts towards neutrosophic sets and developed neutrosophic topological spaces. Levine [4] gave a clearcut idea on generated closed set which leads to the developement of the generalized topological spaces by P. Sivagami and D. Sivaraj [9] had a great impact in the field of topology. Belatedly P. Sivagami, G. Helen Rajapushpam, G. Hari Siva Annam [8] introduced Intuitionistic generalized closed sets in generalised intuitionistic topological space which provokes my thoughts into μ_N Topological spaces. In this write up we launch new initiatives to the generalized topological spaces using neutrosophic sets.

2. Preliminaries

Here, we bring back the ideas which are already exists in the field of neutrosophy.

Definition 2.1. *[6] Let X be a non-empty fixed set. A Neutrosophic set [NS for short] A is an object having the form* $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ *where* $\mu_A(x)$,$\sigma_A(x)$ *and* $\gamma_A(x)$ *which represents the degree of membership function , the degree of indeterminacy and the degree of non-membership function respectively of each element* $x \in X$ *to the set A.*

Remark 2.1. *[6] Every intuitionistic fuzzy set A is a non empty set in X is obviously on Neutrosophic sets having the form* $A = \{< \mu_A(x), 1 - \mu_A(x) + \sigma_A(x), \gamma_A(x) >: x \in X\}$*.In order to construct the tools for developing Neutrosophic Set and Neutrosophic topology, here we introduce the neutrosophic sets* 0_N *and* 1_N *in X as follows:*

0_N *may be defined as follows*

$(0_1) 0_N = \{< x, 0, 0, 1 >: x \in X\}$

$(0_2) 0_N = \{< x, 0, 1, 1 >: x \in X\}$

$(0_3) 0_N = \{< x, 0, 1, 0 >: x \in X\}$

$(0_1) 0_N = \{< x, 0, 0, 0 >: x \in X\}$

1_N *may be defined as follows*

$(1_1) 1_N = \{< x, 1, 0, 0 >: x \in X\}$

$(1_2) 1_N = \{< x, 1, 0, 1 >: x \in X\}$

$(1_3) 1_N = \{< x, 1, 1, 0 >: x \in X\}$

$(1_4) 1_N = \{< x, 1, 1, 1 >: x \in X\}$

Definition 2.2. *[6] Let $A = \{< \mu_A, \sigma_A, \gamma_A >\}$ be a NS on X , then the complement of the set A [$C(A)$ for short] may be defined in three ways as follows:*

$(C_1) C(A) = A = \{< x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x) >: x \in X\}$

$(C_2) C(A) = A = \{< x, \gamma_A(x), \sigma_A(x), \mu_A(x) >: x \in X\}$

$(C_3) C(A) = A = \{< x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x) >: x \in X\}$

Definition 2.3. *[6] LetX be a non-empty set and neutrosophic sets A and B in the form $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$ and $B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$. Then we may consider two possibilities for definitions for subsets($A \subseteq B$). $A \subseteq B$ may be defined as :*

$(A \subseteq B) \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$

$(A \subseteq B) \iff \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x), \gamma_A(x) \geq \gamma_B(x), \forall x \in X$

Proposition 2.1. *[6] For any neutrosophic set A, the following conditions holds: $0_N \subseteq A, 0_N \subseteq 0_N$, $A \subseteq 1_N, 1_N \subseteq 1_N$*

Definition 2.4. *[6] Let X be a non empty set and $A = \{< x, \mu_A(x), \sigma_A(x), \gamma_A(x) >: x \in X\}$, $B = \{< x, \mu_B(x), \sigma_B(x), \gamma_B(x) >: x \in X\}$ are NSs.*

Then $A \cap B$ may be defined as :

$(I_1) A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

$(I_2) A \cap B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

$A \cup B$ may be defined as :

$(I_1) A \cup B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

$(I_2) A \cup B = < x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \gamma_A(x) \vee \gamma_B(x) >$

Proposition 2.2. *[6] For all A and B are two neutrosophic sets then the following conditions are true: $C(A \cup B) = C(A) \cap C(B)$;$C(A \cap B) = C(A) \cup C(B)$.*

Definition 2.5. *[6] A neutrosophic topology [NT for Short] is a non-empty set X is a family τ_N of neutrosophic subsets in X satisfying the following axioms: $(NT_1) 0_N, 1_N \in \tau_N, (NT_2) G_1 \cap G_2 \in \tau_N$ for any $G_1, G_2 \in \tau_N$*

$(NT_3) \cup G_{i\in\tau_N}$ for every $\{G_i : i \in J\} \subseteq \tau_N$

The pair (X,τ_N) is called neutrosophic topological space [NTS for short]. The elements of τ_N are called neutrosophic open sets [NOS for short].

A neutrosophic set F is called neutrosophic closed if and only if $C(F)$ is neutrosophic open.

Definition 2.6. *[6] The complement of A $[C(A)]$ of NOS is called a neutrosophic closed set [NCS for short] in X.*

Definition 2.7. *[6] Let (X,τ_N) be NTS and $A = \{<\mu_A,\sigma_A,\gamma_A>\}$ be a Neutrosophic set in X. Then the neutrosophic Closure and Neutrosophic Interior of A are defined by*

$NCl(A)$ is the intersection of Neutrosophic closed super sets of A.

$NInt(A)$ is the union of Neutrosophic open subsets of A.

Definition 2.8. *[5] The intersection of all NFPs of A is called a neutrosophic frontier of A and is denoted by $NFr(A)$. That is, $NFr(A) = NCl(A) \cap NCl(C(A))$.*

3. μ_N Topological Spaces

In this part of the article we introduce the new concept named as μ_N Topological space.

