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# Monitoring the Variability in the Process Using Neutrosophic Statistical Interval Method

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**Abstract:** Existing variance control charts are designed under the assumptions that no uncertain, fuzzy and imprecise observations or parameters are in the population or the sample. Neutrosophic statistics, which is the extension of classical statistics, has been widely used when there is uncertainty in the data. In this paper, we will originally design  $S^2$  control chart under the neutrosophic interval methods. The complete structure of the neutrosophic  $S^2$  control chart will be given. The necessary measures of neutrosophic  $S^2$  will be given. The neutrosophic coefficient of  $S^2$  control chart will be determined through the neutrosophic algorithm. Some tables are given for practical use. The efficiency of the proposed control chart is shown over the  $S^2$  control chart designed under the classical statistics in neutrosophic average run length (NARL). A real example is also added to illustrate the proposed control chart. From the comparison in the simulation study and case study, it is concluded that the proposed control chart performs better than the existing control chart under uncertainty.

**Keywords:** classical statistics; neutrosophic statistics; neutrosophic average run length; fuzzy; uncertainty

## 1. Introduction

In this modern era, the customers demand high-quality products or services to fulfill their needs. High quality in a product can be only achieved if the manufacturing process meets the given specifications limits. To the manufacturer to produce the defect-free product, the variation in the process should be controlled. Generally, the manufacturing process moves away from control limits due to two types of variation, which are called the natural variation and special variation. Therefore, to produce the high-quality product that meets the given standard the elimination of variation is necessary. The control chart is one of the important tools that have been widely used in the industry for the monitoring of the process. The control chart immediately informs the engineers if any problem occurs that can shift the process from its target. According to Abbas et al. [1], “Dispersion charts are used to monitor within samples variability while location charts are used to monitor between samples variability. So, it is preferable to monitor the process dispersion before the location of the process”. Among the control chart, the Shewhart [2]  $S^2$  control chart is easy to apply in the industry. The Shewhart [2]  $S^2$  control chart has an upper control limit (UCL), a lower control limit (LCL) and central limit (CL). There are many other signals, such as two points out of three points to be out-of-control and four points out of five points to be out-of-control, which determine if the process is “out-of-control” or not. A process is declared to be out-of-control if the plotting statistic lies beyond the UCL or LCL. But, the first point to be out-of-control follows the geometric distribution. This criterion

has been widely used to find the average run length to assess the performance of the control chart. The  $S^2$  control charts under classical statistics have been studied by many authors, for example, Khoo [3] proposed  $S^2$  control chart for the double sampling. Zhang et al. [4] presented the design of  $S^2$  control chart. Khoo [5] studied the modified form of dispersion chart. Lee et al. [6] worked for this chart under the interval sampling. Riaz [7] presented a dispersion chart based on Interquartile range. Guo and Wang [8] studied the dispersion chart when the variance is estimated. Some details on the dispersion control chart can be seen in references [1,9].

The fuzzy approach has been widely applied in the uncertainty environment, see Zadeh [10]. According to Senturk and Erginel [11] “observations include human judgments, and evaluations and decisions, a continuous random variable of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. These variability causes create vagueness in the measurement system”. Therefore, the fuzzy control charts have been widely applied in a situation when there is uncertainty. Several authors worked on the designing of control chart using the fuzzy approach. The authors of [12,13] proposed fuzzy control charts for the Statistical Process Control (SPC) zone rules. Senturk and Erginel [11] proposed fuzzy dispersion control charts. Mojtaba et al. [14] proposed the fuzzy chart using triangle fuzzy random variable. Shu et al. [15] proposed the fuzzy chart using data-adoptability approach. Afshari and Gildeh [16] designed a fuzzy control chart using the multiple dependent state (MDS) sampling. Fadaei and Pooya [17] proposed the fuzzy U control chart. More detail on the control charts using the fuzzy approach can be seen in references [15,18].

