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Interval Complex Neutrosophic Graph of Type 1

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Abstract

The neutrosophic set theory, proposed by smarandache, can be used as a general mathematical tool for dealing with indeterminate and inconsistent information. By applying the concept of neutrosophic sets on graph theory, several studies of neutrosophic models have been presented in the literature. In this paper, the concept of complex neutrosophic graph of type 1 is extended to interval complex neutrosophic graph of type 1(ICNG1). We have proposed a representation of ICNG1 by adjacency matrix and studied some properties related to this new structure. The concept of ICNG1 generalized the concept of generalized fuzzy graphs of type 1 (GFG1), generalized single valued neutrosophic graphs of type 1 (GSVNG1) generalized interval valued neutrosophic graphs of type 1 (GIVNG1) and complex neutrosophic graph type 1(CNG1).

Keywords

Neutrosophic set; complex neutrosophic set; interval complex neutrosophic set; interval complex neutrosophic graph of type 1; adjacency matrix.

1 Introduction

Crisp set, fuzzy sets [14] and intuitionisitic fuzzy sets [13] already acts as a mathematical tool. But Smarandache [5, 6] gave a momentum by introducing

the concept of neutrosophic sets (NSs in short). Neutrosophic sets came as a glitter in this field as their vast potential to intimate imprecise, incomplete, uncertainty and inconsistent information of the world. Neutrosophic sets associates a degree of membership (T) , indeterminacy(I) and non- membership (F) for an element each of which belongs to the non-standard unit interval]−0, 1+[. Due to this characteristics, the practical implement of NSs becomes difficult. So, for this reason, Smarandache [5, 6] and Wang et al. [10] introduced the concept of a single valued Neutrosophic sets (SVNS), which is an instance of a NS and can be used in real scientific and engineering applications. Wang et al. [12] defined the concept of interval valued neutrosophic sets as generalization of SVNS. In [11], the readers can found a rich literature on single valued neutrosophic sets and their applications in divers fields.

Graph representations are widely used for dealing with structural information, in different domains such as networks, image interpretation, pattern recognition operations research. In a crisp graphs two vertices are either related or not related to each other, mathematically, the degree of relationship is either 0 or 1. While in fuzzy graphs, the degree of relationship takes values from [0, 1].In [1] Atanassov defined the concept of intuitionistic fuzzy graphs (IFGs) with vertex sets and edge sets as IFS. The concept of fuzzy graphs and their extensions have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

Fuzzy graphs and their extensions such as hesitant fuzzy graph, intuitionistic fuzzy graphs ..etc , deal with the kinds of real life problems having some uncertainty measure. All these graphs cannot handle the indeterminate relationship between object. So, for this reason, Smaranadache [3,9]defined a new form of graph theory called neutrosophic graphs based on literal indeterminacy (I) to deal with such situations. The same author[4]initiated a new graphical structure of neutrosphic graphs based on (T, I, F) components and proposed three structures of neutrosophic graphs such as neutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. In [8] Smarandache defined a new classes of neutrosophic graphs including neutrosophic offgraph, neutrosophic bipolar/tripola/ multipolar graph. Single valued neutrosphic graphs with vertex sets and edge sets as SVN were first introduced by Broumi [33] and defined some of its properties. Also, Broumi et al.[34] defined certain degrees of SVNG and established some of their properties. The same author proved a necessary and sufficient condition for a single valued neutrosophic graph to be an isolated-SVNG [35]. In addition, Broumi et al. [47] defined the concept of the interval valued neutrosophic graph as a generalization of SVNG and analyzed some properties of it. Recently, Several extension of

single valued neutrosophic graphs, interval valued neutrosophic graphs and their application have been studied deeply [17-19, 21-22, 36-45, 48-49,54-56].

In [7] Smarandache initiated the idea of removal of the edge degree restriction of fuzzy graphs, intuitionistic fuzzy graphs and single valued neutrosophic graphs. Samanta et al [53] discussed the concept of generalized fuzzy graphs (GFG) and studied some properties of it . The authors claim that fuzzy graphs and their extension defined by many researches are limited to represented for some systems such as social network. Employing the idea initiated by smarandache [7], Broumi et al. [46, 50,51]proposed a new structures of neutrosophic graphs such as generalized single valued neutrosophic graph of type1(GSVNG1), generalized interval valued neutrosophic graph of type1(GIVNG1), generalized bipolar neutrosophic graph of type 1, all these types of graphs are a generalization of generalized fuzzy graph of type1[53]. In [2], Ramot defined the concept of complex fuzzy sets as an extension of the fuzzy set in which the range of the membership function is extended from the subset of the real number to the unit disc. Later on, some extensions of complex fuzzy set have been studied well in the litteratur e[20,23,26,28,29,58-68].In [15],Ali and Smarandache proposed the concept of complex neutrosophic set in short CNS. The concept of complex neutrosophic set is an extension of complex intuitionistic fuzzy sets by adding by adding complex-valued indeterminate membership grade to the definition of complex intuitionistic fuzzy set. The complex-valued truth membership function, complex-valued indeterminacy membership function, and complex-valued falsity membership function are totally independent. The complex fuzzy set has only one extra phase term, complex intuitionistic fuzzy set has two additional phase terms while complex neutrosophic set has three phase terms. The complex neutrosophic sets (CNS) are used to handle the information of uncertainty and periodicity simultaneously. When the values of the membership function indeterminacy-membership function and the falsity-membership function in a CNS are difficult to be expressed as exact single value in many real-world problems, interval complex neutrosophic sets can be used to characterize the uncertain information more sufficiently and accurately. So for this purpose, Ali et al [16] defined the concept of interval complex neutrosophic sets (ICNs) and examined its characteristics. Recently, Broumi et al.[52]defined the concept of complex neutrosophic graphs of type 1 with vertex sets and edge sets as complex neutrosophic sets.

