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MAGDM for agribusiness in the environment of various cubic m-polar fuzzy averaging aggregation operators

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Abstract. In multi-attribute group decision-making (MAGDM) problems, there exist some multi-polarity for the attributes and criteria. Sometimes in real life situations, we deal with the both membership and non-membership grades for the attributes in the presence of multi-polarity. For this purpose, we change verbally stated information into mathematical language with the help of uncertain linguistic variables to deal with the ambiguities and uncertainties. In that case, we construct some extensions from the existing hybrid structures of fuzzy set to handle these types of problems. That's why from the prevailing concepts of cubic set and m-polar fuzzy set, we innovate the concept of cubic m-polar fuzzy set (CMPFS). We investigate its numerous operations with the help of examples. With the enthusiasm of CMPFS, we establish certain aggregation operators based on cubic m-polar fuzzy numbers (CMPFNs) namely Cubic m-polar fuzzy weighted averaging (CMPFWA), Cubic m-polar fuzzy ordered weighted averaging (CMPFOWA) and Cubic m-polar fuzzy hybrid averaging (CMPFHA) operators corresponding to *R*-order and *P*-order, simultaneously. Using the score function and accuracy function a relation is proposed, through which we can compare the CMPFNs. This manuscript presents a novel approach for treating ambiguities based on the application of land selection using linguistic variables in CMPF decision theory. An algorithm based on MAGDM is intended for a given agricultural project, which will produce results according to the proposed operators one by one. Furthermore, a comparative analysis is listed to demonstrate the difference, advantages, validity, simplicity, flexibility and superiority to the proposed operators.

Keywords: Cubic m-polar fuzzy set (CMPFS), membership degrees, Cubic m-polar fuzzy weighted averaging (CMPFWA) operator, Cubic m-polar fuzzy ordered weighted averaging (CMPFOWA) operator and Cubic m-polar fuzzy hybrid averaging (CMPFHA) operator, MAGDM for agricultural purpose

1. Introduction

Multi attribute group decision-making (MAGDM) is widely employed in real world problems of different areas such as technology, economics, psychology, social sciences, management and medical diagnosis. It is regarded as the intellectual process which results the selection of a belief or a class of activity among various alternative possibilities according to diverse standards. If we amass the data and deduce the result without handling ambiguities, then given results will be undefined and equivocal. Sometimes, we have to change verbally stated information into mathematical language with the help of uncertain linguistic variables to deal with these ambiguities and uncertainties. For this purpose a fuzzy set (FS) was established by Zadeh [49], which is an imperative precise erection to epitomize an assembling of items whose boundary is ambiguous. After that, more hybrid models

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of FS have been presented and investigated such as, intuitionistic fuzzy set (IFS) [5], single valued neutrosophic set (SVNS) [38, 39], m-polar fuzzy set (MPFS) [7], interval valued fuzzy set (IVFS) [50] and cubic fuzzy set (CFS) [11].

Aggregation means the creation of a numeral of things into a cluster or a bunch of things that have come or been taken together. In the past few years, aggregation operators based on FS and its various hybrid structures have made very much attention and become popular. We can use them in various practical areas of different domains. Xu [44] introduced the concept of intuitionistic fuzzy aggregation operators. Xu and Cai, in their book [45], presented the theory and applications of intuitionistic fuzzy information aggregation. Xu, in his book [46], presented hesitant fuzzy sets theory and various types of hesitant fuzzy aggregation operators. Ye [47] introduced interval-valued hesitant fuzzy prioritized weighted aggregation (IVHFPWA) operators and their application in MADM. Ye [48] introduced linguistic neutrosophic cubic numbers and their application in multiple attribute decision-making. Kaur and Garg [13] established aggregation operators on CIFNs and use these operators in decision-making approach for job selection. Jose and Kuriaskose [12] investigated aggregation operators with the corresponding score function for MCDM in the context of IFNs. Mahmood et al. [16] established generalized aggregation operators for CHFNs and use it into MCDM. Riaz and Hashmi [24–28] investigated certain applications of FPFS-sets, FPFS-topology and FPFS-compact spaces. They developed fixed point theorems of FNS-mapping with its decision-making. Riaz et al. [29, 30] introduced soft rough topology with multi-attribute group decision-making problems (MAGDM). Riaz et al. [31] introduced N-soft topology with multi-criteria group decision-making problems (MCGDM). Riaz and Tahrim [32–36] established the idea of bipolar fuzzy soft topology, TOPSIS method based on bipolar neutrosophic topology and cubic bipolar fuzzy ordered weighted geometric aggregation operators and their application using internal and external cubic bipolar fuzzy data. They presented various illustrations and decision-making applications of these concepts by using different algorithms. Akram et al. [1–3] presented certain applications of m-polar fuzzy sets and neutrosophic incidence fuzzy graphs in decision-making problems. Ali [4] write a note on soft, rough soft and fuzzy soft sets. Qurashi and Shabir [23] presented generalized approximations of $(\in, \in \vee q)$-fuzzy ideals in quantales. Shabir and Ali [40] established some properties of soft ideals and generalized fuzzy ideals in semigroups. Shabir and Naz [41] introduced soft topological space. Xueling et al. [43] introduced decision-making methods based on various hybrid soft sets. Feng et al. [8–10] introduced properties of soft sets combined with fuzzy and rough sets and MADM models in the environment of generalized IF soft set and fuzzy soft set. Boran et al. [6] use TOPSIS decision-making method for the supplier selection in the context of IFS. Liu et al. [14] worked on hesitant IF linguistic operators and presented its MAGDM problem. Wei et al. [42] established hesitant triangular fuzzy operators in MADGDM problems. Pamucar et al. [18–22] established normalized weighted geometric Bonferroni mean operator of interval rough numbers and presented their application in interval rough DEMATEL-COPRAS. They introduced a novel approach for the selection of power generation technology using an linguistic neutrosophic combinative distance-based assessment (CODAS) method. They also worked on integration of interval rough AHP and interval rough MABAC methods for the evaluation of university web pages. They presented modification of the Best-Worst and MABAC methods by using interval-valued fuzzy-rough numbers. They presented an application of multi-criteria decision-making of sensitivity analysis. Mukhametzyanov and Pamucar [17] established a sensitivity analysis in multi-criteria decision-making problems by using statistical methods. Liu et al. [15] worked on multi-criteria model for the selection of the transport service provider by constructing the single valued neutrosophic DEMATEL multi-criteria model. Zhan et al. [51, 52] presented the concepts of rough soft hemirings, soft rough covering and its applications to multi-criteria group decision-making (MCGDM) problems.

Many mathematicians did not focus on important and useful real life applications of their proposed research methodologies. The concepts behind their proposed algebra are valid and strong, but the application area is not well defined. Fuzzy set theory is more useful and applicable in many real life problems due to its vast concept than classical algebra. We can handle uncertainties and ambiguities by using fuzzy set theory. Various mathematical concepts have been redefined using fuzzy sets and its extensions such as neutrosopic, bipolar, m-polar, soft, intuitionistic and cubic set. These extensions are used when fuzzy theory is not enough to elaborate the logics and ideas of real life problems. To fill this gap, we propose this novel idea of CMPFS with its application in MAGDM problem.

One important question arises here that, why we introduce the CMPFS? As cubic set (CS) [11] is an abstraction of IFS [5]. In IFS we deal with the membership and non-membership grades, while in CS we have a fuzzy interval for membership grade, so we have more options for choosing the grades for different alternatives. If $\inf[\mathfrak{A}^-, \mathfrak{A}^+] = \sup[\mathfrak{A}^-, \mathfrak{A}^+]$ for every alternative in CS then it reduces to IFS. Sometimes in real life situations, we have to deal with the both membership and non-membership grades for the attributes in the presence of multi-polarity. The existing hybrid structures of fuzzy sets are not enough to deal with these type of input data in real life situations. In that case, we have to construct some extensions from the existing hybrid structures of fuzzy set to handle these types of problems. For this purpose, we introduce a new idea of CMPFS with the combination of CS [11] and MPFS [7]. The idea of Kaur and Garg [13] with Xu [44] and Ye [47] helps us to get some extended examples of aggregation operators based on the new concept of cubic m-polar fuzzy set (CMPFS). With the reference of previous theories, we construct the interval valued MPFS (IVMPFS) which is based on m intervals for every alternative of the universal set, where every interval is the subset of [0, 1]. Next combining CS and IVMPFS we establish CMPFS, where m intervals and m degrees of memberships exists for every single alternative of the reference set. We explain our contributions to this research field briefly in every section of this article. We elaborate CMPFS with the R-order and P-order operations corresponding to the examples. We define averaging aggregation operators of R-order and P-order such as CMPFWA, CMPFOWA, CMPFHA operator in the context of CMPFNs. Lastly, we establish an innovative application of land chosen for an agricultural labor under the superintendence of an international agricultural firm. The MAGDM method is utilized to establish the decision by using the defined operators. A brief comparison analysis and discussion is also given to elaborate the significance of proposed approach.

The motivation of this extended and hybrid work is given step by step in the whole manuscript. We show that other hybrid structures of fuzzy sets become special cases of CMPFS under some suitable conditions. We discuss about the validity, flexibility, simplicity and superiority of our proposed model. The constructed model is use to collect data at a large scale and covers many hybrid fuzzy structures. In CMPFS, we deal with “m” number of criteria or “m” properties of the attributes given in the sample space. For each criteria, there exists both truth and falsity grades to deal with both aspects of attributes. The novelty of our proposed approach is given in Section 4. In this section, we can see that by using CMPF input data, we can handle MAGDM problems in diverse fields of life. The purpose and significance of constructed model is given in each part of this manuscript, specially in the application of selection of the land for an agricultural project.

This manuscript has various objectives. The first objective is to improve the methodology for treating ambiguities in the field of the multi-attribute group decision-making (MAGDM) , and the techniques for selecting the best and suitable alternative through a novel approach in the uncertainty treatment based on cubic m-polar fuzzy numbers and cubic m-polar fuzzy aggregation operators. The second goal of the research is to arrange the criteria and form a new model that would enable an objective, scientifically based approach to the selection of optimal choice. The third objective of this paper is to bridge the gap that exists in the methodology for the decision-making through a new approach to the treatment of uncertainty that is based on proposed model.

The layout of this paper is systematized as follows: Section 2 implies a novel idea of Cubic m-polar fuzzy set (CMPFS). We establish some of its operations, score function and accuracy function with the addition of related illustrations. In Section 3, we use CMPFS to establish novel averaging aggregation operators for R-order and P-order, respectively. To elaborate the mathematical notations and calculations of these operators various illustrations are listed in this section. In Section 4, we establish a method for the solution of MAGDM problem using defined aggregated operators by the suggested algorithm. This model demonstrates the feasibility and advantages of the proposed approaches. In the sequence, we make a brief comparison analysis of all the proposed operators with the help of graphs and table. Finally, the conclusion of this research is summarized in Section 5.

2. Cubic m-polar fuzzy set (CMPFS)

2.1. Basic concepts

In this section, we discuss about some basic ideas which will be used for the construction of CMPFS.

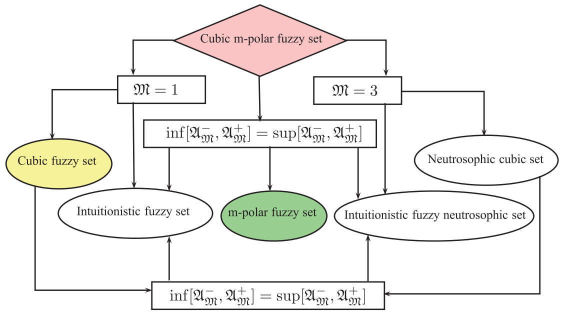

Fig. 1. Relationship between CMPFS and other hybrid structures of fuzzy set.

Fig. 2. Score values of C4PFNs.

In the whole paper, we use $\mathcal{Q}$ as a fixed sample space or as the universal circle.

Definition 2.1. [7] An abstraction of bipolar fuzzy set was inaugurated by Chen named as m-polar fuzzy set (MPFS). An MPFS $\mathfrak{C}$ on a non-empty universal set $\mathcal{Q}$ is a mathematical function $\mathfrak{C} : \mathcal{Q} \to [0, 1]^m$, symbolized by

$$\mathfrak{C} = \{\langle \varsigma, P_i o \Lambda(\varsigma)\rangle : \varsigma \in \mathcal{Q}; i = 1, 2, 3, ..., m\}$$

where and $P_i : [0, 1]^m \to [0, 1]$ is the i-th projection mathematical function ($i \in m$).

Definition 2.2. [11] A cubic set or (CS) $\mathfrak{S}$ on $\mathcal{Q}$ is denoted as $\mathfrak{S} = \langle \mathfrak{A}_{\mathcal{F}}, \eta_{\mathcal{F}}\rangle$ and can be scripted as

$$\mathfrak{S} = \{\langle \varsigma, \mathfrak{A}_{\mathcal{F}}(\varsigma), \eta_{\mathcal{F}}(\varsigma)\rangle : \varsigma \in \mathcal{Q}\}$$

where $\mathfrak{A}_{\mathcal{F}}(\varsigma) = [\mathfrak{A}^-(\varsigma), \mathfrak{A}^+(\varsigma)]$ and $\eta_{\mathcal{F}}(\varsigma)$ are interval valued fuzzy set (IVFS) and fuzzy set (FS), respectively on $\mathcal{Q}$.

Definition 2.3. [11] The complement of CS is defined as

$$\mathfrak{S}^c = \{\langle \varsigma, \mathfrak{A}^c_{\mathcal{F}}(\varsigma), 1 - \eta_{\mathcal{F}}(\varsigma)\rangle : \varsigma \in \mathcal{Q}\}$$

where $\mathfrak{A}^c_{\mathcal{F}}(\varsigma) = [1 - \mathfrak{A}^+(\varsigma), 1 - \mathfrak{A}^-(\varsigma)]$ and $\eta_{\mathcal{F}}(\varsigma)$ are IVFS and FS on $\mathcal{Q}$.

Definition 2.4. [11] Suppose that we have an assembling of CFSs
$\mathfrak{S}_{\wp} = \{\langle \varsigma, \mathfrak{A}_{\wp}(\varsigma), \eta_{\wp}(\varsigma)\rangle : \varsigma \in \mathcal{Q}, \wp \in \Delta\}$ where Δ is an indexing set, then we can define the following:
(i) P-union: ${}^P\bigcup_{\wp\in\Delta} \mathfrak{S}_{\wp} =$
$\{\langle \varsigma, (\bigcup_{\wp\in\Delta} \mathfrak{A}_{\wp})(\varsigma), (\bigvee_{\wp\in\Delta} \eta_{\wp})(\varsigma)\rangle : \varsigma \in \mathcal{Q}, \wp \in \Delta\}$,
(ii) P-intersection: ${}^P\bigcap_{\wp\in\Delta} \mathfrak{S}_{\wp} =$
$\{\langle \varsigma, (\bigcap_{\wp\in\Delta} \mathfrak{A}_{\wp})(\varsigma), (\bigwedge_{\wp\in\Delta} \eta_{\wp})(\varsigma)\rangle : \varsigma \in \mathcal{Q}, \wp \in \Delta\}$,
(iii) R-union: ${}^R\bigcup_{\wp\in\Delta} \mathfrak{S}_{\wp} =$
$\{\langle \varsigma, (\bigcup_{\wp\in\Delta} \mathfrak{A}_{\wp})(\varsigma), (\bigwedge_{\wp\in\Delta} \eta_{\wp})(\varsigma)\rangle : \varsigma \in \mathcal{Q}, \wp \in \Delta\}$,
(iv) R-intersection: ${}^R\bigcap_{\wp\in\Delta} \mathfrak{S}_{\wp} =$
$\{\langle \varsigma, (\bigcap_{\wp\in\Delta} \mathfrak{A}_{\wp})(\varsigma), (\bigvee_{\wp\in\Delta} \eta_{\wp})(\varsigma)\rangle : \varsigma \in \mathcal{Q}, \wp \in \Delta\}$.