The authors of [19] mentioned that fuzzy logic is the special case of the neutrosophic logic. Smarandache [20] introduced some basic work in the neutrosophic statistics. According to [20], the neutrosophic statistics can be applied when the observations or the parameters are imprecise, indeterminate and fuzzy. Recently, the authors of [21,22] worked on rock study issues using the neutrosophic statistics. References [23,24] introduced the neutrosophic sampling plan the first time.

A rich literature on control chart for the monitoring of the variation or shift in the dispersion parameter is available under classical statistics. For the monitoring of the dispersion parameter, the  $S^2$  control chart has been widely used in the industry. The existing literature of  $S^2$  control charts is designed under the assumption that there is no uncertainty, indeterminacy, imprecise and fuzzy observations/parameter in the data. In practice, due to the measurement process, it may not possible to record data having all observations determined. So, when the observations are imprecise and uncertain, the  $S^2$  control charts under the classical statistics cannot be applied. The neutrosophic statistics, which is the generalization of the classical statistics, deal with the situations when the observations or parameters are fuzzy. The neutrosophic statistics is used when the population or the sample has some uncertain observations.

According to the best of our knowledge, there is no study on the design of  $S^2$  control chart using the neutrosophic statistics. In this paper, our objective is to originally design a  $S^2$  control chart under the neutrosophic interval methods. The efficiency of the proposed chart will be compared with the chart under the classical statistics in terms of neutrosophic average run length. The design of the proposed control chart is given in Section 2. In Section 3, the advantages of the proposed control chart are discussed. A case study is given in Section 4 and some concluding remarks are given in the last section.

## 2. Design of Proposed Control Chart

A random sample selected from such a population or the sample having indeterminacy in observations is called a neutrosophic random sample. Suppose a neutrosophic random number  $X_{Ni} \in \{X_L, X_U\}$ ;  $i = 1, 2, 3, \dots, n_N$ , where  $X_L$  is a determinate part and  $X_U$  is an indeterminate part. Let  $\mu_N = \sum_{i=1}^{n_N} X_N / n_N$ ;  $\mu_N \in \{\mu_L, \mu_U\}$  represent the mean of population having indeterminate observations; where  $\mu_L$  and  $\mu_U$  are the means of determinate part and indeterminate parts, respectively

and  $\sigma_N^2 = \sum_{i=1}^{n_N} (X_N - \mu_N)^2 / N_N - 1$ ;  $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$  represents the neutrosophic population variance, where  $\sigma_L^2$  and  $\sigma_U^2$  are the variance of determinate part and indeterminate parts, respectively. Let  $\bar{X}_N = \sum_{i=1}^{n_N} X_N / n_N$ ;  $\bar{X}_N \in \{\bar{X}_L, \bar{X}_U\}$  and  $S_N^2 = \sum_{i=1}^{n_N} (X_N - \bar{X}_N)^2 / n_N - 1$ ;  $S_N^2 \in \{S_L^2, S_U^2\}$  be the neutrosophic sample mean and neutrosophic variance of  $X_{Ni} \in \{X_L, X_U\}$ . More detail can be seen in [20]. In this section, we will propose the following  $S^2$  control chart under the neutrosophic statistical interval method. The proposed chart emphasized that variable inspection (measuring the quality of interest) is used to monitor the variance of the process.

**Step-1:** Select a random sample of size  $n_N$  from the production process and compute  $S_N^2$ .

**Step-2:** Declare the process is in-control state if  $LCL_N \leq S_N^2 \leq UCL_N$ ; where  $LCL_N \in \{LCL_L, LCL_U\}$  and  $UCL_N \in \{UCL_L, UCL_U\}$  are neutrosophic interval control limits.

The neutrosophic control limits  $LCL_N \in \{LCL_L, LCL_U\}$  and  $UCL_N \in \{UCL_L, UCL_U\}$  are given by

$$UCL_N = \sigma_N^2 + k_N \sqrt{2(\sigma_N^2)^2 / (n_N - 1)}; \sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}, k_N \in \{k_L, k_U\} \tag{1}$$

$$LCL_N = \sigma_N^2 - k_N \sqrt{2(\sigma_N^2)^2 / (n_N - 1)}; \sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}, k_N \in \{k_L, k_U\} \tag{2}$$

where  $k_N \in \{k_L, k_U\}$  is a neutrosophic control chart coefficient and will be determined through the neutrosophic algorithm.

