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Monitoring Non-Conforming Products Using Multiple Dependent State Sampling Under Indeterminacy-An Application to Juice Industry

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ABSTRACT In this article a new *np* control chart for the multiple dependent state sampling using the neutrosophic statistics has been introduced for the efficient monitoring of the number of defective items in any production process or the customer services agencies. The coefficients of the control limits of the proposed control chart have been determined using the neutrosophic algorithms. The efficiency of its quick performance has been determined by computing the neutrosophic average run lengths with respect to different false alarm rates under different process settings at different process shift levels. The comparison with the existing counterparts for the quick detecting ability of the proposed chart has also been conducted. The practical application of the proposed chart has been elucidated using a real-world example.

INDEX TERMS Attribute control chart, multiple dependent state sampling, neutrosophic statistics, average run length, indeterminacy.

I. INTRODUCTION

The control chart is historic and broadly utilized the methodology of the statistical process control to control the production of defective items or the monitoring of bad repute of any services providing company [1]. Quality characteristics for any type of goods and services can be broadly classified as an attribute and the variable items. Attribute quality characteristic is adopted for the production process when the produced items are bifurcated on the criteria as the good or defective, life or dead, go or not go, conforming or not conforming satisfy or not satisfy, etc. Firstly, the attribute quality characteristic is practically easy to use and secondly, a smaller sample size provides sufficient information to reach the decision [2]. Chan *et al.* [3] developed a monitoring scheme for nonconforming items using the cumulative counts by calculating optimal probabilities of false alarm. Epprecht, *et al.* [4] developed an adaptive attribute chart for one, two, or three parameters to choose an effective model. Rudisill, *et al.* [5] investigated the attribute charts using the Poisson distribution for observing the smaller number of nonconforming items. Chiu and Kuo [6] proposed an attribute chart for multivariate Poisson distribution. Niaki and Abbasi [7] developed the multi-attribute control chart when these attributes are correlated. Lee, *et al.* [8] investigated the np chart using the large subgroup size with executed statistical constraints.

The aforementioned chart is well-presented using the multiple dependent state sampling (MDS) instead of the conventional single sampling scheme Hsu, *et al.* [9]. Due to its broad spectrum of the production process, it has attracted the attention of several researchers during the last few years. Conventional sampling schemes are applied only to observe the current status of the product whether it is defective or non-defective but the MDS sampling scheme keeps the history of the product to declare the process about its production of defective or non-defective items. Zhou, *et al.* [10] suggested that MDS sampling improves the performance of the *np* chart because the Shewhart type charts have the tendency of lacking the responsiveness towards small changes in the production process. Reference [11] claimed that the efficiency of the MDS sampling plan is manifold as compared to a single sampling scheme as the lot sentencing

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is based upon not only the current lot information but also considers the information of the already submitted lots. Several SPC researchers explored the MDS sampling and declared it an efficient scheme including Wu and Wang [11], Aslam, *et al.* [12], [13]–[20].

It is not very uncommon for the researchers to face the uncertain data in real-life situations..Smarandache [21] float the concept of neutrosophic statistics, which is the generalization of the traditional statistics, to analyze the uncertain, unclear, vague, and incomplete data. Smarandache [22] enhanced the fuzzy logic due to its limitations and suggested a neutrosophic set to deal with the doubted situations. Khan, *et al.* [23] designed the dispersion chart using the standard deviation for the neutrosophic statistics Aslam and Albassam [24] presented the post hoc tested for the neutrosophic statistics. Albassam, *et al.* [25] designed a technique to study the village population of the USA under the neutrosophic statistics. Zhang, *et al.* [26] expressed the general symmetry of singular neutrosophic extended group under the neutrosophic logic. Aslam, *et al.* [27] developed a chart for the neutrosophic statistics under the gamma distribution. Aslam, *et al.* [28] developed a new attribute Shewhart chart under the neutrosophic interval method. Abdel-Basset *et al.* [29] developed criteria for project selection using a hybrid neutrosophic environment. Rich material is available for further reading on neutrosophic statistics for instance [30]–[46]. Motivated by the neutrosophic statistics the current article is aimed to develop an attribute np chart using the MDS sampling scheme.

In this paper, we will explain an efficient monitoring scheme for attribute data using the neutrosophic interval method to determine the control chart coefficients and to calculate the neutrosophic average run lengths under different process settings. The rest of the paper is organized as the design of the suggested chart is explained in the second section. In section three the advantages of the proposed methodology have been discussed. The comparison of the proposed chart with the existing *np* attribute neutrosophic chart has been given in section four. A simulated example has been included and discussed in section five.

