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Monitoring circuit boards products in the presence of indeterminacy

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ABSTRACT

In this article a repetitive group sampling control has been introduced for the neutrosophic statistics under the Conway-Maxwell-Poisson (COM-Poisson) distribution. The suggested chart has been compared with the existing plan using simulated data generated from neutrosophic COM-Poisson distribution. The practical implementation of the suggested chart has also been expounded using the data from the manufacturing of the electric circuit boards. Overall, the results demonstrate that the suggested chart will be a proficient addition in the control chart literature. It is also observed that the suggested chart is an ideal chart when applied under appropriate conditions.

1. Introduction

Control charts are considered as an effective tool of Statistical Process Control (SPC) used for observing and enhancing the quality of goods and services. Several other process monitoring techniques like histogram, scatter diagram, flow chart, Pareto chart, and fishbone diagram are used very commonly for the certain, crispy and, clear data/observations. There are many situations when the quality control personnel have to face vague, uncertain, indeterminate, unclear data/observations Shu, et al. [1].

Repetitive Group Sampling (RGS) scheme, introduced by Sherman [2], has attracted several researchers of SPC because of its simplicity and efficiency [3]. The methodology of this RGS is similar to a sequential sampling and this sampling plan commits the smallest sample size in addition to the requisite protection to consumers as well as producer. In RGS, a sample is selected from the industrial process to make a decision about the state of the control chart. In the case of indecision, a second sample is selected and the process to select a sample is continued until a decision is made. Moreover, repetitive sampling is more capable than a single and double sampling plan whereas not as capable as the sequential plan. Balamurali, et al. [4] developed variables RGS plan for the disposition of the lot using normal and log-normal distributions. Balamurali and Jun [5] introduced the model of RGS for variables inspection. Ahmad, et al. [6] proposed the RGS control chart for improved monitoring of coal quality. Azam, et al. [7] suggested the hybrid exponentially weighted moving average chart for the repetitive sampling scheme. Ahmad, et al. [8] developed various dispersion charts using repetitive sampling. Aslam, et al. [9] developed the control chart technique for repetitive sampling under the multivariate Poisson distribution. Adeoti and Olaomi [10] proposed a repetitive sampling control chart for the mean monitoring under the process capability index. Aldosari, et al. [11] developed the variance chart using multiple dependent state repetitive sampling. Shafqat, et al. [12] developed an exponentially weighted moving average control chart for nonparametric statistics using the RGS scheme. Al-Marshadi, et al. [13] presented a chart for monitoring customer complaints using the RGS scheme. RGS scheme has been explored by many authors of the industrial statistics including, [14] and [15–21].

In the real-world the researchers often face the presence of incomplete, uncertain, unclear information. Smarandache [22] proposed an alternative approach based upon fuzzy logic to study uncertain observations. The neutrosophic logic is the generalization of the fuzzy logic. The neutrosophic logic deals with the measure of indeterminacy. The theory of neutrosophic statistic introduced by [23] is currently being used extensively to the situations when the results of the traditional statistics are unreliable due to the incomplete, uncertain and vague

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observations [24]. The neutrosophic statistics is presented using the idea of neutrosophic logic and are the extension of the classical statistics and provides information about the measure of indeterminacy. In addition, the neutrosophic statistics can be applied when the data in hand is in the interval, uncertain, and indeterminate. On the other hand, classical statistics can be applied when all observations in the data are determined and précised. Taleb and Limam [25] developed a control chart procedure for fuzzy and linguistic data. Hsieh, et al. [26] presented a control chart for fuzzy theory for monitoring the defects in the IC industry. [27] developed the measurement system to deal with uncertainty. Intaramo and Pongpullponsak [28] developed the control chart using the theory of extreme value of fuzzy observations for non-distributions. Sorooshian [29] presented the improved control chart for considering vagueness and uncertainty of observations. Charongrattanasakul and Pongpullponsak [30] investigated the technique for fuzzy quality control using the Weibull distribution. Panthong and Pongpullponsak [31] presented a variable control chart for non-crispy data using a triangular fuzzy function. Afshari and Sadeghpour Gildeh [32] developed the acceptance sampling plan for the fuzzy data using the multiple deferred state sampling scheme. Shu, et al. [1] proposed a variable control chart for fuzzy data using variable sample sizes. Aslam, et al. [33] developed a reliability chart for Weibull distribution for the uncertainty environment. Aslam, et al. [34] presented the dispersion chart for uncertain data using the neutrosophic interval method. Aslam and Raza [35] designed a neutrosophic multiple deferred state sampling plan for exponentially weighted moving averages for process capability index. Zhang, et al. [36] explored the structural characterizations of the neutrosophic general symmetry. Khan, et al. [37] designed a dispersion chart using neutrosophic statistics and described its application in the production process.