The proposed  $S^2$  control chart under the neutrosophic statistical interval method is the generalization of  $S^2$  control chart under the classical statistics. The proposed chart becomes the existing control chart under the classical statistics when  $k_L = k_U = k$ . The probability that the process is declared to be out-of-control under the neutrosophic interval methods is derived as follows:

$$P_{outN}^{(0)} = P(S_N^2 \geq UCL_N) + P(S_N^2 \leq LCL_N); S_N^2 \in \{S_L^2, S_U^2\} \tag{3}$$

Note here that  $(n_N - 1)S_N^2 / \sigma_N^2$ ;  $S_N^2 \in \{S_L^2, S_U^2\}$ ,  $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$  follows the neutrosophic Chi-square  $\chi_N^2$ ;  $\chi_N^2 \in \{\chi_L^2, \chi_U^2\}$  distribution with neutrosophic degree of freedom  $n_N - 1$ ;  $n_N \in \{n_L, n_U\}$  when the process is in-control state. The hypothesis is that the process is in-control state at  $\sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$ . Suppose  $G_N \in \{G_L, G_U\}$  be the neutrosophic distribution function of  $\chi_N^2 \in \{\chi_L^2, \chi_U^2\}$ . Therefore, for the in-control state,

$$P(S_N^2 \geq UCL_N) = 1 - G_N\left(\frac{(n_N-1)UCL_N}{\sigma_N^2}\right) = 1 - G_N\left((n_N - 1)\left(1 + k_N \sqrt{2/(n_N - 1)}\right)\right); \tag{4}$$

$n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\}$ .

Similarly,

$$P(S_N^2 \leq LCL_N) = G_N\left(\frac{(n_N-1)LCL_N}{\sigma_N^2}\right) = G_N\left((n_N - 1)\left(1 - k_N \sqrt{2/(n_N - 1)}\right)\right); \tag{5}$$

$n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\}$ .

The probability of in-control process under the neutrosophic interval method is given by:

$$P_{outN}^{(0)} = 1 - G_N\left((n_N - 1)\left(1 + k_N \sqrt{2/(n_N - 1)}\right)\right) + G_N\left((n_N - 1)\left(1 - k_N \sqrt{2/(n_N - 1)}\right)\right); \tag{6}$$

$n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\}$

The average run length (ARL) under the classical statistics is an important measure used to assess the performance of the control chart. The ARL indicates that when, on average, the process will be

out-of-control and when it is actually an in-control state. The neutrosophic average run length (NARL) under the neutrosophic interval method is defined by:

$$NARL_{0N} = \frac{1}{P_{outN}^{(0)}}; ARL_{0N} \in \{ARL_{0L}, ARL_{0U}\} \tag{7}$$

Now suppose that the variance of the process has shifted to new target value  $\sigma_{1N}^2 = c\sigma_N^2; \sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$ , where  $c$  denotes the shift constant. The alternative hypothesis for this study is that the process has shifted to a new variance  $\sigma_{1N}^2$ . Note here that  $(n_N - 1)S_N^2 / c\sigma_N^2; S_N^2 \in \{S_L^2, S_U^2\}, \sigma_N^2 \in \{\sigma_L^2, \sigma_U^2\}$  follows the neutrosophic Chi-square  $\chi_N^2; \chi_N^2 \in \{\chi_L^2, \chi_U^2\}$  distribution with neutrosophic degree of freedom  $n_N - 1; n_N \in \{n_L, n_U\}$  when the process is an out-of-control state.