Definition 2.5. An IVMPFS can be represented as $\mathcal{V}_{\mathfrak{M}} = \{\varsigma, \langle[\mathfrak{A}^-_1(\varsigma), \mathfrak{A}^+_1(\varsigma)], [\mathfrak{A}^-_2(\varsigma), \mathfrak{A}^+_2(\varsigma)], ..., [\mathfrak{A}^-_{\mathfrak{M}}(\varsigma), \mathfrak{A}^+_{\mathfrak{M}}(\varsigma)]\rangle : \varsigma \in \mathcal{Q}\}$, where $[\mathfrak{A}^-_{\mathfrak{N}}(\varsigma), \mathfrak{A}^+_{\mathfrak{N}}(\varsigma)] \subseteq [0, 1], \mathfrak{N} = 1, 2, 3, ..., \mathfrak{M}$, i.e all are closed subintervals of [0, 1].

Definition 2.6. A neutrosophic set $\mathfrak{N}$ is defined by $\mathfrak{N} = \{\langle \varsigma, \mathfrak{A}(\varsigma), \mathfrak{S}(\varsigma), \mathfrak{Y}(\varsigma)\rangle, \varsigma \in \mathcal{Q}\}$, where $\mathfrak{A}, \mathfrak{S}, \mathfrak{Y} : \mathcal{Q} \to]^-0, 1^+[$ and $^-0 \leq \mathfrak{A}(\varsigma) + \mathfrak{S}(\varsigma) + \mathfrak{Y}(\varsigma) \leq 3^+$. The neutrosophic set yields the value from real standard or non-standard subsets of $]^-0, 1^+[$.

2.2. CMPFS

Cubic set deals with the membership and non-membership grades of alternatives and for membership grade, we take a fuzzy interval which is a subset of [0, 1]. In MPFS, we discuss about the multi-criteria of alternative with its m-degrees. We establish a hybrid structure of CMPFS with the combination of CS and MPFS. This structure deals with the membership grades as fuzzy intervals and non-membership grade as a fuzzy set. For each grade we have m-criteria to the corresponding alternative of the reference set $\mathcal{Q}$. This is an abstracted model and use to collet data at a large scale.
A relationship between CMPFS and other hybrid structures of fuzzy set is given in Fig. 1, which shows that other structures becomes special cases of

Table 1
Cubic m-polar fuzzy set

$\mathfrak{C}_{\mathfrak{M}}$	*CMPFS*
ς_1	$\big(\langle[\mathfrak{A}_1^-(\varsigma_1), \mathfrak{A}_1^+(\varsigma_1)], [\mathfrak{A}_2^-(\varsigma_1), \mathfrak{A}_2^+(\varsigma_1)], ..., [\mathfrak{A}_{\mathfrak{M}}^-(\varsigma_1), \mathfrak{A}_{\mathfrak{M}}^+(\varsigma_1)]\rangle, \langle\mathfrak{A}_1(\varsigma_1), \mathfrak{A}_2(\varsigma_1), ..., \mathfrak{A}_{\mathfrak{M}}(\varsigma_1)\rangle\big)$
ς_2	$\big(\langle[\mathfrak{A}_1^-(\varsigma_2), \mathfrak{A}_1^+(\varsigma_2)], [\mathfrak{A}_2^-(\varsigma_2), \mathfrak{A}_2^+(\varsigma_2)], ..., [\mathfrak{A}_{\mathfrak{M}}^-(\varsigma_2), \mathfrak{A}_{\mathfrak{M}}^+(\varsigma_2)]\rangle, \langle\mathfrak{A}_1(\varsigma_2), \mathfrak{A}_2(\varsigma_2), ..., \mathfrak{A}_{\mathfrak{M}}(\varsigma_2)\rangle\big)$
...	... ...
$\varsigma_{\mathfrak{N}}$	$\big(\langle[\mathfrak{A}_1^-(\varsigma_{\mathfrak{N}}), \mathfrak{A}_1^+(\varsigma_{\mathfrak{N}})], [\mathfrak{A}_2^-(\varsigma_{\mathfrak{N}}), \mathfrak{A}_2^+(\varsigma_{\mathfrak{N}})], ..., [\mathfrak{A}_{\mathfrak{M}}^-(\varsigma_{\mathfrak{N}}), \mathfrak{A}_{\mathfrak{M}}^+(\varsigma_{\mathfrak{N}})]\rangle, \langle\mathfrak{A}_1(\varsigma_{\mathfrak{N}}), \mathfrak{A}_2(\varsigma_{\mathfrak{N}}), ..., \mathfrak{A}_{\mathfrak{M}}(\varsigma_{\mathfrak{N}})\rangle\big)$

Table 2
Cubic 4-polar fuzzy numbers

$\mathcal{Q}$	*C4PFNs*
ψ_1	$\big(\langle[0.21, 0.51], [0.37, 0.48], [0.47, 0.83], [0.21, 0.38]\rangle, \langle 0.61, 0.51, 0.21, 0.93\rangle\big)$
ψ_2	$\big(\langle[0.34, 0.57], [0.43, 0.78], [0.21, 0.61], [0.53, 0.78]\rangle, \langle 0.25, 0.88, 0.99, 0.13\rangle\big)$

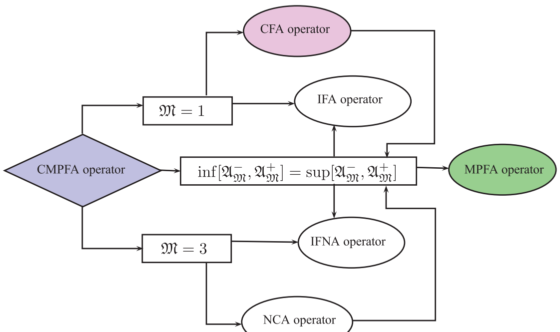

Fig. 3. Relationship between CMPFA operator and other operators.

CMPFS for $\mathfrak{M} = 1$, $\mathfrak{M} = 3$ and for $\inf[\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+] = \sup[\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+]$.

Definition 2.7. A CMPFS $\mathfrak{C}_{\mathfrak{M}}$ is defined on a fixed sample space $\mathcal{Q}$ and defined as an ordered pair

$$\mathfrak{C}_{\mathfrak{M}} = \{\langle\varsigma, \mathcal{V}_{\mathfrak{M}}(\varsigma), \mathfrak{M}(\varsigma)\rangle : \varsigma \in \mathcal{Q}\}$$

where $\mathcal{V}_{\mathfrak{M}} = \{\varsigma, \langle[\mathfrak{A}_1^-(\varsigma), \mathfrak{A}_1^+(\varsigma)], [\mathfrak{A}_2^-(\varsigma), \mathfrak{A}_2^+(\varsigma)], ..., [\mathfrak{A}_{\mathfrak{M}}^-(\varsigma), \mathfrak{A}_{\mathfrak{M}}^+(\varsigma)]\rangle : \varsigma \in \mathcal{Q}\}$ represents IVMPFS and $\mathfrak{M} = \{\varsigma, \langle\mathfrak{A}_1(\varsigma), \mathfrak{A}_2(\varsigma), ..., \mathfrak{A}_{\mathfrak{M}}(\varsigma)\rangle\}$ represent MPFS where $\mathfrak{A}_1(\varsigma), \mathfrak{A}_2(\varsigma), ..., \mathfrak{A}_{\mathfrak{M}}(\varsigma) \in [0, 1]$. This set can be simply denoted as $\psi = \langle\mathcal{V}_{\mathfrak{M}}, \mathfrak{M}\rangle$ where $\mathcal{V}_{\mathfrak{M}} = \langle[\mathfrak{A}_1^-, \mathfrak{A}_1^+], [\mathfrak{A}_2^-, \mathfrak{A}_2^+], ..., [\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+]\rangle$ and $\mathfrak{M} = \langle\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_{\mathfrak{M}}\rangle$ and this is called cubic m-polar fuzzy number (CMPFN).
Tabular representation of CMPFS is given in Table 1.

Definition 2.8. We define some operations for CMPFNs $\psi = \big(\langle[\mathfrak{A}_1^-, \mathfrak{A}_1^+], [\mathfrak{A}_2^-, \mathfrak{A}_2^+], ..., [\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_{\mathfrak{M}}\rangle\big)$ and $\psi_\wp = \big(\langle[{}^\wp\mathfrak{A}_1^-, {}^\wp\mathfrak{A}_1^+], [{}^\wp\mathfrak{A}_2^-, {}^\wp\mathfrak{A}_2^+], ..., [{}^\wp\mathfrak{A}_{\mathfrak{M}}^-, {}^\wp\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle{}^\wp\mathfrak{A}_1, {}^\wp\mathfrak{A}_2, ..., {}^\wp\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$ given as:

(i): $\psi^c = \big(\langle[1 - \mathfrak{A}_1^+, 1 - \mathfrak{A}_1^-], [1 - \mathfrak{A}_2^+, 1 - \mathfrak{A}_2^-], ..., [1 - \mathfrak{A}_{\mathfrak{M}}^+, 1 - \mathfrak{A}_{\mathfrak{M}}^-]\rangle, \langle 1 - \mathfrak{A}_1, 1 - \mathfrak{A}_2, ..., 1- \mathfrak{A}_{\mathfrak{M}}\rangle\big)$.

(ii): $\psi_1 = \psi_2 \Leftrightarrow [{}^1\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+] = [{}^2\mathfrak{A}_1^-, {}^2\mathfrak{A}_1^+]$, $[{}^1\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+] = [{}^2\mathfrak{A}_2^-, {}^2\mathfrak{A}_2^+]$,..., $[{}^1\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+] = [{}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^2\mathfrak{A}_{\mathfrak{M}}^+]$ and ${}^1\mathfrak{A}_1 = {}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 = {}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} = {}^2\mathfrak{A}_{\mathfrak{M}}$.

(iii): (P-order)
$\psi_1 \subseteq_P \psi_2 \Leftrightarrow [{}^1\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+] \subseteq [{}^2\mathfrak{A}_1^-, {}^2\mathfrak{A}_1^+]$, $[{}^1\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+] \subseteq [{}^2\mathfrak{A}_2^-, {}^2\mathfrak{A}_2^+]$,..., $[{}^1\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+] \subseteq [{}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^2\mathfrak{A}_{\mathfrak{M}}^+]$ and ${}^1\mathfrak{A}_1 =\leq {}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 \leq {}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} \leq {}^2\mathfrak{A}_{\mathfrak{M}}$.

(iv): (R-order)
$\psi_1 \subseteq_R \psi_2 \Leftrightarrow [{}^1\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+] \subseteq [{}^2\mathfrak{A}_1^-, {}^2\mathfrak{A}_1^+]$, $[{}^1\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+] \subseteq [{}^2\mathfrak{A}_2^-, {}^2\mathfrak{A}_2^+]$,..., $[{}^1\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+] \subseteq [{}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^2\mathfrak{A}_{\mathfrak{M}}^+]$ and ${}^1\mathfrak{A}_1 =\geq {}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 \geq {}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} \geq {}^2\mathfrak{A}_{\mathfrak{M}}$.

(v): (P-union)
$\bigcup_P \psi_\wp = \big(\langle \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_1^-, {}^\wp\mathfrak{A}_1^+], \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_2^-, {}^\wp\mathfrak{A}_2^+], ..., \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_{\mathfrak{M}}^-, {}^\wp\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle \sup\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_1, \sup\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_2, ..., \sup\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$.

(vi): (P-intersection)
$\bigcap_P \psi_\wp = \big(\langle \inf\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_1^-, {}^\wp\mathfrak{A}_1^+], \inf\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_2^-, {}^\wp\mathfrak{A}_2^+], ..., \inf\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_{\mathfrak{M}}^-, {}^\wp\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_1, \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_2, ..., \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$.

(vii): (R-union)
$\bigcup_R \psi_\wp = \big(\langle \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_1^-, {}^\wp\mathfrak{A}_1^+], \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_2^-, {}^\wp\mathfrak{A}_2^+], ..., \sup\limits_{\wp\in\Delta}[{}^\wp\mathfrak{A}_{\mathfrak{M}}^-, {}^\wp\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_1, \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_2, ..., \inf\limits_{\wp\in\Delta} {}^\wp\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$.

(viii): (R-intersection)

Table 3
Cubic 3-polar fuzzy numbers

$\mathcal{Q}$	$C3PFNs$
ψ_1	$\big(\langle[0.25, 0.57], [0.51, 0.78], [0.31, 0.45]\rangle, \langle 0.33, 0.45, 0.38\rangle\big)$
ψ_2	$\big(\langle[0.51, 0.77], [0.61, 0.98], [0.11, 0.43]\rangle, \langle 0.63, 0.88, 0.22\rangle\big)$
ψ_3	$\big(\langle[0.41, 0.77], [0.61, 0.83], [0.33, 0.45]\rangle, \langle 0.66, 0.71, 0.41\rangle\big)$
ψ_4	$\big(\langle[0.11, 0.45], [0.81, 0.91], [0.28, 0.45]\rangle, \langle 0.35, 0.89, 0.31\rangle\big)$

$\bigcap_R \psi_\wp = \big(\langle \inf\limits_{\wp\in\Delta} [{}^{\wp}\mathfrak{A}_1^-, {}^{\wp}\mathfrak{A}_1^+], \inf\limits_{\wp\in\Delta} [{}^{\wp}\mathfrak{A}_2^-, {}^{\wp}\mathfrak{A}_2^+], ..., \inf\limits_{\wp\in\Delta} [{}^{\wp}\mathfrak{A}_{\mathfrak{M}}^-, {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle \sup\limits_{\wp\in\Delta} {}^{\wp}\mathfrak{A}_1, \sup\limits_{\wp\in\Delta} {}^{\wp}\mathfrak{A}_2, ..., \sup\limits_{\wp\in\Delta} {}^{\wp}\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$.

