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Abstract

In this paper, inspired by the concept of generalized single-valued neutrosophic graphs(GSVNG) of the first type, we define yet another generalization of neutrosophic graph called the generalized interval- valued neutrosophic graph of 1 type (GIVNG1) in addition to our previous work on complex neutrosophic graph (CNG1) in [47]. We will also show a matrix representation for this new generalization. Many of the fundamental properties and characteristics of this new concept is also studied. Like the concept CNG1 in [47], the concept of GIVNG1 is another extension of generalized fuzzy graphs 1 (GFG1) and GSVNG1.

Subject Classification: *(2000) 03E72, 05C72, 68R10*

Keywords: *Interval-Valued Neutrosophic Graph, Generalized Interval Valued Neutrosophic Graph Of First Type, Matrix Representation; Neutrosophic Graph*

1. Introduction

In order to efficiently handle real life scenarios that conatins uncertain information,neutrosophic set(NS) theory, established by Smarandache [32], is put forward from the perspective of philosophical standpoints through regarding the degree of indeterminacy or neutrality as an independent element. As a result, many extended forms of fuzzy sets such as classical fuzzy sets [45], intuitionistic fuzzy sets [3-4], interval-valued fuzzy sets [40] and interval-valued intuitionistic fuzzy sets [5] could be seen as reduced forms of NS theory. In a NS, a true membership degree T, an indeterminacy membership degree I and a falsity membership degree F constitute the whole independent membership degrees owned by each element. However, it is noticed that the range of T , I and F falls within a real standard or nonstandard unit interval$]^{-}0, 1^{+}[$, hence it is difficult in applying NSs to many kinds of real world situations due to the limitation of T , I and F . Therefore, an updated form called single

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valued neutrosophic sets (SVNSs) was designed by Smarandache firstly [32]. Then, several properties in terms of SVNSs were further explored by Wang et al. [43]. In addition, it is relatively tough for experts to provide the three membership degrees with exact values, sometimes the form of interval numbers outperform the exact values in many practical situations. Inspired by this issue, Wang et al. [43] constructed interval neutrosophic sets concept (INSs) that performs better in precision and flexibility. Thus, INSs could be regarded as an extension of SVNSs. Moreover, some recent works about NSs, INSs and SVNSs along with their applications could be found in [13-15, 22,35, 53-59].

To studying the relationship between objects or events, the concept of Graph is thus created. In classical crisp graph theory, each of the two vertices (representing object or event) can assign two crisp value, 0 (not related/connected) or 1 (related/connected). The approach of fuzzy graph is a generalization the classical graph by allowing the degree of relationship (i.e. the membership value) to be anywhere in [0,1] for the edges, and it also assign membership values for the vertices. In the context of fuzzy graph, there is a rule that must be satisfied by all the edges and vertices, as follows:

the membership value of an edge must always be less than or equal to both the membership values of its two adjacent vertices. (*)

In over one hundred research papers, the further generalization of fuzzy graphs were studied, such as intuitionistic graphs, interval valued fuzzy graphs [7, 25, 28, 29]and interval-valued intuitionistic fuzzy graphs [24].However, such generalization still preserve (*) that was established since the period of fuzzy graphs.

As a result, Samanta et al. [39]analysed the concept of generalized fuzzy graphs (GFG), which was derived from the concept of fuzzy graph while removing the confinement of (*) . He had also studied some major advantages of GFG, such as completeness and regularity, by some proven facts. These authors had further developed GFG into two types, namely: generalized fuzzy graphs of first type (GFG1), generalized fuzzy graphs based on second type (GFG2). Each type of GFG can likewise be created by matrices just as in the case of some fuzzy graphs. The authors had also justified that the concept of fuzzy graphs on previous literatures are limited to representing some very particular systems such as social network, and therefore GFG is claimed to be capable to put to use on a much wider range of different scenario.

On the other hand, when the description of an object or a relation is both indeterminate and uncertain, it may be handled by fuzzy[23], intuitionistic fuzzy, interval-valued fuzzy, interval-valued intuitionistic fuzzy graphs and Set-valued graphs [2]. So, for this purpose, another new concept: neutrosophic graphs based on literal indeterminacy (I),were proposed by Smarandache [34]to deal with such situations. Such concept was published in a book by the same author collaborating with Vasantha et al.[42]. Later on, Smarandache[30-31] further introduced yet a new concept for neutrosophic graph theory, this time using the neutrosophic truth-values (*T,I,F*). He also gave various characterization on neutrosophicgraph, such as theneutrosophic edge graphs, neutrosophic vertex graphs and neutrosophic vertex-edge graphs. Later on[33], Smarandache himself further generalized the concept of neutrosophic graphs, and yield even more new structures such as neutrosophic offgraph, neutrosophic bipolar graphs, neutrosophic tripolar graphs and neutrosophic multipolar graphs. After which, the study of neutrosophic vertex-edge graphs has captured the attention of most researchers, and thus having more generalizations derived from it.

In 2016, using the concepts of SVNSs, Broumi et al.[8] investigated on the concept of single-valued neutrosophic graphs, and formulated certain types of single-valued neutrosophic graphs (SVNGs). After that, Broumi et al.introduced in [9,10,16,17,36]: the necessity of neighbourhood degree of a vertices and closed neighborhood degree of vertices in single-valued neutrosophic graph, isolated-SVNGs, Bipolar-SVNGs, complete bipolar-SVNGs, regular bipolar-SVNGs, uniform-SVNGs. In[11-12,18], also they studied the concept of interval-valued neutrosophic graphs and the importance of strong interval-valued neutrosophic graph, where different methods such as union, join, intersection and complement have been further investigated. In [35], Broumi et al. proposed some computing procedure in Matlab for neutrosophic operational matrices. Broumi et al.[37] developed a Matlab toolbox for interval valued neutrosophic matrices for computer applications. Akram and Shahzadi [6] introduced a new version of SVNGs that are different from those proposed in [8,36],and studied some of their properties. Ridvan[20] presented a new approach to neutrosophic graph theory with applications. Malarvizhi and Divya[38] presented the the ideas of antipodal single valued neutrosophic graph. Karaaslan and Davvaz[21] explore some interesting properties of single-valued neutrosophic graphs.Krishnarajet al.[1] introduced the concept of perfect and status in single valued neutrosophic graphs and investigated some of their properties.