II. DESIGN AND METHODOLOGY OF THE PROPOSED CHART

In this section, the methodology of the *np* chart using the MDS sampling under the neutrosophic statistics is elaborated. Let the neutrosophic probability $p_N \epsilon [p_L, p_U]$ be the neutrosophic proportion of non-conforming items which fail to meet the specified standard criteria. Further, suppose that the neutrosophic sample size be denoted by $n_N \epsilon [n_L, n_U]$. We are dealing with the data consisting of uncertainty or fuzziness so the parameters of the attribute control chart can be determined using the neutrosophic statistics. Here we suppose that the neutrosophic probability that $p_N \epsilon [p_L, p_U]$ is the same for all $n_N \epsilon [n_L, n_U]$ and further, suppose that the non-conforming numbers are denoted by $X_N \epsilon [X_L, X_U]$. According to [47] the above-mentioned conditions support to use the well-known binomial probability distribution based upon the neutrosophic statistics which can be defined as

$$\mathrm{P_N}(x_N; n_N, \mathrm{p_N}) = \binom{n_N}{x_N} \mathrm{p_N}^{x_N} (1-\mathrm{p_N})^{n_N - x_N}, \quad x_N = 0, 1, \ldots, n_N \qquad (1)$$

TABLE 1. The NARL of the proposed chart for $n\epsilon$[50, 50] and $m\epsilon$[2, 4].

k_{1N}	[4.377,5.773]	[3.779,4.647]	[3.69,3.995]
k_{2N}	[1.794,2.201]	[2.083,2.151]	[2.16,2.307]
p_{0N}	[0.202,0.211]	[0.224,0.238]	[0.21,0.221]
c	NARL	NARL	NARL
0	[200.29,204.47]	[300.37,310.48]	[380.07,376.74]
0.01	[188.72,190.69]	[292.55,294.69]	[354.57,344.48]
0.02	[176.3,176.11]	[281.12,275.7]	[327.49,311.78]
0.05	[137.71,132.45]	[232.28,210.03]	[245.96,220.78]
0.08	[102.86,95.17]	[176.76,148.33]	[176.1,150.04]
0.10	[83.49,75.39]	[142.99,114.93]	[138.99,114.92]
0.20	[28.94,23.95]	[44.55,31.44]	[42.52,31.99]
0.30	[11.50,9.26]	[15.46,10.64]	[15.28,11.25]
0.40	[5.53,4.53]	[6.64,4.76]	[6.78,5.13]
0.50	[3.18,2.73]	[3.52,2.72]	[3.66,2.94]
0.60	[2.13,1.93]	[2.23,1.88]	[2.33,2.02]
0.70	[1.61,1.53]	[1.63,1.48]	[1.70,1.57]
0.80	[1.34,1.31]	[1.33,1.26]	[1.38,1.33]
0.90	[1.19,1.18]	[1.18,1.14]	[1.21,1.19]
0.95	[1.14,1.13]	[1.13,1.10]	[1.16,1.14]
1.00	[1.10,1.1]	[1.09,1.07]	[1.11,1.1]

TABLE 2. The NARL of the proposed chart for $n\epsilon$[100, 100] and $m\epsilon$[2, 4].

k_{1N}	[3.562,6.14]	[5.291,6.826]	[4.934,6.559]
k_{2N}	[2.06,2.067]	[2.044,2.142]	[1.939,2.303]
p_{0N}	[0.104,0.128]	[0.114,0.117]	[0.111,0.115]
c	NARL	NARL	NARL
0	[205.61,200.06]	[304.86,301.31]	[375.74,373.58]
0.01	[193.16,180.71]	[275.71,270.93]	[346.88,343.97]
0.02	[180.05,162.03]	[247.92,241.96]	[317.62,313.64]
0.05	[140.49,113.19]	[175.91,167.53]	[234.65,227.31]
0.08	[105.52,77.24]	[122.49,113.58]	[167.12,157.94]
0.10	[86.19,59.75]	[96.04,87.52]	[132.03,122.61]
0.20	[31.12,18.29]	[30.62,26.1]	[41.63,35.92]
0.30	[12.78,7.27]	[11.95,9.94]	[15.58,12.99]
0.40	[6.27,3.74]	[5.75,4.85]	[7.15,5.99]
0.50	[3.63,2.37]	[3.32,2.91]	[3.94,3.41]
0.60	[2.42,1.75]	[2.22,2.04]	[2.54,2.3]
0.70	[1.81,1.43]	[1.68,1.61]	[1.85,1.75]
0.80	[1.48,1.25]	[1.39,1.37]	[1.49,1.45]
0.90	[1.29,1.15]	[1.22,1.22]	[1.28,1.28]
0.95	[1.22,1.11]	[1.17,1.17]	[1.22,1.22]
1.00	[1.17,1.08]	[1.13,1.14]	[1.17,1.17]

TABLE 3. The NARL of the proposed chart for $n\epsilon[150, 150]$ and $m\epsilon[2, 4]$.