Definition 3.1. *A μ_N topology is a non - empty set X is a family of neutrosophic subsets in X satisfying the following axioms:*

$(\mu_{N_1})0_N \in \mu_N$

$(\mu_{N_2})G_1 \cup G_2 \in \mu_N$ for any $G_1, G_2 \in \mu_N$.

Throughout this article, the pair of(X,μ_N) is known as μ_N Topological Space[μ_N TS for short].

Remark 3.1. *The elements of μ_N are μ_N open sets and their complement is called μ_N closed sets.*

Definition 3.2. *Let (X,μ_N) be a μ_N TS and $A =< x,\mu_A(x),\sigma_A(x),\gamma_A(x) >$ be a neutrosophic set in X. Then the μ_N-Closure is the intersection of all μ_N closed sets containing A.*

Definition 3.3. *Let (X,μ_N) be a μ_N TS and $A =< x,\mu_A(x),\sigma_A(x),\gamma_A(x) >$ be a neutrosophic set in X. Then the μ_N-Interior is the union of all μ_N open sets contained in A.*

Example 3.1. *Let* $X = \{a,b\}$ *with* $\mu_N = \{0_N, A, B, C\}$ *where* $A = \{<0.2,0.4,0.6><0.1,0.2,0.3>\}$, $B = \{<0.6,0.8,0.5><0.3,0.2,0.5>\}$ *and* $C = \{<0.6,0.4,0.5><0.3,0.2,0.3>\}$ *be a* μ_N *TS. Here* $\mu_N cl(0_N) = \{<0.5,0.6,0.6><0.3,0.8,0.3>\}$ *and* $\mu_N Int(1_N) = \{<0.6,0.4,0.5><0.3,0.2,0.5>\}$

Remark 3.2. *Intersection of* μ_N *closed sets is again* μ_N *closed.*

Proof. Let A and B be two μ_N closed sets then $C(A)$ and $C(B)$ are μ_N open sets. We have $C(A \cap B) = C(A) \cup C(B)$ which is a μ_N open. Hence, $A \cap B$ is μ_N closed. Thus, Intersection of any two μ_N closed sets is again μ_N closed.We can extend the above proof for any number of μ_N closed sets.

□

Theorem 3.1. *Let A be a subset of X then A is* μ_N *open iff* $A = \mu_N IntA$.

Proof. Suppose A is μ_N open set then $\mu_N Int(A)$ is the union of all μ_N open sets contained in A.Hence $A = \mu_N IntA$ .Conversely, Assume $A = \mu_N IntA \Rightarrow \mu_N Int(A) = \cup\{G/G \subseteq A, G$ is μ_N open$\}$ we know that arbitrary union of μ_N open sets is μ_N open. Hence A is μ_N open. □

Theorem 3.2. *Let A be a subset of X then A is* μ_N *closed iff* $A = \mu_N Cl(A)$.

Proof. Suppose A is μ_N closed set then $\mu_N Cl(A)$ is the intersection of all closed sets containing A. Hence $A = \mu_N Cl(A)$.Conversely, Assume $A = \mu_N Cl(A)$ which implies us that $\mu_N Cl(A) = \cap\{F/A \subseteq F, F$ is μ_N closed$\}$ we know that arbitrary intersection of μ_N closed sets is μ_N closed. Hence A is μ_N closed. □

4. PROPERTIES OF μ_N CLOSURE & μ_N INTERIOR

In this section we will be discussing about some of the properties of μ_N closure and μ_N Interior.

Result 4.1. $\mu_N Cl(0_N) \neq 0_N$; $\mu_N Cl(1_N) = 1_N$

Theorem 4.1. *Enhancing Property of* μ_N *closure:*

Statement: $A \subseteq \mu_N Cl(A)$

Proof. Since $\mu_N Cl$ is the intersection of μ_N closed sets containing A.Hence $A \subseteq \mu_N Cl(A)$. □

Theorem 4.2. *μ_N closure is monotone $A \subseteq B \Rightarrow \mu_N ClA \subseteq \mu_N ClB$*

Proof. Suppose $A \subseteq B$. Let $x \notin \mu_N ClB \Rightarrow x \notin \cap F; F$ is μ_N closed super sets of B and $B \subseteq F. \Rightarrow$ $x \notin F$; for some μ_N closed super sets of $B, B \subseteq F$. Since , $A \subseteq B, x \notin \cap F; F$ is μ_N closed super set of A. Then, $x \notin \mu_N Cl(A)$. Hence the proof. □

Theorem 4.3. *μ_N Closure is idempotent.i.e ,$\mu_N Cl(\mu_N ClA) = \mu_N ClA$.*

Proof. By theorem 4.1 we get :$A \subseteq \mu_N Cl(A)$ which implies us that $\mu_N Cl(A) \subseteq \mu_N Cl(\mu_N Cl(A))$. Let $B = \mu_N Cl(A)$. Let $x \notin B \Rightarrow x \notin \cap F; F$ is μ_N closed sets and $B \subseteq F \Rightarrow x \notin F$; for some μ_N closed set $F, B \subseteq F$. Hence $x \notin \mu_N cl(F)$; for some μ_N closed sets of $\mu_N cl(F), \mu_N cl(B) \subseteq \mu_N cl(F)$(by theorem 3.2).Hence $x \notin \mu_N cl(B)$. Hence, $\mu_N Cl(\mu_N ClA) = \mu_N ClA$.