Definition 2.9. In the process of mathematical modeling for multi-attribute group decision-making problems via CMPFNs, it is necessary to rank these numbers for the appropriate decision. For this we have to define the score function corresponding to CMPFN $\psi = \big(\langle[\mathfrak{A}_1^-, \mathfrak{A}_1^+], [\mathfrak{A}_2^-, \mathfrak{A}_2^+], ..., [\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_{\mathfrak{M}}\rangle\big)$ will be defined as:

$$\mathcal{L}(\psi) = \frac{1}{2\mathfrak{M}}\left(\sum_{\wp=1}^{\mathfrak{M}}(\mathfrak{A}_\wp^- + \mathfrak{A}_\wp^+ + \mathfrak{A}_\wp) - \mathfrak{M}\right)$$

where $\mathcal{V}_\wp = [\mathfrak{A}_\wp^-, \mathfrak{A}_\wp^+] \in \mathcal{V}_{\mathfrak{M}}$ and $\mathfrak{A}_\wp \in \mathfrak{M}$.

If $\sum\limits_{\wp=1}^{\mathfrak{M}} (\mathfrak{A}_\wp^- + \mathfrak{A}_\wp^+ + \mathfrak{A}_\wp) = \mathfrak{M}$ then $\mathcal{L}(\psi) = 0$,

If $\sum\limits_{\wp=1}^{\mathfrak{M}} (\mathfrak{A}_\wp^- + \mathfrak{A}_\wp^+ + \mathfrak{A}_\wp) = 3\mathfrak{M}$ then $\mathcal{L}(\psi) = 1$.

Definition 2.10. Let ψ_1 and ψ_2 be two CMPFNs, then we can define an order relation between these numbers by using score function as follows:
(a): $\mathcal{L}(\psi_1) < \mathcal{L}(\psi_2) \Rightarrow \psi_1 \prec \psi_2$,
(b): $\mathcal{L}(\psi_1) > \mathcal{L}(\psi_2) \Rightarrow \psi_1 \succ \psi_2$,
(c): If $\mathcal{L}(\psi_1) = \mathcal{L}(\psi_2)$ then,
(i): $\mathcal{H}(\psi_1) < \mathcal{H}(\psi_2) \Rightarrow \psi_1 \prec \psi_2$,
(ii): $\mathcal{H}(\psi_1) > \mathcal{H}(\psi_2) \Rightarrow \psi_1 \succ \psi_2$.
Where $\mathcal{H}(\psi)$ is the accuracy function given as;

$$\mathcal{H}(\psi) = \left[\frac{1}{\mathfrak{M}}\sum_{\wp=1}^{\mathfrak{M}}(\mathfrak{A}_\wp^- + \mathfrak{A}_\wp^+ + \mathfrak{A}_\wp - \mathcal{L}(\psi))^2\right]^{1/2}$$

Example 2.11. Consider two C4PFNs ψ_1 and ψ_2 given in tabular form as Table 2. $\mathcal{L}(\psi_1) = 0.215$ and $\mathcal{L}(\psi_2) = 0.3125$. This shows that $\mathcal{L}(\psi_1) < \mathcal{L}(\psi_2)$ so $\psi_2 \succ \psi_1$, means that ψ_2 is the best alternative and can be seen graphically in Fig. 2.

Table 4
Cubic 2-polar fuzzy numbers

$\mathcal{Q}$	$C2PFNs$
ψ_1	$\big(\langle[0.51, 0.87], [0.63, 0.91]\rangle, \langle 0.75, 0.85\rangle\big)$
ψ_2	$\big(\langle[0.37, 0.62], [0.71, 0.89]\rangle, \langle 0.53, 0.79\rangle\big)$
ψ_3	$\big(\langle[0.89, 0.99], [0.79, 0.98]\rangle, \langle 0.88, 0.97\rangle\big)$

Table 5
Ordered Cubic 2-polar fuzzy numbers

$\mathcal{Q}$	Ordered C2PFNs
$\psi_{\wr(1)}$	$\big(\langle[0.89, 0.99], [0.79, 0.98]\rangle, \langle 0.88, 0.97\rangle\big)$
$\psi_{\wr(2)}$	$\big(\langle[0.51, 0.87], [0.63, 0.91]\rangle, \langle 0.75, 0.85\rangle\big)$
$\psi_{\wr(3)}$	$\big(\langle[0.37, 0.62], [0.71, 0.89]\rangle, \langle 0.53, 0.79\rangle\big)$

Table 6
Cubic 3-polar fuzzy numbers

$\mathcal{Q}$	$C3PFNs$
ψ_1	$\big(\langle[0.35, 0.56], [0.78, 0.93], [0.51, 0.97]\rangle, \langle 0.48, 0.89, 0.87\rangle\big)$
ψ_2	$\big(\langle[0.21, 0.37], [0.41, 0.76], [0.68, 0.87]\rangle, \langle 0.29, 0.67, 0.73\rangle\big)$
ψ_3	$\big(\langle[0.44, 0.78], [0.87, 0.99], [0.79, 0.89]\rangle, \langle 0.69, 0.97, 0.87\rangle\big)$

Definition 2.12. Let $\psi = \big(\langle[\mathfrak{A}_1^-, \mathfrak{A}_1^+], [\mathfrak{A}_2^-, \mathfrak{A}_2^+], ..., [\mathfrak{A}_{\mathfrak{M}}^-, \mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle\mathfrak{A}_1, \mathfrak{A}_2, ..., \mathfrak{A}_{\mathfrak{M}}\rangle\big)$ be an arbitrary CMPFN and $\psi_\wp = \big(\langle[{}^{\wp}\mathfrak{A}_1^-, {}^{\wp}\mathfrak{A}_1^+], [{}^{\wp}\mathfrak{A}_2^-, {}^{\wp}\mathfrak{A}_2^+], ..., [{}^{\wp}\mathfrak{A}_{\mathfrak{M}}^-, {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^{\wp}\mathfrak{A}_1, {}^{\wp}\mathfrak{A}_2, ..., {}^{\wp}\mathfrak{A}_{\mathfrak{M}}\rangle : \wp \in \Delta\big)$ be an assembling of CMPFNs then we can define some operations laws on CMPFNs with an arbitrary real number $\eta > 0$ given as follows:

(a) For R-order:

(i): $\psi_1 \oplus_R \psi_2 = \big([\langle {}^1\mathfrak{A}_1^- + {}^2\mathfrak{A}_1^- - {}^1\mathfrak{A}_1^- {}^2\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+ + {}^2\mathfrak{A}_1^+ - {}^1\mathfrak{A}_1^+ {}^2\mathfrak{A}_1^+], [{}^1\mathfrak{A}_2^- + {}^2\mathfrak{A}_2^- - {}^1\mathfrak{A}_2^- {}^2\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+ + {}^2\mathfrak{A}_2^+ - {}^1\mathfrak{A}_2^+ {}^2\mathfrak{A}_2^+], ..., [{}^1\mathfrak{A}_{\mathfrak{M}}^- + {}^2\mathfrak{A}_{\mathfrak{M}}^- - {}^1\mathfrak{A}_{\mathfrak{M}}^- {}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+ + {}^2\mathfrak{A}_{\mathfrak{M}}^+ - {}^1\mathfrak{A}_{\mathfrak{M}}^+ {}^2\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^1\mathfrak{A}_1 {}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 {}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} {}^2\mathfrak{A}_{\mathfrak{M}}\rangle\big)$.

(ii): $\psi_1 \otimes_R \psi_2 = \big(\langle[{}^1\mathfrak{A}_1^- {}^2\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+ {}^2\mathfrak{A}_1^+], [{}^1\mathfrak{A}_2^- {}^2\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+ {}^2\mathfrak{A}_2^+], ..., [{}^1\mathfrak{A}_{\mathfrak{M}}^- {}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+ {}^2\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^1\mathfrak{A}_1 + {}^2\mathfrak{A}_1 - {}^1\mathfrak{A}_1 {}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 + {}^2\mathfrak{A}_2 - {}^1\mathfrak{A}_2 {}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} + {}^2\mathfrak{A}_{\mathfrak{M}} - {}^1\mathfrak{A}_{\mathfrak{M}} {}^2\mathfrak{A}_{\mathfrak{M}}\rangle\big)$

(iii): $\eta\psi = \big(\langle[1 - (1 - \mathfrak{A}_1^-)^\eta, 1 - (1 - \mathfrak{A}_1^+)^\eta], [1 - (1 - \mathfrak{A}_2^-)^\eta, 1 - (1 - \mathfrak{A}_2^+)^\eta], ..., [1 - (1 - \mathfrak{A}_{\mathfrak{M}}^-)^\eta, 1 - (1 - \mathfrak{A}_{\mathfrak{M}}^+)^\eta]\rangle, \langle \mathfrak{A}_1^\eta, \mathfrak{A}_2^\eta, ..., \mathfrak{A}_{\mathfrak{M}}^\eta\rangle\big) : \eta > 0$.

(iv): $\psi^\eta = \big(\langle[(\mathfrak{A}_1^-)^\eta, (\mathfrak{A}_1^+)^\eta], [(\mathfrak{A}_2^-)^\eta, (\mathfrak{A}_2^+)^\eta], ..., [(\mathfrak{A}_{\mathfrak{M}}^-)^\eta, (\mathfrak{A}_{\mathfrak{M}}^+)^\eta]\rangle, \langle 1 - (1 - \mathfrak{A}_1)^\eta, 1 - (1 - \mathfrak{A}_2)^\eta, ..., 1 - (1 - \mathfrak{A}_{\mathfrak{M}})^\eta\rangle\big); \eta > 0$

(b) For P-order:

(i): $\psi_1 \oplus_P \psi_2 = \big([\langle {}^1\mathfrak{A}_1^- + {}^2\mathfrak{A}_1^- - {}^1\mathfrak{A}_1^- {}^2\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+ + {}^2\mathfrak{A}_1^+ - {}^1\mathfrak{A}_1^+ {}^2\mathfrak{A}_1^+], [{}^1\mathfrak{A}_2^- + {}^2\mathfrak{A}_2^- - {}^1\mathfrak{A}_2^- {}^2\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+ +$

Table 7
Cubic 3-polar fuzzy numbers

$\mathcal{Q}$	$\dot{\psi}_{\wp} = 3\zeta_{\wp}\psi_{\wp}$
$\dot{\psi}_1$	$\big(\langle [0.5394, 0.7718], [0.9344, 0.9916], [0.7230, 0.9981] \rangle, \langle 0.2668, 0.8107, 0.7782 \rangle\big)$
$\dot{\psi}_2$	$\big(\langle [0.1318, 0.2421], [0.2713, 0.5752], [0.4952, 0.7059] \rangle, \langle 0.4758, 0.7864, 0.8279 \rangle\big)$
$\dot{\psi}_3$	$\big(\langle [0.2938, 0.5968], [0.7059, 0.9369], [0.6079, 0.7340] \rangle, \langle 0.8004, 0.9818, 0.9198 \rangle\big)$

Table 8
Ordered Cubic 3-polar fuzzy numbers

$\mathcal{Q}$	Ordered $\dot{\psi}_{\wr(\wp)}$
$\dot{\psi}_{\wr(1)}$	$\big(\langle [0.5394, 0.7718], [0.9344, 0.9916], [0.7230, 0.9981] \rangle, \langle 0.2668, 0.8107, 0.7782 \rangle\big)$
$\dot{\psi}_{\wr(2)}$	$\big(\langle [0.2938, 0.5968], [0.7059, 0.9369], [0.6079, 0.7340] \rangle, \langle 0.8004, 0.9818, 0.9198 \rangle\big)$
$\dot{\psi}_{\wr(3)}$	$\big(\langle [0.1318, 0.2421], [0.2713, 0.5752], [0.4952, 0.7059] \rangle, \langle 0.4758, 0.7864, 0.8279 \rangle\big)$

${}^2\mathfrak{A}_2^+ - {}^1\mathfrak{A}_2^{+2}\mathfrak{A}_2^+], ..., \quad [{}^1\mathfrak{A}_{\mathfrak{M}}^- + {}^2\mathfrak{A}_{\mathfrak{M}}^- - {}^1\mathfrak{A}_{\mathfrak{M}}^{-2}\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+ + {}^2\mathfrak{A}_{\mathfrak{M}}^+ - {}^1\mathfrak{A}_{\mathfrak{M}}^{+2}\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \quad \langle {}^1\mathfrak{A}_1 + {}^2\mathfrak{A}_1 - {}^1\mathfrak{A}_1{}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2 + {}^2\mathfrak{A}_2 - {}^1\mathfrak{A}_2{}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}} + {}^2\mathfrak{A}_{\mathfrak{M}} - {}^1\mathfrak{A}_{\mathfrak{M}}{}^2\mathfrak{A}_{\mathfrak{M}}\rangle\big)$

(ii): $\psi_1 \otimes_P \psi_2 = \big(\langle [{}^1\mathfrak{A}_1^{-2}\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^{+2}\mathfrak{A}_1^+], [{}^1\mathfrak{A}_2^{-2}\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^{+2}\mathfrak{A}_2^+], ..., [{}^1\mathfrak{A}_{\mathfrak{M}}^{-2}\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^{+2}\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^1\mathfrak{A}_1{}^2\mathfrak{A}_1, {}^1\mathfrak{A}_2{}^2\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}}{}^2\mathfrak{A}_{\mathfrak{M}}\rangle\big)$

(iii): $\eta\psi = \big(\langle [1 - (1 - \mathfrak{A}_1^-)^\eta, 1 - (1 - \mathfrak{A}_1^+)^\eta], [1 - (1 - \mathfrak{A}_2^-)^\eta, 1 - (1 - \mathfrak{A}_2^+)^\eta], ..., [1 - (1 - \mathfrak{A}_{\mathfrak{M}}^-)^\eta, 1 - (1 - \mathfrak{A}_{\mathfrak{M}}^+)^\eta]\rangle, \langle 1 - (1 - \mathfrak{A}_1)^\eta, 1 - (1 - \mathfrak{A}_2)^\eta, ..., 1 - (1 - \mathfrak{A}_{\mathfrak{M}})^\eta\rangle\big) : \eta > 0.$

(iv): $\psi^\eta = \big(\langle [(\mathfrak{A}_1^-)^\eta, (\mathfrak{A}_1^+)^\eta], [(\mathfrak{A}_2^-)^\eta, (\mathfrak{A}_2^+)^\eta], ..., [(\mathfrak{A}_{\mathfrak{M}}^-)^\eta, (\mathfrak{A}_{\mathfrak{M}}^+)^\eta]\rangle, \langle \mathfrak{A}_1^\eta, \mathfrak{A}_2^\eta, ..., \mathfrak{A}_{\mathfrak{M}}^\eta\rangle\big); \eta > 0$

Theorem 2.13. *For two CPMFNs ψ_1 and ψ_2 we can show that $\psi_1 \oplus_R \psi_2, \psi_1 \otimes_R \psi_2, \psi_1 \oplus_P \psi_2, \psi_1 \otimes_P \psi_2, \eta\psi$(For P-order and R-order) and ψ^η (For P-order and R-order) are also CMPFNs.*

Theorem 2.14. *Let ψ_1, ψ_2 and ψ be three CMPFNs and $\eta_1, \eta_2, \eta_3 > 0$ are real numbers then some results holds:*

(i): $\eta(\psi_1 \oplus_R \psi_2) = \eta\psi_1 \oplus_R \eta\psi_2$,
(ii): $(\psi_1^\eta \otimes_R \psi_2^\eta) = (\psi_1 \otimes_R \psi_2)^\eta$,
(iiii): $\eta(\psi_1 \oplus_P \psi_2) = \eta\psi_1 \oplus_P \eta\psi_2$,
(iv): $(\psi_1^\eta \otimes_P \psi_2^\eta) = (\psi_1 \otimes_P \psi_2)^\eta$,
(v): $(\eta_1\eta_2)(\psi) = \eta_1(\eta_2\psi)$ *(For P-order and R-order),*
(vi): $\psi^{\eta_1\eta_2} = (\psi^{\eta_2})^{\eta_1}$ *(For P-order and R-order).*

3. Aggregation operators

Aggregation means the creation of a numeral of things into a cluster or a bunch of things that have come or been taken together. They are used to aggregate different values for the given input data. We utilize them in decision-making problems and for the ranking of alternatives. In this section, we use CMPFS to establish novel averaging aggregation operators for R-order and P-order respectively. Figure 3 shows that CMPFA operators are superior to other models, because all the listed models become special cases of CMPFA operators with some suitable conditions. All the defined operators holds the properties of idempotency, boundedness and commutativity.