In this paper, an extended version of complex neutrosophic graph of type 1(ICNG1) is introduced. To the best of our knowledge, there is no research on interval complex neutrosophic graph of type 1 in literature at present.

The remainder of this paper is organized as follows. In Section 2, some fundamental and basic concepts regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1 are presented. In Section 3, ICNG1 is proposed and provided by a numerical example. In section 4 a representation matrix of ICNG1 is introduced and finally we draw conclusions in section 5.

2 Fundamental and Basic Concepts

In this section we give some definitions regarding neutrosophic sets, single valued neutrosophic sets, complex neutrosophic set, interval complex neutrosophic set and complex neutrosophic graphs of type 1

Definition 2.1 [5, 6]

Let ζ be a space of points and let x $\in \zeta$. A neutrosophic set A $\in \zeta$ is characterized by a truth membership function T, an indeterminacy membership function I, and a falsity membership function F. The values of T, I, F are real standard or nonstandard subsets of $]^-0,1^+[$, and T, I, F: $\zeta \rightarrow]^-0,1^+[$. A neutrosophic set can therefore be represented as

$$A = \{ (x, T_A(x), I_A(x), F_A(x)) : x \in \zeta \} \quad (1)$$

Since $T, I, F \in [0, 1]$, the only restriction on the sum of T, I, F is as given below:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+. \quad (2)$$

From philosophical point of view, the NS takes on value from real standard or non-standard subsets of $]^-0,1^+[$. However, to deal with real life applications such as engineering and scientific problems, it is necessary to take values from the interval [0, 1] instead of $]^-0,1^+[$.

Definition 2.2 [10]

Let ζ be a space of points (objects) with generic elements in ζ denoted by x. A single valued neutrosophic set A (SVNS A) is characterized by truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. For each point x in ζ, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. The SVNS A can therefore be written as

$$A = \{ (x, T_A(x), I_A(x), F_A(x)) : x \in \zeta \} \quad (3)$$

Definition 2.3 [15]

A complex neutrosophic set A defined on a universe of discourse X, which is characterized by a truth membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$ that assigns a complex-valued membership grade to $T_A(x), I_A(x), F_A(x)$ for any $x \in X$. The values of $T_A(x), I_A(x), F_A(x)$ and their sum may be any values within a unit circle

in the complex plane and is therefore of the form $T_A(x) = p_A(x)e^{i\mu_A(x)}$, $I_A(x) = q_A(x)e^{i\nu_A(x)}$, and $F_A(x) = r_A(x)e^{i\omega_A(x)}$. All the amplitude and phase terms are real-valued and $p_A(x), q_A(x), r_A(x) \in [0,1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x) \in (0, 2\pi]$, such that the condition.

$$0 \leq p_A(x) + q_A(x) + r_A(x) \leq 3 \tag{4}$$

is satisfied. A complex neutrosophic set A can thus be represented in set form as:

$$A = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F \rangle : x \in X\}, \tag{5}$$

Where $T_A: X \to \{a_T : a_T \in C, |a_T| \leq 1\}$, $I_A: X \to \{a_I : a_I \in C, |a_I| \leq 1\}$, $F_A: X \to \{a_F : a_F \in C, |a_F| \leq 1\}$, and also

$$|T_A(x) + I_A(x) + F_A(x)| \leq 3. \tag{6}$$

Let A and B be two CNSs in X, which are as defined as follow $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ and $B = \{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$.

Definition 2.4 [15]

Let A and B be two CNSs in X. The union, intersection and complement of two CNSs are defined as:

The union of A and B denoted as $A \cup_N B$, is defined as:

$$A \cup_N B = \{(x, T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)) : x \in X\}, \tag{7}$$

Where, $T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x)$ are given by

$$T_{A\cup B}(x) = \max(p_A(x), p_B(x)) \cdot e^{i\,(\mu_A(x)\cup\mu_B(x))}$$

$$I_{A\cup B}(x) = \min(q_A(x), q_B(x)) \cdot e^{i\,(\nu_A(x)\cup\nu_B(x))},$$

$$F_{A\cup B}(x) = \min(r_A(x), r_B(x)) \cdot e^{i\,(\omega_A(x)\cup\omega_B(x))}.$$

The intersection of A and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \{(x, T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)) : x \in X\}, \tag{8}$$

Where $T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x)$ are given by

$$T_{A\cup B}(x) = \min(p_A(x), p_B(x)) \cdot e^{i\,(\mu_A(x)\cap\mu_B(x))} \tag{9}$$

$$I_{A\cup B}(x) = \max(q_A(x), q_B(x)) \cdot e^{i\,(\nu_A(x)\cap\nu_B(x))}, \tag{10}$$

$$F_{A\cup B}(x) = \max(r_A(x), r_B(x)) \cdot e^{i\,(\omega_A(x)\cap\omega_B(x))}. \tag{11}$$