□

Theorem 4.4. *$\mu_N Cl(A \cap B) \subseteq \mu_N Cl(A) \cap \mu_N ClB$.*

Proof. We have $A \cap B \subseteq A$ and $A \cap B \subseteq B$ which together implies us that $\mu_N Cl(A \cap B) \subseteq \mu_N Cl\ A$ and $\mu_N Cl(A \cap B) \subseteq \mu_N Cl\ B$. From the above two inclusion we get :$\mu_N Cl(A \cap B) \subseteq \mu_N Cl(A) \cap \mu_N Cl(B)$. In this inequality the inclusions may be strict. It is given in the upcoming example

□

Example 4.1. *Let $X = \{a\}, \mu_N = \{0_N, A, C, \}$ where $A = \{< 0.7, 0.8, 0.9 >\}$,$B = \{< 0.3, 0.4, 0.6 > \}$,$C = \{< 0.9, 0.7, 0.6 >\}$; $A \cap B = \{< 0.3, 0.8, 0.9 >\}$;$\mu_N Cl(A \cap B) = \{< 0.6, 0.3, 0.9 >\}$; $\mu_N Cl(A) \cap \mu_N ClB = \{< 0.9, 0.2, 0.7 >\} \Rightarrow \mu_N Cl(A \cap B) \subset \mu_N Cl(A) \cap \mu_N ClB$.*

Example 4.2. *Let $X = a, \mu_N = \{0_N, A, B, C, \}$ where $A = \{< 1, 0.8, 0.6 >\}$,$B = \{< 0.4, 0.6, 0.8 > \}$,$C = \{< 1, 0.6, 0.6 >\}$. Here $\mu_N Cl(A \cap B) = \{< 0.8, 0.4, 0.4 >\}$; $\mu_N Cl(A) = \{< 1, 0, 0 >\}$; $\mu_N Cl(B) = \{< 0.8, 0.4, 0.4 >\}$. Hence $\mu_N Cl(A) \cap \mu_N ClB = \{< 0.8, 0.4, 0.4 >\} = \mu_N Cl(A \cap B)$*

Theorem 4.5. *$\mu_N Cl(A \cup B) \supseteq \mu_N Cl(A) \cup \mu_N Cl(B)$.*

Proof. We have $A \cup B \supseteq A \Rightarrow \mu_N Cl(A \cup B) \supseteq \mu_N ClA$ and $A \cup B \supseteq B \Rightarrow \mu_N Cl(A \cup B) \supseteq \mu_N ClB$. Thus we get $\mu_N Cl(A \cup B) \supseteq \mu_N Cl(A) \cup \mu_N Cl(B)$. □

Result 4.2. $\mu_N Int(0_N) = 0_N$; $\mu_N Int(1_N) \neq 1_N$.

Theorem 4.6. *Contraction Property of μ_N Interior*

Statement : $\mu_N Int(A) \subseteq A$

Proof. Since $\mu_N Int(A)$ is the union of μ_N open sets contained in A .Hence, $\mu_N Int(A) \subseteq A$. Also $\mu_N Int(A)$ is the largest μ_N open sets contained in A.

□

Theorem 4.7. *Monotonicity of μ_N Interior.*

Statement: $A \subseteq B \Rightarrow \mu_N IntA \subseteq \mu_N IntB$

Proof. Suppose $A \subseteq B$.By previous the theorem 4.6,$\mu_N Int(A) \subseteq A$. Hence $\mu_N Int(A) \subseteq B$. But $\mu_N Int(B)$ is the largest μ_N open sets contained in B. Hence,$\mu_N IntA \subseteq \mu_N IntB$.

□

Theorem 4.8. *Idempotency of μ_N Interior. i.e. ,$\mu_N Int(\mu_N IntA) = \mu_N IntA$*

Proof. By theorem 4.6 we get : $\mu_N Int(A) \subseteq A \Rightarrow \mu_N Int(\mu_N IntA) \subseteq \mu_N IntA$. Let $B = \mu_N IntA$. Let $x \notin \mu_N IntB \Rightarrow x \notin \cup G; G$ is μ_N open sets contained in B. By theorem 4.6 $x \notin G$; for all μ_N open sets contained in A. $\Rightarrow \mu_N IntA \subseteq \mu_N Int(\mu_N IntA)$. Hence the proof. □

Theorem 4.9. $\mu_N Int(A \cap B) \subseteq \mu_N Int(A) \cap \mu_N IntB$

Proof. We have $A \cap B \subseteq A \Rightarrow \mu_N Int(A \cap B) \subseteq \mu_N IntA$ and $A \cap B \subseteq B \Rightarrow \mu_N Int(A \cap B) \subseteq \mu_N IntB$. Thus we get $\mu_N Int(A \cap B) \subseteq \mu_N Int(A) \cap \mu_N IntB$. □

Example 4.3. *Let $X = \{a\}, \mu_N = \{0_N, A, C, E\}$ where $A = \{< 0.3, 0.4, 0.5 >\}$, $B = \{< 0.3, 0, 0.1 > \}$, $C = \{< 0.4, 0.6, 0.8 >\}$, $D = \{< 0.4, 0, 0.1 >\}$, $E = \{< 0.4, 0.4, 0.5 >\}$. Here $B \cap C = \{< 0.3, 0.6, 0.8 >\}$, $\mu_N Int(B \cap C) = \{< 0, 1, 1 >\}$ and $\mu_N Int(B) = \{< 0.3, 0.4, 0.5 >\}$; $\mu_N Int(C) = \{< 0.4, 0.6, 0.8 >\}$, $\mu_N Int(B \cap C) = \{< 0, 1, 1 >\}$.*
Hence $\mu_N Int(B) \cap \mu_N Int(C) = \{< 0.3, 0.6, 0.8 >\} \Rightarrow \mu_N Int(B \cap C) \subset \mu_N Int(B) \cap \mu_N Int(C)$