The Conway-Maxwell-Poisson (COM-Poisson) distribution is a two parametric generalized form of the conventional Poisson distribution, promptly growing distribution in several areas of research used to study the count data for under-dispersion and over-dispersion scenario [38]. Due to some nice properties of the COM-Poisson distribution such as it is the special case of Bernoulli and geometric distribution, it has attracted the consideration of several authors during the last 20 years. Shmueli, et al. [39] described its statistical and the probabilistic properties for the count data and suggested three methods for estimation of parameters. COM-Poisson distribution has more applications in queuing modeling, quality control, reliability, modeling ionization statistics if the data is of the form log-concave and flexible enough to model under and over dispersed count data. Aslam, et al. [40] developed a multiple dependent state sampling chart under COM-Poisson distribution. Aslam, et al. [41] designed a chart for exponentially weighted moving averages using the COM-Poisson distribution. Mashuri [42] developed a control chart for fuzzy bivariate data.

Aslam and Al-Marshadi [43] proposed the control chart for COM-Poisson distribution under neutrosophic statistics using the single sampling scheme. By exploring the literature and according to the best of our knowledge, there is no work on the control chart for COM-Poisson distribution under neutrosophic statistics using the RGS. The main aim is to propose an efficient control chart to minimize the non-conforming product from the production process. The present manuscript is constructed for describing a new repetitive sampling chart technique for the neutrosophic statistics using the COM-Poisson distribution. We will compare the efficiency of the proposed control chart with the existing chart in terms of neutrosophic average run length. It is expected that the proposed chart will perform better than the existing chart under uncertain environment. This paper is organized as: the design of repetitive control chart for NCOM-Poisson distribution is described in Section 2. The advantages of the suggested chart have been described in Section 3. In Section 4 a simulation study is provided. The practical application of the suggested chart has been discussed in Section 5 and in the end, concluding remarks have been added in Section 6.

2. Repetitive control chart based on NCOM-Poisson distribution

Sellers, et al. [38] proposed a conventional COM-Poisson distribution and the generalization to conventional COM-Poisson distribution is developed by Aslam and Al-Marshadi [43] called as neutrosophic COM-Poisson (NCOM-Poisson) distribution. The neutrosophic probability mass function (npmf) of NCOM-Poisson distribution is defined by (see [43]:

$$P(Y_N = y_N; \mu_N, \nu_N) = \frac{\mu_N^{y_N}}{(Y_N!)^{\nu} Z_N(\mu_N, \nu_N)}; Y_N \in \{Y_L, Y_U\} \quad \text{for } y_N = [0,0], [1,1], [2,2], \ldots \tag{1}$$

where $\mu_N \in \{\mu_L, \mu_U\}$ is a neutrosophic scale parameter and $\nu_N \in \{\nu_L, \nu_U\}$ is the neutrosophic dispersion parameter, while $z_N(\mu_N, \nu_N) = \sum_{j_N}^{\infty} \frac{\mu_N^{y_N}}{(j_N!)^{\nu_N}}$ where $j_N = [0,0], [1,1], [2,2], \ldots$ is a neutrosophic normalization constant. The NCOM-Poisson distribution tends to the classical COM-Poisson distribution when no uncertainty is present in the sample of the population. The NCOM-Poisson distribution reduced to neutrosophic Poisson distribution if $\nu_N \in \{1,1\}$, neutrosophic geometric distribution if $\nu_N \in \{0,0\}$ and Bernoulli distribution if $\nu_N \in \{\infty,\infty\}$. For more information on neutrosophic discrete distributions, readers should see [44]. The mean and variance for the NCOM-Poisson distribution can be stated as

$$\mu_{Y_N} = \mu_N^{1/\nu_N} - \frac{\nu_N - 1}{2\nu_N}; \mu_N \in \{\mu_L, \mu_U\}, \nu_N \in \{\nu_L, \nu_U\} \tag{2}$$

$$\sigma_{Y_N}^2 = \frac{\mu_N^{1/\nu_N}}{\nu_N}; \mu_N \in \{\mu_L, \mu_U\}, \nu_N \in \{\nu_L, \nu_U\} \tag{3}$$

The following repetitive chart using the neutrosophic statistic can be stated as:

Step 1: Choose an item randomly from the manufacturing process with the quality characteristic $Y_N \in \{Y_L, Y_U\}$ and record the number of non-conformities say $Y_{tN} \in \{Y_{tL}, Y_{tU}\}$.