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu_{A\cup B}(x) = \mu_A(x) + \mu_B(x), \tag{12}$$

$$\nu_{A\cup B}(x) = \nu_A(x) + \nu_B(x), \tag{13}$$

$$\omega_{A \cup B}(x) = \omega_A(x) + \omega_B(x). \tag{14}$$

Max:

$$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x)), \tag{15}$$

$$\nu_{A \cup B}(x) = \max(\nu_A(x), \nu_B(x)), \tag{16}$$

$$\omega_{A \cup B}(x) = \max(\omega_A(x), \omega_B(x)). \tag{17}$$

Min:

$$\mu_{A \cup B}(x) = \min(\mu_A(x), \mu_B(x)), \tag{18}$$

$$\nu_{A \cup B}(x) = \min(\nu_A(x), \nu_B(x)), \tag{19}$$

$$\omega_{A \cup B}(x) = \min(\omega_A(x), \omega_B(x)). \tag{20}$$

"The game of winner, neutral, and loser":

$$\mu_{A \cup B}(x) = \begin{cases} \mu_A(x) & if \quad p_A > p_B \\ \mu_B(x) & if \quad p_B > p_A \end{cases}, \tag{21}$$

$$\nu_{A \cup B}(x) = \begin{cases} \nu_A(x) & if \quad q_A < q_B \\ \nu_B(x) & if \quad q_B < q_A \end{cases}, \tag{22}$$

$$\omega_{A \cup B}(x) = \begin{cases} \omega_A(x) & if \quad r_A < r_B \\ \omega_B(x) & if \quad r_B < r_A \end{cases}. \tag{23}$$

Definition 2.5 [16]

An interval complex neutrosophic set A defined on a universe of discourse ζ, which is characterized by an interval truth membership function $\tilde{T}_A(x) = [T_A^L(x), T_A^U(x)]$, an interval indeterminacy-membership function $\tilde{I}_A(x)$, and an interval falsity-membership function $\tilde{F}_A(x)$ that assigns a complex-valued membership grade to $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ for any $x \in \zeta$. The values of $\tilde{T}_A(x), \tilde{I}_A(x), \tilde{F}_A(x)$ and their sum may be any values within a unit circle in the complex plane and is therefore of the form

$$\tilde{T}_A(x) = [p_A^L(x), p_A^U(x)].e^{i[\mu_A^L(x), \ \mu_A^U(x)]}, \tag{24}$$

$$\tilde{I}_A(x) = [q_A^L(x), q_A^U(x)].e^{i[\nu_A^L(x), \ \nu_A^U(x)]} \tag{25}$$

$$\text{and } \tilde{F}_A(x) = [r_A^L(x), r_A^U(x)].e^{i[\omega_A^L(x), \ \omega_A^U(x)]} \tag{26}$$

All the amplitude and phase terms are real-valued and $p_A^L(x), p_A^U(x), q_A^L(x), q_A^U(x), r_A^L(x) \text{ and } r_A^U(x) \in [0,1]$, whereas $\mu_A(x), \nu_A(x), \omega_A(x) \in (0, 2\pi]$, such that the condition

$$0 \leq p_A^U(x) + q_A^U(x) + r_A^U(x) \leq 3 \tag{27}$$

is satisfied. An interval complex neutrosophic set $\tilde{A}$ can thus be represented in set form as:

$$\tilde{A} = \{\langle x, T_A(x) = a_T, I_A(x) = a_I, F_A(x) = a_F \rangle : x \in \zeta\}, \tag{28}$$

Where$T_A: \zeta. \rightarrow \{a_T: a_T \in C, |a_T| \leq 1\}, I_A: \zeta. \rightarrow \{a_I: a_I \in C, |a_I| \leq 1\}, F_A: \zeta. \rightarrow$
$\{a_F: a_F \in C, |a_F| \leq 1\},$ and also $\left|T_A^U(x) + I_A^U(x) + F_A^U(x)\right| \leq 3.$ (29)

Definition 2.6 [16]

Let A and B be two ICNSs in ζ. The union, intersection and complement of two ICNSs are defined as:

The union of A and B denoted as $A \cup_N B$, is defined as:

$$A \cup_N B = \left\{\left(x, \tilde{T}_{A \cup B}(x), \tilde{I}_{A \cup B}(x), \tilde{F}_{A \cup B}(x)\right): x \in X\right\},$$ (30)

Where, $\tilde{T}_{A \cup B}(x), \tilde{I}_{A \cup B}(x), \tilde{F}_{A \cup B}(x)$ are given by

$$T_{A \cup B}^L(x) = \left[(p_A^L(x) \vee p_B^L(x))\right].e^{j.\mu_{T_{A \cup B}}^L(x)},$$

$$T_{A \cup B}^U(x) = \left[(p_A^U(x) \vee p_B^U(x))\right].e^{j.\mu_{T_{A \cup B}}^U(x)}$$ (31)

$$I_{A \cup B}^L(x) = \left[(q_A^L(x) \wedge q_B^L(x))\right].e^{j.\mu_{I_{A \cup B}}^L(x)},$$

$$I_{A \cup B}^U(x) = \left[(q_A^U(x) \wedge q_B^U(x))\right].e^{j.\mu_{I_{A \cup B}}^U(x)},$$ (32)

$$F_{A \cup B}^L(x) = \left[(r_A^L(x) \wedge r_B^L(x))\right].e^{j.\mu_{F_{A \cup B}}^L(x)},$$

$$F_{A \cup B}^U(x) = \left[(r_A^U(x) \wedge r_B^U(x))\right].e^{j.\mu_{F_{A \cup B}}^U(x)}$$ (33)

The intersection of A and B denoted as $A \cap_N B$, is defined as:

$$A \cap_N B = \left\{\left(x, \tilde{T}_{A \cap B}(x), \tilde{I}_{A \cap B}(x), \tilde{F}_{A \cap B}(x)\right): x \in X\right\},$$ (34)