Example 4.4. *Let $X = \{a\}, \mu_N = \{0_N, A, B, C, \}$ where $A = \{< 0.3, 0.3, 0.5 >\}$, $B = \{< 0.1, 0.2, 0.3 > \}$, $C = \{< 0.3, 0.2, 0.3 >\}$, $D = \{< 0.3, 0.6, 0.2 >\}$, $E = \{< 0.3, 0.8, 0.5 >\}$. Here $\mu_N Int(D) =$*

$\{< 0,1,1 >\}$ *and* $\mu_N Int(E) = \{< 0,1,1 >\}$, $\mu_N Int(D \cap E) = \{< 0,1,1 >\}$. *Hence* $\mu_N Int(A) \cap \mu_N Int B = \mu_N Int(A \cap B)$.

Theorem 4.10. $\mu_N Int(A \cup B) \supseteq \mu_N Int(A) \cup \mu_N Int B$

Proof. Since, We know $A \cup B \supseteq A$ and $A \cup B \supseteq B$ which implies us that $\mu_N Int(A \cup B) \supseteq \mu_N Int(A)$ and $\mu_N Int(A \cap B) \supseteq \mu_N Int(B)$. Thus we obtain $\mu_N Int(A \cup B) \supseteq \mu_N Int(A) \cup \mu_N Int B$. □

5. PROPERTIES OF μ_N CLOSURE AND μ_N INTERIOR USING COMPLEMENTS

Theorem 5.1. *Let* (X, μ_N) *be a* μ_N *topological space then the following statements hold:*

a) $\mu_N Cl(C(A)) = C(\mu_N Int(A))$.

b) $\mu_N Int(C(A)) = C(\mu_N Cl(A))$.

c) $C(\mu_N Cl(C(A)) = \mu_N Int(A)$.

d) $C(\mu_N Int(C(A))) = \mu_N Cl(A)$.

Proof. a) Let $x \in \mu_N Cl(C(A))$ then $x \in \cap F, F$ is μ_N closed sets and $C(A) \subseteq F$ which yields that $x \in F$, for each μ_N closed sets F such that $C(A) \subseteq F$. Hence, we get $x \notin X - F$ for all μ_N open sets $X - F$ such that $X - F \subseteq A$. Then $x \notin \mu_N Int(A)$. Hence $X \in C(\mu_N Int(A)) \Rightarrow \mu_N Cl(C(A)) \subseteq C(\mu_N Int(A))$. Suppose $x \notin \mu_N Cl(C(A)) \Rightarrow x \notin \cap F, F$ is μ_N closed sets and $C(A) \subseteq F$ which implies that $x \notin F$, for some μ_N closed sets contains $C(A)$. Therefore,$x \in X - F$ for some μ_N open set $X - F$ such that $X - F \subseteq A$ and hence $x \in \mu_N Int(A)$ which implies that $x \notin C(\mu_N Int(A))$. Henceforth, $C(\mu_N Int(A)) \subseteq \mu_N Cl(C(A))$. Hence, we get $\mu_N Cl(C(A)) = C(\mu_N Int(A))$.

b) The proof is similar to a).

c) The proof follows by taking complement in a).

d) The proof can be implemented by replacing A by $(\overline{A})$ in a).

□

6. μ_N-EXTERIOR & μ_N- FRONTIER

Definition 6.1. *If* $\mu_N - Ext(A) = \mu_N - Int((C(A))$ *then it will be called as* μ_N*−Exterior of A.*

Remark 6.1. $(i)\mu_N - Ext(1_N) = 0_N$ $(ii)\mu_N - Ext(0_N) \notin 1_N$

Proof. i)

$$\begin{aligned} \mu_N - Ext(1_N) &= \mu_N Int(C(1_N)) \\ &= \mu_N Int(0_N) \\ &= 0_N \end{aligned}$$

ii)

$$\begin{aligned} \mu_N - Ext(0_N) &= \mu_N Int(C(0_N)) \\ &= \mu_N Int(1_N) \\ &\neq \mu_N \end{aligned}$$

□

Example 6.1. *In this example let us show that* $(i)\mu_N - Ext(1_N) = 0_N$ $(ii)\mu_N - Ext(0_N) \notin 1_N$. *Let* $X = \{a,b\}, \mu_N = \{0_N, A, B, C, \}$*where* $A = \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\}$, $B = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >\}$, $C = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\}$. *Here* $Ext(1_N) = \{< 0, 1, 1 >< 0, 1, 1 >\} = 0_N$ & $\mu_N - Ext(0_N) = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\} \neq 1_N$

Theorem 6.1. $\mu_N - Ext(A) = C(\mu_N Cl(A))$

Proof.

$$\begin{aligned} \mu_N - Ext(A) &= \mu_N Int(C(A)) \\ &= \mu_N Int(X - A) \\ &= X - \mu_N Cl(A) \\ &= C(\mu_N Cl(A)). \end{aligned}$$

□

Theorem 6.2. $\mu_N - Ext(A \cup B) \subseteq \mu_N - Ext(A) \cap \mu_N - Ext(B)$

Proof.

$$\begin{aligned}
\mu_N - Ext(A \cup B) &= \mu_N Int(C(A \cup B)) \\
&= \mu_N Int(C(A) \cap C(B)) \\
&\subseteq \mu_N IntC(A) \cap \mu_N IntC(B)) \\
&= \mu_N - Ext(A) \cap \mu_N - Ext(B).
\end{aligned}$$

□

Theorem 6.3. $\mu_N - Ext(A \cap B) \supseteq \mu_N - Ext(A) \cup \mu_N - Ext(B)$.