Step 2: If $Y_{tN} \geqslant UCL_{1N}$ or $Y_{tN} \leqslant LCL_{1N}$ declare the process as out-of-control and if $LCL_{2N} \leqslant Y_{tN} \leqslant UCL_{2N}$; $Y_{tN} \in \{Y_{tL}, Y_{tU}\}$ then declare the process as in-control. Otherwise, repeat the process.

Where LCL_{1N} and UCL_{1N} are lower and upper outer control limits; whereas, LCL_{2N} and UCL_{2N} are lower and upper inner control limits for the neutrosophic statistical interval method accordingly.

The outer control limits are given by

$$LCL_{1N} = \mu_{Y_N} - k_{1N}\sigma_{Y_N} = \left(\mu_N^{1/\nu_N} - \frac{\nu_N - 1}{2\nu_N}\right) - k_{1N}\left(\frac{\mu_N^{1/\nu_N}}{\nu_N}\right)^{1/2}; k_{1N} \in \{k_{1L}, k_{1U}\} \tag{4}$$

$$UCL_{1N} = \mu_{Y_N} + k_{1N}\sigma_{Y_N} = \left(\mu_N^{1/\nu_N} - \frac{\nu_N - 1}{2\nu_N}\right) + k_{1N}\left(\frac{\mu_N^{1/\nu_N}}{\nu_N}\right)^{1/2}; k_{1N} \in \{k_{1L}, k_{1U}\} \tag{5}$$

Also,

$$LCL_{2N} = \mu_{Y_N} - k_{2N}\sigma_{Y_N} = \left(\mu_N^{1/\nu_N} - \frac{\nu_N - 1}{2\nu_N}\right) - k_{2N}\left(\frac{\mu_N^{1/\nu_N}}{\nu_N}\right)^{1/2}; k_{2N} \in \{k_{2L}, k_{2U}\} \quad (6)$$

$$UCL_{2N} = \mu_{Y_N} + k_{2N}\sigma_{Y_N} = \left(\mu_N^{1/\nu_N} - \frac{\nu_N - 1}{2\nu_N}\right) + k_{2N}\left(\frac{\mu_N^{1/\nu_N}}{\nu_N}\right)^{1/2}; k_{2N} \in \{k_{2L}, k_{2U}\} \quad (7)$$

where $k_{1N} \in \{k_{1L}, k_{1U}\}$ and $k_{2N} \in \{k_{2L}, k_{2U}\}$ are the neutrosophic control chart coefficient and are obtained by using in-control neutrosophic average run lengths (NARLs).

The suggested chart for NCOM-Poisson distribution for neutrosophic statistics is a generalization of the NCOM-Poisson distribution under the neutrosophic statistics and reduces to single sampling plan when $k_{1N} = k_{2N} = k_N$. The chart based upon NCOM-Poisson distribution using neutrosophic statistics given by Aslam and Al-Marshadi [43] is a generalization of classical chart which utilized COM-Poisson distribution developed by Sellers, et al. [38].

Assume that the neutrosophic mean of the in-control process is $\mu_{0N} \in \{\mu_{0L}, \mu_{0U}\}$. The process is said to be out-of-control if $Y_{tN} \geqslant UCL_{1N}$ or $Y_{tN} \leqslant LCL_{1N}$. The probability of the out-of-control the process when actually it is not out-of-control, then

$$P_{0N}^{out} = P\{Y_{tN} \geqslant UCL_{1N} | \mu = \mu_{0N}\} + P\{Y_{tN} \leqslant LCL_{1N} | \mu = \mu_{0N}\}$$

$$= P\{(Y_{tN} - \mu_{Y_N})/\sigma_{Y_N} \geqslant (UCL_{1N} - \mu_{Y_N})/\sigma_{Y_N}\} + P\{(Y_{tN} - \mu_{Y_N})/\sigma_{Y_N} \leqslant (LCL_{1N} - \mu_{Y_N})/\sigma_{Y_N}\}$$

$$= P\{Z_{tN} \geqslant k_{1N}\} + P\{Z_{tN} \leqslant -k_{1N}\}$$

$$= 2[1 - \Phi(k_{1N})] \quad (8)$$

where $\Phi()$ is the neutrosophic cumulative standard normal distribution and $Z_{tN} = (Y_{tN} - \mu_{Y_N})/\sigma_{Y_N}$ is the neutrosophic standard normal variable, for more details see [22] and [45].