Where, $\tilde{T}_{A \cap B}(x), \tilde{I}_{A \cap B}(x), \tilde{F}_{A \cap B}(x)$ are given by

$$T_{A \cap B}^L(x) = \left[(p_A^L(x) \wedge p_B^L(x))\right].e^{j.\mu_{T_{A \cup B}}^L(x)},$$

$$T_{A \cap B}^U(x) = \left[(p_A^U(x) \wedge p_B^U(x))\right].e^{j.\mu_{T_{A \cup B}}^U(x)}$$ (35)

$$I_{A \cap B}^L(x) = \left[(q_A^L(x) \vee q_B^L(x))\right].e^{j.\mu_{I_{A \cup B}}^L(x)},$$

$$I_{A \cap B}^U(x) = \left[(q_A^U(x) \vee q_B^U(x))\right].e^{j.\mu_{I_{A \cup B}}^U(x)},$$ (36)

$$F_{A \cap B}^L(x) = \left[(r_A^L(x) \vee r_B^L(x))\right].e^{j.\mu_{F_{A \cup B}}^L(x)},$$

$$F_{A \cap B}^U(x) = \left[(r_A^U(x) \vee r_B^U(x))\right].e^{j.\mu_{F_{A \cup B}}^U(x)}$$ (37)

The union and the intersection of the phase terms of the complex truth, falsity and indeterminacy membership functions can be calculated using any one of the following operations:

Sum:

$$\mu_{A\cup B}^{L}(x) = \mu_{A}^{L}(x) + \mu_{B}^{L}(x),$$
$$\mu_{A\cup B}^{U}(x) = \mu_{A}^{U}(x) + \mu_{B}^{U}(x), \tag{38}$$
$$\nu_{A\cup B}^{L}(x) = \nu_{A}^{L}(x) + \nu_{B}^{L}(x),$$
$$\nu_{A\cup B}^{U}(x) = \nu_{A}^{U}(x) + \nu_{B}^{U}(x), \tag{39}$$
$$\omega_{A\cup B}^{L}(x) = \omega_{A}^{L}(x) + \omega_{B}^{L}(x),$$
$$\omega_{A\cup B}^{U}(x) = \omega_{A}^{U}(x) + \omega_{B}^{U}(x), \tag{40}$$

Max:

$$\mu_{A\cup B}^{L}(x) = \max(\mu_{A}^{L}(x), \mu_{B}^{L}(x)),$$
$$\mu_{A\cup B}^{U}(x) = \max(\mu_{A}^{U}(x), \mu_{B}^{U}(x)), \tag{41}$$
$$\nu_{A\cup B}^{L}(x) = \max(\nu_{A}^{L}(x), \nu_{B}^{L}(x)),$$
$$\nu_{A\cup B}^{U}(x) = \max(\nu_{A}^{U}(x), \nu_{B}^{U}(x)), \tag{42}$$
$$\omega_{A\cup B}^{L}(x) = \max(\omega_{A}^{L}(x), \omega_{B}^{L}(x)),$$
$$\omega_{A\cup B}^{U}(x) = \max(\omega_{A}^{U}(x), \omega_{B}^{U}(x)), \tag{43}$$

Min:

$$\mu_{A\cup B}^{L}(x) = \min(\mu_{A}^{L}(x), \mu_{B}^{L}(x)),$$
$$\mu_{A\cup B}^{U}(x) = \min(\mu_{A}^{U}(x), \mu_{B}^{U}(x)), \tag{44}$$
$$\nu_{A\cup B}^{L}(x) = \min(\nu_{A}^{L}(x), \nu_{B}^{L}(x)),$$
$$\nu_{A\cup B}^{U}(x) = \min(\nu_{A}^{U}(x), \nu_{B}^{U}(x)), \tag{45}$$
$$\omega_{A\cup B}^{L}(x) = \min(\omega_{A}^{L}(x), \omega_{B}^{L}(x)),$$
$$\omega_{A\cup B}^{U}(x) = \min(\omega_{A}^{U}(x), \omega_{B}^{U}(x)), \tag{46}$$

"The game of winner, neutral, and loser":

$$\mu_{A\cup B}(x) = \begin{cases} \mu_A(x) & if & p_A > p_B \\ \mu_B(x) & if & p_B > p_A \end{cases}, \tag{47}$$

$$\nu_{A\cup B}(x) = \begin{cases} \nu_A(x) & if & q_A < q_B \\ \nu_B(x) & if & q_B < q_A \end{cases}, \tag{48}$$

$$\omega_{A\cup B}(x) = \begin{cases} \omega_A(x) & if & r_A < r_B \\ \omega_B(x) & if & r_B < r_A \end{cases}. \tag{49}$$

Definition 2.7 [52]

Consider V be a non-void set. Two function are considered as follows:

$\rho=(\rho_T, \rho_I, \rho_F):V \to [\,0,1]^3$ and

$\omega= (\omega_T, \omega_I, \omega_F):VxV \to [\,0,1]^3$. We suppose

A= $\{(\rho_T(x),\rho_T(y))\,|\,\omega_T(x,y) \ge 0\}$, $\qquad\qquad\qquad$ (50)

B= $\{(\rho_I(x),\rho_I(y))\,|\omega_I(x,y) \ge 0\}$, $\qquad\qquad\qquad$ (51)

C= $\{(\rho_F(x),\rho_F(y))\,|\omega_F(x,y) \ge 0\}$, $\qquad\qquad\qquad$ (52)

considered ω_T, ω_I and $\omega_F \ge 0$ for all set A,B, C since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be complex neutrosophic graph of type 1 (CNG1) if there are functions

α:A$\to [\,0,1]$, β:B$\to [\,0,1]$ and δ:C$\to [\,0,1]$ such that

$\omega_T(x,y) = \alpha((\rho_T(x),\rho_T(y)))$ $\qquad\qquad\qquad\qquad$ (53)

$\omega_I(x,y) = \beta((\rho_I(x),\rho_I(y)))$ $\qquad\qquad\qquad\qquad$ (54)

$\omega_F(x,y) = \delta((\rho_F(x),\rho_F(y)))$ where x, y$\in$ V. $\qquad\qquad$ (55)

For each $\rho(x)= (\rho_T(x), \rho_I(x), \rho_F(x))$,,x$\in$ V are called the complex truth, complex indeterminacy and complex falsity-membership values, respectively, of the vertex x. likewise for each edge (x, y) : ω(x,y)=(ω_T(x,y), ω_I(x,y), ω_F(x,y)) are called the complex membership, complex indeterminacy membership and complex falsity values of the edge.