3.1. CMPFA operators based on operations of R-Order and P-Order

Definition 3.1. Let $\psi_{\wp}(\wp = 1, 2, 3, ..., \mathfrak{N})$ ba an assembling of CMPFNs and $\text{CMPFWA}_R : \mho^{\mathfrak{N}} \to \mho$, if $\text{CMPFWA}_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \zeta_1\psi_1 \oplus_R \zeta_2\psi_2 \oplus_R ... \oplus_R \zeta_{\mathfrak{N}}\psi_{\mathfrak{N}}$ where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the weight vector of $\psi_{\wp}$ such that $\zeta_{\wp} > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_{\wp} = 1$ and $\mho$ is the collection of CMPFNs, then CMPFWA_R is said to be cubic m-polar fuzzy weighted averaging operator.

Theorem 3.2. *Let $\psi_{\wp}(\wp = 1, 2, 3, ..., \mathfrak{N})$ be an assembling of CMPFNs, then CMPFWA operator can also be represented as:*

$$\mathit{CMPFWA}_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \Bigg(\Bigg\langle \Big[1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_1^-)^{\zeta_{\wp}}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_1^+)^{\zeta_{\wp}}\Big], \Big[1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_2^-)^{\zeta_{\wp}}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_2^+)^{\zeta_{\wp}}\Big], ..., \Big[1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_{\wp}}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_{\wp}}\Big] \Bigg\rangle, \Bigg\langle \prod_{\wp=1}^{\mathfrak{N}} ({}^{\wp}\mathfrak{A}_1)^{\zeta_{\wp}}, \prod_{\wp=1}^{\mathfrak{N}} ({}^{\wp}\mathfrak{A}_2)^{\zeta_{\wp}}, ..., \prod_{\wp=1}^{\mathfrak{N}} ({}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\zeta_{\wp}} \Bigg\rangle\Bigg) \quad (A)$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ *is the weight vector of* $\psi_\wp$ *such that* $\zeta_\wp > 0$ *with* $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$.

Proof. We have to show that the equation (A) holds for CMPFWA operator. For this we will follow mathematical induction. As we know that

$\psi_\wp = (\langle[{}^\wp\mathfrak{A}_1^-, {}^\wp\mathfrak{A}_1^+], [{}^\wp\mathfrak{A}_2^-, {}^\wp\mathfrak{A}_2^+], ..., [{}^\wp\mathfrak{A}_{\mathfrak{M}}^-, {}^\wp\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^\wp\mathfrak{A}_1, {}^\wp\mathfrak{A}_2, ..., {}^\wp\mathfrak{A}_{\mathfrak{M}}\rangle)$, $\wp = 1, 2, 3, ..., \mathfrak{N}$ is an assembling of CMPFNs then we can follow the given steps of mathematical induction,

Step 1: For $\mathfrak{N} = 2$, $\psi_1 = \big(\langle[{}^1\mathfrak{A}_1^-, {}^1\mathfrak{A}_1^+], [{}^1\mathfrak{A}_2^-, {}^1\mathfrak{A}_2^+], ..., [{}^1\mathfrak{A}_{\mathfrak{M}}^-, {}^1\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^1\mathfrak{A}_1, {}^1\mathfrak{A}_2, ..., {}^1\mathfrak{A}_{\mathfrak{M}}\rangle\big)$ and
$\psi_2 = \big(\langle[{}^2\mathfrak{A}_1^-, {}^2\mathfrak{A}_1^+], [{}^2\mathfrak{A}_2^-, {}^2\mathfrak{A}_2^+], ..., [{}^2\mathfrak{A}_{\mathfrak{M}}^-, {}^2\mathfrak{A}_{\mathfrak{M}}^+]\rangle, \langle {}^2\mathfrak{A}_1, {}^2\mathfrak{A}_2, ..., {}^2\mathfrak{A}_{\mathfrak{M}}\rangle\big)$ and for ζ_1 and ζ_2 we can write that

$$\zeta_1\psi_1 = \Bigg(\Bigg\langle [1-(1-{}^1\mathfrak{A}_1^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_1^+)^{\zeta_1}], [1-(1-{}^1\mathfrak{A}_2^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_2^+)^{\zeta_1}], ..., [1-(1-{}^1\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_1}] \Bigg\rangle, \Big\langle ({}^1\mathfrak{A}_1)^{\zeta_1}, ({}^1\mathfrak{A}_2)^{\zeta_1}, ..., ({}^1\mathfrak{A}_{\mathfrak{M}})^{\zeta_1} \Big\rangle\Bigg)$$

and

$$\zeta_2\psi_2 = \Bigg(\Bigg\langle [1-(1-{}^2\mathfrak{A}_1^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_1^+)^{\zeta_2}], [1-(1-{}^2\mathfrak{A}_2^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_2^+)^{\zeta_2}], ..., [1-(1-{}^2\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_2}] \Bigg\rangle, \Big\langle ({}^2\mathfrak{A}_1)^{\zeta_2}, ({}^2\mathfrak{A}_2)^{\zeta_2}, ..., ({}^2\mathfrak{A}_{\mathfrak{M}})^{\zeta_2} \Big\rangle\Bigg)$$

Now $\text{CMPFWA}_R(\psi_1, \psi_2) = \zeta_1\psi_1 \oplus_R \zeta_2\psi_2$

$$\text{CMPFWA}_R(\psi_1, \psi_2) = \Bigg(\Bigg\langle [1-(1-{}^1\mathfrak{A}_1^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_1^+)^{\zeta_1}], [1-(1-{}^1\mathfrak{A}_2^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_2^+)^{\zeta_1}], ..., [1-(1-{}^1\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_1}, 1-(1-{}^1\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_1}] \Bigg\rangle, \Big\langle ({}^1\mathfrak{A}_1)^{\zeta_1}, ({}^1\mathfrak{A}_2)^{\zeta_1}, ..., ({}^1\mathfrak{A}_{\mathfrak{M}})^{\zeta_1} \Big\rangle\Bigg) \oplus_R \Bigg(\Bigg\langle [1-(1-{}^2\mathfrak{A}_1^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_1^+)^{\zeta_2}], [1-(1-{}^2\mathfrak{A}_2^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_2^+)^{\zeta_2}], ..., [1-(1-{}^2\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_2}, 1-(1-{}^2\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_2}] \Bigg\rangle, \Big\langle ({}^2\mathfrak{A}_1)^{\zeta_2}, ({}^2\mathfrak{A}_2)^{\zeta_2}, ..., ({}^2\mathfrak{A}_{\mathfrak{M}})^{\zeta_2} \Big\rangle\Bigg)$$

$$\text{CMPFWA}_R(\psi_1, \psi_2) = \Bigg(\Bigg\langle [1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_1^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_1^+)^{\zeta_\wp}], [1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_2^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_2^+)^{\zeta_\wp}], ..., [1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{2}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle \prod_{\wp=1}^{2}({}^\wp\mathfrak{A}_1)^{\zeta_\wp}, \prod_{\wp=1}^{2}({}^\wp\mathfrak{A}_2)^{\zeta_\wp}, ..., \prod_{\wp=1}^{2}({}^\wp\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp} \Bigg\rangle\Bigg)$$

This shows that result (A) holds for $\mathfrak{N} = 2$.

Step 2: Let (A) holds for $\mathfrak{N} = \mathcal{Z}$.

Step 3: Now we will show that it holds for $\mathfrak{N} = \mathcal{Z} + 1$.

Consider $\text{CMPFWA}_R(\psi_1, \psi_2, ..., \psi_{\mathcal{Z}+1}) = \left(\bigoplus_{\wp=1}^{\mathcal{Z}} \zeta_\wp\psi_\wp\right) \oplus_R (\zeta_{\mathcal{Z}+1}\psi_{\mathcal{Z}+1})$

$$\text{CMPFWA}_R(\psi_1, \psi_2, ..., \psi_{\mathcal{Z}+1}) = \Bigg(\Bigg\langle [1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_1^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_1^+)^{\zeta_\wp}], [1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_2^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_2^+)^{\zeta_\wp}], ..., [1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle \prod_{\wp=1}^{\mathcal{Z}}({}^\wp\mathfrak{A}_1)^{\zeta_\wp}, \prod_{\wp=1}^{\mathcal{Z}}({}^\wp\mathfrak{A}_2)^{\zeta_\wp}, ..., \prod_{\wp=1}^{\mathcal{Z}}({}^\wp\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp} \Bigg\rangle\Bigg) \oplus_R \Bigg(\Bigg\langle [1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_1^-)^{\zeta_{\mathcal{Z}+1}}, 1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_1^+)^{\zeta_{\mathcal{Z}+1}}], [1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_2^-)^{\zeta_{\mathcal{Z}+1}}, 1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_2^+)^{\zeta_{\mathcal{Z}+1}}], ..., [1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_{\mathcal{Z}+1}}, 1-(1-{}^{\mathcal{Z}+1}\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_{\mathcal{Z}+1}}] \Bigg\rangle, \Big\langle ({}^{\mathcal{Z}+1}\mathfrak{A}_1)^{\zeta_{\mathcal{Z}+1}}, ({}^{\mathcal{Z}+1}\mathfrak{A}_2)^{\zeta_{\mathcal{Z}+1}}, ..., ({}^{\mathcal{Z}+1}\mathfrak{A}_{\mathfrak{M}})^{\zeta_{\mathcal{Z}+1}} \Big\rangle\Bigg)$$

$$\text{CMPFWA}_R(\psi_1, \psi_2, ..., \psi_{\mathcal{Z}+1}) = \Bigg(\Bigg\langle [1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_1^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_1^+)^{\zeta_\wp}], [1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_2^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_2^+)^{\zeta_\wp}], ..., [1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_\wp}, 1-\prod_{\wp=1}^{\mathcal{Z}+1}(1-{}^\wp\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle \prod_{\wp=1}^{\mathcal{Z}+1}({}^\wp\mathfrak{A}_1)^{\zeta_\wp}, \prod_{\wp=1}^{\mathcal{Z}+1}({}^\wp\mathfrak{A}_2)^{\zeta_\wp}, ..., \prod_{\wp=1}^{\mathcal{Z}+1}({}^\wp\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp} \Bigg\rangle\Bigg)$$

Thus equation (A) holds for $\mathfrak{N} = \mathcal{Z} + 1$. Hence by mathematical induction (A) is true for all values of $\mathfrak{N}$.

Remark. The aggregated value by using CMPFWA operator is again CMPFN. □

Example 3.3. Consider C3PFNs ψ_1, ψ_2, ψ_3 and ψ_4 with the $\zeta = (0.3, 0.4, 0.2, 0.1)^T$ as $\sum_{\wp=1}^{4} \zeta_\wp = 1$. Table 3 represent the C3PFNs ψ_1, ψ_2, ψ_3 and ψ_4. Then by using (A) for $\mathfrak{M} = 3$ we obtain $CMPFWA_R(\psi_1, \psi_2, \psi_3, \psi_4) = \big(\langle[0.3868, 0.6973], [0.6115, 0.9268], [0.2375, 0.4422]\rangle, \langle 0.4937, 0.6899, 0.3037\rangle\big)$.

Definition 3.4. Let $\psi_\wp(\wp = 1, 2, 3, ..., \mathfrak{N})$ ba an assembling of CMPFNs and $CMPFOWA_R : \mho^{\mathfrak{N}} \to \mho$, if $CMPFOWA_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \zeta_1\psi_{\wr(1)} \oplus_R \zeta_2\psi_{\wr(2)} \oplus_R ... \oplus_R \zeta_{\mathfrak{N}}\psi_{\wr(\mathfrak{N})}$ where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the weight vector of $\psi_\wp$ such that $\zeta_\wp > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$ and $\mho$ is the collection of CMPFNs. Here $(\wr(1), \wr(2), ..., \wr(\mathfrak{N}))$ is a permutation of $(1, 2, ..., \mathfrak{N})$ such that $\psi_{\wr(\wp-1)} \geq \psi_{\wr(\wp)}, \forall \wp = 2, 3, ..., \mathfrak{N}$, then CMPFOWA is called cubic m-polar fuzzy ordered weighted averaging operator.

Theorem 3.5. *Let $\psi_\wp(\wp = 1, 2, 3, ..., \mathfrak{N})$ be an assembling of CMPFNs, then CMPFOWA operator can also be represented as:*

$$CMPFOWA_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \Bigg(\Bigg\langle[1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^-_{\wr(1)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^+_{\wr(1)})^{\zeta_\wp}], [1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^-_{\wr(2)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^+_{\wr(2)})^{\zeta_\wp}], ..., \Bigg[1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^-_{\wr(\mathfrak{M})})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\mathfrak{A}^+_{\wr(\mathfrak{M})})^{\zeta_\wp}\Bigg]\Bigg\rangle, \Bigg\langle\prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_{\wr(1)})^{\zeta_\wp}, \prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_{\wr(2)})^{\zeta_\wp}, ..., \prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\mathfrak{A}_{\wr(\mathfrak{M})})^{\zeta_\wp}\Bigg\rangle\Bigg) \qquad (B)$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the weight vector of $\psi_\wp$ such that $\zeta_\wp > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$.

Example 3.6. Consider C2PFNs ψ_1, ψ_2 and ψ_3 with $\zeta = (0.3, 0.2, 0.5)^T$ as $\sum_{\wp=1}^{3} \zeta_\wp = 1$. In tabular form ψ_1, ψ_2 and ψ_3 can be written as Table 4. The score functions of C2PFNs is calculated by using Definition 2.9 and given as: $\pounds(\psi_1) = 0.63$, $\pounds(\psi_2) = 0.4775$, $\pounds(\psi_3) = 0.875$
This shows that $\psi_3 \succ \psi_1 \succ \psi_2$. So ordered C2PFNs are given in Table 5.
Now using equation (B) we can write that $CMPFOWA_R(\psi_1, \psi_2, \psi_3) = \big(\langle[0.6452, 0.8971], [0.7237, 0.9367]\rangle, \langle 0.6613, 0.8525\rangle\big)$.