The probability that the process is declared to be out-of-control for the shifted process under the neutrosophic interval methods is derived as follows

$$P_{outN}^{(1)} = P(S_N^2 \geq UCL_N | \sigma_{1N}^2) + P(S_N^2 \leq LCL_N | \sigma_{1N}^2); S_N^2 \in \{S_L^2, S_U^2\} \tag{8}$$

Therefore, for the out-of-control state at  $\sigma_{1N}^2$  is given by

$$P(S_N^2 \geq UCL_N | \sigma_{1N}^2) = 1 + G_N\left(\frac{n_N - 1}{c} \left(1 - k_N \sqrt{\frac{2}{n_N - 1}}\right)\right); n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\}. \tag{9}$$

Similarly,

$$P(S_N^2 \leq LCL_N | \sigma_{1N}^2) = G_N\left(\frac{n_N - 1}{c} \left(1 - k_N \sqrt{\frac{2}{n_N - 1}}\right)\right); n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\}. \tag{10}$$

The probability of an out-of-control process at  $\sigma_{1N}^2$  under the neutrosophic interval method is given by:

$$P_{outN}^{(1)} = 1 + G_N\left(\frac{n_N - 1}{c} \left(1 - k_N \sqrt{\frac{2}{n_N - 1}}\right)\right) + G_N\left(\frac{n_N - 1}{c} \left(1 - k_N \sqrt{\frac{2}{n_N - 1}}\right)\right); n_N \in \{n_L, n_U\}, k_N \in \{k_L, k_U\} \tag{11}$$

The NARL for the shifted process is given by:

$$NARL_{1N} = \frac{1}{P_{outN}^{(1)}}; ARL_{1N} \in \{ARL_{1L}, ARL_{1U}\} \tag{12}$$

Let  $r_{0N}$  denote the specified values of  $NARL_{0N}$ . The values of NARL for various subgroup size  $n_N \in \{n_L, n_U\}$  and shift  $c$  are presented in Tables 1–3. In Table 1, the values of NARL are given when  $n_N \in \{3, 4\}$  and  $r_{0N} = 300$  and 370. In Table 2, the values of NARL are given when  $n_N \in \{4, 6\}$  and  $r_{0N} = 300$  and 370. In Table 3, the values of NARL are given when  $n_N \in \{3, 4\}$  and  $r_{0N} = 300$  and 370. From Tables 1–3, the following trends in NARL can be noted.

1. For the fixed values of  $n_N \in \{n_L, n_U\}$  and  $c$ , the range in indeterminacy interval of NARL increases as  $NARL_{0N}$  decreases from 300 to 370.
2. For the fixed values of  $NARL_{0N}$  and  $c$ , the range in indeterminacy interval of NARL decreases as  $n_N \in \{n_L, n_U\}$  increases.

**Table 1.** The neutrosophic plan parameters when  $n_N \in \{3, 4\}$  and  $r_{0N} = 300, 370$ .

Parameters	$r_{0N} = 300$	$r_{0N} = 370$
$n_N$	[3,4]	[3,4]
$k_N$	[4.716,4.784]	[4.921,4.925]
$c$	$NARL_{1N}$	
1	[303.571,482.268]	[372.916,567.398]
1.1	[180.551,257.614]	[217.685,298.395]
1.2	[117.098,153.218]	[138.998,175.183]
1.3	[81.177,98.952]	[95.096,111.902]
1.4	[59.298,68.163]	[68.685,76.363]
1.5	[45.169,49.433]	[51.809,54.931]
1.6	[35.597,37.374]	[40.482,41.237]
1.7	[28.85,29.239]	[32.562,32.06]
1.8	[23.935,23.534]	[26.833,25.662]
1.9	[20.251,19.399]	[22.567,21.048]
2	[17.423,16.317]	[19.311,17.625]
3	[6.721,5.593]	[7.198,5.874]
4	[4.174,3.353]	[4.394,3.474]