3 Interval Complex Neutrosophic Graph of Type 1

In this section, based on the concept of complex neutrosophic graph of type 1 [52], we define the concept of interval complex neutrosophic graph of type 1 as follows:

Definition 3.1.

Consider V be a non-void set. Two function are considered as follows:

$\rho=([\rho_T^L,\rho_T^U],[\rho_I^L,\rho_I^U], [\rho_F^L,\rho_F^U]):V \to [\,0,1]^3$ and

$\omega=(\,[\omega_T^L,\omega_T^U], [\omega_I^L,\omega_I^U], [\omega_F^L,\omega_F^U]):VxV \to [\,0,1]^3$. We suppose

A= $\{([\rho_T^L(x),\rho_T^U(x)], [\rho_T^L(y),\rho_T^U(y)])\,|\omega_T^L(x,y) \ge 0$

and $\omega_T^U(x,y) \ge 0\ \}$, $\qquad\qquad\qquad\qquad\qquad\qquad$ (56)

B= $\{([\rho_I^L(x),\rho_I^U(x)], [\rho_I^L(y),\rho_I^U(y)])\,|\omega_I^L(x,y) \ge 0$

and $\omega_I^U(x,y) \ge 0\}$, $\qquad\qquad\qquad\qquad\qquad\qquad$ (57)

C= $\{([\rho_F^L(x),\rho_F^U(x)], [\rho_F^L(y),\rho_F^U(y)])\,|\omega_F^L(x,y) \ge 0$

and $\omega_F^U(x,y) \ge 0\}$, $\qquad\qquad\qquad\qquad\qquad\qquad$ (58)

We have considered ω_T, ω_I and $\omega_F \ge 0$ for all set A,B, C , since its is possible to have edge degree = 0 (for T, or I, or F).

The triad (V, ρ, ω) is defined to be an interval complex neutrosophic graph of type 1 (ICNG1) if there are functions

α:A$\rightarrow$ [0, 1] , β:B$\rightarrow$ [0, 1] and δ:C$\rightarrow$ [0, 1] such that

$$\omega_T(x,y)=[\ \omega_T^L(x,y),$$
$$\omega_T^U(x,y)]= \alpha([\rho_T^L(x),\rho_T^U(x)],[\rho_T^L(y),\rho_T^U(y)]) \tag{59}$$
$$\omega_I(x,y)=[\ \omega_I^L(x,y),$$
$$\omega_I^U(x,y)]= \beta([\rho_I^L(x),\rho_I^U(x)],[\rho_I^L(y),\rho_I^U(y)]) \tag{60}$$
$$\omega_F(x,y)=[\ \omega_F^L(x,y),$$
$$\omega_F^U(x,y)]= \delta([\rho_F^L(x),\rho_F^U(x)],[\rho_F^L(y),\rho_F^U(y)]) \text{ where x, y}\in \text{V}. \tag{61}$$

For each $\rho(x)=([\rho_T^L(x),\rho_T^U(x)], [\rho_I^L(x),\rho_I^U(x)],[\rho_F^L(x),\rho_F^U(x)]),x\in$ V are called the interval complex truth-membership, interval complex indeterminacy and interval complex falsity-membership values, respectively, of the vertex x. likewise for each edge(x, y) :$\omega(x,y)=(\omega_T(x,y), \omega_I(x,y), \omega_F(x,y))$ are called the interval complex membership, interval complex indeterminacy membership and interval complex falsity values of the edge.

Example 3.2

Consider the vertex set be V={x, y, z, t} and edge set be E= {(x, y),(x, z),(x, t),(y, t)}

	x	y	z	t
$[\rho_T^L,\rho_T^U]$	$[0.5, 0.6]e^{j.\pi[0.8,0.9]}$	$[0.9\ , 1]e^{j.\pi[0.7,0.8]}$	$[0.3, 0.4]e^{j.\pi[0.2,0.5]}$	$[0.8, 0.9]e^{j.\pi[0.1,0.3]}$
$[\rho_I^L,\rho_I^U]$	$[0.3, 0.4]e^{j.\pi[0.1,0.2]}$	$[0.2, 0.3]e^{j.\pi[0.5,0.6]}$	$[0.1, 0.2]e^{j.\pi[0.3,0.6]}$	$[0.5, 0.6]e^{j.\pi[0.2,0.8]}$
$[\rho_F^L,\rho_F^U]$	$[0.1, 0.2]e^{j.\pi[0.5,0.7]}$	$[0.6, 0.7]e^{j.\pi[0.2,0.3]}$	$[0.8, 0.9]e^{j.\pi[0.2,0.4]}$	$[0.4, 0.5]e^{j.\pi[0.3,0.7]}$

Table 1. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the vertex set.