k_{1N}	[3.597,4.558]	[3.042,3.092]	[3.49,4.8]
k_{2N}	[1.927,2.084]	[2.222,2.43]	[2.11,2.272]
p_{0N}	[0.108,0.144]	[0.132,0.137]	[0.12,0.142]
c	NARL	NARL	NARL
0	[202.79,205.27]	[303.28,302.26]	[370.74,374.8]
0.01	[185.84,187.87]	[274.32,265.99]	[336.56,331.61]
0.02	[168.5,168.73]	[245.12,231.91]	[301.54,288.51]
0.05	[119.84,112.55]	[166.93,148.87]	[205.31,177.61]
0.08	[81.91,70.24]	[110.27,94.06]	[133.85,104.54]
0.10	[63.09,50.82]	[83.47,69.47]	[99.94,73.38]
0.20	[18.44,11.87]	[22.91,17.68]	[25.39,15.44]
0.30	[6.97,4.28]	[8.10,6.25]	[8.56,5.14]
0.40	[3.44,2.28]	[3.75,3.07]	[3.88,2.56]
0.50	[2.13,1.58]	[2.21,1.95]	[2.26,1.69]
0.60	[1.56,1.28]	[1.57,1.47]	[1.60,1.33]
0.70	[1.29,1.13]	[1.28,1.24]	[1.30,1.16]
0.80	[1.15,1.06]	[1.14,1.12]	[1.15,1.08]
0.90	[1.08,1.02]	[1.06,1.05]	[1.07,1.03]
0.95	[1.05,1.02]	[1.04,1.04]	[1.05,1.02]
1.00	[1.04,1.01]	[1.03,1.02]	[1.03,1.01]

Therefore, in this article, the following steps have been adopted to develop the *np* chart using neutrosophic statistics denoted by *Nnp* chart:

Step-I Select a random sample of size $n_N\epsilon\,[n_L, n_U]$ from any production process at each subgroup then count the defective items $D_N\epsilon\,[D_L, D_U]$.

Step-II If $\mathrm{LCL2_N} \le D_N \le UC\mathrm{L2_N}$; $\mathrm{UCL2_N}\epsilon[UCL2_L, UCL2_U]$ and $\mathrm{LCL2_N}\epsilon\,[LCL2_L, LCL2_U]$ then declare the process as in-control, otherwise, the process is straightforwardly declared as out-of-control if m_N subgroups are out-of-control.

In this way, we can state that the proposed chart is the generalization of the traditional Shewhart *np* chart. The two control limits $\mathrm{LCL1_N}\epsilon\,[LCL1_L, LCL1_U]$ is the lower control limit and $\mathrm{UCL1_N}\epsilon\,[UCL1_L, UCL1_U]$ is known as the upper control can be defined as

$$CL1_N = np_{N0} + k_{1N}\sqrt{np_{N0}\,(1-p_{N0})};\quad p_{N0}\epsilon\,[p_{L0}, p_{U0}]\,,\, k_{1N}\epsilon\,[k_{1L}, k_{1U}] \tag{2}$$

$$LCL1_N = max[0, np_{N0} - k_{1N}\sqrt{np_{N0}\,(1-p_{N0})}];\quad p_{N0}\epsilon\,[p_{L0}, p_{U0}]\,,\, k_{1N}\epsilon\,[k_{1L}, k_{1U}] \tag{3}$$

$$UCL2_N = np_{N0} + k_{2N}\sqrt{np_{N0}\,(1-p_{N0})};\quad p_{N0}\epsilon\,[p_{L0}, p_{U0}]\,,\, k_{2N}\epsilon\,[k_{2L}, k_{2U}] \tag{4}$$

$$LCL2_N = max[0, np_{N0} - k_{2N}\sqrt{np_{N0}\,(1-p_{N0})}];\quad p_{N0}\epsilon\,[p_{L0}, p_{U0}]\,,\, k_{2N}\epsilon\,[k_{2L}, k_{2U}] \tag{5}$$

where $p_{N0}\epsilon\,[p_{L0}, p_{U0}]$ is termed as the probability of the in-control process based upon the neutrosophic statistics and the term $k_{1N}\epsilon\,[k_{1L}, k_{1U}]$ and $k_{2N}\epsilon\,[k_{2L}, k_{2U}]$ are neutrosophic control limits coefficients. Then the probability-based upon the neutrosophic statistics under the in-control process at $p_N = p_{N0}\epsilon\,[p_{L0}, p_{U0}]$ is calculated as

$$\begin{aligned} P^0_{Nin}|p_{N0} &= P\,(LCL2_N \le D_N \le UCL2_N) \\ &\quad + \{P\,(LCL1_N \le D_N \le LCL2_N) \\ &\quad + P\,(UCL2_N \le D_N \le UCL1_N)\} \\ &\quad \times [P\,(LCL2_N \le D_N \le UCL2_N)]^{m_N} \end{aligned} \tag{6}$$

The probability of the in-control is given in Eq. (6) can be rewritten as

$$\begin{aligned} &P^0_{Nin}|p_{N0} \\ &= \sum\nolimits_{d_N=|LCL2_N|+1}^{|UCL2_N|} \binom{n}{d_N} p_{N0}^{d_N}\,(1-p_{N0})^{n-d_N} \\ &\quad + \left\{\sum\nolimits_{d_N=|LCL1_N|+1}^{|LCL2_N|} \binom{n}{d_N} p_{N0}^{d_N}\,(1-p_{N0})^{n-d_N} \right. \\ &\quad \left. + \sum\nolimits_{d_N=|UCL2_N|+1}^{|UCL1_N|} \binom{n}{d_N} p_{N0}^{d_N}\,(1-p_{N0})^{n-d_N} \right\} \\ &\quad \times \left[\sum\nolimits_{d_N=|LCL2_N|+1}^{|UCL2_N|} \binom{n}{d_N} p_{N0}^{d_N}\,(1-p_{N0})^{n-d_N}\right]^{m_N} \end{aligned} \tag{7}$$