Krishnaraj et al. [26] also analysed the concepts self-centered single valued neutrosophic graphs and discussed the properties of this concept with various examples, while Mohmed Ali et al.[41]extended it further to interval valued neutrosophic graphs[11].Kalyan and Majumdar [27] introduce the concept of single valued neutrosophic digraphs and implemented it in solving a multicriterion decision making problems.

The interval-valued neutrosophic graphs studied in the literature [11, 12], like the concept of fuzzy graph, is nonetheless bounded with the following condition familiar to (*):

The edge membership value is lessser than the minimum of its end vertex values, whereas the edge indeterminacy-membership value is lesser than the maximum of its end vertex values or greater than the maximum of its end vertex values. Also the edge non-membership value is lesser than the minimum of its end vertex values or is greater than the maximum of its end vertex values. (**)

Broumi et al.[19]had thus followed the approach of Samanta et al. [39], by suggesting the removal of (**) and presented the logic of generalized single-valued neutrosophic graph of type1 (GSVNG1). This is also a generalization from generalized fuzzy graph of type1 [39].

The main goal of this work is to further generalize the method of GSVNG1 to interval-valued neutrosophic graphs of first type (GIVNG1), for which all the true, indeterminacy, and false membership values, are inconsistent. Similarly, the appropriate matrix representation of GIVNG1 will also be given.

The results in this article is further derived from a conference paper [46] that we have published one year ago in IEEE. On the other hand, we have just published a paper on complex neutrosophic graph (CNG1), which is another extension of GFG1 and GSVNG1 in [47]. The approach ofGIVNG1 and CNG1, however, are distint from one another. This is becausethe concept of CNG1 extends the existing theory by generalizing real numbers into complex numbers, while all the entries remain single valued; whereas in this paper,the concept of GIVNG1 extends the existing theory by generalizing the single valued entriesinto inter-valued entries,while all those inter-valued entries remains as real numbers

Thus, following the format of our recent conference paper [46], this paper has been aligned likewise: In Section 2, the concept on neutrosophic sets, single- valued neutrosophic sets, interval valued neutrosophic graph and generalized single-valued neutrosophic graphs of type 1are described in detail, which serves as cornerstones for all the contents in later parts of the article. In Section 3, we present the ideas of GIVNG1 illustrated with an example. Section 4 gives the appropriate way to represent the matrix of GIVNG1.

2. Some preliminary results

In this part, we briefly include some basic definitions in [19, 32, 43,47] related to NS, SVNSs, interval- valued neutrosophic graphs(IVNG) and generalized single-valued neutrosophic graphs of type 1(GSNG1).

Definition 2.1 [32]. Let X be a series of points with basic elements in X presented by x; then the neutrosophic set(NS) A (is an object in the form A = {< x: $T_A(x)$, $I_A(x)$, $F_A(x)$ >, $x \in X$}, defines the functions $T, I, F: X \rightarrow$ $]^-0,1^+[$denoted by the truth-membership, indeterminacy-membership, and falsity-membership of the element x X to the set A showing the condition:

$$^-0 \leq T_A(x) + I_A(x) + F_A(x) 3^+. \quad (1)$$

The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are absolute standard or non-standard subsets of $]^-0,1^+[$.

As it is very complex i applying NSs to real issues, Smarandache [32] developed the notion of a SVNS, which is an occurrence of a NS and can be employed in practical scientific and engineering applications.

Definition 2.2 [43]. Let X be a series of points (objects) with basic elements in X presented by x. A single valued neutrosophic set A ($SVNS$ A) is characterized by truth-membership $T_A(x)$, an indeterminacy-membership $I_A(x)$, and a falsity-membership $F_A(x)$.$\forall x \in X$, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. A SVNS A can be rewritten as

$$A = \{< x: T_A(x), I_A(x), F_A(x) >, x \in X\} \quad (2)$$

Definition 2.3 [19] Suppose the following conditions are expected:

a) V is a null-void set.

b) $\rho_T, \rho_I, \rho_F : \mathrm{V} \rightarrow [0,1]$

c) $\mathrm{E} = \{(\rho_T(u), \rho_T(v)) \mid u, v \in V\}$,
$\mathrm{F} = \{(\rho_I(u), \rho_I(v)) \mid u, v \in V\}$,
$\mathrm{G} = \{(\rho_F(u), \rho_F(v)) \mid u, v \in V\}$.

d) $\alpha: \mathrm{E} \rightarrow [0, 1]$, $\beta: \mathrm{F} \rightarrow [0, 1]$, $\delta: \mathrm{G} \rightarrow [0, 1]$ are three functions.

e) $\rho = (\rho_T, \rho_I, \rho_F)$; and
$\omega = (\omega_T, \omega_I, \omega_F)$ with
$\omega_T(u, v) = \alpha((\rho_T(x), \rho_T(v)))$,
$\omega_I(u, v) = \beta((\rho_I(x), \rho_I(v)))$,
$\omega_F(u, v) = \delta((\rho_F(x), \rho_F(v)))$, $\forall u, v \in V$.

Then:

i) The structure $\xi = < V, \rho, \omega >$ is considered to a GSVNG1.

Remark: ρ which depends on ρ_T, ρ_I, ρ_F. And ω which depends on α, β. Hence there are 7 mutually alone parameters in total which make up a CNG1: V, ρ_T, ρ_I, ρ_F, α, β, δ.

ii) $\forall \in V$, x is considered to bea *vertex* of ξ. The whole set V is termed as the *vertex set* of ξ.

iii) $\forall\, u, v \in V$, (u, v) is considered to be a *directed edge* of ξ.
In special, (u, v) is considered to be a *loop* of ξ.

iv) For all vertex : $\rho_T(v)$, $\rho_I(v)$, $\rho_F(v)$ are considered to be the T, I, and F *membership value*, respectively of that vertex v . Moreover, if $\rho_T(v) = \rho_I(v) = \rho_F(v) = 0$, then v is supposed to be a *null* vertex.

v) Correspondingly, for all edge (u, v) : $\omega_T(u, v)$, $\omega_I(u, v)$, $\omega_F(u, v)$ considered to have T, I, and F respectively membership value, of that directed edge (u, v). In addition, if $\omega_T(u, v) = \omega_I(u, v) = \omega_F(u, v) = 0$,then (u, v) is considered to be a *null* directed edge.