Then the probability of repetition (P_{0N}^{rep}) is given as

$$P_{0N}^{rep} = P\{UCL_{2N} \leqslant Y_{tN} \leqslant UCL_{1N} | \mu = \mu_{0N}\} + P\{LCL_{1N} \leqslant Y_{tN} \leqslant LCL_{2N} | \mu = \mu_{0N}\}$$

$$= P\{Z_{tN} \leqslant k_{1N}\} - P\{Z_{tN} \leqslant k_{2N}\} + P\{Z_{tN} \leqslant -k_{2N}\} - P\{Z_{tN} \leqslant -k_{1N}\}$$

$$= \Phi(k_{1N}) - \Phi(k_{2N}) + \Phi(-k_{2N}) - \Phi(-k_{1N})$$

$$= 2[\Phi(k_{1N}) - \Phi(k_{2N})]. \quad (9)$$

Therefore, the probability that it is out-of-control ($P_{0N.out}$) may be defined as

$$P_{0N.out} = \frac{P_{0N}^{out}}{1 - P_{0N}^{rep}}. \quad (10)$$

In the above expressions, the superscript 0 shows in-control process. According to the literature of the control chart, the performance of any suggested chart can be evaluated by calculating its Average Run Length (ARL). The ARL may be defined as the average number of samples before the process indicates an out-of-control or the deteriorated signal. The ARL of the suggested control chart can be calculated using the formula.

$$ARL_{0N} = \frac{1}{P_{0N.out}}; ARL_{0N} \in \{ARL_{0L}, ARL_{0U}\}. \quad (11)$$

Assume that the process neutrosophic parameter $\mu_{0N} \in \{\mu_{0L}, \mu_{0U}\}$ is shifted to $\mu_{1N} = \delta\mu_{0N}$ where δ be a shift constant and $\mu_{1N} \in \{\mu_{1L}, \mu_{1U}\}$. Then, the probability of out-of-control process for a single sample when a shift of moderate size is introduced may be given as

$$P_{1N}^{out} = P\{Y_{tN} \geqslant UCL_{1N} | \mu = \mu_{1N}\} + P\{Y_{tN} \leqslant LCL_{1N} | \mu = \mu_{1N}\}$$

Table 1
The NARLs of proposed chart when \; $\nu_N \in [0.4, 0.6]$ and $\mu_N \in [2.5, 3.5]$.

k_{1N}	[2.9507,3.2403]	[3.0408,3.1851]	[3.0498,3.0709]
k_{2N}	[0.8980,0.3022]	[1.0480,0.5720]	[1.4214,1.2464]
c	NARL		
1.0000	[200.01,200.00]	[300.01,300.00]	[308.37, 327.44]
1.0125	[167.71,174.15]	[249.39,262.44]	[249.79, 284.91]
1.0250	[136.78,148.79]	[201.89,225.38]	[198.07, 244.44]
1.0375	[109.25,125.15]	[159.55,190.62]	[154.86, 207.28]
1.0500	[86.04,103.99]	[124.75,159.32]	[120.64, 174.86]
1.0625	[67.22,85.68]	[95.57,132.01]	[92.82, 146.26]
1.0750	[52.35,70.19]	[73.32,108.77]	[34.56, 70.01]
1.1250	[19.83,31.31]	[26.68,49.46]	[21.02, 49.87]
1.1500	[12.71,21.21]	[16.60,33.72]	[9.82, 25.08]
1.2000	[5.85,10.34]	[7.45,16.49]	[4.47,14.62]
1.2500	[3.18,5.56]	[3.85,8.72]	[2.62,8.30]
1.3000	[2.05,3.35]	[2.36,5.07]	[2.00,5.74]
1.3500	[1.28,2.28]	[1.36,3.24]	[1.53,3.95]
1.4000	[1.08,1.72]	[1.11,2.29]	[1.30,2.55]
1.4500	[1.02,1.42]	[1.03,1.77]	[1.17,2.03]
1.5000	[1.00,1.25]	[1.00,1.47]	[1.05,1.45]
1.6000	[1.00,1.10]	[1.00,1.19]	[1.00,1.10]
1.8000	[1.00,1.02]	[1.00,1.03]	[1.00,1.05]
1.9000	[1.00,1.00]	[1.00,1.01]	[1.00,1.02]
2.0000	[1.00,1.00]	[1.00,1.00]	[308.37, 327.44]

Table 2
The NARLs of proposed chart when \; $\nu_N \in [0.9, 1.1]$ and $\mu_N \in [3.5, 4.5]$.