Given the following functions
$$\alpha(m,n)=[m_T^L(u) \vee n_T^L(u), m_T^U(u) \vee n_T^U(u)] . e^{j.\pi\mu_{A\cup B}(u)} \tag{62}$$
$$\beta(m,n)=[m_I^L(u) \wedge n_I^L(u), m_I^U(u) \wedge n_I^U(u)] . e^{j.\pi\nu_{A\cup B}(u)} \tag{63}$$
$$\delta(m,n)= [m_F^L(u) \wedge n_F^L(u), m_F^U(u) \wedge n_F^U(u)] . e^{j.\pi\omega_{A\cup B}(u)} \tag{64}$$
Here,
A={$([0.5,\ 0.6]e^{j.\pi[0.8,0.9]}$, $[0.9,\ 1]e^{j.\pi[0.7,0.8]})$, $([0.5,\ 0.6]e^{j.\pi[0.8,0.9]}$, $[0.3, 0.4]e^{j.\pi[0.2,0.5]})$, $([0.5,\ 0.6]e^{j.\pi[0.8,0.9]}$, $[0.8, 0.9]e^{j.\pi[0.1,0.3]})$,$([0.9, 1.0]e^{j.\pi[0.7,0.8]}$, $[0.8, 0.9]e^{j.\pi[0.1,0.3]})$}

B={$([0.3,\ 0.4]e^{j.\pi[0.1,0.2]}$, $[0.2,\ 0.3]e^{j.\pi[0.5,0.6]})$, $([0.3, 0.4]e^{j.\pi[0.1,0.2]}$, $[0.1,$

$0.2]e^{j.\pi[0.3,0.6]}$), ([0.3, 0.4]$e^{j.\pi[0.1,0.2]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$), ([0.2, 0.3]$e^{j.\pi[0.5,0.6]}$, [0.5, 0.6]$e^{j.\pi[0.2,0.8]}$)}

C={([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.6, 0.7]$e^{j.\pi[0.2,0.3]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.8, 0.9]$e^{j.\pi[0.2,0.4]}$), ([0.1, 0.2]$e^{j.\pi[0.5,0.7]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$), ([0.6, 0.7]$e^{j.\pi[0.2,0.3]}$, [0.4, 0.5]$e^{j.\pi[0.3,0.7]}$)}.

Then

ω	(x,y)	(x,z)	(x,t)	(y,t)
$[\omega_T^L,\omega_T^U]$	[0.9, 1]$e^{j.\pi[0.8,0.9]}$	[0.5, 0.6]$e^{j.\pi[0.8,0.9]}$	[0.8,0.9]$e^{j.\pi[0.8,0.9]}$	[0.9,1]$e^{j.\pi[0.8,0.9]}$
$[\omega_I^L,\omega_I^U]$	[0.2,0.3]$e^{j.\pi[0.5,0.6]}$	[0.1,0.2]$e^{j.\pi[0.3,0.6]}$	[0.3,0.4]$e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]$e^{j.\pi[0.5,0.8]}$
$[\omega_F^L,\omega_F^U]$	[0.1, 0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.1,0.2]$e^{j.\pi[0.5,0.7]}$	[0.4,0.5]$e^{j.\pi[0.5,0.7]}$

Table 2. Interval Complex truth-membership, indeterminacy-membership and falsity-membership of the edge set.

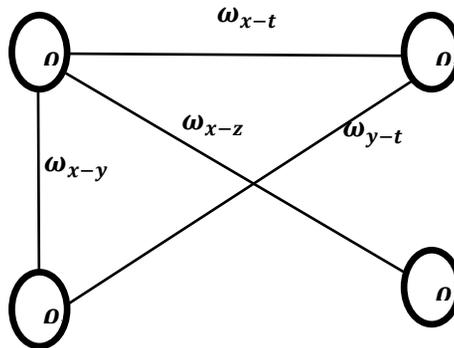The figure 2 show the interval complex neutrosophic graph of type 1

Fig 2. Interval complex neutrosophicgraph of type 1.

In classical graph theory, any graph can be represented by adjacency matrices, and incident matrices. In the following section ICNG1 is represented by adjacency matrix.

4 Representation of interval complex neutrosophic graph of Type 1 by adjacency matrix

In this section, interval truth-membership, interval indeterminate-membership and interval false-membership are considered independents. Based on the representation of complex neutrosophic graph of type 1 by adjacency matrix [52],

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we propose a matrix representation of interval complex neutrosophic graph of type 1 as follow:

The interval complex neutrosophic graph (ICNG1) has one property that edge membership values (T, I, F) depends on the membership values (T, I, F) of adjacent vertices. Suppose $\xi=(V, \rho, \omega)$ is a ICNG1 where vertex set $V=\{v_1,v_2,\ldots,v_n\}$. The functions

$\alpha :A\rightarrow (0,1]$ is taken such that

$\omega_T^L(x,y) = \alpha((\rho_T^L(x),\rho_T^L(y))), \omega_T^U(x,y) = \alpha((\rho_T^U(x),\rho_T^U(y)))$, where x, y$\in$ V and

A= $\{([\rho_T^L(x),\rho_T^U(x)], [\rho_T^L(y),\rho_T^U(y)]) \,|\omega_T^L(x, y) \geq 0$ and $\omega_T^U(x, y) \geq 0 \}$,

$\beta :B\rightarrow (0,1]$ is taken such that

$\omega_I^L(x,y) = \beta((\rho_I^L(x),\rho_I^L(y))), \omega_I^U(x,y) = \beta((\rho_I^U(x),\rho_I^U(y)))$, where x, y$\in$ V and

B= $\{([\rho_I^L(x),\rho_I^U(x)], [\rho_I^L(y),\rho_I^U(y)]) \,|\omega_I^L(x, y) \geq 0$ and $\omega_I^U(x, y) \geq 0 \}$

and

$\delta :C\rightarrow (0,1]$ is taken such that

$\omega_F^L(x,y) = \delta((\rho_F^L(x),\rho_F^L(y))), \omega_F^U(x,y) = \delta((\rho_F^U(x),\rho_F^U(y)))$, where x, y$\in$ V and

C= $\{([\rho_F^L(x),\rho_F^U(x)], [\rho_F^L(y),\rho_F^U(y)]) \,|\omega_F^L(x, y) \geq 0$ and $\omega_F^U(x, y) \geq 0 \}$.