Remark : It obeys that: $V \times V \rightarrow [0,1]$.

3. Concepts related to Generalized Interval Valued Neutrosophic Graph of First Type

In the modelling of real life scenarios with neutrosophic system (i.e. neutrosophic sets, neutrosophic graphs, etc), the truth-membership value, indeterminate-membership value, and false-membership value are often taken to mean the *ratio out of a population* who find reasons to "agree", "be neutral" and "disagree". It can also by any 3 analogous descriptions, such as "seek excitement" "loft around" and "relax". However, there are real life situations where even such ratio out of the population are subject to conditions. One typical example will be having the highest and the lowest value. For example "It is expected that 20% to 30% of the population of country X will disagree with the Prime Minister's decision".

To model such an event, therefore, we generalize Definition 2.3 so that the truth-membership value, indeterminate-membership value, and false-membership value can be any closed subinterval of [0,1], instead of a single number from [0,1].Such generalization is further derived from [46], which is a conference paper that we have just published on this topic.

Note: For all the other parts of this work, we will define:

$$\Delta_1 = \{[x, y]: 0 \leq x \leq y \leq 1\}$$

Definition 3.1 [46]. Let the statements below holds good:

a) V is considered as a non-empty set.

b) $\tilde{\rho}_T, \tilde{\rho}_I, \tilde{\rho}_F$ are three functions, eachfrom V to Δ_1.

c) $E = \{(\tilde{\rho}_T(u), \tilde{\rho}_T(v)) \mid u, v \in V\}$,
$F = \{(\tilde{\rho}_I(u), \tilde{\rho}_I(v)) \mid u, v \in V\}$,
$G = \{(\tilde{\rho}_F(u), \tilde{\rho}_F(v)) \mid u, v \in V\}$.

d) $\alpha: E \to \Delta_1$, $\beta: F \to \Delta_1$, $\delta: G \to \Delta_1$ are three functions.

e) $\tilde{\rho} = (\tilde{\rho}_T, \tilde{\rho}_I, \tilde{\rho}_F)$; and
$\tilde{\omega} = (\tilde{\omega}_T, \tilde{\omega}_I, \tilde{\omega}_F)$ with
$\tilde{\omega}_T(u, v) = \alpha((\tilde{\rho}_T(x), \tilde{\rho}_T(v)))$,
$\tilde{\omega}_I(u, v) = \alpha((\tilde{\rho}_I(x), \tilde{\rho}_I(v)))$,,
$\tilde{\omega}_F(u, v) = \alpha((\tilde{\rho}_F(x), \tilde{\rho}_F(v)))$,,
for every $u, v \in V$.

Then:

i) The structure $\xi = <V, \tilde{\rho}, \tilde{\omega}>$is said to be a *generalized interval-valued neutrosophic graph of type* 1 (GIVNG1) .

ii) For each $\in V$, x is termed to be a *vertex* of ξ. The spanned set V is named the *vertex set* of ξ.

iii) $\forall u, v \in V$, (u, v) is termed to be a *directed edge* of ξ In particular, (u, v) is said to be a *loop* of ξ .

iv) $\forall$ vertex : $\tilde{\rho}_T(v)$, $\tilde{\rho}_I(v)$, $\tilde{\rho}_F(v)$ are said to be the *truth-membership value*, *indeterminate-membership value*, and *false-membership value*, respectively, of that vertex v. Moreover, if $\tilde{\rho}_T(v) = \tilde{\rho}_I(v) = \tilde{\rho}_F(v) = [0,0]$, then v is deemed as *void* vertex.

v) Similarly, for each edge (u, v) : $\tilde{\omega}_T(u, v)$, $\tilde{\omega}_I(u, v)$, $\tilde{\omega}_F(u, v)$ are said to be the *T*, *I*, and *F membership value* respectively of that directed edge (u, v). Moreover, if $\tilde{\omega}_T(u, v) = \tilde{\omega}_I(u, v) = \tilde{\omega}_F(u, v) = [0,0]$,then (u, v) is said to be a *void* directed edge.

Remark : It follows that : $V \times V \to \Delta_1$.

Note that every vertex v in a GIVNG1 have a single, undirected loop, whether void or not. Also each of the distinct vertices u, v in a GIVNG1possses *two* directed edges, resulting from (u, v) and (v, u), whether void or not.

We study that in classical graph theory, we handle ordinary (or undirected) graphs, and also some simple graphs. Further we relate our GIVNG1 with it, we now give the below definition.

Definition 3.2. [46] Given ξ = <V, $\tilde{\rho}$, $\tilde{\omega}$> be a GIVNG1.

a) If $\tilde{\omega}_T(a,b) = \tilde{\omega}_T(b,a)$, $\tilde{\omega}_I(a,b) = \tilde{\omega}_I(b,a)$ and $\tilde{\omega}_F(a,b) = \tilde{\omega}_F(b,a)$, then $\{a, b\} = \{(a, b), (b, a)\}$ is said to be an (*ordinary*) *edge* of ξ. Moreover, $\{a, b\}$ is said to be a *void* (ordinary) edge if both (a, b) and (b, a) are void.

b) If $\tilde{\omega}_T(u,v) = \tilde{\omega}_T(v,u), \tilde{\omega}_I(u,v) = \tilde{\omega}_I(v,u)$ and $\tilde{\omega}_F(u,v) = \tilde{\omega}_F(v,u)$ holds good for all $v \in V$, then ξ is considered to be *ordinary* (or *undirected*), else it is considered to be *directed*.

c) When all the loops of ξ are becoming void, then ξ is considered to be *simple*.

In the following section, we discuss a real life scenario, for which GSVNG1 is insufficient to model it - it can only be done by using GIVNG1.

Example 3.3. Part 3.3.1 The scenario

Country X has 4 cities $\{a, b, c, d\}$. The cities are connected with each other by some roads, there are villages along the four roads (all of them are two way) $\{a, b\}$, $\{c, b\}$, $\{a, c\}$ and $\{d, b\}$. As for the other roads, such as $\{c, b\}$, they are either non-exitsant, or there are no population living along them (e.g. industrial area, national park, or simply forest).The legal driving age of Country X is 18.The prime minister of Country X would like to suggest an amendment of the legal driving age from 18 to 16. Before conducting a countrywide survey involving all the citizens, the prime minister discuss with all members of the parliament about the expected outcomes.