**Table 2.** The neutrosophic plan parameters when  $n_N \in \{4, 6\}$  and  $r_{0N} = 300, 370$ .

Parameters	$r_{0N} = 300$	$r_{0N} = 370$
$n_N$	[4,6]	[4,6]
$k_N$	[4.37095,4.38408]	[4.56277,4.60236]
$c$	$NARL_{1N}$	
1	[300.03049,490.72292]	[373.89462,521.39944]
1.1	[167.75455,236.62553]	[204.67051,261.86897]
1.2	[103.63777,130.01298]	[124.23386,148.39858]
1.3	[69.11578,78.91434]	[81.62574,92.2282]
1.4	[48.93744,51.76218]	[57.06262,61.60346]
1.5	[36.34426,36.10675]	[41.91335,43.57537]
1.6	[28.05528,26.46593]	[32.04372,32.28357]
1.7	[22.35463,20.20032]	[25.31664,24.84157]
1.8	[18.28768,15.94213]	[20.55525,19.72459]
1.9	[15.29499,12.93761]	[17.07608,16.07855]
2	[13.03379,10.74914]	[14.46377,13.40067]
3	[4.84897,3.5816]	[5.18032,4.44486]
4	[3.02663,2.21472]	[3.17374,2.68331]

**Table 3.** The neutrosophic plan parameters when  $n_N \in \{9, 10\}$  and  $r_{0N} = 300, 370$ .

Parameters	$r_{0N} = 300$	$r_{0N} = 370$
$n_N$	[9,10]	[9,10]
$k_N$	[3.77774,3.87857]	[3.90143,3.92448]
$c$	$NARL_{1N}$	
1	[310.11015,398.93766]	[374.85685,429.14801]
1.1	[140.42988,169.87263]	[166.07453,181.17044]
1.2	[73.89048,85.19126]	[85.81325,90.2116]
1.3	[43.56058,48.35266]	[49.82352,50.89613]
1.4	[28.04099,30.19755]	[31.65868,31.6243]
1.5	[19.345,20.33186]	[21.59801,21.19929]
1.6	[14.10561,14.53629]	[15.59629,15.09889]
1.7	[10.75721,10.90994]	[11.79341,11.29453]
1.8	[8.51099,8.52027]	[9.26122,8.79481]
1.9	[6.94206,6.87633]	[7.504,7.0795]
2	[5.80835,5.70385]	[6.24147,5.85883]
3	[2.16011,2.05723]	[2.24008,2.0842]
4	[1.48785,1.42243]	[1.51913,1.43261]

The following neutrosophic algorithm is applied to determine the indeterminacy interval of  $k_N \in \{k_L, k_U\}$ .

**Step-1:** Specify the indeterminacy interval of  $n_N \in \{n_L, n_U\}$  and  $c$ .

**Step-2:** Determine the indeterminacy interval of  $k_N \in \{k_L, k_U\}$  such that  $NARL_{0N} \geq r_{0N}$ .

**Step-3:** Find indeterminacy interval of  $NARL_{1N}$  using  $k_N \in \{k_L, k_U\}$  selected in Step-2.

### 3. Comparison Studies

In this section, we will compare the efficiency of the proposed  $S^2$  control chart under the neutrosophic interval method with  $S^2$  control chart under the classical statistics.

#### 3.1. Comparison by NARL

We will present the comparison in NARL when  $n_N \in \{4, 6\}$  and  $r_{0N} = 300$  in Table 4. According to the authors of [22], a method which provides the parameters in indeterminacy interval rather than the determined value under the uncertainty environment, is said to be more effective and adequate. From Table 4, it can be noted that the proposed control chart under the neutrosophic statistics has NARL in indeterminacy interval, while the existing chart under the classical statistics provides the determined value. For example, when  $c = 1.1$ , the indeterminacy interval of the proposed chart is  $ARL_{1N} \in \{167.75455, 236.62553\}$  while  $ARL = 167$  from the existing control chart. From the comparison, it can be concluded that the existing control chart cannot be applied when some observations/parameters are uncertain. Therefore, by following the theory proposed in [22], the proposed control chart is more effective and adequate than control chart under classical statistics.