Proof.

$$\begin{aligned}
\mu_N - Ext(A \cap B) &= \mu_N Int(C(A \cap B)) \\
&= \mu_N Int(C(A) \cup C(B)) \\
&\supseteq \mu_N Int(C(A)) \cup \mu_N Int(C(B)) \\
&= \mu_N - Ext(A) \cup \mu_N - Ext(B)
\end{aligned}$$

□

Theorem 6.4. $\mu_N - Ext(\mu_N - Ext(A)) = \mu_N Int(\mu_N Cl(A)) \supseteq \mu_N Int(A)$.

Proof.

$$\begin{aligned}
\mu_N - Ext(\mu_N - Ext(A)) &= \mu_N - Ext(\mu_N Int(C(A))) \\
&= \mu_N IntC(\mu_N Int(C(A)) \\
&= \mu_N Int(\mu_N Cl(A)) \\
&\supseteq \mu_N Int(A)
\end{aligned}$$

□

Theorem 6.5. *If* $A \subseteq B$, *then* $\mu_N - Ext(B) \subseteq \mu_N - Ext(A)$

Proof. Suppose $A \subseteq B$, then

$$\begin{aligned} \mu_N - Ext(B) &= \mu_N Int(C(B)) \\ &\subseteq \mu_N Int(C(A)) \\ &= \mu_N - Ext(A) \end{aligned}$$

□

Definition 6.2. *If A is a neutrosophic subset of μ_N Topological space X then μ_N Frontier of A is defined as $\mu_N Fr(A) = \mu_N Cl(A) \cap \mu_N Cl(C(A))$.*

Remark 6.2. *If A is a neutrosophic subset of μ_N Topological space X then μ_N Frontier of A is always closed.*

Theorem 6.6. *If A is a neutrosophic open subset of μ_N Topological space X then $A \setminus \mu_N Fr(A) \subseteq A$.*

Proof. Since A is μ_N open ,$C(A)$ is μ_N closed. We have

$$\begin{aligned} \mu_N Fr(A) &= \mu_N Cl(A) \cap \mu_N Cl(C(A)) \\ &= \mu_N Cl(A) \cap C(A) \end{aligned}$$

Now

$$\begin{aligned} C(\mu_N Fr(A)) &= C(\mu_N Cl(A) \cap (C(A))) \\ &= C(\mu_N Cl(A)) \cup C(C(A)) \\ &= C(\mu_N Cl(A)) \cup A \end{aligned}$$

Now

$$\begin{aligned} A \setminus \mu_N Fr(A) &= A \cap C(\mu_N Fr(A)) \\ &= A \cap (C(\mu_N Cl(A)) \cup A) \\ &= (A \cap C(\mu_N Cl(A))) \cup (A \cup A) \\ &\subseteq (A \cap C(A)) \cup A \\ &= \varphi \cup A \\ &= A \end{aligned}$$

□

Example 6.2. *Let* $X = \{a\}, \mu_N = \{0_N, A, B, C, \}$ *where* $A = \{< 0.3, 0.3, 0.5 >\}$*,*$B = \{< 0.1, 0.2, 0.3 >\}$*,*$C = \{< 0.3, 0.2, 0.3 >\}$*,*$D = \{< 0.3, 0.6, 0.2 >\}$*,*$E = \{< 0.3, 0.8, 0.5 >\}$*. Here,* $0_N, A, B, C$ *are* μ_N *open.* $\mu_N Fr\varphi = \{< 0.3, 0.8, 0.3 >\}; \mu_N FrA = \{< 0.5, 0.7, 0.3 >\}$*;* $\mu_N FrB = \{< 0.3, 0.8, 0.1 >\}$*;* $\mu_N FrC = \{< 0.3, 0.8, 0.3 >\}$*. Now,* $\varphi \setminus \mu_N Fr\varphi = \{< 0, 1, 1 >\} = \varphi; A \setminus \mu_N FrA = \{< 0.3, 0.3, 0.5 >\} = A$*;* $B \setminus \mu_N FrB = \{< 0.1, 0.2, 0.3 >\} = B$*;* $C \setminus \mu_N FrC = \{< 0.3, 0.2, 0.3 >\} = C$*.*

Theorem 6.7. *If A is a not a neutrosophic open subset of* μ_N *Topological space X then* $A \setminus \mu_N Fr(A) \subseteq A$*.*

Proof. $A \setminus \mu_N Fr(A) = A \cap (C(\mu_N Fr(A)))$ which implies that $A \cap U \subseteq A$ where $U = C(\mu_N Fr(A))$ which is μ_N open since by remark 6.2 . Hence , $A \setminus \mu_N Fr(A) \subseteq A$. □

The inclusion may be strict both the cases are discussed in the below example.

Example 6.3. *Let* $X = \{a\}, \mu_N = \{0_N, A, B, C, \}$ *where* $A = \{< 0.3, 0.3, 0.5 >\}$*,*$B = \{< 0.1, 0.2, 0.3 >\}$*,*$C = \{< 0.3, 0.2, 0.3 >\}$*,*$D = \{< 0.3, 0.6, 0.2 >\}$*,*$E = \{< 0.3, 0.8, 0.5 >\}$*. Here* $D, E, 1_N$ *are not* μ_N *open.* $\mu_N FrD = \{< 1, 0, 0 >\}$*;* $\mu_N FrE = \{< 0.3, 0.8, 0.3 >\}$*;* $\mu_N Fr1_N = \{< 0.3, 0.8, 0.3 >\}$*. Now* $D \setminus \mu_N FrD = \{< 0, 1, 1 >\} \subset D$*;* $E \setminus \mu_N FrE = \{< 0.3, 0.8, 0.5 >\} = E$*;*$1_N \setminus \mu_N Fr1_N = \{< 0.3, 0.8, 0.3 >\} \subset 1_N$.