The culture and living standard of all the cities and villages differ from one another. In particular:

The public transport in c is so developed that few will have to drive. The people are rich enough to buy even air tickets. People in d tend to be more open minded in culture. Moreover, sports car exhibitions and shows are commonly held there. A fatal road accident just happened along {c,b}, claiming the lives of five unlicensed teenagers racing at 200km/h. {a, c} is governed by an opposition leader who is notorious for being very uncooperative in all parliament affairs.

Eventually the parliament meeting was concluded with the following predictions:

		Expected percentage of citizens that will -					
		support		be neutral		Against	
		at least	at most	at least	at most	at least	at most
Cities	*a*	0.1	0.4	0.2	0.6	0.3	0.7
	b	0.3	0.5	0.2	0.5	0.2	0.5
	c	0.1	0.2	0.0	0.3	0.1	0.2
	d	0.5	0.7	0.2	0.4	0.1	0.2
Villages along the roads	{*a*,*b*}	0.2	0.3	0.1	0.4	0.4	0.7
	{*c*,*b*}	0.1	0.2	0.1	0.2	0.5	0.8
	{*a*,*c*}	0.1	0.7	0.1	0.8	0.1	0.7
	{*d*,*b*}	0.2	0.3	0.3	0.6	0.2	0.5

Without loss of generality: It is either $\{c, d\}$ does not exist, or there are no people living there, so all the six values – support (least, most), neutral (least, most), against(least, most), are all zero.

Part 3.3.2 Representing with GIVNG1

When we start from step a to e in def. 3.1 , to illustrate the schema with a special GIVNG1

a) Take$V_0 = \{a, b, c, d\}$

b) In line with the scenario, present the three functions $\tilde{\rho}_T, \tilde{\rho}_I, \tilde{\rho}_F$, as illustrated in the following table.

	a	*b*	*c*	*d*
$\tilde{\rho}_T$	[0.1,0.4]	[0.3,0.5]	[0.1,0.2]	[0.5,0.7]
$\tilde{\rho}_I$	[0.2,0.6]	[0.2,0.5]	[0.0,0.3]	[0.2,0.4]
$\tilde{\rho}_F$	[0.3,0.7]	[0.2,0.5]	[0.1,0.2]	[0.1,0.2]

c) By statement c) from Definition 3.1: Let

$E_0 = \{(\tilde{\rho}_T(u), \tilde{\rho}_T(v)) \mid u, v \in \{a, b, c, d\}\}$

$F_0 = \{(\tilde{\rho}_I(u), \tilde{\rho}_I(v)) \mid u, v \in \{a, b, c, d\}\}$

$G_0 = \{(\tilde{\rho}_F(u), \tilde{\rho}_F(v)) \mid u, v \in \{a, b, c, d\}\}$

d) In accordance with the scenario, define

$\alpha : \mathrm{E}_0 \to \Delta_1, \beta : \mathrm{F}_0 \to \Delta_1, \delta : \mathrm{G}_0 \to \Delta_1,$

as illustrated in the following tables.

$\alpha\,((\tilde{\rho}_T(u), \tilde{\rho}_T(v)))$:

v / u	a	b	c	d
a	0	[0.2,0.3]	[0.1,0.7]	0
b	[0.2,0.3]	0	[0.1,0.2]	[0.2,0.3]
c	[0.1,0.7]	[0.1,0.2]	0	0
d	0	[0.2,0.3]	0	0

$\alpha\,((\tilde{\rho}_I(u), \tilde{\rho}_I(v)))$:

v / u	a	b	c	d
a	0	[0.1,0.4]	[0.1,0.8]	0
b	[0.1,0.4]	0	[0.1,0.2]	[0.3,0.6]
c	[0.1,0.8]	[0.1,0.2]	0	0
d	0	[0.3,0.6]	0	0

$\alpha\,((\tilde{\rho}_F(u), \tilde{\rho}_F(v)))$:

v / u	a	b	c	d
a	0	[0.4,0.7]	[0.1,0.7]	0
b	[0.4,0.7]	0	[0.5,0.8]	[0.2,0.5]
c	[0.1,0.7]	[0.5,0.8]	0	0
d	0	[0.2,0.5]	0	0

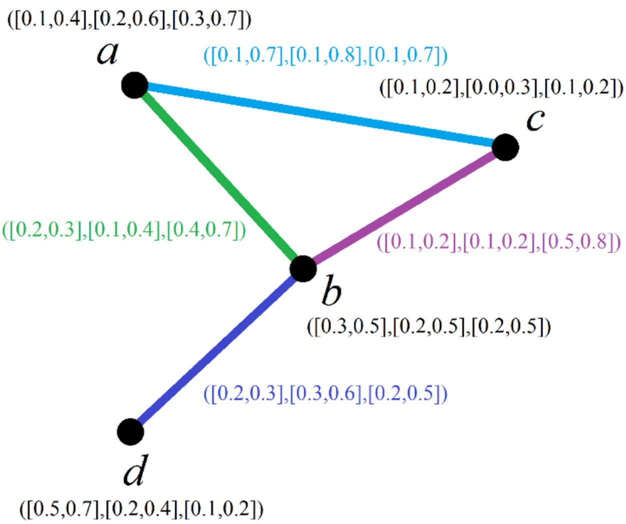

Figure 1

e) By statement e) from Definition 3.1, let

$\tilde{\rho}_0 = (\tilde{\rho}_T, \tilde{\rho}_I, \tilde{\rho}_F)$; and

$\tilde{\omega}_0=(\tilde{\omega}_T, \tilde{\omega}_I, \tilde{\omega}_F)$ with

$\tilde{\omega}_T(u, v) = \alpha((\tilde{\rho}_T(u), \tilde{\rho}_T(v)))$,

$\tilde{\omega}_T(u, v) = \beta((\tilde{\rho}_I(u), \tilde{\rho}_I(v)))$,

$\tilde{\omega}_T(u, v) = \delta((\tilde{\rho}_F(u), \tilde{\rho}_F(v)))$,

for all $u, v \in V_0$. We now have formed $<V_0, \tilde{\rho}_0, \tilde{\omega}_0>$, which is a GIVNG1.

The way of showing the concepts of $<V_0, \tilde{\rho}_0, \tilde{\omega}_0>$ is by exerting a diagram that is similar with graphs as in classical graph theory, as given in the figure 1 below

That is to say, only the non-void edges (whether directed or ordinary) and vertices been drawn in the picture shown above.