TABLE 4. Comparison of the proposed chart with Shewhart *np* attribute under neutrosophic statistics and traditional Shewhart *np* chart when $n = 150$ and $NARL_{N0} = 370$.

c	Shewhart *np* chart	Shewhart *np* attribute under neutrosophic statistics	The proposed chart
	NARL		
0	370.133	[372.021,380.802]	[370.74,374.8]
0.02	302.992	[320.879,306.17]	[301.54,288.51]
0.05	219.395	[246.195,218.741]	[205.31,177.61]
0.08	157.863	[184.106,156.677]	[133.85,104.54]
0.10	127.151	[151.004,126.101]	[99.94,73.38]
0.20	47.041	[58.374,46.673]	[25.39,15.44]
0.30	20.50	[25.634,20.216]	[8.56,5.14]
0.40	10.357	[12.816,10.085]	[3.88,2.56]
0.50	5.94	[7.19,5.694]	[2.26,1.69]
0.60	3.795	[4.464,3.584]	[1.6,1.33]
0.70	2.660	[3.03,2.481]	[1.30,1.16]
0.80	2.015	[2.224,1.866]	[1.15,1.08]
0.90	1.630	[1.747,1.507]	[1.07,1.03]
0.95	1.497	[1.584,1.386]	[1.05,1.02]
1.00	1.392	[1.455,1.292]	[1.03,1.01]

The average run length (ARL) is used to evaluate the performance of the developed control chart which may be defined as the average number of samples before the process is declared as out-of-control. In this article, the ARL calculated for the neutrosophic statistics are denoted as NARL. Thus, the NARL is computed as

$$NARL_{N0} = \frac{1}{1 - P^{0}_{Nin}|p_{N0}}; \quad NARL_{N0} \epsilon \left[NARL_{N0L}, NARL_{N0U}\right] \tag{8}$$

The shift is considered as the natural inherent phenomena which may provoke any time in the running process. Therefore, let we suppose that the fraction of the defectives may exist in the process in which this fraction is shifted from $p_N = p_{N0} \epsilon \left[p_{L0}, p_{U0}\right]$ to $p_N = p_{N1} \epsilon \left[p_{L1}, p_{U1}\right]$. Then, the neutrosophic probability of the shifted the process when it is termed as in-control is written as

$$\begin{aligned} P^{1}_{Nin}|p_{N1} &= P\left(LCL2_N \leq D_N \leq UCL2_N\right) \\ &\quad + \left\{P\left(LCL1_N \leq D_N \leq LCL2_N\right) + P\left(UCL2_N \leq D_N \leq UCL1_N\right)\right\} \\ &\quad \times \left[P\left(LCL2_N \leq D_N \leq UCL2_N\right)\right]^{m_N} \end{aligned} \tag{9}$$

where $p_{N1} = p_{N0} + cp_{N0}$; $p_{N0} \epsilon \left[p_{L0}, p_{U0}\right]$.

The probability $P^{1}_{Nin}|p_{N1}$ can be written as

$$\begin{aligned} P^{1}_{Nin}|p_{N1} &= \sum_{d_N=|LCL2_N|+1}^{|UCL2_N|} \binom{n}{d_N} p_{N1}^{d_N}(1-p_{N1})^{n-d_N} \\ &\quad + \left\{\sum_{d_N=|LCL1_N|+1}^{|LCL2_N|} \binom{n}{d_N} p_{N1}^{d_N}(1-p_{N1})^{n-d_N} + \sum_{d_N=|UCL2_N|+1}^{|UCL1_N|} \binom{n}{d_N} p_{N1}^{d_N}(1-p_{N1})^{n-d_N}\right\} \\ &\quad \times \left[\sum_{d_N=|LCL2_N|+1}^{|UCL2_N|} \binom{n}{d_N} p_{N1}^{d_N}(1-p_{N1})^{n-d_N}\right]^{m_N} \end{aligned} \tag{10}$$

Then, the NARL denoted by $NARL_{N1}$ for the shifted process it is calculated as

$$NARL_{N1} = \frac{1}{1 - P^{1}_{Nin}|p_{N1}}; \quad NARL_{N1} \epsilon \left[NARL_{N1L}, NARL_{N1U}\right] \tag{11}$$

An R-code program was written using the above-mentioned methodology under different parametric settings. The control chart coefficients $k_{1N} \epsilon \left[k_{1L}, k_{1U}\right]$ and $k_{2N} \epsilon \left[k_{2L}, k_{2U}\right]$ have been determined and mentioned in Tables 1-3 under different specified prameters. For example in Table 1 the control chart coefficient k_{1N} is estimated as [4.377,5.773], [3.779,4.647]

TABLE 5. The simulated data for the proposed chart.