Theorem 6.8. *If A is a neutrosophic subset of μ_N Topological space X then* $\mu_N Fr(A) = \mu_N Fr(C(A))$.

Proof.

$$\begin{aligned} \mu_N Fr(A) &= \mu_N Cl(A) \cap \mu_N Cl(C(A)) \\ &= \mu_N Cl(X \setminus A) \cap \mu_N Cl(X \setminus C(A)). \\ &= \mu_N Fr(C(A)). \end{aligned}$$

□

Theorem 6.9. *Let A be a neutrosophic subsets of μ_N Topological space X then* $\mu_N Fr(A) = \mu_N Cl(A) - \mu_N Int(A)$.

Proof. By Theorem 5.1 we have $C(\mu_N Cl(C(A))) = \mu_N IntA$ and by the definition of $\mu_N Fr(A), \mu_N Fr(A) = \mu_N Cl(A) \cap \mu_N Cl(C(A))$ which is equivalent to $\mu_N Cl(A) - C(\mu_N Cl(C(A)))$ since we have $A - B = A \cap C(B)$. Thus,$\mu_N Fr(A) = \mu_N Cl(A) - \mu_N Int(A)$. □

Theorem 6.10. *For each $A \in \mu_N$ $TS(X)$, $A \cup \mu_N Fr(A) \subseteq \mu_N Cl(A)$.*

Proof. Let A be a neutrosophic subsets of μ_N Topological space X.

$$\begin{aligned} A \cup \mu_N Fr(A) &= A \cup (\mu_N Cl(A) \cap \mu_N Cl(C(A))) \\ &= (A \cup (\mu_N Cl(A))) \cap (A \cup (\mu_N Cl(C(A)))) \\ &= \mu_N Cl(A) \cap (A \cup (\mu_N Cl(C(A)))) \\ &\subseteq \mu_N Cl(A). \end{aligned}$$

□

The inclusion may be strict both the cases are discussed in the below example.

Example 6.4. *Let* $X = \{a\}, \mu_N = \{0_N, A, B, C,\}$ *where* $A = \{< 0.3, 0.3, 0.5 >\}$,$B = \{< 0.1, 0.2, 0.3 >\}$,$C = \{< 0.3, 0.2, 0.3 >\}$,$D = \{< 0.3, 0.6, 0.2 >\}$,$E = \{< 0.3, 0.8, 0.5 >\}$. *Here* ,$\mu_N Fr\varphi = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N FrA = \{< 0.5, 0.7, 0.3 >\}$; $\mu_N FrB = \{< 0.3, 0.8, 0.1 >\}$; $\mu_N FrC = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N FrD = \{< 1, 0, 0 >\}$; $\mu_N FrE = \{< 0.3, 0.8, 0.3 >\}$; $\mu_N Fr1_N = \{<$

$0.3, 0.8, 0.3 >\}$. *Also* $\mu_N Cl\varphi = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N ClA = \{< 1, 0, 0 >\}$; $\mu_N ClB = \{< 1, 0, 0 >\}$; $\mu_N ClC = \{< 1, 0, 0 >\}$, $\mu_N ClD = \{< 1, 0, 0 >\}$; $\mu_N ClE = \{< 0.3, 0.8, 0.3 >\}$; $\mu_N Cl1_N = \{< 1, 0, 0 >\}$. *Now* ,$\varphi \cup \mu_N Fr(\varphi) = \{< 0.3, 0.8, 0.3 >\} = \mu_N Cl\varphi$; $A \cup \mu_N Fr(A) = \{< 0.5, 0.3, 0.3 >\} \subset \{< 1, 0, 0 >\} = \mu_N Cl(A)$; $B \cup \mu_N Fr(B) = \{< 0.3, 0.2, 0.1 >\} \subset \mu_N Cl(B)$; $C \cup \mu_N Fr(C) = \{< 0.3, 0.2, 0.3 >\} \subset \{< 1, 0, 0 >\} = \mu_N Cl(C)$; $D \cup \mu_N Fr(D) = \{< 1, 0, 0 >\} = \mu_N Cl(D)$; $E \cup \mu_N Fr(E) = \{< 0.3, 0.8, 0.3 >\} = \mu_N Cl(E)$; $1_N \cup \mu_N Fr(1_N) = \{< 1, 0, 0 >\} = \mu_N Cl(1_N)$.

Theorem 6.11. *For a neutrosophic subsets A in the* μ_N *Topological space X,* $\mu_N Fr(\mu_N Int(A)) \subseteq \mu_N Fr(A)$.

Proof. Let A be the neutrosophic subsets in the μ_N Topological space X.

$$\begin{aligned}
\mu_N Fr(\mu_N Int(A)) &= \mu_N Cl(\mu_N Int(A)) \cap \mu_N Cl(C(\mu_N Int(A))) \\
&= \mu_N Cl(\mu_N Int(A)) \cap \mu_N Cl(\mu_N Cl(C(A)) \\
&= \mu_N Cl(\mu_N Int(A)) \cap \mu_N Cl(\mu_N Cl(C(A)) \\
&\subseteq \mu_N Cl(A) \cap \mu_N Cl(C(A)) \\
&= \mu_N Fr(A)
\end{aligned}$$

Hence, $\mu_N Fr(\mu_N Int(A)) \subseteq \mu_N Fr(A)$. □

Remark 6.3. *The converse of the above theorem is not true as shown by the following example Let* $X = \{a\}, \mu_N = \{0_N, A, B, C, \}$ *where* $A = \{< 0.3, 0.3, 0.5 >\}, B = \{< 0.1, 0.2, 0.3 >\}, C = \{< 0.3, 0.2, 0.3 >\}, D = \{< 0.3, 0.6, 0.2 >\}, E = \{< 0.3, 0.8, 0.5 >\}$. $\mu_N Fr(D) = \{< 1, 0, 0 >\}$, $\mu_N Fr(\mu_N IntD) = \mu_N Fr\{< 0, 1, 1 >\} = \{< 0.3, 0.8, 0.3 >\}$. *Here* $\mu_N Fr(D) \not\subseteq \mu_N Fr(\mu_N IntD)$.