Also, understanding the fact that, in classical graph theory GT, a graph isdenoted by adjacency matrix, for which the entries are either a positive integer (connected) or 0 (which is not connected).

This motivates us to present a GIVNG1, by a matrix as well. However, instead of a single value which defines the value that is either 0 or 1, there are *three* values to handle: $\tilde{\omega}_T, \tilde{\omega}_I, \tilde{\omega}_F$, with each of them being elements of Δ_1. Moreover, each of the vertices themselves also contains $\tilde{\rho}_T, \tilde{\rho}_I, \tilde{\rho}_F$, which should be taken into account as well.

4. Illustration of GIVNG1by virtue adjacency matrix

Section 4.1 Algorithms representing GIVNG1

In light of two ways that are similar to other counterparts, the focal point of interest in the following part is to express the notion of GIVNG1.

Suppose $\xi = <V, \tilde{\rho}, \tilde{\omega}>$ is a GIVNG1 where V=$\{v_1, v_2, \ldots, v_n\}$ denotes the vertex set (i.e. GIVNG1 has finite vertices). Remember that GIVNG1has its edge membership values (T,I,F) depending on the membership values (T,I,F) of adjacent vertices, in accordance with the functions α, β, δ.

Furthermore:

$\tilde{\omega}_T(u, v) = \alpha((\tilde{\rho}_T(u), \tilde{\rho}_T(v)))$ for all $v \in V$, where

$\alpha : \mathrm{E} \to \Delta_1$, and $E = \{(\tilde{\rho}_T(u), \tilde{\rho}_T(v)) \mid u, v \in V\}$,

$\tilde{\omega}_I(u, v) = \beta((\tilde{\rho}_I(u), \tilde{\rho}_I(v)))$ for all $u, v \in V$, where

$\beta : \mathrm{F} \to \Delta_1$, and $\mathrm{F} = \{(\tilde{\rho}_I(u), \tilde{\rho}_I(v)) \mid u, v \in V\}$,

$\tilde{\omega}_F(u, v) = \delta((\tilde{\rho}_F(u), \tilde{\rho}_F(v)))$ for all $u, v \in V$, where

$\delta : \mathrm{G} \to \Delta_1$, and $\mathrm{G} = \{(\tilde{\rho}_F(u), \tilde{\rho}_F(v)) \mid u, v \in V\}$.

First we will form an $n \times n$ matrix as presented

$$\tilde{\mathbf{S}} = \left[\tilde{\mathbf{a}}_{i,j}\right]_n = \begin{pmatrix} \tilde{\mathbf{a}}_{1,1} & \tilde{\mathbf{a}}_{1,2} & \cdots & \tilde{\mathbf{a}}_{1,n} \\ \tilde{\mathbf{a}}_{2,1} & \tilde{\mathbf{a}}_{2,2} & & \tilde{\mathbf{a}}_{2,n} \\ \vdots & & \ddots & \vdots \\ \tilde{\mathbf{a}}_{n,1} & \tilde{\mathbf{a}}_{n,2} & \cdots & \tilde{\mathbf{a}}_{n,n} \end{pmatrix},$$

For each i, j, $\tilde{\mathbf{a}}_{i,j} = (\tilde{\omega}_T(v_i, v_j), \tilde{\omega}_I(v_i, v_j), \tilde{\omega}_F(v_i, v_j))$

That is to say, for an element of the matrix $\tilde{S}$, different from taking numbers 0 or 1 according to classical literatures, we usually take the element as an ordered set involving 3 closed subintervals of [0,1].

Remark : Due to the fact that ξ could only have undirected loops, the dominating diagonal elements of $\tilde{S}$ is not multiplied by 2, which is shown as adjacency matrices from classical literatures. It is noted that 0 represents void, 1 for directed ones and 2 for undirected ones.

At the same time, considering $\tilde{\rho}_T$, $\tilde{\rho}_I$, $\tilde{\rho}_F$ is included in ξ, which also deserves to be considered.

Therefore another matrix $\tilde{\boldsymbol{R}}$ is given in the following part.

$$\tilde{\mathbf{R}} = \left[\tilde{\mathbf{R}}_i\right]_{n,1} = \begin{pmatrix} \tilde{\mathbf{r}}_1 \\ \tilde{\mathbf{r}}_2 \\ \vdots \\ \tilde{\mathbf{r}}_n \end{pmatrix},$$

Where

$$\begin{aligned} \tilde{\mathbf{r}}_i &= (\tilde{\rho}_T(v_i), \tilde{\rho}_I(v_i), \tilde{\rho}_F(v_i)) \\ &= \left([\rho_T^L(v_i), \rho_T^U(v_i)], [\rho_I^L(v_i), \rho_I^U(v_i)], [\rho_F^L(v_i), \rho_F^U(v_i)]\right)\forall. \end{aligned}$$

In order to complete the task of describing the whole ξ in our way, the matrix $\tilde{\boldsymbol{R}}$ is augmented with $\tilde{S}$. Then $[\tilde{\boldsymbol{R}} \,|\, \tilde{S}]$ is represented as an *adjacency matrix of* GIVNG, which is presented below.

$$[\mathbf{R} \,|\, \mathbf{S}] = \begin{pmatrix} \tilde{\mathbf{r}}_1 & \tilde{\mathbf{a}}_{1,1} & \tilde{\mathbf{a}}_{1,2} & \cdots & \tilde{\mathbf{a}}_{1,n} \\ \tilde{\mathbf{r}}_2 & \tilde{\mathbf{a}}_{2,1} & \tilde{\mathbf{a}}_{2,2} & & \tilde{\mathbf{a}}_{2,n} \\ & \vdots & & \ddots & \vdots \\ \tilde{\mathbf{r}}_n & \tilde{\mathbf{a}}_{n,1} & \tilde{\mathbf{a}}_{n,2} & \cdots & \tilde{\mathbf{a}}_{n,n} \end{pmatrix},$$

where $\tilde{\mathbf{a}}_{i,j} = (\tilde{\omega}_T(v_i, v_j), \tilde{\omega}_I(v_i, v_j), \tilde{\omega}_F(v_i, v_j))$,

and $\tilde{\mathbf{r}}_i = ([\rho_T^L(v_i), \rho_T^U(v_i)], [\rho_I^L(v_i), \rho_I^U(v_i)], [\rho_F^L(v_i), \rho_F^U(v_i)])$, $\forall i$ and j.