Definition 3.7. Let $\psi_\wp(\wp = 1, 2, 3, ..., \mathfrak{N})$ be an assembling of CMPFNs and $CMPFHA_R : \mho^{\mathfrak{N}} \to \mho$, if $CMPFHA_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \theta_1\dot{\psi}_{\wr(1)} \oplus_R \theta_2\dot{\psi}_{\wr(2)} \oplus_R ... \oplus_R \theta_{\mathfrak{N}}\dot{\psi}_{\wr(\mathfrak{N})}$ where $\theta = (\theta_1, \theta_2, ..., \theta_{\mathfrak{N}})^T$ is the weight vector of $\psi_\wp$ such that $\theta_\wp > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \theta_\wp = 1$ and $\mho$ is the collection of CMPFNs. Here $(\wr(1), \wr(2), ..., \wr(\mathfrak{N}))$ is a permutation of $(1, 2, ..., \mathfrak{N})$ such that $\dot{\psi}_{\wr(\wp-1)} \geq \dot{\psi}_{\wr(\wp)}, \forall \wp = 2, 3, ..., \mathfrak{N}$ and $\dot{\psi}_{\wr(\wp)}$ is the $\wp th$ weighted CMPFN given by $\dot{\psi}_\wp = \mathfrak{N}\zeta_\wp\psi_\wp$, where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the associated weight vector with $\psi_\wp$ such that $\zeta_\wp > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$, then CMPFHA is called cubic m-polar fuzzy hybrid averaging operator.

Theorem 3.8. *Let $\psi_\wp(\wp = 1, 2, 3, ..., \mathfrak{N})$ be an assembling of CMPFNs, then CMPFHA operator can also be represented as:*

$$CMPFHA_R(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \Bigg(\Bigg\langle[1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^-_{\wr(1)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^+_{\wr(1)})^{\zeta_\wp}], [1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^-_{\wr(2)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^+_{\wr(2)})^{\zeta_\wp}], ..., [1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^-_{\wr(\mathfrak{M})})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}}(1 - {}^{\wp}\dot{\mathfrak{A}}^+_{\wr(\mathfrak{M})})^{\zeta_\wp}]\Bigg\rangle, \Bigg\langle\prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\dot{\mathfrak{A}}_{\wr(1)})^{\zeta_\wp}, \prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\dot{\mathfrak{A}}_{\wr(2)})^{\zeta_\wp}, ..., \prod_{\wp=1}^{\mathfrak{N}}({}^{\wp}\dot{\mathfrak{A}}_{\wr(\mathfrak{M})})^{\zeta_\wp}\Bigg\rangle\Bigg) \qquad (C)$$

where $\zeta = (\zeta_1, \zeta_2, ..., \zeta_{\mathfrak{N}})^T$ is the weight vector of $\psi_\wp$ such that $\zeta_\wp > 0$ with $\sum_{\wp=1}^{\mathfrak{N}} \zeta_\wp = 1$.

Example 3.9. Consider three C3PFNs ψ_1, ψ_2 and ψ_3 with $\zeta = (0.6, 0.2, 0.2)^T$ is the weight vector then Table 6 represent these C3PFNs.

Now calculating $\dot{\psi}_{\wp} = 3\zeta_{\wp}\psi_{\wp}$ we get C3PFNs written in Table 7.

Next we calculate the score of $\dot{\psi}_1$, $\dot{\psi}_2$ and $\dot{\psi}_3$ by using Definition 2.9 given as

$\pounds(\dot{\psi}_1) = 0.6356$, $\pounds(\dot{\psi}_2) = 0.2519$, $\pounds(\dot{\psi}_3) = 0.5062$ Which clearly shows that $\dot{\psi}_1 \succ \dot{\psi}_3 \succ \dot{\psi}_2$. It is easy to see the order of C3PFNs in Table 8.

The considered C3PFNs are attributes of the reference set and present some specific object. For the decision, we set three experts of the related problem. These experts choose the values of associated weight vector according to the uncertain linguistic variables. For example if we set "Low(L) $\rightarrow 0 \leq L \leq 0.3$", "Medium(M) $\rightarrow 0.3 < M \leq 0.6$" and "High(H) $\rightarrow 0.6 < H \leq 1$". Then according to these linguistic terms we can see that the weight vector $\theta = (0.5, 0.1, 0.4)^T$ represents $\theta = (M, L, M)^T$.

Let $\theta = (0.5, 0.1, 0.4)^T$ be the associated weight vector according to the experts choice. By using equation (C) we can now find the aggregated value of C3PFNs as $\text{CMPFHA}_R(\psi_1, \psi_2, \psi_3) = \big(\big\langle[0.3807, 0.6096], [0.8004, 0.9507], [0.6355, 0.9767]\big\rangle, \big\langle 0.3752, 0.8162, 0.8110\big\rangle\big)$

Definition 3.10. By using the same input as above for operators of R-order we can also define operators for p-order given as

$$\text{CMPFWA}_P(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \zeta_1\psi_1 \oplus_P \zeta_2\psi_2 \oplus_P ... \oplus_P \zeta_{\mathfrak{N}}\psi_{\mathfrak{N}} = \Bigg(\Bigg\langle [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_1^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_1^+)^{\zeta_\wp}], [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_2^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_2^+)^{\zeta_\wp}], ..., [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\mathfrak{M}}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_1)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_2)^{\zeta_\wp}, ..., 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\mathfrak{M}})^{\zeta_\wp} \Bigg\rangle\Bigg) \quad \text{(D)}$$

$$\text{CMPFOWA}_P(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \zeta_1\psi_{\wr(1)} \oplus_P \zeta_2\psi_{\wr(2)} \oplus_P ... \oplus_P \zeta_{\mathfrak{N}}\psi_{\wr(\mathfrak{N})} = \Bigg(\Bigg\langle [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(1)}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(1)}^+)^{\zeta_\wp}], [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(2)}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(2)}^+)^{\zeta_\wp}], ..., [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(\mathfrak{M})}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(\mathfrak{M})}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(1)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(2)})^{\zeta_\wp}, ..., 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\mathfrak{A}_{\wr(\mathfrak{M})})^{\zeta_\wp} \Bigg\rangle\Bigg) \quad \text{(E)}$$

$$\text{CMPFHA}_P(\psi_1, \psi_2, ..., \psi_{\mathfrak{N}}) = \theta_1\dot{\psi}_{\wr(1)} \oplus_P \theta_2\dot{\psi}_{\wr(2)} \oplus_P ... \oplus_P \theta_{\mathfrak{N}}\dot{\psi}_{\wr(\mathfrak{N})} = \Bigg(\Bigg\langle [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(1)}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(1)}^+)^{\zeta_\wp}], [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(2)}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(2)}^+)^{\zeta_\wp}], ..., [1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(\mathfrak{M})}^-)^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(\mathfrak{M})}^+)^{\zeta_\wp}] \Bigg\rangle, \Bigg\langle 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(1)})^{\zeta_\wp}, 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(2)})^{\zeta_\wp}, ..., 1 - \prod_{\wp=1}^{\mathfrak{N}} (1 - {}^{\wp}\dot{\mathfrak{A}}_{\wr(\mathfrak{M})})^{\zeta_\wp} \Bigg\rangle\Bigg) \quad \text{(F)}$$

are the CMPFWA, CMPFOWA and CMPFHA operators for p-order respectively.

4. MAGDM using proposed operators

The occupation of farming output is called agribusiness. Goldberg and Davis brought out this term in 1957. It contains agrichemicals, breeding, crop production (agricultural and agreement farming), supply, farm technology, processing, and seed supply, as well as advertising and marketing sales. In that respect there are different elements that affect crop yield. There are two types of these components: **(i):** Internal factors, **(ii):** External factors.

We will see the categorization of these factors through the following flow chart diagram given in Fig. 4.

The multi-attribute group decision-making is a process in which a committee cooperatively take a decision from an assembling of different attributes of the reference set. The decision obtained by the panel is strong and authentic as compared to the decision of a single person. The MAGDM is useful and strong when the problems involve uncertainties and ambiguities. In short it has great importance in real life decision-making problems. From the presented algorithm and authorization of ranking credibility, we can conclude that the proposed approach exploits the hesitations that arise in the multi-attribute group decision-making process. This study helps an international firm for the selection of a land and suggest the following benefits: The first benefit of this research is developing the case study for MAGDM problem and the selection of suitable criteria based on a comprehensive literature review. The second benefit is not only in selecting the best attribute, but also analysis of algorithm based on proposed operators that give ranking results and helpful in diverse fields of life.

Algorithm:

Input:

Step 1: Input CMPF-data for suitable number of alternatives $\psi_{\wp'}; (\wp' = 1, 2, 3, ..., m)$ under the effect of different criteria $\mathfrak{Y}_{\wp}; (\wp = 1, 2, 3, ..., n)$. This input table represents verbally stated information into mathematical language in the form of CMPFNs. We discuss the behavior of each alternative under every individual criteria by using CMPF linguistic variables.

Step 2: Normalize the input CMPF-data:

$$\psi_{\wp'} = \begin{cases} \left(\langle[\mathfrak{A}^{+}_{\mathfrak{M}}, \mathfrak{A}^{-}_{\mathfrak{M}}]\rangle, \langle\mathfrak{A}_{\mathfrak{M}}\rangle\right); \text{for same type} \\ \left(\langle[1-\mathfrak{A}^{+}_{\mathfrak{M}}, 1-\mathfrak{A}^{-}_{\mathfrak{M}}]\rangle, \langle 1-\mathfrak{A}_{\mathfrak{M}}\rangle\right); \\ \quad \text{for different type} \end{cases}$$

It is necessary to normalize the input information before further calculations to obtain the best and precise solutions. If the type is same for all attributes, then there is no need to normalize the information. In our given application all the alternatives are of same types and then we do not normalize our input and instantly use it for our deliberations.

Step 3: According to the experts obtain associated weight vector $\zeta = (\zeta_1, \zeta_2, ..., \zeta_m)$ for further calculations.

Calculations:

Step 4: Compute the aggregated values of alternatives $\psi_{\wp'}; (\wp' = 1, 2, 3, ..., m)$ corresponding to the different criteria $\mathfrak{Y}_{\wp}; (\wp = 1, 2, 3, ..., n)$ by using C3PFWA$_R$, C3PFOWA$_R$, C3PFHA$_R$, C3PFWA$_P$, C3PFOWA$_P$ and C3PFHA$_P$ operators given in equations (A), (B), (C), (D), (E) and (F) respectively, and hence the evaluated aggregated values are given by $\mathcal{O}_{\wp'}; (\wp' = 1, 2, 3, ..., m)$.

Output:

Step 5: Using $\mathcal{O}_{\wp'}; (\wp' = 1, 2, 3, ..., m)$ for every operator separately calculate score values by using Definition 2.9.

Step 6: We rank the alternatives on the basis of score values according to the remark stated below Definition 2.9.

Table 9
Uncertain Linguistic Variables for each criteria

Linguistic variable	Numerical range
Low(L)	$0 \leq L < 0.20$
Medium Low(ML)	$0.20 \leq ML < 0.35$
Medium(M)	$0.35 \leq M < 0.55$
Medium High(MH)	$0.55 \leq MH < 0.75$
High(H)	$0.75 \leq H \leq 1$

Step 7: Choose the alternative with the maximum score calculated through the purposed method.

Numerical Example:

Consider that an international agricultural firm wants to buy a land for an agricultural project. They are proceeding to invest a large sum of money for this project and it also holds shares of different societies. To get to their country economically strong and to improve the competitive capability of their company they are interested to purchase a suitable state for this project with which they can get maximum and healthy production of the crop with the desirable investment. For the scientific decision, some experts are selected by the expert team of the company. These experts analyze only the external factors for this decision because internal factors are less influenced for the selection of land.

Let $\mathcal{Q} = \{\psi_1, \psi_2, \psi_3, \psi_4\}$ be the set of available lands through which they want to select the suitable land. There are five external factors $\mathfrak{Y}_1, \mathfrak{Y}_2, \mathfrak{Y}_3, \mathfrak{Y}_3$ and $\mathfrak{Y}_5$ which are used by experts to input CMPF-data for the given set $\mathcal{Q}$. Where,

$\mathfrak{Y}_1$ = Climate factors, $\mathfrak{Y}_2$ = Edaphic factors,
$\mathfrak{Y}_3$ = Biotic factors, $\mathfrak{Y}_4$ = Phsiographic factors,
$\mathfrak{Y}_5$ = Socio-economic factors.

There are three experts so for each $\psi \in \mathcal{Q}$ the input data consists of C3PFNs corresponding to every attribute $\mathfrak{Y}_{\wp}; (\wp = 1, 2, 3, 4, 5)$. Since the problem is a real decision-making and performed by the real decision makers. We explain, how we assign these values. The input values selected by experts are chosen from the set of uncertain linguistic variables for each criteria. The range of these variables is given in Table 9.

In tabular form the C3PF-data is given as Table 10.

We have three experts so we use input data for $\mathfrak{M} = 3$, which shows the opinion of three experts. For each expert interval shows that how much the given criteria $\mathfrak{Y}_{\wp}$ is present in the corresponding alternative $\psi_{\wp'}$. Non-membership grades shows that how much the criteria $\mathfrak{Y}_{\wp}$ is not found

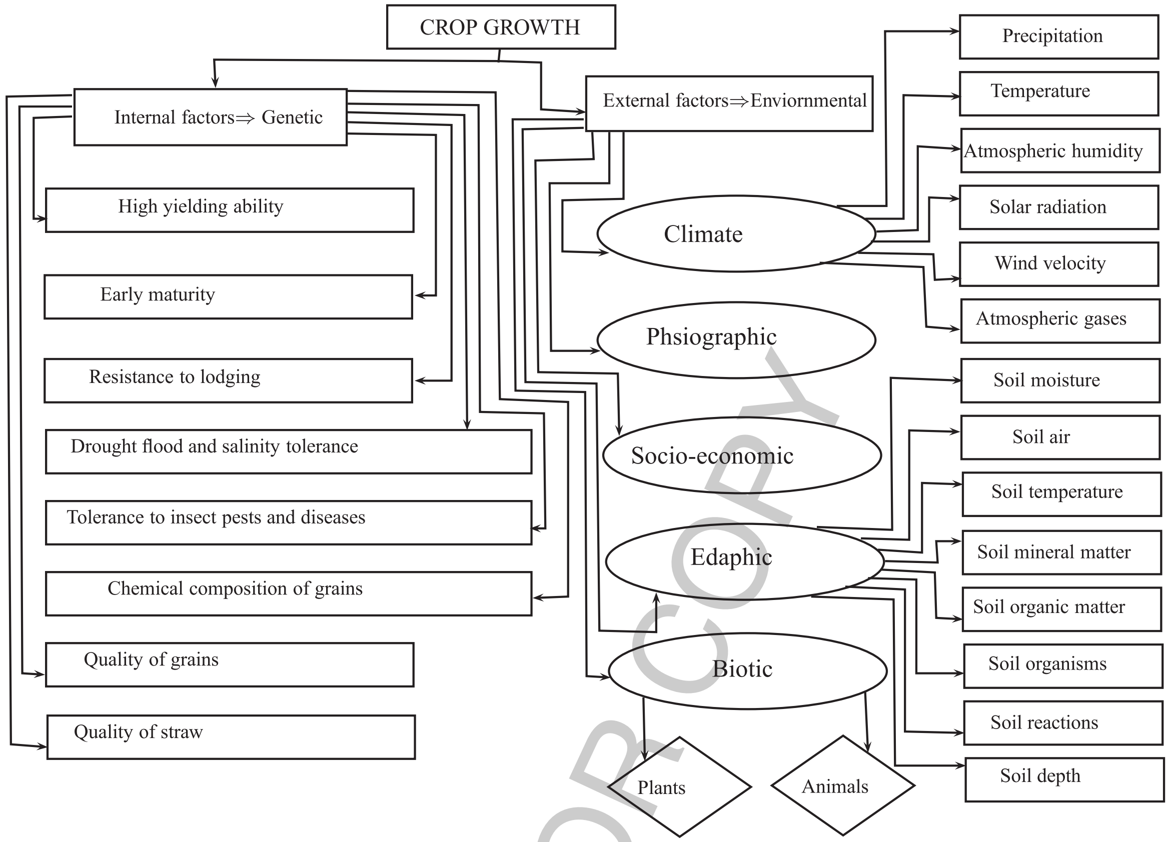

Fig. 4. Factors affecting crop production.