Sr#	1	2	3	4	5	6	7	8	9	10
DN	[25,23]	[14,20]	[22,27]	[14,20]	[22,24]	[26,15]	[19,22]	[20,19]	[15,19]	[15,13]
Sr#	11	12	13	14	15	16	17	18	19	20
DN	[16,20]	[18,28]	[17,27]	[20,21]	[14,24]	[23,23]	[20,26]	[15,18]	[17,24]	[15,20]
Sr#	21	22	23	24	25	26	27	28	29	30
DN	[19,27]	[21,30]	[18,35]	[27,29]	[22,26]	[25,21]	[15,28]	[23,29]	[16,25]	[20,37]
Sr#	31	32	33	34	35	36	37	38	39	40
DN	[21,21]	[22,23]	[19,21]	[22,42]	[19,28]	[28,24]	[19,18]	[20,19]	[20,33]	[23,25]

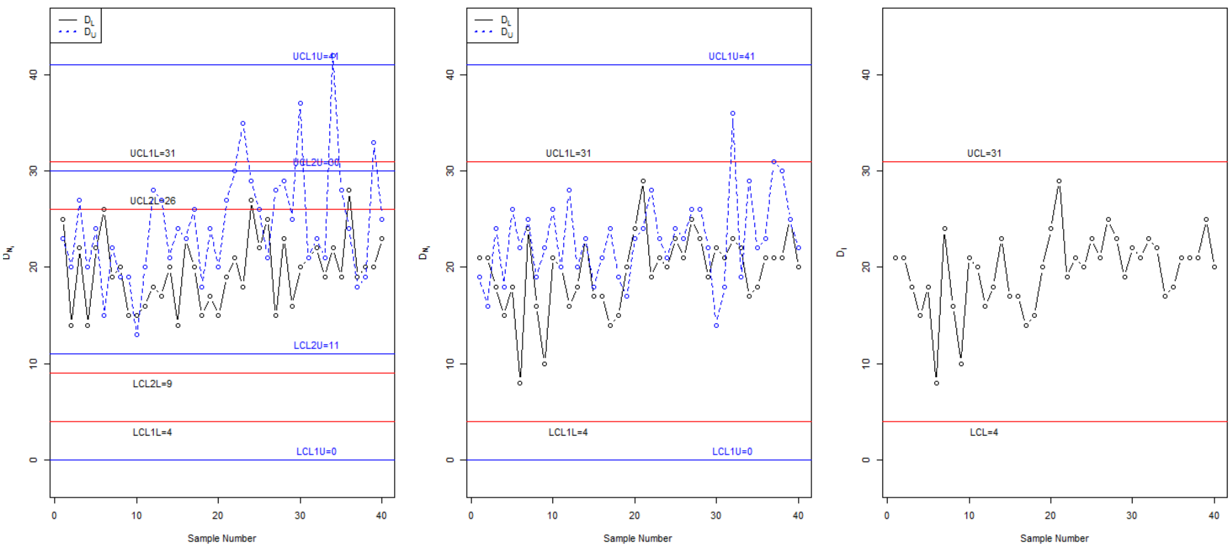

FIGURE 1. The control chart display of the proposed chart, chart by [28] and traditional Shewhart *np* chart, respectively.

and [3.69,3.995] and k_{2N} is estimated as [1.794,2.201], [2.083,2.151] and [2.16,2.307] with p_{N0} = [0.202,0.211], [0.224,0.238] and [0.21,0.221 respectively. Likewise, the NARL are determined for different specific $NARL_0 = 200$, 300 and 370 and the shifted $NARL_1$ are generated for several shift levels as 0, 0.01, 0.02, 0.05, 0.08, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 0.95 and 1.00 under different samples as 50, 100 and 150 respectively. It is worth mentioning here that the smaller $NARL_1$ values show that the process is quickly announced or declared an out-of-control process. From Table 2, for example, the proposed chart detected the out-of-control process in just [3.63, 2.37], [3.32, 2.91] and [3.94, 3.41] samples for a shift of 0.05 when specified $NARL_0$ are 200, 300 and 370 respectively.

From Table 1 through 3 it can be observed that

1. As the shift size increases the values of $NARL_1$ are going to decrease.
2. As the specified $NARL_0$ increases the values $NARL_1$ also increases for all shift levels.

III. COMPARISON IN NARL

In this section, the comparison of the proposed *Nnp* attribute chart for the MDS sampling using the neutrosophic statistics has been discussed. Table 4 has been generated for the comparison of the proposed chart with the traditional Shewhart *np* attribute chart and Shewhart *np* attribute chart based upon neutrosophic statistics proposed by [28]. From Table 4 it can be observed very easily that the proposed chart is efficient in quick detection of the out-of-control process. For instance, the proposed chart detects a shift of 0.10 from 73rd sample to 99th sample. The Shewhart *np* attribute under neutrosophic statistics detects the shift from 126th sample to 151st sample. On the other hand, the traditional Shewhart under classical statistics detects the shift at 127th sample. The same pattern can be observed at all other levels of different shifts. So it can be stated very clearly that the proposed chart is efficient in quickly detecting the out-of-control process as compared to the existing control charts.

