



## How we can extend the standard deviation notion with neutrosophic interval and quadruple neutrosophic numbers

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### Abstract

During scientific demonstrating of genuine specialized framework we can meet any sort and rate model vulnerability. Its reasons can be incognizance of modelers or information mistake. In this way, characterization of vulnerabilities, as for their sources, recognizes aleatory and epistemic ones. The aleatory vulnerability is an inalienable information variety related with the researched framework or its condition. Epistemic one is a vulnerability that is because of an absence of information on amounts or procedures of the framework or the earth [7]. Right now, we examine fourfold neutrosophic numbers and their potential application for practical displaying of physical frameworks, particularly in the unwavering quality evaluation of engineering structures. Contribution: we propose to extend the notion of standard deviation to by using symbolic quadruple operator.

**Keywords:** Standard deviation, Neutrosophic Interval, Quadruple Neutrosophic Numbers.

### 1. Introduction

We all know about uncertainty modelling of various systems, which usually is represented by:

$$X = x' + 1.64s \quad (1)$$

Or

$$X = x' + 1.96s \quad (2)$$

Here, the constants 1.64 or 1.96 can be replaced with  $k$ . What we mean is a constant corresponding to bell curve, the number is usually assumed to be 1.96 for 95% acceptance, or 1.64 for 90% acceptance, respectively.

But since  $s$  only takes account statistical uncertainty, there is lack of measure for indeterminacy. That is why we suggest to extend from

$$X = x' + k. s \quad (3)$$

To become neutrosophic quadruple numbers.

Before we move to next section, first we would mention other possibility, i.e. by expressing the relation as follow

$$(X_L + X_U I_N) = k. (\sigma_L + \sigma_U I_N), \text{ where } I_N \text{ is a measure of indeterminacy} \quad (4)$$

Actually, we we need to add some results for various  $I_N$ , for example  $I_N=0,0.1,0.2,0.3,0.4$  etc. Nonetheless, because this paper is merely suggesting a conceptual framework, we don't explore it further here. Interested readers are suggested to consult ref. [1-2].

## 2. A short review on quaternions

We all know the quaternions, but quadruple neutrosophic numbers are different. In quaternions,  $a+bi + cj + dk$  you have  $i^2 = j^2 = k^2 = -1 = ijk$ , while on quadruple neutrosophic numbers we have:[3]

$$N = a + bT + cI + dF \text{ one has: } T^2 = T, I^2 = I, F^2 = F, \quad (5)$$

where  $a =$  known part of  $N$ ,  $bT+cI+dF =$  unknown part of  $N$ , with  $T =$  degree of truth-membership,  $I =$  degree of indeterminate-membership, and  $F =$  degree of false-membership, and  $a, b, c, d$  are real (or complex) numbers, and an absorption law defined depending on expert and on application (so it varies); if we consider for example the neutrosophic order  $T > I > F$ , then the stronger absorbs the weaker, i.e.

$$TI = T, TF = T, \text{ and } IF = I, TIF = T. \quad (6)$$

Other orders can also be employed, for example  $T < I < F$ : (see book [1], at page 186.) Other interpretations can be given to  $T, I, F$  upon each application.

## 3. Application: statistical uncertainty and beyond

Designers must arrangement with dangers and vulnerabilities as a piece of their expert work and, specifically, vulnerabilities are intrinsic to building models. Models assume a focal job in designing. Models regularly speak to a dynamic and admired rendition of the scientific properties of an objective. Utilizing models, specialists can explore and gain comprehension of how an article or wonder will perform under specified conditions.[8]

Furthermore, according to Murphy & Gardoni & Harris Jr, which can be rephrased as follows: "For engineers, managing danger and vulnerability is a significant piece of their expert work. Vulnerabilities are associated with understanding the normal world, for example, knowing whether a specific occasion will happen, and in knowing the presentation of building works, for example, the conduct and reaction of a structure or foundation, the fluctuation in material properties (e.g., attributes of soil, steel, or solid), geometry, and outer limit conditions (e.g., loads or physical

limitations). Such vulnerabilities produce dangers. In the standard record chance is the result of a lot of potential outcomes and their related probabilities of event (Kaplan and Gerrick 1981), where the probabilities measure the probability of event of the potential outcomes considering the hidden vulnerabilities. One significant utilization of models in designing danger investigation is to measure the probability or likelihood of the event of specific occasions or a lot of outcomes. Such models are regularly alluded to as probabilistic models to feature their specific capacity to represent and measure vulnerabilities.”[8]

Uncertainties come in many forms, for example:

“The uncertainties in developing a model are:

- **Model Inexactness.** This kind of vulnerability emerges when approximations are presented in the plan of a model. There are two basic issues that may emerge: blunder as the model (e.g., a straight articulation is utilized when the real connection is nonlinear), and missing factors (i.e., the model contains just a subset of the factors that influence the amount of intrigue). ...
- **Mistaken Assumptions.** Models depend on a series of expectations. Vulnerabilities may be related with the legitimacy of such suspicions (e.g., issues emerge when a model accept typicality or homoskedasticity when these suppositions are disregarded).
- **Measurement Error.** The parameters in a model are commonly aligned utilizing an example of the deliberate amounts of intrigue and the fundamental factors considered in the model. These watched qualities, in any case, could be inaccurate because of blunders in the estimation gadgets or systems, which at that point prompts mistakes in the alignment procedure. ...
- **Statistical Uncertainty.** Factual vulnerability emerges from the scantiness of information used to align a model. Specifically, the exactness of one's derivations relies upon the perception test size. The littler the example size, the bigger is the vulnerability in the evaluated estimations of the parameters. ... However, the confidence in the model would probably increment on the off chance that it was adjusted utilizing one thousand examples. The factual vulnerability catches our level of confidence in a model considering the information used to adjust the model.”[8]

With regards to statistical uncertainty, according to Ditlevsen and Madsen, which can rephrased as follows: “It is the reason for any estimating technique to produce data about an amount identified with the object of estimation. In the event that the amount is of a fluctuating nature with the goal that it requires a probabilistic model for its depiction, the estimating technique must make it conceivable to define quantitative data about the parameters of the picked probabilistic model. Clearly a deliberate estimation of a solitary result of a non-degenerate arbitrary variable  $X$  just is sufficient for giving a rough gauge of the mean estimation of  $X$  and is insufficient for giving any data about the standard deviation of  $X$ . In any case, if an example of  $X$  is given, that is, whenever estimated estimations of a specific number of freely produced results of  $X$  are given, these qualities can be utilized for figuring gauges for all parameters of the model. The reasons that such an estimation from an example of  $X$  is conceivable and bodes well are to be found in the numerical likelihood hypothesis. The most rudimentary ideas and rules of the hypothesis of insights are thought to be known to the peruser. To delineate the job of the measurable ideas in the unwavering quality examination it is beneficial to rehash the most fundamental highlights of the depiction of the data that an example of  $X$  of size  $n$  contains

about the mean worth  $E[X]$ . It is sufficient for our motivation to make the streamlining supposition that  $X$  has a known standard deviation  $D[X] = \sigma$ .”[5]

Now, it seems possible to extend it further to include not only statistical uncertainty but also modelling error etc. It can be a good application of Quadruple Neutrosophic Numbers.

#### 4. Towards an improved model of standard deviation

Few days ago, we just got an idea regarding application of symbolic Neutrosophic quadruple numbers, where we can use it to extend the notion of *standard deviation*.

As we know usually people wrote:

$$X' = x + k.\sigma \quad (7)$$

Where  $X$  mean observation,  $\sigma$  standard deviation, and  $k$  is usually a constant to be determined by statistical bell curve, for example 1.64 for 95% accuracy.

We can extend it by using symbolic quadruple operator:

$$X' = x \pm (k.\sigma + m.i + n.f) \quad (8)$$

Where  $X'$  stands for actual prediction from a set of observed  $x$  data,  $\sigma$  is standard deviation,  $i$  is indeterminacy and  $f$  falsehood. That way modelling error (falsehood) and indeterminacy can be accounted for.

Alternatively, one can write a better expression:

$$X' = x \pm (T.\sigma + I.\sigma + F.\sigma) \quad (9)$$

where  $T$  = the truth degree of  $s$  (standard deviation),  $I$  = degree of indeterminacy about  $s$ , and  $F$  = degree of falsehood about  $s$ .

A slightly more general expression is the following:

$$X' = x \pm a (T.\sigma + I.\sigma + F.\sigma) \quad (10)$$

where  $T$  = the truth degree of  $s$  (standard deviation),  $I$  = degree of indeterminacy about  $s$ , and  $F$  = degree of falsehood about  $s$ .

Or

$$X' = x \pm (a.T.\sigma + b.I.\sigma + c.F.\sigma) \quad (11)$$

where  $T$  = the truth degree of  $s$  (standard deviation),  $I$  = degree of indeterminacy about  $s$ , and  $F$  = degree of falsehood about  $s$ , and  $a, b, c$  are constants to be determined.

That way we reintroduce quadruple Neutrosophic numbers into the whole of statistics estimate.

For further use in engineering fields especially in reliability methods, readers can consult [5-7].

## 5. Conclusion

In this paper, we reviewed existing use of standard deviation in various fields of science including engineering, and then we consider a plausible extension of standard deviation based on the notion of quadruple neutrosophic numbers. More investigation is recommended.

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