Fig. 5. Flow chart diagram of proposed algorithm.

Table 10
Cubic 3-polar fuzzy input data

Order	$\mathcal{Q}$	C3PFNs
1	ψ_1	$\mathfrak{Y}_1 : \big(\langle[0.28, 0.57], [0.56, 0.78], [0.67, 0.89]\rangle, \langle 0.48, 0.67, 0.79\rangle\big)$
2	ψ_1	$\mathfrak{Y}_2 : \big(\langle[0.37, 0.84], [0.78, 0.97], [0.21, 0.38]\rangle, \langle 0.77, 0.88, 0.28\rangle\big)$
3	ψ_1	$\mathfrak{Y}_3 : \big(\langle[0.11, 0.41], [0.28, 0.91], [0.35, 0.91]\rangle, \langle 0.34, 0.78, 0.67\rangle\big)$
4	ψ_1	$\mathfrak{Y}_4 : \big(\langle[0.31, 0.48], [0.27, 0.38], [0.44, 0.67]\rangle, \langle 0.38, 0.35, 0.57\rangle\big)$
5	ψ_1	$\mathfrak{Y}_5 : \big(\langle[0.58, 0.77], [0.67, 0.89], [0.88, 0.91]\rangle, \langle 0.66, 0.83, 0.89\rangle\big)$
1	ψ_2	$\mathfrak{Y}_1 : \big(\langle[0.88, 0.91], [0.71, 0.88], [0.87, 0.99]\rangle, \langle 0.90, 0.78, 0.88\rangle\big)$
2	ψ_2	$\mathfrak{Y}_2 : \big(\langle[0.67, 0.77], [0.87, 0.98], [0.81, 0.97]\rangle, \langle 0.68, 0.89, 0.95\rangle\big)$
3	ψ_2	$\mathfrak{Y}_3 : \big(\langle[0.51, 0.61], [0.11, 0.35], [0.43, 0.83]\rangle, \langle 0.58, 0.27, 0.55\rangle\big)$
4	ψ_2	$\mathfrak{Y}_4 : \big(\langle[0.31, 0.58], [0.41, 0.61], [0.51, 0.61]\rangle, \langle 0.47, 0.52, 0.58\rangle\big)$
5	ψ_2	$\mathfrak{Y}_5 : \big(\langle[0.31, 0.77], [0.23, 0.43], [0.17, 0.38]\rangle, \langle 0.46, 0.26, 0.28\rangle\big)$
1	ψ_3	$\mathfrak{Y}_1 : \big(\langle[0.77, 0.89], [0.85, 0.95], [0.68, 0.89]\rangle, \langle 0.83, 0.94, 0.75\rangle\big)$
2	ψ_3	$\mathfrak{Y}_2 : \big(\langle[0.78, 0.98], [0.68, 0.89], [0.56, 0.86]\rangle, \langle 0.88, 0.85, 0.83\rangle\big)$
3	ψ_3	$\mathfrak{Y}_3 : \big(\langle[0.35, 0.58], [0.58, 0.68], [0.43, 0.61]\rangle, \langle 0.44, 0.63, 0.57\rangle\big)$
4	ψ_3	$\mathfrak{Y}_4 : \big(\langle[0.51, 0.83], [0.12, 0.28], [0.34, 0.47]\rangle, \langle 0.58, 0.18, 0.37\rangle\big)$
5	ψ_3	$\mathfrak{Y}_5 : \big(\langle[0.31, 0.47], [0.68, 0.71], [0.28, 0.42]\rangle, \langle 0.42, 0.69, 0.33\rangle\big)$
1	ψ_4	$\mathfrak{Y}_1 : \big(\langle[0.68, 0.86], [0.77, 0.93], [0.81, 0.95]\rangle, \langle 0.72, 0.91, 0.86\rangle\big)$
2	ψ_4	$\mathfrak{Y}_2 : \big(\langle[0.78, 0.87], [0.68, 0.97], [0.81, 0.98]\rangle, \langle 0.81, 0.78, 0.94\rangle\big)$
3	ψ_4	$\mathfrak{Y}_3 : \big(\langle[0.37, 0.48], [0.52, 0.64], [0.74, 0.88]\rangle, \langle 0.41, 0.60, 0.78\rangle\big)$
4	ψ_4	$\mathfrak{Y}_4 : \big(\langle[0.13, 0.34], [0.41, 0.58], [0.27, 0.34]\rangle, \langle 0.26, 0.46, 0.31\rangle\big)$
5	ψ_4	$\mathfrak{Y}_5 : \big(\langle[0.21, 0.38], [0.81, 0.91], [0.67, 0.79]\rangle, \langle 0.30, 0.80, 0.73\rangle\big)$

in the alternative $\psi_{\wp'}$. In Table 10 the C3PFN $\big(\langle [0.28, 0.57], [0.56, 0.78], [0.67, 0.89] \rangle, \langle 0.48, 0.67, 0.79\rangle\big)$ shows that for alternative ψ_1 and criteria $\mathfrak{Y}_1$, we have input data according to three experts under the effect of linguistic terms given as $\big(\langle[ML, MH], [MH, H], [MH, H]\rangle, \langle M, MH, H\rangle\big)$. For land ψ_1 and for criteria $\mathfrak{Y}_1 =$ Climate factors the first fuzzy interval $[0.28, 0.57]$ and first fuzzy value 0.48 shows the input according to the first expert. On the same pattern, we can see the physical sense of remaining alternatives and attributes. Experts assign some weights according to the given factors and associated weight vector is given as $\zeta = (0.2, 0.4, 0.1, 0.2, 0.1)$ such that $\sum_{\wp=1}^{5} \zeta_{\wp} = 1$. This weigh vector is constructed according to the requirement of company for each criteria.

• **Calculations for** C3PFWA$_R$**:**
By using equation (A) on Table 10, we get the C3PFN $\mathcal{O}_1$ given as
$\mathcal{O}_1 = \big(\langle[0.3451, 0.7084], [0.6234, 0.896], [0.497, 0.7372]\rangle, \langle 0.5519, 0.6806, 0.4865\rangle\big)$
Similarly we can calculate $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$ for ψ_2, ψ_3 and ψ_4 given as
$\mathcal{O}_2 = \big(\langle[0.6502, 0.7733], [0.701, 0.8974], [0.7247, 0.9353]\rangle, \langle 0.6322, 0.6109, 0.7102\rangle\big)$
$\mathcal{O}_3 = \big(\langle[0.6746, 0.9189], [0.6541, 0.8323], [0.5174, 0.7777]\rangle, \langle 0.6933, 0.6043, 0.6077\rangle\big)$
$\mathcal{O}_4 = \big(\langle[0.606, 0.7548], [0.6654, 0.9138], [0.7289, 0.9269]\rangle, \langle 0.5331, 0.7068, 0.7078\rangle\big)$
The score values of above aggregated C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.4210$, $\pounds(\mathcal{O}_2) = 0.6058$,
$\pounds(\mathcal{O}_3) = 0.5467$, $\pounds(\mathcal{O}_4) = 0.5905$.
This shows that $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$. Hence ψ_2 is the best land for their agricultural project. Graphically it can be seen in Fig. (6)

• **Calculations for** C3PFOWA$_R$:
First of all we will calculate the score values of our C3PF-data form Table 10.

Now the ordered C3PF-data for every $\mathfrak{Y}_{\wp}$ and $\psi_{\wp'}$; $(\wp' = 1, 2, 3, 4)$, $(\wp = 1, 2, 3, 4, 5)$ is given in Table 11.

By using equation (B) for C3PFOWA$_R$ we get the C3PFNs $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3, \mathcal{O}_4$. The score values of these aggregated C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.4574$, $\pounds(\mathcal{O}_2) = 0.6003$,
$\pounds(\mathcal{O}_3) = 0.5597$, $\pounds(\mathcal{O}_4) = 0.6214$. This shows that $\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$. Hence ψ_4 is the best land for

Table 11
Ordered cubic 3-polar fuzzy numbers

Order	$\mathcal{Q}$	Ordered C3PFNs
1	$\psi_{\wr(1)}$	$\mathfrak{Y}_5 : \left(\langle[0.58, 0.77], [0.67, 0.89], [0.88, 0.91]\rangle, \langle 0.66, 0.83, 0.89\rangle\right)$
2	$\psi_{\wr(1)}$	$\mathfrak{Y}_1 : \left(\langle[0.28, 0.57], [0.56, 0.78], [0.67, 0.89]\rangle, \langle 0.48, 0.67, 0.79\rangle\right)$
3	$\psi_{\wr(1)}$	$\mathfrak{Y}_2 : \left(\langle[0.37, 0.84], [0.78, 0.97], [0.21, 0.38]\rangle, \langle 0.77, 0.88, 0.28\rangle\right)$
4	$\psi_{\wr(1)}$	$\mathfrak{Y}_3 : \left(\langle[0.11, 0.41], [0.28, 0.91], [0.35, 0.91]\rangle, \langle 0.34, 0.78, 0.67\rangle\right)$
5	$\psi_{\wr(1)}$	$\mathfrak{Y}_4 : \left(\langle[0.31, 0.48], [0.27, 0.38], [0.44, 0.67]\rangle, \langle 0.38, 0.35, 0.57\rangle\right)$
1	$\psi_{\wr(2)}$	$\mathfrak{Y}_1 : \left(\langle[0.88, 0.91], [0.71, 0.88], [0.87, 0.99]\rangle, \langle 0.90, 0.78, 0.88\rangle\right)$
2	$\psi_{\wr(2)}$	$\mathfrak{Y}_2 : \left(\langle[0.67, 0.77], [0.87, 0.98], [0.81, 0.97]\rangle, \langle 0.68, 0.89, 0.95\rangle\right)$
3	$\psi_{\wr(2)}$	$\mathfrak{Y}_4 : \left(\langle[0.31, 0.58], [0.41, 0.61], [0.51, 0.61]\rangle, \langle 0.47, 0.52, 0.58\rangle\right)$
4	$\psi_{\wr(2)}$	$\mathfrak{Y}_3 : \left(\langle[0.51, 0.61], [0.11, 0.35], [0.43, 0.83]\rangle, \langle 0.58, 0.27, 0.55\rangle\right)$
5	$\psi_{\wr(2)}$	$\mathfrak{Y}_5 : \left(\langle[0.31, 0.77], [0.23, 0.43], [0.17, 0.38]\rangle, \langle 0.46, 0.26, 0.28\rangle\right)$
1	$\psi_{\wr(3)}$	$\mathfrak{Y}_1 : \left(\langle[0.77, 0.89], [0.85, 0.95], [0.68, 0.89]\rangle, \langle 0.83, 0.94, 0.75\rangle\right)$
2	$\psi_{\wr(3)}$	$\mathfrak{Y}_2 : \left(\langle[0.78, 0.98], [0.68, 0.89], [0.56, 0.86]\rangle, \langle 0.88, 0.85, 0.83\rangle\right)$
3	$\psi_{\wr(3)}$	$\mathfrak{Y}_3 : \left(\langle[0.35, 0.58], [0.58, 0.68], [0.43, 0.61]\rangle, \langle 0.44, 0.63, 0.57\rangle\right)$
4	$\psi_{\wr(3)}$	$\mathfrak{Y}_5 : \left(\langle[0.31, 0.47], [0.68, 0.71], [0.28, 0.42]\rangle, \langle 0.42, 0.69, 0.33\rangle\right)$
5	$\psi_{\wr(3)}$	$\mathfrak{Y}_4 : \left(\langle[0.51, 0.83], [0.12, 0.28], [0.34, 0.47]\rangle, \langle 0.58, 0.18, 0.37\rangle\right)$
1	$\psi_{\wr(4)}$	$\mathfrak{Y}_2 : \left(\langle[0.78, 0.87], [0.68, 0.97], [0.81, 0.98]\rangle, \langle 0.81, 0.78, 0.94\rangle\right)$
2	$\psi_{\wr(4)}$	$\mathfrak{Y}_1 : \left(\langle[0.68, 0.86], [0.77, 0.93], [0.81, 0.95]\rangle, \langle 0.72, 0.91, 0.86\rangle\right)$
3	$\psi_{\wr(4)}$	$\mathfrak{Y}_5 : \left(\langle[0.21, 0.38], [0.81, 0.91], [0.67, 0.79]\rangle, \langle 0.30, 0.80, 0.73\rangle\right)$
4	$\psi_{\wr(4)}$	$\mathfrak{Y}_3 : \left(\langle[0.37, 0.48], [0.52, 0.64], [0.74, 0.88]\rangle, \langle 0.41, 0.60, 0.78\rangle\right)$
5	$\psi_{\wr(4)}$	$\mathfrak{Y}_4 : \left(\langle[0.13, 0.34], [0.41, 0.58], [0.27, 0.34]\rangle, \langle 0.26, 0.46, 0.31\rangle\right)$