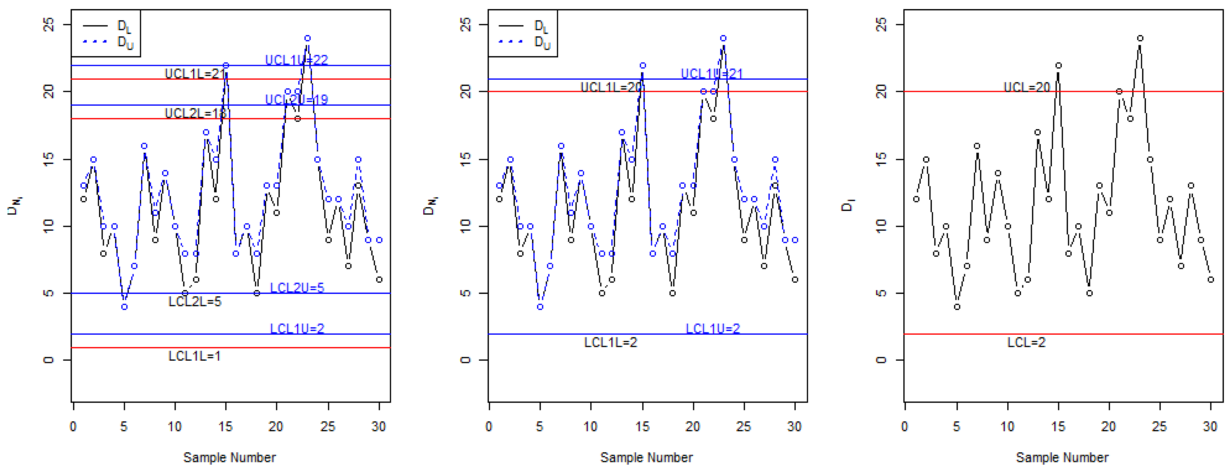

FIGURE 2. The control chart display of the proposed chart, chart by [28] and traditional Shewhart *np* chart for a real data, respectively.

TABLE 6. Neutrosophic data of juice company.

Sample number	No. of nonconforming units D_i	Sample number	No. of nonconforming units
1	[12,13]	16	[8,8]
2	[15,15]	17	[10,10]
3	[8,10]	18	[5,8]
4	[10,10]	19	[13,13]
5	[4,4]	20	[11,13]
6	[7,7]	21	[20,20]
7	[16,16]	22	[18,20]
8	[9,11]	23	[24,24]
9	[14,14]	24	[15,15]
10	[10,10]	25	[9,12]
11	[5,8]	26	[12,12]
12	[6,8]	27	[7,10]
13	[17,17]	28	[13,15]
14	[12,15]	29	[9,9]
15	[22,22]	30	[6,9]

IV. COMPARISON BY SIMULATION

The application of the proposed chart has been given using a simulated example in this section. We have used $n_N \epsilon$ [150, 150], $p_{N0}\epsilon$ [0.1195, 0.1418], $K_{1N}\epsilon$[3.4894, 4.7995] and $K_{2N}\epsilon$[2.165, 2.2717] for shift $c = 0.20$ and the specified $NARL_{N0} = 370$. The simulated values for these settings have been placed in Table 5. Figure 1 shows the three control charts. On observing Figure 1 it can be seen that the conventional chart is unable to detect the out-of-control situation. Shewhart *np* attribute under neutrosophic statistics by [28] shows only one observation at sample number 32 which falls outside the upper limit. The proposed control chart indicates that several points are outside the control limits. In addition, it can be seen that the proposed control chart shows out-of-control earlier than the two existing control charts. From this simulation, it is concluded that the proposed chart is efficient in detecting the shift in the process earlier as compared to the existing control charts.

V. APPLICATION

In this section, we will present a real-world example for the illustration and the practical application of the proposed chart. For this purpose, we used the data of the orange juice. According to [48] "Frozen orange juice concentrate is packed in 6-oz cardboard cans. These cans are formed on a machine by spinning them from cardboard stock and attaching a metal bottom panel. By inspection of a can, we may determine whether, when filled, it could possibly leak either on the side seam or around the bottom joint. Such a nonconforming can have an improper seal on either the side seam or the bottom panel''. The data has been shown in Table 6 for ready reference. For this data set the $D_N \epsilon [D_L, D_U]$ values are plotted in Figure 2. On observing Figure 2 it can be seen that the proposed chart indicates that several points are beyond the control limits. The control chart proposed by [28] shows only 4 observations in the outside of the upper control limit. The traditional Shewhart *np* shows that three points are beyond the upper limit. From this study, it is concluded that the proposed control chart is the most efficient chart as several observations fall outside the upper control limit. By comparing the three charts, the proposed control chart clearly indicates some issues in the cans process.

VI. CONCLUSION

In this article, an attribute control chart using the MDS sampling scheme for the neutrosophic statistics was proposed. The control chart coefficients have been determined for different parametric settings under different shift levels. The proposed chart detects shift quickly as compared to the existing chart. The proposed chart is an efficient addition in the literature of the control charts for uncertain and vague observations. The proposed chart can further be extended for the multivariate case as future research.