**Table 4.** Comparison when [4,6] and  $r_{0N} = 300$ .

$c$	Proposed Chart	Existing Chart
	NARL <sub>1N</sub>	
1	[300.03049,490.72292]	300.0044
1.1	[167.75455,236.62553]	167.7414
1.2	[103.63777,130.01298]	103.6304
1.3	[69.11578,78.91434]	69.11124
1.4	[48.93744,51.76218]	48.93448
1.5	[36.34426,36.10675]	36.34222
1.6	[28.05528,26.46593]	28.05381
1.7	[22.35463,20.20032]	22.35353
1.8	[18.28768,15.94213]	18.28684
1.9	[15.29499,12.93761]	15.29432
2	[13.03379,10.74914]	13.03325
3	[4.84897,3.5816]	4.84884
4	[3.02663,2.21472]	3.02657

#### 3.2. Comparison by Simulation

We now compare the efficiency of the proposed control with the control chart under the classical statistics by using the simulation data. The data is generated from the neutrosophic normal distribution with  $\mu_N \in \{0, 0\}$  and neutrosophic variance  $\sigma_N^2 \in \{4, 6.25\}$ . The first 20 neutrosophic observations are generated from the in-control process and next 20 neutrosophic observations from the shifted process with  $c = 1.8$ . For the simulation study, let  $n_N \in \{5, 5\}$  and  $r_{0N} = 370$ . We plotted the neutrosophic statistic  $S_N^2$  on Figure 1. It is expected that the proposed chart should detect a shift in the indeterminacy interval  $ARL_{1N} \in \{20.55, 19.72\}$ . From Figure 1, it can be noted that the proposed plan detect a shift in the process at the 19<sup>th</sup> sample. The same values statistic  $S^2$  under the classical statistic is plotted in Figure 2. Figure 2 indicates that the process is an in-control state. By comparing Figure 1 with Figure 2, it is concluded that the proposed control chart has the ability to detect a shift in the process. Also, the proposed control chart is more effective in the uncertainty environment.

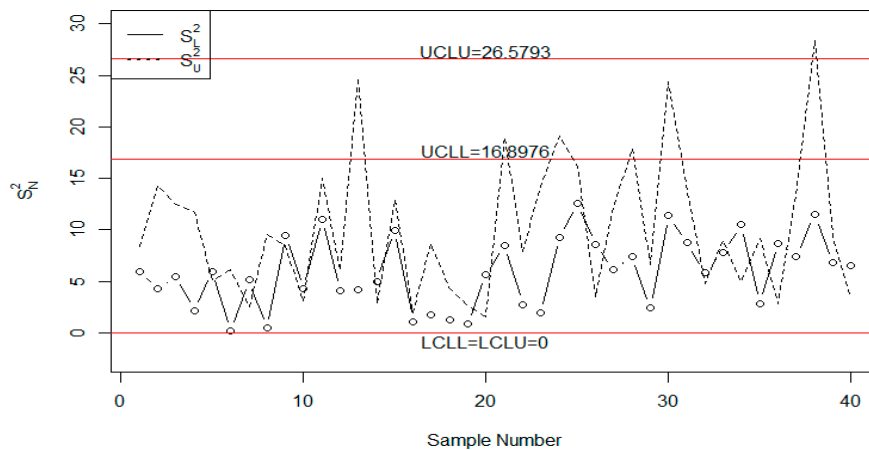


Figure 1. The proposed chart under the neutrosophic interval statistical method.