Theorem 6.12. *For a neutrosophic subsets A in* μ_N *topological space X,*
$\mu_N Fr(\mu_N Cl(A)) \subseteq \mu_N Fr(A)$.

Proof. Let A be a NS in the μ_N Topological space.Then by definition

$$\begin{aligned} \mu_N Fr(\mu_N Cl(A)) &= \mu_N Cl(\mu_N Cl(A)) \cap \mu_N Cl(C(\mu_N Cl(A))) \\ &= \mu_N Cl(A) \cap \mu_N Cl(\mu_N Int(C(A))) \\ &\subseteq \mu_N Cl(A) \cap \mu_N Cl(C(A)) \\ &= \mu_N Fr(A). \end{aligned}$$

□

Remark 6.4. *The converse of the above theorem is not true which is explained in the following example . Let* $X = \{a\}, \mu_N = \{0_N, A, B, C\}$ *where* $A = \{< 0.3, 0.3, 0.5 >\}$,$B = \{< 0.1, 0.2, 0.3 >\}$,$C = \{< 0.3, 0.2, 0.3 >\}$,$D = \{< 0.3, 0.6, 0.2 >\}$,$E = \{< 0.3, 0.8, 0.5 >\}$. *Here,* $\mu_N Fr(E) = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N Fr(\mu_N Cl(E)) = \mu_N Fr\{< 0.3, 0.8, 0.3 >\} = \{< 0, 1, 1 >\}$. *Here,* $\mu_N Fr(E) \not\subseteq \mu_N Fr(\mu_N Cl(E))$

Remark 6.5. *In General Topology, the following conditions are hold.*

(i) $NFr(A) \cap NInt(A) = 0_N$

(ii) $NInt(A) \cup NFr(A) = Ncl(A)$

(iii) $NInt(A) \cup NInt(C(A)) \cup NFr(A) = 1_N$

But here in μ_N *Topological space we provide counter-examples (11-13) to show that the above conditions may not be hold in general.*

Example 6.5. *Let* $X = \{a, b\}$ *and* $\mu_N = \{0_N, A, B, C\}$ *Then* (X, μ_N) *be a* μ_N *Topological space and* $A = \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\}$,$B = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >\}$,$C = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\}$. *Here,* $\mu_N Fr(A) \cap \mu_N Int(A) = \{< 0.6, 0.6, 0.2 >< 0.3, 0.8, 0.1 >\} \cap \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\} = \{< 0.2, 0.6, 0.6 >< 0.1, 0.8, 0.3 >\} \neq 0_N$. *Thus,* $\mu_N Fr(A) \cap \mu_N Int(A) \neq 0_N$.

Example 6.6. *Let* $X = \{a, b\}$ *and* $\mu_N = \{0_N, A, B, C\}$ *Then* (X, μ_N) *be a* μ_N *Topological space and* $A = \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\}$,$B = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >\}$,$C = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\}$. *Here,* $\mu_N Int(B) \cup \mu_N Fr(B) = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >$

$\} \cup \{< 0.5, 0.2, 0.6 >< 0.5, 0.8, 0.3 >\} = \{< 0.6, 0.2, 0.5 >< 0.5, 0.2, 0.3 >\} \neq \mu_N Cl(B) = \{< 1, 0, 0 >< 1, 0, 0 >\}$.

Example 6.7. *Let $X = \{a, b\}$ and $\mu_N = \{0_N, A, B, C\}$ Then (X, μ_N) be a μ_N Topological space and $A = \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\}$,$B = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >\}$,$C = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\}$.*
Here,$\mu_N Int(C) \cup \mu_N Int(C(C)) \cup \mu_N Fr(C) = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\} \cup \{< 0, 1, 1 >< 0, 1, 1 >\} \cup \{< 0.5, 0.6, 0.6 >< 0.3, 0.8, 0.3 >\} = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\} \neq 1_N$.

Remark 6.6. *In μ_N Topological space X,$\mu_N Fr(A \cap B)$ and $\mu_N Fr(A) \cap \mu_N Fr(B)$ are independent. It is established in the following example. Let $X = \{a, b\}$ and $\mu_N = \{0_N, A, B, C\}$. Then (X, μ_N) be a μ_N Topological space and $A = \{< 0.2, 0.4, 0.6 >< 0.1, 0.2, 0.3 >\}$,$B = \{< 0.6, 0.8, 0.5 >< 0.3, 0.2, 0.5 >\}$,$C = \{< 0.6, 0.4, 0.5 >< 0.3, 0.2, 0.3 >\}$. $\mu_N Fr(A \cap B) = \{< 0.6, 0.2, 0.2 >< 0.5, 0.8, 0.1 >\}$, $\mu_N Fr(A) \cap \mu_N Fr(B) = \{< 0.5, 0.6, 0.6 >< 0.3, 0.8, 0.3 >\}$ From this we say that $\mu_N Fr(A \cap B) \not\subseteq \mu_N Fr(A) \cap \mu_N Fr(B)$ and also $\mu_N Fr(A) \cap \mu_N Fr(B) \not\subseteq \mu_N Fr(A \cap B)$.*