The ICNG1 can be represented by (n+1) x (n+1) matrix $M_{G_1}^{T,I,F}=[a^{T,I,F}(i, j)]$ as follows:

The interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the vertices are provided in the first row and first column. The (i+1, j+1)- th-entry are the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge (x_i,x_j), i, j=1,...,n if i$\neq$j.

The (i, i)-th entry is $\rho(x_i)=(\rho_T(x_i), \rho_I(x_i), \rho_F(x_i))$, where i=1,2,...,n. the interval complex truth membership (T), interval complex indeterminacy-membership (I) and the interval complex falsity-membership (F) values of the edge can be computed easily using the functions α, β and δ which are in (1,1)-position of the matrix. The matrix representation of ICNG1, denoted by $M_{G_1}^{T,I,F}$, can be written as three matrix representation $M_{G_1}^T, M_{G_1}^I$ and $M_{G_1}^F$. For convenience representation $v_i(\rho_T(v_i)) =[\rho_T^L(v_i), \rho_T^U(v_i)], v_i(\rho_I(v_i)) =[\rho_I^L(v_i), \rho_I^U(v_i)]$ and $v_i(\rho_F(v_i)) =[\rho_F^L(v_i), \rho_F^U(v_i)]$, for i=1, ..., n

The $M_{G_1}^T$ can be therefore represented as follows

α	$v_1(\rho_T(v_1))$	$v_2(\rho_T(v_2))$	$v_n(\rho_T(v_n))$
$v_1(\rho_T(v_1))$	$[\rho_T^L(v_1), \rho_T^U(v_1)]$	$\alpha(\rho_T(v_1),\rho_T(v_2))$	$\alpha(\rho_T(v_1),\rho_T(v_n))$
$v_2(\rho_T(v_2))$	$\alpha(\rho_T(v_2),\rho_T(v_1))$	$[\rho_T^L(v_2), \rho_T^U(v_2)]$	$\alpha(\rho_T(v_2),\rho_T(v_2))$
…	….	…	…
$v_n(\rho_T(v_n))$	$\alpha(\rho_T(v_n),\rho_T(v_1))$	$\alpha(\rho_T(v_n),\rho_T(v_2))$	$[\rho_T^L(v_n), \rho_T^U(v_n)]$

Table 3. Matrix representation of T-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

β	$v_1(\rho_I(v_1))$	$v_2(\rho_I(v_2))$	$v_n(\rho_I(v_n))$
$v_1(\rho_I(v_1))$	$[\rho_I^L(v_1), \rho_I^U(v_1)]$	$\beta(\rho_I(v_1),\rho_I(v_2))$	$\beta(\rho_I(v_1),\rho_I(v_n))$
$v_2(\rho_I(v_2))$	$\beta(\rho_I(v_2),\rho_I(v_1))$	$[\rho_I^L(v_2), \rho_I^U(v_2)]$	$\beta(\rho_I(v_2),\rho_I(v_2))$
…	….	…	…
$v_n(\rho_I(v_n))$	$\beta(\rho_I(v_n),\rho_I(v_1))$	$\beta(\rho_T(v_n),\rho_I(v_2))$	$[\rho_I^L(v_n), \rho_I^U(v_n)]$

Table 4. Matrix representation of I-ICNG1

The $M_{G_1}^I$ can be therefore represented as follows

δ	$v_1(\rho_F(v_1))$	$v_2(\rho_F(v_2))$	$v_n(\rho_F(v_n))$
$v_1(\rho_F(v_1))$	$[\rho_F^L(v_1), \rho_F^U(v_1)]$	$\delta(\rho_F(v_1),\rho_F(v_2))$	$\delta(\rho_F(v_1),\rho_F(v_n))$
$v_2(\rho_F(v_2))$	$\delta(\rho_F(v_2),\rho_F(v_1))$	$[\rho_F^L(v_2), \rho_F^U(v_2)]$	$\delta(\rho_F(v_2),\rho_F(v_2))$
…	….	…	…
$v_n(\rho_F(v_n))$	$\delta(\rho_F(v_n),\rho_F(v_1))$	$\delta(\rho_F(v_n),\rho_F(v_2))$	$[\rho_F^L(v_n), \rho_F^U(v_n)]$

Table 5. Matrix representation of F-ICNG1

Here the Interval complex neutrosophic graph of first type (ICNG1) can be represented by the matrix representation depicted in table 9. The matrix representation can be written as three interval complex matrices one containing the entries as T, I, F (see table 6, 7 and 8).

$\alpha = \max(x, y)$	x([0.5, 0.6] $e^{j.\pi[0.8,0.9]}$)	y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)
x([0.5, 0.6].$e^{j.\pi[0.8,0.9]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	[0.8, 0.9]. $e^{j.\pi[0.8\ 0.9]}$
y([0.9 , 1]. $e^{j.\pi[0.7,0.8]}$)	[0.9 , 1]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$
z([0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$)	[0.5, 0.6].$e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.3, 0.4]. $e^{j.\pi[0.2,0.5]}$	**[0, 0]**
t([0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$)	[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$	[0.9 , 1]. $e^{j.\pi[0.7,0.8]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.1,0.3]}$

Table 6: Lower and upper Truth- matrix representationof ICNG1

$\beta = \min(x, y)$	x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)
x([0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$)	[0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	[0.3, 0.4]. $e^{j.\pi[0.3,0.6]}$
y([0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$)	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$	**[0, 0]**	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$
z([0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**	[0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$	**[0, 0]**
t([0.5, 0.6]. $e^{j.\pi[0.2,0.8]}$)	[0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$	[0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$	**[0, 0]**	[0.5 0.6]. $e^{j.\pi[0.2,0.8]}$