It is worth noticing $[\tilde{R}\,|\,\tilde{S}]$ is not a square matrix ($n \times (n+1)$matrix), thus this kind of representation will aid us to save another *divided* ordered set to denote the values of vertices as $\tilde{\rho}_T$, $\tilde{\rho}_I$, $\tilde{\rho}_F$.

For both edges and vertices, it is imperative to separately handle each of three kinds of membership values in several situations. Consequently, by means of three $n\times(n+1)$ matrices, we aim to give a brand-new way for expressing the whole ξ, denoted as $[\tilde{R}\,|\,\tilde{S}]_T$, $[\tilde{R}\,|\,\tilde{S}]_I$ and $[\tilde{R}\,|\,\tilde{S}]_F$,each of them is resulted from $[\tilde{R}\,|\,\tilde{S}]$ through taking a single kind of membership values from the corresponding elements.

$$[\tilde{\mathbf{R}}\,|\,\tilde{\mathbf{S}}]_T = [\tilde{\mathbf{R}}_T\,|\,\tilde{\mathbf{S}}_T] = \begin{pmatrix} \tilde{\rho}_T(v_1) & \tilde{\omega}_T(v_1,v_1) & \tilde{\omega}_T(v_1,v_2) & & \tilde{\omega}_T(v_1,v_n) \\ \tilde{\rho}_T(v_2) & \tilde{\omega}_T(v_2,v_1) & \tilde{\omega}_T(v_2,v_2) & \cdots & \tilde{\omega}_T(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \tilde{\rho}_T(v_n) & \tilde{\omega}_T(v_n,v_1) & \tilde{\omega}_T(v_n,v_2) & \cdots & \tilde{\omega}_T(v_n,v_n) \end{pmatrix},$$

$$[\tilde{\mathbf{R}}\,|\,\tilde{\mathbf{S}}]_I = [\tilde{\mathbf{R}}_I\,|\,\tilde{\mathbf{S}}_I] = \begin{pmatrix} \tilde{\rho}_I(v_1) & \tilde{\omega}_I(v_1,v_1) & \tilde{\omega}_I(v_1,v_2) & & \tilde{\omega}_i(v_1,v_n) \\ \tilde{\rho}_I(v_2) & \tilde{\omega}_I(v_2,v_1) & \tilde{\omega}_I(v_2,v_2) & \cdots & \tilde{\omega}_I(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \tilde{\rho}_I(v_n) & \tilde{\omega}_I(v_n,v_1) & \tilde{\omega}_I(v_n,v_2) & \cdots & \tilde{\omega}_I(v_n,v_n) \end{pmatrix},$$

$$[\tilde{\mathbf{R}}\,|\,\tilde{\mathbf{S}}]_F = [\tilde{\mathbf{R}}_F\,|\,\tilde{\mathbf{S}}_F] = \begin{pmatrix} \tilde{\rho}_F(v_1) & \tilde{\omega}_F(v_1,v_1) & \tilde{\omega}_F(v_1,v_2) & & \tilde{\omega}_F(v_1,v_n) \\ \tilde{\rho}_F(v_2) & \tilde{\omega}_F(v_2,v_1) & \tilde{\omega}_F(v_2,v_2) & \cdots & \tilde{\omega}_F(v_2,v_n) \\ & \vdots & & \ddots & \vdots \\ \tilde{\rho}_F(v_n) & \tilde{\omega}_F(v_n,v_1) & \tilde{\omega}_F(v_n,v_2) & \cdots & \tilde{\omega}_F(v_n,v_n) \end{pmatrix}.$$

$[\tilde{R}\,|\,\tilde{S}]_T$, $[\tilde{R}\,|\,\tilde{S}]_I$ and $[\tilde{R}\,|\,\tilde{S}]_F$ should be stated respectively with the *true adjacency matrix*, the *indeterminate adjacency matrix*, and *false adjacency matrix* of ξ .

Remark 1 : If $[\tilde{R}\,|\,\tilde{S}]_I = [\tilde{R}\,|\,\tilde{S}]_F = [[0, 0]]_{n,\,n+1}$, $\tilde{R}_T = [[1, 1]]_{n,1}$, all the entries of $\tilde{S}_T$ are either [1, 1] or [0, 0] , then ξ is reduced to a graph in classical literature. Moreover, if that $\tilde{S}_T$ is symmetric and the main diagonal elements are being 0, we have ξ is further condensed to a simple ordinary graph in literature.

Remark 2 : If $[\tilde{R}\,|\,\tilde{S}]_I = [\tilde{R}\,|\,\tilde{S}]_F = [[0, 0]]_{n,\,n+1}$, and all the entries of $[\tilde{R}\,|\,\tilde{S}]_T = [[a_{i,j}, a_{i,j}]]_{n,\,n+1}$, then ξ is reduced to a generalized fuzzy graph type 1 (GFG1).

Remark 3 : If $[\tilde{R}\,|\,\tilde{S}]_T = [[a_{i,j}, a_{i,j}]]_{n,\,n+1}$, $[\tilde{R}\,|\,\tilde{S}]_T = [[b_{i,j}, b_{i,j}]]_{n,\,n+1}$, $[\tilde{R}\,|\,\tilde{S}]_T = [[c_{i,j}, c_{i,j}]]_{n,\,n+1}$, then ξ is thus reduced to GSVNG1.