Remark 6.7. *For any neutrosophic subsets A and B in the Neutrosophic Topological space X, then $NFr(A \cap B) \subseteq NFr(A) \cup NFr(B)$.But it may not hold in μ_N Topological space, we provide a counter-example to explain the scenario.*

Example 6.8. *Let $X = \{a\}$,$\mu_N = \{0_N, A, B, C\}$ where $A = \{< 0.3, 0.3, 0.5 >\}$,$B = \{< 0.1, 0.2, 0.3 >\}$,$C = \{< 0.3, 0.2, 0.3 >\}$,$D = \{< 0.3, 0.6, 0.2 >\}$,$E = \{< 0.3, 0.8, 0.5 >\}$. Here, $\mu_N Fr\varphi = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N FrA = \{< 0.5, 0.7, 0.3 >\}$; $\mu_N FrB = \{< 0.3, 0.8, 0.1 >\}$; $\mu_N FrC = \{< 0.3, 0.8, 0.3 >\}$, $\mu_N FrD = \{< 1, 0, 0 >\}$; $\mu_N FrE = \{< 0.3, 0.8, 0.3 >\}$; $\mu_N Fr1_N = \{< 0.3, 0.8, 0.3 >\}$. Here, $A \cap B = \{< 0.1, 0.3, 0.5 >\}$; $\mu_N Fr(A \cap B) = \{< 1, 0, 0 >\}$; $\mu_N Fr(A) \cup \mu_N Fr(B) = \{< 0.5, 0.7, 0.1 >\}$ which says that $\mu_N Fr(A \cap B) \supseteq \mu_N Fr(A) \cup \mu_N Fr(B)$. On the other hand while considering the sets D and E, $D \cap E = \{< 0.3, 0.8, 0.5 >\}$; $\mu_N Fr(D \cap E) = \{< 0.3, 0.8, 0.3 >\}$; $\mu_N Fr(D) \cup \mu_N Fr(E) = \{< 1, 0, 0 >\}$ in this case we get $\mu_N Fr(D \cap E) \subseteq \mu_N Fr(D) \cup \mu_N Fr(E)$*

Theorem 6.13. *For any neutrosophic subsets A in the μ_N Topological space X,* *(i)* $\mu_N Fr(\mu_N Fr(A)) \subseteq \mu_N Fr(A)$

(ii) $\mu_N Fr(\mu_N Fr(\mu_N Fr(A))) \subseteq \mu_N Fr(\mu_N Fr(A))$

Proof. Let A be the μ_N Topological space X

(i)

$$\begin{aligned}
\mu_N Fr(\mu_N Fr(A)) &= (\mu_N Fr(\mu_N Fr(A))) \\
&= (\mu_N Cl(\mu_N Cl(A))) \cap (\mu_N Cl(\mu_N Cl(C(A)))) \\
&\qquad \cap (\mu_N Cl(C(\mu_N Cl(A)))) \cap (\mu_N Cl((C(A))) \\
&\subseteq (\mu_N Cl(\mu_N Cl(A))) \cap (\mu_N Cl(\mu_N Cl(C(A)))) \\
&\qquad \cap (\mu_N Cl(\mu_N Int(C(A)))) \cup (\mu_N Int(A)) \\
&= (\mu_N Cl(A) \cap \mu_N Cl(C(A))) \cap (\mu_N Cl(\mu_N Int(C(A)))) \\
&\qquad \cup (\mu_N Cl(\mu_N Int(A))) \\
&\subseteq \mu_N Cl(A) \cap \mu_N Cl(C(A)) \\
&= \mu_N Fr(A)
\end{aligned}$$

Hence,$\mu_N Fr(\mu_N Fr(A)) \subseteq \mu_N Fr(A)$.

(ii)

$$\begin{aligned}
\mu_N Fr(\mu_N Fr(\mu_N Fr(A))) &= (\mu_N Cl(\mu_N Fr(\mu_N Fr(A)))) \cap \\
&\qquad (\mu_N Cl(C(\mu_N Fr(\mu_N Fr(A))))) \\
&\subseteq (\mu_N Cl(\mu_N Fr(A))) \cap (\mu_N Cl(C(\mu_N Fr(A)))) \\
&\subseteq \mu_N Fr(\mu_N Fr(A))
\end{aligned}$$

Hence,$\mu_N Fr(\mu_N Fr(\mu_N Fr(A))) \subseteq \mu_N Fr(\mu_N Fr(A))$

□

Remark 6.8. *From the above , The converse of (i) need not be true as shown in the example.From example 14,* $\mu_N FrD = \{< 1,0,0 >\}$*;* $\mu_N Fr(\mu_N Fr(D)) = \mu_N Fr\{< 1,0,0 >\} = \{< 0.3,0.8,0.3 >\} \not\supseteq \{< 1,0,0 >\}$ *which implies* $\mu_N Fr(A) \not\subseteq \mu_N Fr(\mu_N Fr(A))$.

Remark 6.9. *No counter example could be brought out to estabilish the irreversibility of the inclusion in (ii).*

7. CONCLUSION

In this paper , we studied the behaviour of generalized topological spaces using neutrosophic sets and some of their properties were discussed. Also, we study about the operators in μ_N Topological spaces. In future we plan to extend our research towards μ_N continuous, μ_N connected, μ_N Compact and also some new μ_N open sets are to be introduced.

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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