k_{1N}	[3.1682,3.2172]	[3.0897,2.9829]	[3.1170,3.0570]
k_{2N}	[0.3923,0.3286]	[0.8398,1.4527]	[0.9821,1.3554]
c	NARL		
1.0000	[200.00,200.00]	[300.00,300.00]	[370.00,370.00]
1.0125	[183.72,186.22]	[276.85,282.53]	[341.18,347.41]
1.0250	[167.82,172.56]	[254.13,265.00]	[312.91,324.82]
1.0375	[152.54,159.22]	[232.16,247.66]	[285.61,302.54]
1.0500	[138.05,146.36]	[211.22,230.71]	[259.61,280.84]
1.0625	[124.49,134.09]	[191.51,214.31]	[235.15,259.92]
1.0750	[111.92,122.52]	[173.13,198.59]	[212.39,239.96]
1.1250	[71.85,83.81]	[113.63,144.02]	[138.86,171.33]
1.1500	[57.36,68.93]	[91.68,121.98]	[111.82,143.98]
1.2000	[36.80,46.66]	[59.97,87.43]	[72.88,101.61]
1.2500	[24.08,31.94]	[39.88,63.12]	[48.30,72.28]
1.3000	[16.19,22.27]	[27.13,46.17]	[32.75,52.14]
1.3500	[11.22,15.87]	[18.94,34.32]	[22.78,38.25]
1.4000	[8.03,11.57]	[13.58,25.95]	[16.26,28.56]
1.4500	[5.94,8.65]	[10.00,19.96]	[11.91,21.72]
1.5000	[4.54,6.63]	[7.55,15.61]	[8.95,16.80]
1.6000	[2.91,4.18]	[4.65,10.02]	[5.43,10.57]
1.8000	[1.66,2.18]	[2.33,4.91]	[2.62,5.01]
1.9000	[1.41,1.76]	[1.86,3.71]	[2.04,3.74]
2.0000	[1.27,1.51]	[1.57,2.93]	[1.69,2.93]

Table 3
The NARLs of proposed chart when \; $\nu_N \in [0.36, 0.37]$ and $\mu_N \in [2.85, 2.90]$.

k_{1N}	[2.9443,2.9170]	[3.0635,3.0194]	[3.2757,3.2942]
k_{2N}	[0.9228,1.0434]	[0.9423,1.1681]	[0.5084,0.4735]
c	NARL		
1.0000	[200.00,200.00]	[300.00,300.00]	[370.00,370.00]
1.0125	[161.79,163.61]	[239.04,242.65]	[285.58,287.18]
1.0250	[123.46,126.76]	[179.00185.40]	[205.55,208.46]
1.0375	[90.61,94.67]	[128.68,136.33]	[141.49,144.88]
1.0500	[65.11,69.24]	[90.54,98.18]	[95.26,98.47]
1.0625	[46.4350.23]	[63.24,70.14]	[63.7466.47]
1.0750	[33.16,36.45]	[44.27,50.16]	[42.82,45.00]
1.1250	[9.59,11.06]	[11.87,14.40]	[10.00,10.73]
1.1500	[5.67,6.62]	[6.76,8.37]	[5.45,5.86]
1.2000	[2.52,2.92]	[2.80,3.46]	[2.22,2.36]
1.2500	[1.55,1.72]	[1.62,1.91]	[1.38,1.43]
1.3000	[1.21,1.29]	[1.23,1.36]	[1.13,1.15]
1.3500	[1.08,1.72]	[1.09,1.15]	[1.04,1.05]
1.4000	[1.03,1.29]	[1.03,1.06]	[1.01,1.02]
1.4500	[1.01,1.12]	[1.01,1.02]	[1.00,1.00]
1.5000	[1.00,1.05]	[1.00,1.01]	[1.00,1.00]
1.6000	[1.00,1.02]	[1.00,1.00]	[1.00,1.00]
1.8000	[1.00,1.01]	[1.00,1.00]	[1.00,1.00]
1.9000	[1.00,1.00]	[1.00,1.00]	[1.00,1.00]
2.0000	[1.00,1.00]	[1.00,1.00]	[1.00,1.00]

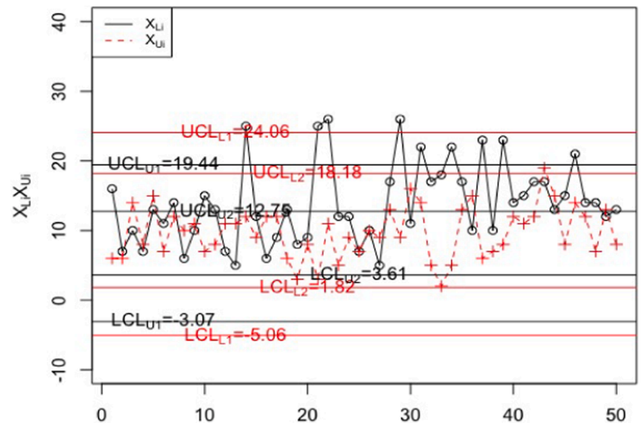

Fig. 1. Proposed chart for simulation data set.