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REFERENCES

[1] C. D. Montgomery, *Introduction to Statistical Quality Control*, 6th ed. New York, NY, USA: Wiley, 2009.
[2] S. Balamurali and C.-H. Jun, "Repetitive group sampling procedure for variables inspection," *J. Appl. Statist.*, vol. 33, no. 3, pp. 327–338, Apr. 2006.
[3] L. Y. Chan, C. D. Lai, M. Xie, and T. N. Goh, "A two-stage decision procedure for monitoring processes with low fraction nonconforming," *Eur. J. Oper. Res.*, vol. 150, no. 2, pp. 420–436, Oct. 2003.
[4] E. K. Epprecht, A. F. B. Costa, and F. C. T. Mendes, "Adaptive control charts for attributes," *IIE Trans.*, vol. 35, no. 6, pp. 567–582, Jun. 2003.
[5] F. Rudisill, L. A. Litteral, and D. Walter, "Modified u charts for monitoring and controlling Poisson attribute processes," *Qual. Eng.*, vol. 16, no. 4, pp. 637–642, Jan. 2004.
[6] J.-E. Chiu and T.-I. Kuo, "Attribute control chart for multivariate Poisson distribution," *Commun. Statist.-Theory Methods*, vol. 37, no. 1, pp. 146–158, Dec. 2007.
[7] S. T. A. Niaki and B. Abbasi, "Bootstrap method approach in designing multi-attribute control charts," *Int. J. Adv. Manuf. Technol.*, vol. 35, nos. 5–6, pp. 434–442, Nov. 2007.
[8] M. H. Lee, M. B. C. Khoo, X. Chew, and P. H. H. Then, "Economic-statistical design of synthetic np chart with estimated process parameter," *PLoS ONE*, vol. 15, no. 4, Apr. 2020, Art. no. e0230994.
[9] B.-M. Hsu, M.-H. Shu, and T.-C. Wang, "Variables adjustable multiple dependent state sampling plans with a loss-based capability index," *Int. J. Adv. Manuf. Technol.*, vol. 107, nos. 5–6, pp. 2163–2175, Mar. 2020.
[10] W. Zhou, Q. Wan, Y. Zheng, and Y.-W. Zhou, "A joint-adaptive np control chart with multiple dependent state sampling scheme," *Commun. Statist.-Theory Methods*, vol. 46, no. 14, pp. 6967–6979, Jul. 2017.
[11] C.-W. Wu and Z.-H. Wang, "Developing a variables multiple dependent state sampling plan with simultaneous consideration of process yield and quality loss," *Int. J. Prod. Res.*, vol. 55, no. 8, pp. 2351–2364, Apr. 2017.
[12] M. Aslam, S. Balamurali, and C.-H. Jun, "A new multiple dependent state sampling plan based on the process capability index," *Commun. Statist.-Simul. Comput.*, 2019, doi: 10.1080/03610918.2019.1588307.
[13] M. S. Aldosari, M. Aslam, and C.-H. Jun, "A new attribute control chart using multiple dependent state repetitive sampling," *IEEE Access*, vol. 5, pp. 6192–6197, 2017.
[14] A. Yan, S. Liu, and X. Dong, "Designing a multiple dependent state sampling plan based on the coefficient of variation," *SpringerPlus*, vol. 5, no. 1, p. 1447, Dec. 2016.
[15] S. Balamurali, P. Jeyadurga, and M. Usha, "Designing of Bayesian multiple deferred state sampling plan based on Gamma–Poisson distribution," *Amer. J. Math. Manage. Sci.*, vol. 35, no. 1, pp. 77–90, Jan. 2016.
[16] C.-W. Wu, S.-W. Liu, and A. H. Lee, "Design and construction of a variables multiple dependent state sampling plan based on process yield," *Eur. J. Ind. Eng.*, vol. 9, no. 6, pp. 819–838, 2015.
[17] M. Aslam, A. Nazir, and C.-H. Jun, "A new attribute control chart using multiple dependent state sampling," *Trans. Inst. Meas. Control*, vol. 37, no. 4, pp. 569–576, Apr. 2015.
[18] M. Aslam, M. Azam, N. Khan, and C.-H. Jun, "A control chart for an exponential distribution using multiple dependent state sampling," *Qual. Quantity*, vol. 49, no. 2, pp. 455–462, Mar. 2015.
[19] M. Aslam, C.-H. Yen, C.-H. Chang, and C.-H. Jun, "Multiple dependent state variable sampling plans with process loss consideration," *Int. J. Adv. Manuf. Technol.*, vol. 71, nos. 5–8, pp. 1337–1343, Mar. 2014.
[20] M. Aslam, N. Khan, and C.-H. Jun, "A multiple dependent state control chart based on double control limits," *Res. J. Appl. Sci., Eng. Technol.*, vol. 7, no. 21, pp. 4490–4493, Jun. 2014.
[21] F. Smarandache, *Neutrosophy, Neutrosophic Probability, Set and Logic*. Rehoboth, DE, USA: American Research Press, 1998.
[22] F. Smarandache, *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*. Rehoboth, DE, USA: American Research Press, 1999.
[23] Z. Khan, M. Gulistan, R. Hashim, N. Yaqoob, and W. Chammam, "Design of S-control chart for neutrosophic data: An application to manufacturing industry," *J. Intell. Fuzzy Syst.*, vol. 38, no. 4, pp. 4743–4751, Apr. 2020.
[24] M. Aslam and M. Albassam, "Presenting post hoc multiple comparison tests under neutrosophic statistics," *J. King Saud Univ.-Sci.*, vol. 32, no. 6, pp. 2728–2732, Sep. 2020.
[25] M. Albassam, N. Khan, and M. Aslam, "The W/S test for data having neutrosophic numbers: An application to USA village population," *Complexity*, vol. 2020, May 2020, Art. no. 3690879.
[26] X. Zhang, X. Wang, F. Smarandache, T. G. Jaíyéolá, and T. Lian, "Singular neutrosophic extended triplet groups and generalized groups," *Cognit. Syst. Res.*, vol. 57, pp. 32–40, Oct. 2019.
[27] M. Aslam, R. A. R. Bantan, and N. Khan, "Design of a control chart for gamma distributed variables under the indeterminate environment," *IEEE Access*, vol. 7, pp. 8858–8864, 2019.
[28] M. Aslam, R. A. R. Bantan, and N. Khan, "Design of a new attribute control chart under neutrosophic statistics," *Int. J. Fuzzy Syst.*, vol. 21, no. 2, pp. 433–440, Mar. 2019.
[29] M. Abdel-Basset, A. Atef, and F. Smarandache, "A hybrid neutrosophic multiple criteria group decision making approach for project selection," *Cognit. Syst. Res.*, vol. 57, pp. 216–227, Oct. 2019.
[30] X. Peng and J. Dai, "Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function," *Neural Comput. Appl.*, vol. 29, no. 10, pp. 939–954, May 2018.
[31] X. Peng and J. Dai, "A bibliometric analysis of neutrosophic set: Two decades review from 1998 to 2017," *Artif. Intell. Rev.*, vol. 53, pp. 199–255, Aug. 2018.
[32] M. Aslam, N. Khan, and M. Khan, "Monitoring the variability in the process using neutrosophic statistical interval method," *Symmetry*, vol. 10, no. 11, p. 562, Nov. 2018.
[33] M. Aslam, N. Khan, and M. Albassam, "Control chart for failure-censored reliability tests under uncertainty environment," *Symmetry*, vol. 10, no. 12, p. 690, Dec. 2018.
[34] M. Aslam and O. Arif, "Testing of grouped product for the weibull distribution using neutrosophic statistics," *Symmetry*, vol. 10, no. 9, p. 403, Sep. 2018.
[35] M. Aslam and A. Al-Marshadi, "Design of sampling plan using regression estimator under indeterminacy," *Symmetry*, vol. 10, no. 12, p. 754, Dec. 2018.