Table 12
Hybrid cubic 3-polar fuzzy data

Order	$\mathcal{Q}$	$\mathfrak{Y}_{\wp}$	Hybrid C3PFNs
1	$\dot{\psi}_1$	$\mathfrak{Y}_1$	$\left(\langle[0.28, 0.57], [0.56, 0.78], [0.67, 0.89]\rangle, \langle 0.48, 0.67, 0.79\rangle\right)$
2	$\dot{\psi}_1$	$\mathfrak{Y}_2$	$\left(\langle[0.6031, 0.9744], [0.9516, 0.9991], [0.3759, 0.6156]\rangle, \langle 0.5929, 0.7744, 0.0784\rangle\right)$
3	$\dot{\psi}_1$	$\mathfrak{Y}_3$	$\left(\langle[0.0566, 0.2318], [0.1514, 0.7], [0.1937, 0.7]\rangle, \langle 0.5830, 0.8831, 0.8185\rangle\right)$
4	$\dot{\psi}_1$	$\mathfrak{Y}_4$	$\left(\langle[0.31, 0.48], [0.27, 0.38], [0.44, 0.67]\rangle, \langle 0.38, 0.35, 0.57\rangle\right)$
5	$\dot{\psi}_1$	$\mathfrak{Y}_5$	$\left(\langle[0.3519, 0.5204], [0.4255, 0.6683], [0.6535, 0.7]\rangle, \langle 0.8124, 0.9110, 0.9433\rangle\right)$
1	$\dot{\psi}_2$	$\mathfrak{Y}_1$	$\left(\langle[0.88, 0.91], [0.71, 0.88], [0.87, 0.99]\rangle, \langle 0.90, 0.78, 0.88\rangle\right)$
2	$\dot{\psi}_2$	$\mathfrak{Y}_2$	$\left(\langle[0.8911, 0.9471], [0.9831, 0.9996], [0.9639, 0.9991]\rangle, \langle 0.4624, 0.7921, 0.9025\rangle\right)$
3	$\dot{\psi}_2$	$\mathfrak{Y}_3$	$\left(\langle[0.3, 0.3755], [0.0566, 0.1937], [0.2450, 0.5876]\rangle, \langle 0.7615, 0.5196, 0.7416\rangle\right)$
4	$\dot{\psi}_2$	$\mathfrak{Y}_4$	$\left(\langle[0.31, 0.58], [0.41, 0.61], [0.51, 0.61]\rangle, \langle 0.47, 0.52, 0.58\rangle\right)$
5	$\dot{\psi}_2$	$\mathfrak{Y}_5$	$\left(\langle[0.1693, 0.5204], [0.1225, 0.2450], [0.0889, 0.2125]\rangle, \langle 0.6782, 0.5099, 0.5291\rangle\right)$
1	$\dot{\psi}_3$	$\mathfrak{Y}_1$	$\left(\langle[0.77, 0.89], [0.85, 0.95], [0.68, 0.89]\rangle, \langle 0.83, 0.94, 0.75\rangle\right)$
2	$\dot{\psi}_3$	$\mathfrak{Y}_2$	$\left(\langle[0.9516, 0.9996], [0.8976, 0.9879], [0.8064, 0.9804]\rangle, \langle 0.7744, 0.7225, 0.6889\rangle\right)$
3	$\dot{\psi}_3$	$\mathfrak{Y}_3$	$\left(\langle[0.1937, 0.3519], [0.3519, 0.4343], [0.2450, 0.3755]\rangle, \langle 0.6633, 0.7937, 0.7549\rangle\right)$
4	$\dot{\psi}_3$	$\mathfrak{Y}_4$	$\left(\langle[0.51, 0.83], [0.12, 0.28], [0.34, 0.47]\rangle, \langle 0.58, 0.18, 0.37\rangle\right)$
5	$\dot{\psi}_3$	$\mathfrak{Y}_5$	$\left(\langle[0.1693, 0.2719], [0.4343, 0.4616], [0.1514, 0.2384]\rangle, \langle 0.6480, 0.8306, 0.5744\rangle\right)$
1	$\dot{\psi}_4$	$\mathfrak{Y}_1$	$\left(\langle[0.68, 0.86], [0.77, 0.93], [0.81, 0.95]\rangle, \langle 0.72, 0.91, 0.86\rangle\right)$
2	$\dot{\psi}_4$	$\mathfrak{Y}_2$	$\left(\langle[0.9516, 0.9831], [0.8976, 0.9991], [0.9639, 0.9996]\rangle, \langle 0.6561, 0.6084, 0.8836\rangle\right)$
3	$\dot{\psi}_4$	$\mathfrak{Y}_3$	$\left(\langle[0.2062, 0.2788], [0.3071, 0.4], [0.4900, 0.6535]\rangle, \langle 0.5099, 0.6782, 0.5567\rangle\right)$
4	$\dot{\psi}_4$	$\mathfrak{Y}_4$	$\left(\langle[0.13, 0.34], [0.41, 0.58], [0.27, 0.34]\rangle, \langle 0.26, 0.46, 0.31\rangle\right)$
5	$\dot{\psi}_4$	$\mathfrak{Y}_5$	$\left(\langle[0.1111, 0.2125], [0.5641, 0.7], [0.4255, 0.5417]\rangle, \langle 0.5477, 0.8944, 0.8544\rangle\right)$

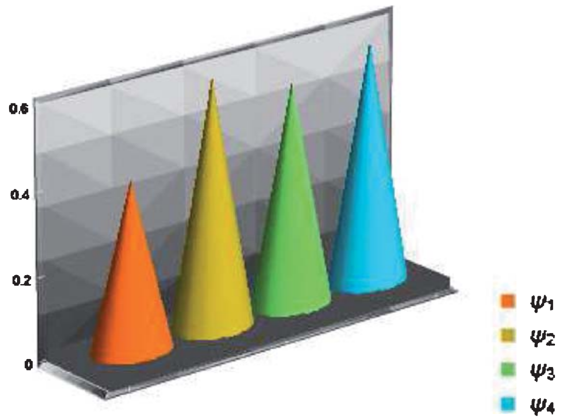

Fig. 6. Ranking of C3PFNs for C3PFWA$_R$.

Fig. 7. Ranking of C3PFNs for C3PFOWA$_R$.

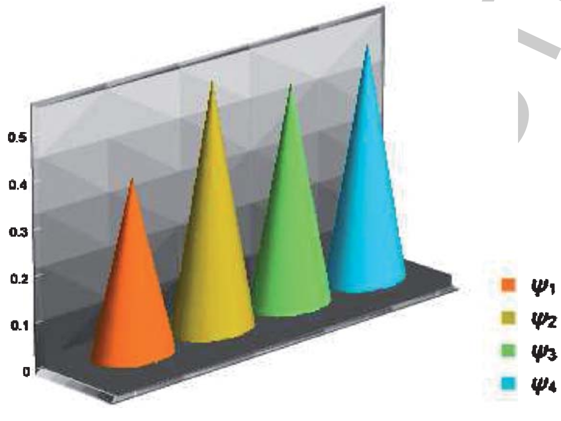

Fig. 8. Ranking of C3PFNs for C3PFHA$_R$.

their agricultural project. Graphically it can be seen in Fig. (7)

• **Calculations for** C3PFHA$_R$:

For C3PFHA$_R$ experts will define another weight vector according to the some related parameters for their choice to hybridize the given data. This hybridization is another approach to select the best alternative and can be used to find the more accurate result for our decision. The weight vector is given as $\theta = (0.2, 0.1, 0.3, 0.2, 0.2)$ with $\sum_{\wp=1}^{5} \theta_{\wp} = 1$. For aggregated C3PFNs we will first calculate $\psi_{\wp'} = \mathfrak{N}\psi_{\wp'}\zeta_{\wp}$, where $\mathfrak{N} = 5$ given in Table 12.

The score values of these aggregated C3PFNs from Table 12 are calculated by using the Definition 2.9, then we get ordered hybrid C3PF data given in Table 13.

By using equation (C) for Table 13, we get the C3PFNs $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$ and $\mathcal{O}_4$. The score values of these aggregated hybrid C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.3979$, $\pounds(\mathcal{O}_2) = 0.5543$,
$\pounds(\mathcal{O}_3) = 0.5015$, $\pounds(\mathcal{O}_4) = 0.5404$.
This shows that $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$. Hence ψ_2 is the best land for their agricultural project. Graphically it can be seen in Fig. (8). Now we use aggregated operators for P-order in the similar manner as for R-order.

• **Calculations for** C3PFWA$_P$:

By using equation (D) for C3PFWA$_P$ we get the C3PFNs $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$ and $\mathcal{O}_4$ and the score values of above aggregated C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.4683$, $\pounds(\mathcal{O}_2) = 0.6636$,
$\pounds(\mathcal{O}_3) = 0.6070$, $\pounds(\mathcal{O}_4) = 0.6444$.
This shows that $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$. Hence ψ_2 is the best land for their agricultural project.

• **Calculations for** C3PFOWA$_P$:

By using equation (E) for C3PFOWA$_P$ we get the C3PFNs $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$ and $\mathcal{O}_4$ and the score values of above aggregated C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.4842$, $\pounds(\mathcal{O}_2) = 0.6621$,
$\pounds(\mathcal{O}_3) = 0.6120$, $\pounds(\mathcal{O}_4) = 0.6598$.
This shows that $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$. Hence ψ_2 is the best land for their agricultural project.

• **Calculations for** C3PFHA$_P$:

By using equation (F) for C3PFHA$_P$ we get the C3PFNs $\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3$ and $\mathcal{O}_4$ and the score values of above aggregated C3PFNs are calculated by using the Definition 2.9 given as,
$\pounds(\mathcal{O}_1) = 0.4566$, $\pounds(\mathcal{O}_2) = 0.6130$,
$\pounds(\mathcal{O}_3) = 0.5438$, $\pounds(\mathcal{O}_4) = 0.5463$.
This shows that $\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$. Hence ψ_2 is the best land for their agricultural project.

Table 13
Ordered hybrid cubic 3-polar fuzzy data

Order	$\mathcal{Q}$	$\mathfrak{Y}_{\wp}$	Ordered hybrid C3PFNs
1	$\dot{\psi}_{\wr(1)}$	$\mathfrak{Y}_5$	$\big(\langle[0.3519, 0.5204], [0.4255, 0.6683], [0.6535, 0.7]\rangle, \langle 0.8124, 0.9110, 0.9433\rangle\big)$
2	$\dot{\psi}_{\wr(1)}$	$\mathfrak{Y}_2$	$\big(\langle[0.6031, 0.9744], [0.9516, 0.9991], [0.3759, 0.6156]\rangle, \langle 0.5929, 0.7744, 0.0784\rangle\big)$
3	$\dot{\psi}_{\wr(1)}$	$\mathfrak{Y}_1$	$\big(\langle[0.28, 0.57], [0.56, 0.78], [0.67, 0.89]\rangle, \langle 0.48, 0.67, 0.79\rangle\big)$
4	$\dot{\psi}_{\wr(1)}$	$\mathfrak{Y}_3$	$\big(\langle[0.0566, 0.2318], [0.1514, 0.7], [0.1937, 0.7]\rangle, \langle 0.5830, 0.8831, 0.8185\rangle\big)$
5	$\dot{\psi}_{\wr(1)}$	$\mathfrak{Y}_4$	$\big(\langle[0.31, 0.48], [0.27, 0.38], [0.44, 0.67]\rangle, \langle 0.38, 0.35, 0.57\rangle\big)$
1	$\dot{\psi}_{\wr(2)}$	$\mathfrak{Y}_2$	$\big(\langle[0.8911, 0.9471], [0.9831, 0.9996], [0.9639, 0.9991]\rangle, \langle 0.4624, 0.7921, 0.9025\rangle\big)$
2	$\dot{\psi}_{\wr(2)}$	$\mathfrak{Y}_1$	$\big(\langle[0.88, 0.91], [0.71, 0.88], [0.87, 0.99]\rangle, \langle 0.90, 0.78, 0.88\rangle\big)$
3	$\dot{\psi}_{\wr(2)}$	$\mathfrak{Y}_4$	$\big(\langle[0.31, 0.58], [0.41, 0.61], [0.51, 0.61]\rangle, \langle 0.47, 0.52, 0.58\rangle\big)$
4	$\dot{\psi}_{\wr(2)}$	$\mathfrak{Y}_3$	$\big(\langle[0.3, 0.3755], [0.0566, 0.1937], [0.2450, 0.5876]\rangle, \langle 0.7615, 0.5196, 0.7416\rangle\big)$
5	$\dot{\psi}_{\wr(2)}$	$\mathfrak{Y}_5$	$\big(\langle[0.1693, 0.5204], [0.1225, 0.2450], [0.0889, 0.2125]\rangle, \langle 0.6782, 0.5099, 0.5291\rangle\big)$
1	$\dot{\psi}_{\wr(3)}$	$\mathfrak{Y}_2$	$\big(\langle[0.9516, 0.9996], [0.8976, 0.9879], [0.8064, 0.9804]\rangle, \langle 0.7744, 0.7225, 0.6889\rangle\big)$
2	$\dot{\psi}_{\wr(3)}$	$\mathfrak{Y}_1$	$\big(\langle[0.77, 0.89], [0.85, 0.95], [0.68, 0.89]\rangle, \langle 0.83, 0.94, 0.75\rangle\big)$
3	$\dot{\psi}_{\wr(3)}$	$\mathfrak{Y}_3$	$\big(\langle[0.1937, 0.3519], [0.3519, 0.4343], [0.2450, 0.3755]\rangle, \langle 0.6633, 0.7937, 0.7549\rangle\big)$
4	$\dot{\psi}_{\wr(3)}$	$\mathfrak{Y}_5$	$\big(\langle[0.1693, 0.2719], [0.4343, 0.4616], [0.1514, 0.2384]\rangle, \langle 0.6480, 0.8306, 0.5744\rangle\big)$
5	$\dot{\psi}_{\wr(3)}$	$\mathfrak{Y}_4$	$\big(\langle[0.51, 0.83], [0.12, 0.28], [0.34, 0.47]\rangle, \langle 0.58, 0.18, 0.37\rangle\big)$
1	$\dot{\psi}_{\wr(4)}$	$\mathfrak{Y}_2$	$\big(\langle[0.9516, 0.9831], [0.8976, 0.9991], [0.9639, 0.9996]\rangle, \langle 0.6561, 0.6084, 0.8836\rangle\big)$
2	$\dot{\psi}_{\wr(4)}$	$\mathfrak{Y}_1$	$\big(\langle[0.68, 0.86], [0.77, 0.93], [0.81, 0.95]\rangle, \langle 0.72, 0.91, 0.86\rangle\big)$
3	$\dot{\psi}_{\wr(4)}$	$\mathfrak{Y}_5$	$\big(\langle[0.1111, 0.2125], [0.5641, 0.7], [0.4255, 0.5417]\rangle, \langle 0.5477, 0.8944, 0.8544\rangle\big)$
4	$\dot{\psi}_{\wr(4)}$	$\mathfrak{Y}_3$	$\big(\langle[0.2062, 0.2788], [0.3071, 0.4], [0.4900, 0.6535]\rangle, \langle 0.5099, 0.6782, 0.5567\rangle\big)$
5	$\dot{\psi}_{\wr(4)}$	$\mathfrak{Y}_4$	$\big(\langle[0.13, 0.34], [0.41, 0.58], [0.27, 0.34]\rangle, \langle 0.26, 0.46, 0.31\rangle\big)$

Graphically ranking for P-order operators can be seen in Fig. 9.

The flow chart diagram of above application with the proposed algorithm and all the calculations for all defined operators can be seen in Fig. 5.