Section 4.2 : Case study to illustrate our example in this paper

For our example in the set-up by the last way i.e. with three matrices: $[\tilde{\boldsymbol{R}}\,|\,\tilde{\boldsymbol{S}}]_T$, $[\tilde{\boldsymbol{R}}\,|\,\tilde{\boldsymbol{S}}]_I$ and $[\tilde{\boldsymbol{R}}\,|\,\tilde{\boldsymbol{S}}]_F$:

$$[R\,|\,S]_T = \begin{pmatrix} [0.1,0.4] & [0,0] & [0.2,0.3] & [0.1,0.7] & [0,0] \\ [0.3,0.5] & [0.2,0.3] & [0,0] & [0.1,0.2] & [0.2,0.3] \\ [0.1,0.2] & [0.1,0.7] & [0.1,0.2] & [0,0] & [0,0] \\ [0.1,0.7] & [0,0] & [0.2,0.3] & [0,0] & [0,0] \end{pmatrix}$$

$$[R\,|\,S]_I = \begin{pmatrix} [0.2,0.6] & [0,0] & [0.1,0.4] & [0.1,0.8] & [0,0] \\ [0.2,0.5] & [0.1,0.4] & [0,0] & [0.1,0.2] & [0.3,0.6] \\ [0.0,0.3] & [0.1,0.8] & [0.1,0.2] & [0,0] & [0,0] \\ [0.2,0.4] & [0,0] & [0.3,0.6] & [0,0] & [0,0] \end{pmatrix}$$

$$[R\,|\,S]_F = \begin{pmatrix} [0.3,0.7] & [0,0] & [0.4,0.7] & [0.1,0.7] & [0,0] \\ [0.2,0.5] & [0.4,0.7] & [0,0] & [0.5,0.8] & [0.2,0.5] \\ [0.1,0.2] & [0.1,0.7] & [0.5,0.8] & [0,0] & [0,0] \\ [0.1,0.2] & [0,0] & [0.2,0.5] & [0,0] & [0,0] \end{pmatrix}$$

5. Postulated results on ordinary GIVNG1

We now illustrate some theoretical results that are derived from the definition of ordinary GIVNG1, as well as its indication with adjacency matrix. Since we focus on the basic GIVNG1, all the edges which we will be referring to are termed as ordinary edges.

Definition 5.1 The addition operation + is defined on Δ_1 as follows: $[x\,,y] + [z\,,t] = [\,x+y\,,z+t]$ for all $x, y, z, t \in [0,1]$.

Definition 5.2 Let $\xi = <V, \tilde{\rho}, \tilde{\omega}>$ be an ordinary GIVNG1. Let $V = \{v_1, v_2, \ldots, v_n\}$ to be the vertex set of ξ. Then, $\forall\ i$, the *degree* of v_i, symbolised as $\tilde{D}(v_i)$, is well-defined to be the ordered set

$$(\tilde{D}_T(v_i), \tilde{D}_I(v_i), \tilde{D}_F(v_i)),$$

for which, $\tilde{D}_T(v_i)$ represents the *degree* of v_i and

$$a)\quad \tilde{D}_T(v_i) = \left[\sum_{r=1}^{n}\omega_T^L(v_i,v_r) + \omega_T^L(v_i,v_i), \sum_{r=1}^{n}\omega_T^U(v_i,v_r) + \omega_T^U(v_i,v_i)\right]$$

$$b)\quad \tilde{D}_I(v_i) = \left[\sum_{r=1}^{n}\omega_I^L(v_i,v_r) + \omega_I^L(v_i,v_i), \sum_{r=1}^{n}\omega_I^U(v_i,v_r) + \omega_I^U(v_i,v_i)\right]$$

$$c)\quad \tilde{D}_F(v_i) = \left[\sum_{r=1}^{n}\omega_F^L(v_i,v_r) + \omega_F^L(v_i,v_i), \sum_{r=1}^{n}\omega_F^U(v_i,v_r) + \omega_F^U(v_i,v_i)\right]$$

Remark 1 : In resemblance to classical graph theory, each undirected loop has both its ends connected to the similar vertex and so is counted twice.

Remark 2 : Every value of $\tilde{D}_T(v_i)$, $\tilde{D}_I(v_i)$ and $\tilde{D}_F(v_i)$ are elements of Δ_1 instead of a single number.

Definition 5.3 : Given $\xi = <V, \tilde{\rho}, \tilde{\omega}>$ and $V = \{v_1, v_2, \ldots, v_n\}$ are respectively an ordinary GIVNG1 and the vertex set of ξ . Then, the *quantity of edges in* ξ, represented as E_ξ and we describe the ordered set $(\tilde{E}_T, \tilde{E}_I, \tilde{E}_F)$ for which

a) $$\tilde{E}_T = \left[\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_T^L(v_r, v_s) \, , \sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_T^U(v_r, v_s) \right]$$

b) $$\tilde{E}_I = \left[\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_I^L(v_r, v_s) \, , \sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_I^U(v_r, v_s) \right]$$

c) $$\tilde{E}_F = \left[\sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_F^L(v_r, v_s) \, , \sum_{\{r,s\}\subseteq\{1,2,\ldots,n\}} \omega_F^U(v_r, v_s) \right]$$

Remark 1: We count each edge only once in classical graph theory, as given by $\{r,s\} \subseteq \{1,2,\ldots,n\}$.

For instance, if $\tilde{\omega}_T(v_a, v_b)$ is added, we will not add $\tilde{\omega}_T(v_b, v_a)$ again since $\{a, b\} = \{b, a\}$.

Remark 2 : Each values of $\tilde{E}_T$, $\tilde{E}_I$ and $\tilde{E}_F$ are elements of Δ_1 instead of a single number, and need not be 0 or 1 as in classical graph literature. Consequently, it is called "amount" of edges, instead of the "number" of edges as in the classical reference.

$\tilde{E}_T$, $\tilde{E}_I$, $\tilde{E}_F$ are closed subintervals of [0,1], and $\tilde{D}_T(v_i)$, $\tilde{D}_I(v_i)$, $\tilde{D}_F(v_i)$ are also closed subintervals of [0,1] for each vertex v_i. These give rise to the following lemmas

Lemma 5.4 : *Let* $\xi = <V, \tilde{\rho}, \tilde{\omega}>$ *be an ordinary GIVNG1. Let* $V = \{v_1, v_2, \ldots, v_n\}$ *to be the vertex set of* ξ *. Denote*

a) $\tilde{\omega}_T(v_i, v_j) = [\phi_{T,(i,j)}, \psi_{T,(i,j)}]$

b) $\tilde{\omega}_I(v_i, v_j) = [\phi_{I,(i,j)}, \psi_{I,(i,j)}]$

c) $\tilde{\omega}_F(v_i, v_j) = [\phi_{F,(i,j)}, \psi_{F,(i,j)}], \forall\, i, j$