$$=1-\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}+k_{1N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right)+\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}-k_{1N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right). \quad (12)$$

Again the probability of out-of-control process for repetitive sampling when a shift of moderate size is introduced may be given as $P_{1N}^{rep} = P\{UCL_{2N}\leqslant Y_{tN}\leqslant UCL_{1N}|\mu=\mu_{1N}\}+P\{LCL_{1N}\leqslant Y_{tN}\leqslant LCL_{2N}|\mu=\mu_{1N}\}$

$$=\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}+k_{1N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right)-\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}+k_{2N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right)$$
$$+\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}-k_{2N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right)-\Phi\left(\frac{(1-\delta^{1/\nu_N})\mu_{0N}^{1/\nu_N}-k_{1N}\sqrt{\left(\mu_{0N}^{1/\nu_N}/\nu_N\right)}}{\sqrt{\left(\delta^{1/\nu_N}\mu_{0N}^{1/\nu_N}/\nu_N\right)}}\right). \quad (13)$$

Therefore, the probability under a repetitive sampling of the shifted process when a shift is being used is defined as

$$P_{1N.out}=\frac{P_{1N}^{out}}{1-P_{1N}^{rep}}. \quad (14)$$

The NARL for the shifted process is given as follows:

$$ARL_{1N}=\frac{1}{P_{1N.out}};ARL_{1N}\in\{ARL_{1L},ARL_{1U}\}. \quad (15)$$

3. Advantages of the suggested chart

Under this Section the advantages of the suggested chart have been described. Tables 1–3 have been generated for the above-developed methodology an R-language the program was run to calculate the average run lengths for different process settings νN[0.4,0.6] and μN [2.5,3.5], νN[0.9,1.1] and μN[3.5,4.5] and νN[0.36,0.37] and μN [2.85,2.90] using different shift levels 1.0000, 1.0125, 1.0250, 1.0375,

Table 4
The NARLs of proposed chart when \; $\nu_N \in [0.4, 0.6]$ and $\mu_N \in [2.5, 3.5]$.

existing chart				Proposed chart		
c	NARL					
1.000	[200.02, 200.55]	[300.24, 302.03]	[370.03, 372.73]	[200.01, 200.00]	[300.01, 300.00]	[370.00, 370.00]
1.0125	[172.25, 181.78]	[255.28, 271.45]	[312.54, 333.53]	[167.71, 174.15]	[249.39, 262.44]	[308.37, 327.44]
1.0000	[144.95, 162.56]	[211.69, 240.42]	[257.22, 293.95]	[136.78, 148.79]	[201.89, 225.38]	[249.79, 284.91]
1.0125	[119.88, 143.74]	[172.33, 210.39]	[207.70, 255.84]	[109.25, 125.15]	[159.55, 190.62]	[198.07, 244.44]
1.0250	[98.03, 125.99]	[138.64, 182.38]	[165.69, 220.54]	[86.04, 103.99]	[124.75, 159.32]	[154.86, 207.28]
1.0375	[79.67, 109.73]	[110.85, 157.05]	[131.38, 188.80]	[67.22, 85.68]	[95.57, 132.01]	[120.64, 174.86]
1.0500	[64.63, 95.16]	[88.50, 134.65]	[104.03, 160.92]	[52.35, 70.19]	[73.32, 108.77]	[92.82, 146.26]
1.0625	[28.95, 53.30]	[37.41, 72.18]	[42.70, 84.35]	[19.83, 31.31]	[26.68, 49.46]	[34.56, 70.01]
1.0750	[20.12, 40.26]	[25.36, 53.43]	[28.58, 61.81]	[12.71, 21.21]	[16.60, 33.72]	[21.02, 49.87]
1.1250	[10.26, 23.88]	[12.83, 30.57]	[14.41, 34.72]	[5.85, 10.34]	[7.45, 16.49]	[9.82, 25.08]
1.1500	[6.28, 15.03]	[7.31, 18.65]	[7.92, 20.84]	[3.18,5.56]	[3.85,8.72]	[4.47,14.62]
1.2000	[4.09, 10.02]	[4.63, 12.10]	[4.93, 13.34]	[2.05,3.35]	[2.36,5.07]	[2.62,8.30]
1.2500	[2.90,7.05]	[3.20, 8.31]	[3.37, 9.05]	[1.28,2.28]	[1.36,3.24]	[2.00,5.74]
1.3000	[2.21, 5.20]	[2.38, 6.00]	[2.48, 6.46]	[1.08,1.72]	[1.11,2.29]	[1.53,3.95]
1.3500	[1.78, 4.00]	[1.89, 4.53]	[1.95, 4.83]	[1.02,1.42]	[1.03,1.77]	[1.30,2.55]
1.4000	[1.51, 3.19]	[1.58, 3.55]	[1.62, 3.76]	[1.00,1.25]	[1.00,1.47]	[1.17,2.03]
1.4500	[1.22, 2.23]	[1.25, 2.41]	[1.26, 2.51]	[1.00,1.10]	[1.00,1.19]	[1.05,1.45]
1.5000	[1.03, 1.43]	[1.03, 1.49]	[1.04, 1.52]	[1.00,1.02]	[1.00,1.03]	[1.00,1.10]
1.6000	[1.01, 1.25]	[1.01, 1.29]	[1.01, 1.31]	[1.00,1.00]	[1.00,1.01]	[1.00,1.05]
1.8000	[1.00, 1.15]	[1.00, 1.17]	[1.00, 1.18]	[1.00,1.00]	[1.00,1.00]	[1.00,1.02]