• Discussion:
In this subsection, we discuss about the numerical example and its results obtained from different aggregated operators. The proposed algorithm for numerical example is simple and easy to understand. Firstly, we collect the input data and convert verbally stated information in the form of cubic m-polar fuzzy numbers. We input numerical values for each alternative under the effect of every criteria in the form of cubic 3-polar fuzzy numbers. It is necessary to normalize the input information before further calculations to obtain the best and precise solution. In proposed example the data is same for all attributes, so there is no need to normalize the information. The selection of input data for the problem is according to the opinion of experts under the effect of uncertain linguistic variables. If we observe Table 10 under the effect of Table 9, then we can see that the selected input data is precise and give best solution for our problem. It takes the form of Table 14.

The weight vector is chosen according to the requirement of all decision makers of the company. Then we calculate all the aggregated values by using six aggregated operators given in equations (A),(B),(C),(D),(E) and (F). Then we calculate score of all attributes for each operator by using Definition 2.9. We choose the alternative having maximum score and it is interesting to note that all aggregated operators gives approximately the same result. Due to the difference in numerical techniques and ordering strategies of presented operators, we can see the slightly difference in the ranking of attributes. But the results obtained from all proposed operators are accurate and give suitable preference order for the selection of land.

• Comparison Analysis:
In our proposed research, we defined operators by using the advanced concept of CMPFNs. The impressive point of this model is that we can use it for mathematical modeling at a large scale or $\mathfrak{M}$ numbers of degrees. These degrees basically show the corresponding $\mathfrak{M}$ properties or any $\mathfrak{M}$ criteria about the alternative ψ (for example, when we are dealing with multiple windows or multi-mode phones then we have multiple choices and $\mathfrak{M}$ degrees for the

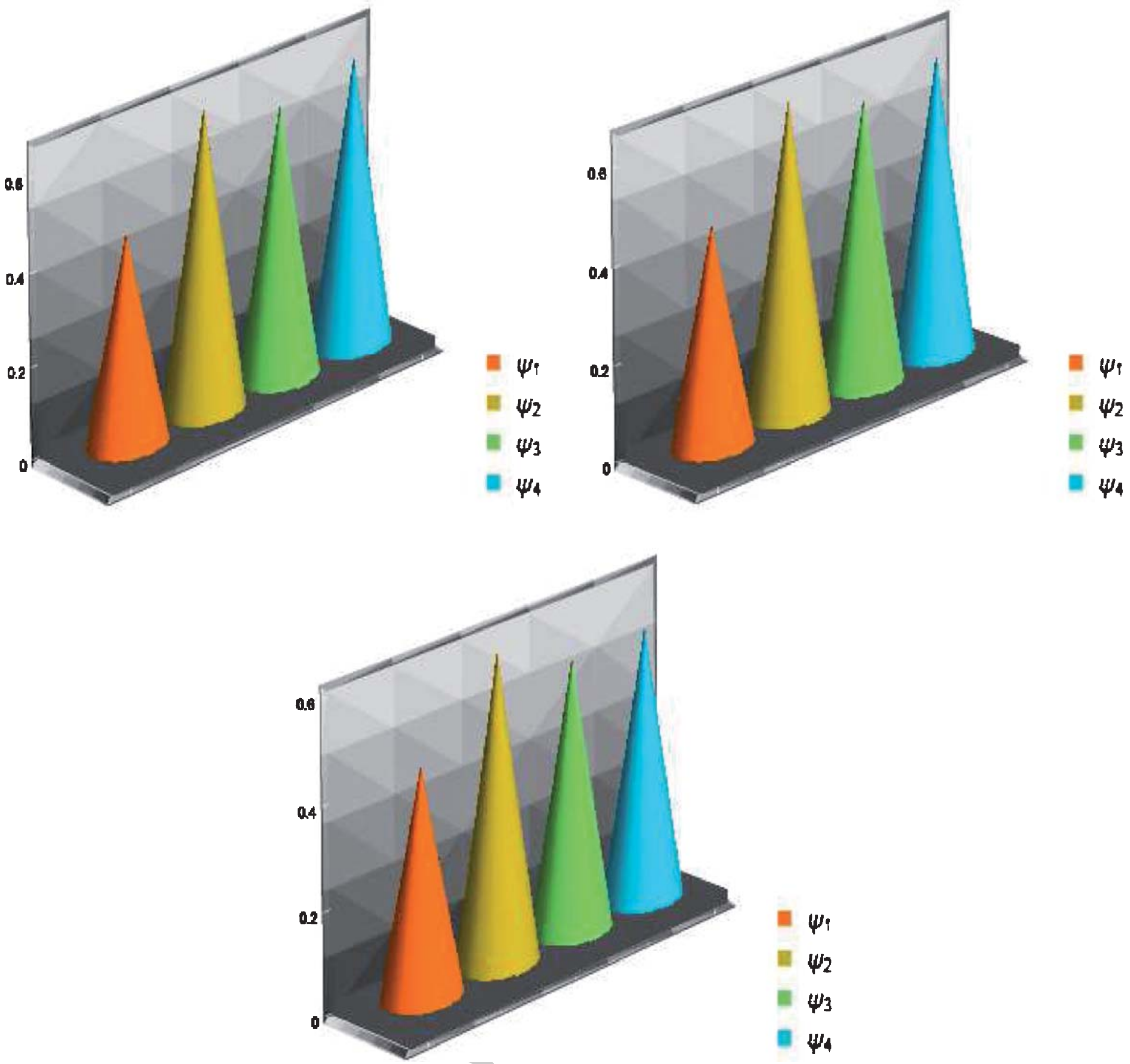

Fig. 9. Ranking of C3PFNs for C3PFWA$_P$, C3PFOWA$_P$ and C3PFHA$_P$.

selected alternative). We can input data at $\mathfrak{M}$ numbers of points for any alternative or an assembling of alternatives. This item proves that it is a hybrid and more generalized model of other approaches. Other sets such as CHFS, CIFS, CNFS, CFS, etc. become the special case of CMPFS with the addition of some suitable conditions. On the same pattern all the operators corresponding to the given sets become the particular cases of our purposed operators for CMPFS for $\mathfrak{M} = 1, 2, 3$. (see Figs. 1 and 3).

In our algorithm, we are going to compare the solutions obtained from different averaging aggregated operators in tabular and graphical variety. Most of the operators produce the same solution for the given problem. Different operators have different ordering strategies so they can afford the slightly different effect according to their deliberations. So on the basis of decision-makers the result can be chosen by comparing the production of various operators given in tabular form as Table 15. Graphically this comparison can be seen as Fig. 10.

From the results, it can be easily seen that the alternative ψ_2 is more preferable and company should choose ψ_2 land for their agricultural project.

• **Advantages of Proposed Approach:**

In this part, we discuss about some advantages of proposed approach based on CMPFSs.

(i) Validity of the method:

The proposed method is valid and suitable for all types of input data. These operators easily deals with

Table 14
Cubic 3-polar fuzzy input data in the form of linguistic terms

Order	$\mathcal{Q}$	C3PFNs
1	ψ_1	$\mathfrak{Y}_1 : \big(\langle [ML, MH], [MH, H], [MH, H]\rangle, \langle M, MH, H\rangle\big)$
2	ψ_1	$\mathfrak{Y}_2 : \big(\langle [M, H], [H, H], [ML, L]\rangle, \langle H, H, ML\rangle\big)$
3	ψ_1	$\mathfrak{Y}_3 : \big(\langle [L, M], [ML, H], [M, H]\rangle, \langle ML, H, MH\rangle\big)$
4	ψ_1	$\mathfrak{Y}_4 : \big(\langle [ML, M], [ML, M], [M, MH]\rangle, \langle M, M, MH\rangle\big)$
5	ψ_1	$\mathfrak{Y}_5 : \big(\langle [MH, H], [MH, H], [H, H]\rangle, \langle MH, H, H\rangle\big)$
1	ψ_2	$\mathfrak{Y}_1 : \big(\langle [H, H], [MH, H], [H, H]\rangle, \langle H, H, H\rangle\big)$
2	ψ_2	$\mathfrak{Y}_2 : \big(\langle [MH, H], [H, H], [H, H]\rangle, \langle MH, H, H\rangle\big)$
3	ψ_2	$\mathfrak{Y}_3 : \big(\langle [M, MH], [L, M], [M, H]\rangle, \langle MH, ML, MH\rangle\big)$
4	ψ_2	$\mathfrak{Y}_4 : \big(\langle [ML, MH], [M, MH], [M, MH]\rangle, \langle M, M, MH\rangle\big)$
5	ψ_2	$\mathfrak{Y}_5 : \big(\langle [ML, H], [ML, M], [L, ML]\rangle, \langle M, ML, ML\rangle\big)$
1	ψ_3	$\mathfrak{Y}_1 : \big(\langle [H, H], [H, H], [MH, H]\rangle, \langle H, H, H\rangle\big)$
2	ψ_3	$\mathfrak{Y}_2 : \big(\langle [H, H], [MH, H], [MH, H]\rangle, \langle H, H, H\rangle\big)$
3	ψ_3	$\mathfrak{Y}_3 : \big(\langle [M, MH], [MH, MH], [M, MH]\rangle, \langle M, MH, MH\rangle\big)$
4	ψ_3	$\mathfrak{Y}_4 : \big(\langle [M, H], [L, ML], [ML, M]\rangle, \langle MH, L, ML\rangle\big)$
5	ψ_3	$\mathfrak{Y}_5 : \big(\langle [ML, M], [MH, MH], [ML, M]\rangle, \langle M, MH, ML\rangle\big)$
1	ψ_4	$\mathfrak{Y}_1 : \big(\langle [MH, H], [H, H], [H, H]\rangle, \langle MH, H, H\rangle\big)$
2	ψ_4	$\mathfrak{Y}_2 : \big(\langle [H, H], [MH, H], [H, H]\rangle, \langle H, H, H\rangle\big)$
3	ψ_4	$\mathfrak{Y}_3 : \big(\langle [M, M], [M, MH], [H, H]\rangle, \langle M, MH, H\rangle\big)$
4	ψ_4	$\mathfrak{Y}_4 : \big(\langle [L, ML], [M, MH], [ML, ML]\rangle, \langle ML, M, ML\rangle\big)$
5	ψ_4	$\mathfrak{Y}_5 : \big(\langle [ML, M], [H, H], MH, H]\rangle, \langle ML, H, H\rangle\big)$

Table 15
Comparison analysis

Proposed operators	ψ_1	ψ_2	ψ_3	ψ_4	Ranking of the alternatives
CMPFWA$_R$	0.4210	0.6058	0.5467	0.5905	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$
CMPFOWA$_R$	0.4574	0.6003	0.5597	0.6214	$\psi_4 \succ \psi_2 \succ \psi_3 \succ \psi_1$
CMPFHA$_R$	0.3979	0.5543	0.5015	0.5404	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$
CMPFWA$_P$	0.4683	0.6636	0.6070	0.6444	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$
CMPFOWA$_P$	0.4842	0.6621	0.6120	0.6598	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$
CMPFHA$_P$	0.4566	0.6130	0.5438	0.5463	$\psi_2 \succ \psi_4 \succ \psi_3 \succ \psi_1$

the flaws appears in the input data and handle the ambiguities and uncertainties. As we can see in Fig. 3 that CMPFA operator covers all the hybrid defined operators so this is most generalized model and use to collect data at a large scale with multiple criteria of alternatives.

(ii) Simplicity dealing with different criteria:
In MAGDM problems we have different types of criteria and input data according to the given situations. The proposed CMPFA operators are simple and easy to understand which can be applied easily at any type of alternatives and criteria.

(iii) Flexibility of aggregation with different inputs and outputs:
The proposed algorithm is flexible and easily variate according to the different situations, inputs and outputs. There is a slightly difference between the ranking of proposed operators because different operators have different ordering strategies so they can afford the slightly different effect according to their deliberations.

(iv) Superiority and sensitivity of proposed method:
From all above discussion we observe that our proposed model and CMPFA operators are superior to others. Figure 3 clearly shows that CFA, IFA, MPFA, NCA, IFNA etc. operators become the special casees of CMPFA operator with the addition of some suitable conditions. So our method is valid flexible simple and superior to others hybrid structures of fuzzy set and operators defined in [10–13, 16, 44, 47].

This flexibility of our algorithm would allow

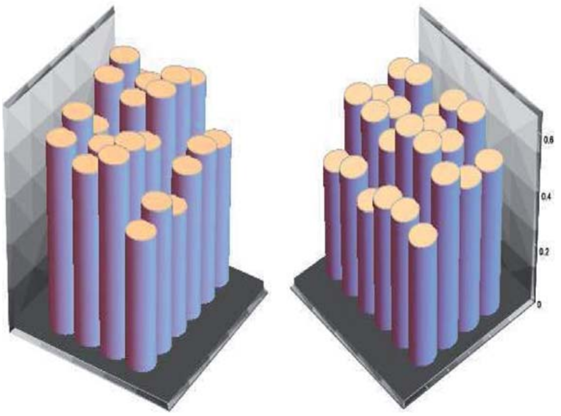

Fig. 10. Comparison of C3PFWA$_R$, C3PFOWA$_R$, C3PFHA$_R$, C3PFWA$_P$, C3PFOWA$_P$ and C3PFHA$_P$ operators.

administration to perform sensitivity analysis at multiple levels and thus obtain more healthy and relevant solutions. The result of this study helps to establish the systematic approach to select the best land within the set of criteria and analyze the most appropriate alternative. Our proposed method is less sensitive and more flexible to different input and outputs. This tool would be acceptable to managers who have to deal with greater magnitudes of uncertainties and vagueness in evaluation of the best choice.

5. Conclusion

The uncertainties present in the multi-attribute group decision-making process are not easy to deal in objective decision-making. This manuscript presents a novel approach for treating these ambiguities based on the application of land selection using linguistic variables in CMPF decision theory. Since this is a new model that has not been considered in the literature so far, the direction of future work should focus on the presented application of MAGDM technique. Our propose research is unique and important in the fiels of aggregation operators. We have established CMPFS with the combination of CFS and IVMPFS. Six averaging aggregated operators in the context of CMPFNs have been determined by using the CMPFS operations with respect to R-order and P-order. Score function and accuracy function has been demonstrated for the comparison of CMPFNs. In the late years, many aggregation operators corresponding to numerous hybrid fuzzy sets have been instituted to deal with the MAGDM problems. We have developed most hybrid averaging aggregation operators based on CMPFNs and use them into MAGDM. Other sets such as CHFS, CIFS, CNFS, CFS, etc. become the special case of CMPFS with the addition of some suitable conditions. On the same pattern all the operators corresponding to the given sets become the particular cases of our purposed operators for CMPFS for $\mathfrak{M} = 1, 2, 3$. Comparative analysis showed that these modified operators can easily deal with the real life problems and decision-making problems. There is slightly difference between the determination of different operators due to their setting up strategies and calculations but most of them conclude the similar results. This approach is more flexible and feasible as compared to other approaches due to its generalization. In future, this work can be gone easily for other approaches and different types of aggregated operators. Researchers will receive beneficial results by exploring and putting through these concepts in the field of MAGDM by using numerous aggregation operators.

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