For each *i* we have:

i) $\tilde{D}_T(v_i) = \left[\sum_{r=1}^{n} \phi_{T,(i,r)} + \phi_{T,(i,i)}, \sum_{r=1}^{n} \psi_{T,(i,r)} + \psi_{T,(i,i)} \right],$

ii) $\tilde{D}_I(v_i) = \left[\sum_{r=1}^{n} \phi_{I,(i,r)} + \phi_{I,(i,i)}, \sum_{r=1}^{n} \psi_{I,(i,r)} + \psi_{I,(i,i)} \right],$

iii) $\tilde{D}_F(v_i) = \left[\sum_{r=1}^{n} \phi_{F,(i,r)} + \phi_{T,(i,i)}, \sum_{r=1}^{n} \psi_{F,(i,r)} + \psi_{F,(i,i)} \right].$

Furthermore:

iv) $\tilde{E}_T = \left[\sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \phi_{T,(r,s)}, \sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \psi_{T,(r,s)} \right],$

v) $\tilde{E}_I = \left[\sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \phi_{I,(r,s)}, \sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \psi_{I,(r,s)} \right],$

vi) $\tilde{E}_F = \left[\sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \phi_{F,(r,s)}, \sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \psi_{F,(r,s)} \right].$

Proof : We can proof it directly by applying Def.5.1, Def. 5.2 and Def. 5.3. In the following two theorems, we introduce two theorems which both as a modified version of the well-known theorem in classical graph theory.

"We know that the sum of the degree of invariably its vertices is twice the number of its edges for any classical graph."

Theorem 5.5 : Let $\xi = <V, \tilde{\rho}, \tilde{\omega}>$ bean ordinary GIVNG1. Then

$$\sum_{r=1}^{n} \tilde{D}(v_r) = 2\tilde{E}_\xi$$

Proof : As $\tilde{D}(v_i) = (\tilde{D}_T(v_i), \tilde{D}_I(v_i), \tilde{D}_F(v_i))$ for all i , and $\tilde{E}_\xi = (\tilde{E}_T, \tilde{E}_I, \tilde{E}_F)$. It is enough to show that $2\tilde{E}_T = \sum_{r=1}^{n} \tilde{D}_T(v_r)$:

$$\tilde{E}_T = \left[\sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \omega_T^L(v_r, v_s), \sum_{\{r,s\} \subseteq \{1,2,\ldots,n\}} \omega_T^U(v_r, v_s) \right]$$

$$= \left[\sum_{\substack{\{r,s\} \subseteq \{1,2,\ldots,n\} \\ r \neq s}} \omega_T^L(v_r, v_s) + \sum_{r=1}^{n} \omega_T^L(v_r, v_r), \right.$$

$$\left. \sum_{\substack{\{r,s\} \subseteq \{1,2,\ldots,n\} \\ r \neq s}} \omega_T^U(v_r, v_s) + \sum_{r=1}^{n} \omega_T^U(v_r, v_r) \right]$$

Since $\{r, s\} = \{s, r\}$ for all s and r ,

$$2\tilde{E}_T = \begin{bmatrix} 2\sum\limits_{\substack{\{r,s\}\subseteq\{1,2,\ldots,n\}\\ r\neq s}} \omega_T^L(v_r,v_s) + 2\sum\limits_{r=1}^{n}\omega_T^L(v_r,v_r), \\ 2\sum\limits_{\substack{\{r,s\}\subseteq\{1,2,\ldots,n\}\\ r\neq s}} \omega_T^U(v_r,v_s) + 2\sum\limits_{r=1}^{n}\omega_T^U(v_r,v_r) \end{bmatrix}$$

$$= \begin{bmatrix} \sum\limits_{\substack{r\in\{1,2,\ldots,n\}\\ s\in\{1,2,\ldots,n\}\\ r\neq s}} \omega_T^L(v_r,v_s) + 2\sum\limits_{r=1}^{n}\omega_T^L(v_r,v_r), \\ \sum\limits_{\substack{r\in\{1,2,\ldots,n\}\\ s\in\{1,2,\ldots,n\}\\ r\neq s}} \omega_T^U(v_r,v_s) + 2\sum\limits_{r=1}^{n}\omega_T^U(v_r,v_r) \end{bmatrix}$$

$$= \begin{bmatrix} \sum\limits_{\substack{r\in\{1,2,\ldots,n\}\\ s\in\{1,2,\ldots,n\}}} \omega_T^L(v_r,v_s) + \sum\limits_{r=1}^{n}\omega_T^L(v_r,v_r), \\ \sum\limits_{\substack{r\in\{1,2,\ldots,n\}\\ s\in\{1,2,\ldots,n\}}} \omega_T^U(v_r,v_s) + \sum\limits_{r=1}^{n}\omega_T^U(v_r,v_r) \end{bmatrix}$$

$$= \begin{bmatrix} \sum\limits_{r=1}^{n}\sum\limits_{s=1}^{n}\omega_T^L(v_r,v_s) + \sum\limits_{r=1}^{n}\omega_T^L(v_r,v_r), \\ \sum\limits_{r=1}^{n}\sum\limits_{s=1}^{n}\omega_T^U(v_r,v_s) + \sum\limits_{r=1}^{n}\omega_T^U(v_r,v_r) \end{bmatrix}$$

$$= \begin{bmatrix} \sum\limits_{r=1}^{n}\left(\sum\limits_{s=1}^{n}\omega_T^L(v_r,v_s) + \omega_T^L(v_r,v_r)\right), \\ \sum\limits_{r=1}^{n}\left(\sum\limits_{s=1}^{n}\omega_T^U(v_r,v_s) + \omega_T^U(v_r,v_r)\right), \end{bmatrix}$$

$$= \sum_{r=1}^{n}\tilde{D}_T(v_r).$$

This finishes the proof. ■

6. Conclusion

The idea of GSVNG1 was extended to introduce the concept of generalized interval-valued neutrosophic graph of type 1(GIVNG1). The matrix representation of GIVNG1 was also introduced. The future direction of this research includes the study of completeness, regularity of GIVNG1, and also denote the notion of generalized interval-valued neutrosophic graphs of type 2.As GIVNG1 (in this paper) and CNG1 (from [47]) are both extensions of the existing concepts of CFG1 and GSVNG1, but in two entirely different directions, the future direction of this research also includes further extensions from GIVNG1 and CNG1, that incorporates *both* the inter-valued entries (as in GIVNG1) and complexity of numbers (as in CNG1), and the study of scenarios that necessitate such extensions [48-52].

Disclosure statement

No potential conflict of interest was reported by the authors.

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