1.0500, 1.0625, 1.0750, 1.1250, 1.1500, 1.2000, 1.2500, 1.3000, 1.3500, 1.4000, 1.4500, 1.5000, 1.6000, 1.8000, 1.9000, and 2.0000. Table 4 has been presented for evaluating the working of the suggested chart with the existing [43]. On comparison, it can be observed that the suggested chart is efficient and quick in indicating the out-of-control situation of the process. For example, a shift level of 1.0500 is detected after 98.03 samples using the existing chart while the same shift is detected after 86.04 samples by the proposed methodology. The same pattern can be observed for other levels of shifts.

4. Simulation study

Generally, control charts are applied for monitoring the unusual changes in the manufacturing process. The best control chart is that which has performed efficiently. So, in this section, the suggested chart performance is discussed for checking the efficiency over the counterpart chart introduced by [43] by using the simulated data created from NCOM-Poisson distribution. Using simulated data, we assume that $ARL_{0N} \in [370, 370]$, $\nu_N \in [0.4, 0.6]$ and $\mu_{0N} \in [2.5, 3.5]$. The 50 observations are generated by using the NCOM-Poisson with designed parameters values, first 20 observations are generated by assuming an in-control process and next 30 observations are created by assuming the process is out-of-control with shifted values 1.2. For calculating the control limits of the proposed chart, the control limits constants are got from Table 1 3.0498, 3.0709$k_{1N} \in []$ and $k_{2N} \in [1.4214, \; 1.2464]$. The calculated NARL from Table 1 is $ARL_{1N} \in [4.47, 14.62]$, so the first out-of-control shift will be detected between the 4th and the 14th sample. We have plotted the values of X_{Ni} for the suggested chart in Fig. 1 and for the existing chart in Fig. 2. On observing Fig. 1, it is noted that the first shift detection is at sample 14th for the proposed chart. The proposed chart detects a total of 4 out-of-control shifts, but the existing chart just detects 2 shifts that can be seen in Fig. 2. The existing chart detects the first out-of-control shift at sample 22nd. After a comparison of both control charts for simulated data set, it is clear that the suggested chart gives out-of-control signal between the values of NARL in the interval of indeterminacy but the existing chart did not show out-of-control signal between the intervals. Although, from Fig. 1, there are some values that lie between the repetitive part and all these values are also need more attention and care of the engineers. Other than that, the suggested chart perceives very quickly the out-of-control signal as compared to the counterpart chart scheme. So the quick recognition ability of any chart assists the engineers to recognize the source of variation which reduced the non-conforming items from the monitoring process.

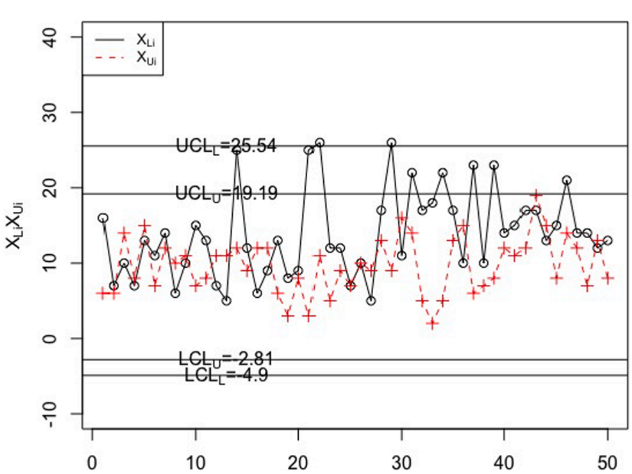

Fig. 2. Existing chart for simulation data set.

5. Applications for circuit boards product

The practical use of the suggested chart is presented in a famous electrical company situated in Saudi Arabia. This electrical group produces circuit boards known as (PCB). The PCB production process is very

Table 5
Data of a well-known electrical company situated in Saudi Arabia.