Table 7: Lower and upper Indeterminacy- matrix representation of ICNG1

$\delta = \min(x, y)$	x([0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$)	y([0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$)	z([0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$)	t([0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)
x([0.1, 0.2].$e^{j.\pi[0.5,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$
y([0.6, 0.7].$e^{j.\pi[0.2,0.3]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.6, 0.7]. $e^{j.\pi[0.2,0.3]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$
z([0.8, 0.9].$e^{j.\pi[0.2,0.4]}$)	[0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$	**[0, 0]**	[0.8, 0.9]. $e^{j.\pi[0.2,0.4]}$	**[0, 0]**
t[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$)	[0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$	**[0, 0]**	[0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$

Table 8: Lower and upper Falsity- matrix representation of ICNG1

The matrix representation of ICNG1 can be represented as follows:

(α, β, δ)	X(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)
X(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>
y(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.9, 0.1]. $e^{j.\pi[0.8,0.9]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>
z(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.1, 0.2]. $e^{j.\pi[0.3,0.6]}$, [0.1, 0.2]. $e^{j.\pi[0.8,0.9]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**
t(<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>)	<[0.8, 0.9]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.2,0.8]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>	<[0.9 1]. $e^{j.\pi[07,0.8]}$, [0.2, 0.3]. $e^{j.\pi[0.5,0.8]}$, [0.4, 0.5]. $e^{j.\pi[0.3,0.7]}$>	**<[0, 0], [0, 0], [0, 0]>**	<[0.5, 0.6]. $e^{j.\pi[0.8,0.9]}$, [0.3, 0.4]. $e^{j.\pi[0.1,0.2]}$, [0.1, 0.2]. $e^{j.\pi[0.5,0.7]}$>

Table 9: Matrix representation of ICNG1.

Remark 1

If $\rho_T^L(x)=\rho_T^U(x)$, $\rho_I^L(x)=\rho_I^U(x)=0$ and $\rho_F^L(x)=\rho_F^U(x)=0$ and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized fuzzy graphs type 1 (GFG1).

Remark 2

If $\rho_T^L(x)=\rho_T^U(x)$, $\rho_I^L(x)=\rho_I^U(x)$ and $\rho_F^L(x)=\rho_F^U(x)$ and the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized single valued graphs type 1 (GSVNG1).

Remark 3

If $\rho_T^L(x)=\rho_T^U(x)$, $\rho_I^L(x)=\rho_I^U(x)$ and $\rho_F^L(x)=\rho_F^U(x)$ the interval complex neutrosophic graphs type 1 is reduced to complex neutrosophic graphs type 1 (CNG1).

Remark 4

If $\rho_T^L(x) \neq \rho_T^U(x)$, $\rho_I^L(x) \neq \rho_I^U(x)$ and $\rho_F^L(x) \neq \rho_F^U(x)$ *and* the interval valued phase terms equals zero, the interval complex neutrosophic graphs type 1 is reduced to generalized interval valued graphs type 1 (GIVNG1).

Theorem 1

Given the $M_{G_1}^T$ be matrix representation of T-ICNG1, then the degree of vertex $D_T(x_k) = [\sum_{j=1, j \neq k}^n a_T^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_T^U(k+1, j+1)], x_k \in V$

or

$$D_T(x_p) = [\sum_{i=1, i \neq p}^n a_T^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_T^U(i+1, p+1) \; x_p \in V.$$

Proof

Similar to that of theorem 1 of [52].

Theorem 2

Given the $M_{G_1}^I$ be a matrix representation of I-ICNG1, then the degree of vertex $D_I(x_k) = [\sum_{j=1, j \neq k}^n a_I^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_I^U(k+1, j+1)], x_k \in V$

or $D_I(x_p) = [\sum_{i=1, i \neq p}^n a_I^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_I^U(i+1, p+1)], x_p \in$ V.

Proof

Similar to that of theorem 1 of [52].

Theorem 3

Given the $M_{G_1}^F$ be a matrix representation of ICNG1, then the degree of vertex

$$D_F(x_k) = [\sum_{j=1, j \neq k}^n a_F^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_F^U(k+1, j+1)], x_k \in \; V$$

or

$$D_F(x_p) = [\sum_{i=1, i \neq p}^n a_F^L(i+1, p+1), \sum_{i=1, i \neq p}^n a_F^U(i+1, p+1)], x_p \in V.$$

Proof

Similar to that of theorem 1 of [52].

Theorem 4

Given the $M_{G_1}^{T,I,F}$ be a matrix representation of ICNG1, then the degree of vertex $D(x_k) = (D_T(x_k), D_I(x_k), D_F(x_k))$ where

$$D_T(x_k) = [\sum_{j=1, j \neq k}^n a_T^L(k+1, j+1), \sum_{j=1, j \neq k}^n a_T^U(k+1, j+1)] , x_k \in V.$$

$$D_I(x_k) == [\sum_{j=1, j \neq k}^{n} a_I^L(k+1, j+1), \sum_{j=1, j \neq k}^{n} a_I^U(k+1, j+1)], x_k \in V.$$

$$D_F(x_k) == [\sum_{j=1, j \neq k}^{n} a_F^L(k+1, j+1), \sum_{j=1, j \neq k}^{n} a_F^U(k+1, j+1)], x_k \in V.$$

Proof

The proof is obvious.

5 Conclusion

In this article, we have introduced the concept of interval complex neutrosophic graph of type1 as generalization of the concept of single valued neutrosophic graph type 1 (GSVNG1), interval valued neutrosophic graph type 1 (GIVNG1) and complex neutrosophic graph of type1(CNG1). Next, we processed to presented a matrix representation of it. In the future works, we plan to study some more properties and applications of ICNG type 1 define the concept of interval complex neutrosophic graphs type 2.

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