[36] M. Aslam, "A new sampling plan using neutrosophic process loss consideration," *Symmetry*, vol. 10, no. 5, p. 132, Apr. 2018.
[37] J. Ye, J. Chen, R. Yong, and S. Du, "Expression and analysis of joint roughness coefficient using neutrosophic number functions," *Information*, vol. 8, no. 2, p. 69, Jun. 2017.
[38] M.-H. Shu, D.-C. Dang, T.-L. Nguyen, B.-M. Hsu, and N.-S. Phan, "Fuzzy x- and s control charts: A data-adaptability and human-acceptance approach," *Complexity*, vol. 2017, pp. 1–17, Mar. 2017.
[39] M. N. P. Fernandez, "Fuzzy theory and quality control charts," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2017, pp. 1–6.
[40] J. Chen, J. Ye, S. Du, and R. Yong, "Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers," *Symmetry*, vol. 9, no. 7, p. 123, Jul. 2017.
[41] C. Panthong and A. Pongpullponsak, "Non-normality and the fuzzy theory for variable parameters control charts," *Thai J. Math.*, vol. 14, no. 1, pp. 203–213, 2016.
[42] S. Sorooshian, "Fuzzy approach to statistical control charts," *J. Appl. Math.*, vol. 2013, pp. 1–6, Aug. 2013.
[43] R. Intaramo and A. Pongpullponsak, "Development of fuzzy extreme value theory control charts using α-cuts for skewed populations," *Appl. Math. Sci.*, vol. 6, no. 117, pp. 5811–5834, 2012.
[44] S. Senturk and N. Erginel, "Development of fuzzy $\tilde{\tilde{X}}-\tilde{R}$ and $\tilde{\tilde{X}}-\tilde{S}$ control charts using α-cuts," *Inf. Sci.*, vol. 179, pp. 1542–1551, Apr. 2009.
[45] K.-L. Hsieh, L.-I. Tong, and M.-C. Wang, "The application of control chart for defects and defect clustering in IC manufacturing based on fuzzy theory," *Expert Syst. Appl.*, vol. 32, no. 3, pp. 765–776, Apr. 2007.
[46] H. Taleb and M. Limam, "On fuzzy and probabilistic control charts," *Int. J. Prod. Res.*, vol. 40, no. 12, pp. 2849–2863, Jan. 2002.
[47] F. Smarandache, "Introduction to neutrosophic statistics: Infinite study," 2014. [Online]. Available: https://arxiv.org/ftp/arxiv/papers/1406/1406.2000.pdf
[48] D. C. Montgomery, *Introduction to Statistical Quality Control*. Hoboken, NJ, USA: Wiley, 2007.

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