Sample No	Non-conformities	Sample No	Non-conformities
1	[1,1]	16	[3,3]
2	[2,2]	17	[5,5]
3	[3,3]	18	[5,5]
4	[3,3]	19	[4,4]
5	[1,1]	20	[6,6]
6	[1,1]	21	[5,5]
7	[8,9]	22	[7,7]
8	[2,2]	23	[5,5]
9	[5,5]	24	[8,8]
10	[11,11]	25	[2,2]
11	[2,3]	26	[5,6]
12	[1,1]	27	[6,6]
13	[0,0]	28	[8,9]
14	[2,2]	29	[3,3]
15	[5,5]	30	[7,7]

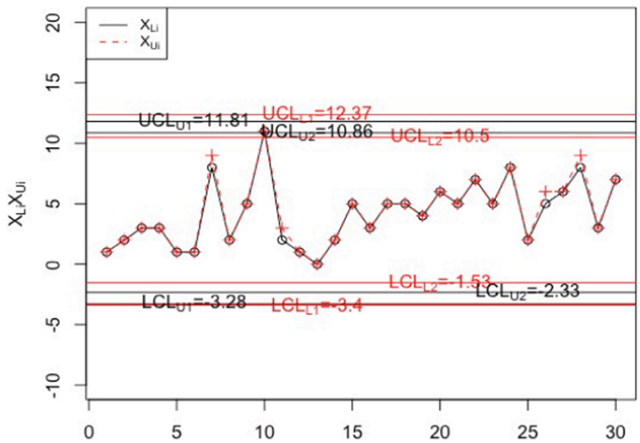

Fig. 3. The proposed chart for real data set.

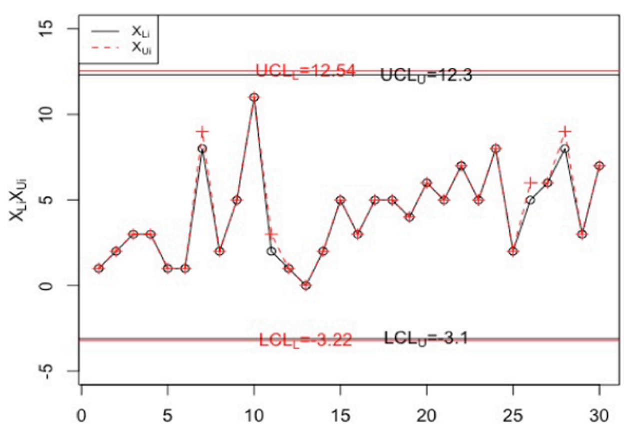

Fig. 4. The existing chart for real data set.

complex, and engineers are unable to find the correct proportion of defective items and also non-conformities in a sample. Due to these conditions the classical control chart is not suitable or not possible to apply for checking the process performance. So, the proposed chart is best for the process of monitoring and detecting the non-conformities. This electrical unit decided to adopt the suggested chart when $ARL_{0N} = [370, 370]$ with 100 sample sizes, then using NCOM-Poisson distribution with\; $\nu_N \in [0.634222, 0.657101]$ and $\mu_{0N} \in [2.5485, 2.5485]$ and is described in Table 5.

The data of non-conformities are plotting in Fig. 3 for the suggested chart and Fig. 4 for the existing chart proposed by [43]. On observing Fig. 3, all observations fall in the portion of in-control but point 10 lies in the repetitive part of the proposed chart and very close to the control limits. At point 10 the PCB non-conformities number is 11 that is very close to the out-of-control the region which is 12.36 and under repetitive control region which is 10.86. While in Fig. 4 all observations fall in the in-control portion and no point is closer to the control limit as compared to the proposed chart. Meanwhile, the counterpart chart doesn't expose that the interval of indeterminacy in the fraction parameter and control limits. So, we can say that the proposed chart's performance is better as compared to the counterpart chart. The proposed chart is more beneficial to be applied and adaptable to be used in an uncertain environment. On comparing the suggested chart and existing chart, it is noticed that the suggested chart shows that there are some issues in monitoring the process that should be identified.

6. Conclusions

In this article a repetitive sample scheme has been presented for the efficient monitoring of uncertain observations for COM-Poisson distribution. Two pairs of control limits have been generated using the neutrosophic interval method by considering specific average run lengths. A comparison study has been presented for describing the benefits of the suggested chart. It is concluded that the suggested chart is efficient, adequate, and flexible when applied under appropriate conditions. The proposed control chart using a cost model can be extended for future research. The proposed control chart using the mixed sampling scheme can be considered as future research.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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