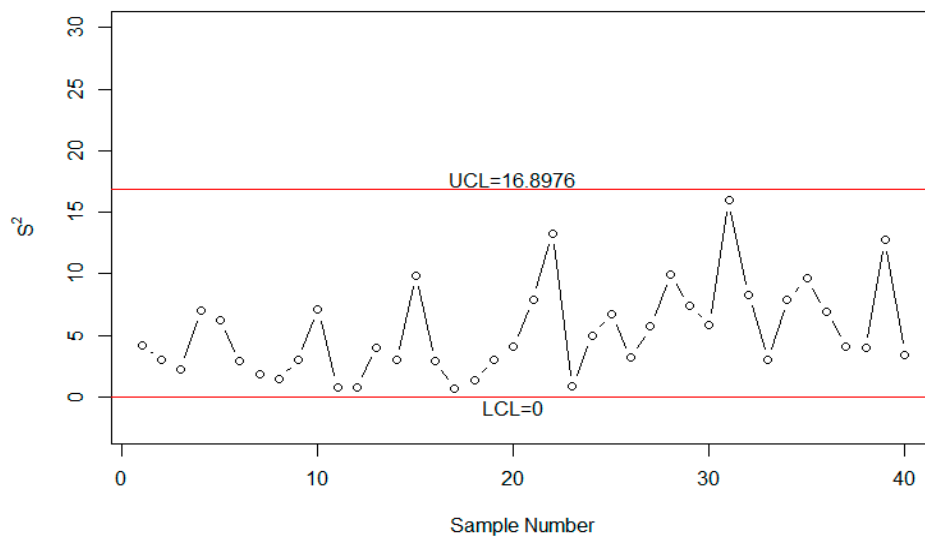


Figure 2. The control chart under the classical statistics.

#### 4. Case Study

The application of the proposed chart is given in the automobile industry. In this industry, the measurement of the inside diameter of engine piston rings is an important variable, see Montgomery [25]. Therefore, the monitoring of this variable is an important task in the automobile industry. The inside diameter is a continuous variable. Due to human subjectivity or measurement devices and environmental conditions, it is possible that some observations are uncertain. In this case, the control chart under classical statistics cannot be applied for the monitoring of the diameter. The data having some uncertain observations are reported in Table 5.

Table 5. Real example data.

Sample No.	Sample Observation					$S_N^2$
1	[74.03,74.03]	[74.002,73.991]	[74.019,74.019]	[73.992,73.992]	[74.008,74.001]	[0.014772,0.017242]
2	[73.995,73.995]	[73.992,74.003]	[74.001,74.001]	[74.011,74.011]	[74.004,74.004]	[0.007503,0.005762]
3	[73.988,74.017]	[74.024,74.024]	[74.021,74.021]	[74.005,74.005]	[74.002,73.995]	[0.014748,0.012116]
4	[74.002,74.002]	[73.996,73.996]	[73.993,73.993]	[74.015,74.015]	[74.009,74.009]	[0.009083,0.009083]
5	[73.992,73.992]	[74.007,74.007]	[74.015,74.015]	[73.989,73.989]	[74.014,73.998]	[0.012219,0.010756]
6	[74.009,74.009]	[73.994,74.001]	[73.997,73.997]	[73.985,73.985]	[73.993,73.993]	[0.008706,0.008944]
7	[73.995,73.998]	[74.006,74.006]	[73.994,73.994]	[74,74]	[74.005,74.005]	[0.005523,0.00498]
8	[73.985,73.985]	[74.003,74.01]	[73.993,73.993]	[74.015,74.015]	[73.988,73.988]	[0.012256,0.01348]
9	[74.008,74.005]	[73.995,73.995]	[74.009,74.009]	[74.005,74.005]	[74.004,74.004]	[0.005541,0.005177]
10	[73.998,73.998]	[74,74]	[73.99,73.99]	[74.007,74.007]	[73.995,73.995]	[0.006285,0.006285]
11	[73.994,73.998]	[73.998,73.998]	[73.994,73.994]	[73.995,73.995]	[73.99,74.001]	[0.002854,0.002775]
12	[74.004,74.004]	[74,74.002]	[74.007,74.005]	[74,74.001]	[73.996,73.996]	[0.004219,0.003507]
13	[73.983,73.993]	[74.002,74.002]	[73.998,73.998]	[73.997,73.997]	[74.012,74.005]	[0.010455,0.004637]
14	[74.006,74.006]	[73.967,73.985]	[73.994,73.994]	[74,74]	[73.984,73.996]	[0.015304,0.007759]
15	[74.012,74.012]	[74.014,74.012]	[73.998,73.998]	[73.999,73.999]	[74.007,74.007]	[0.007314,0.006804]
16	[74,74]	[73.984,73.984]	[74.005,74.005]	[73.998,73.998]	[73.996,73.996]	[0.007797,0.007797]
17	[73.994,73.994]	[74.012,74.012]	[73.986,73.986]	[74.005,74.005]	[74.007,74.007]	[0.010569,0.010569]
18	[74.006,74.006]	[74.01,74.011]	[74.018,74.018]	[74.003,74.003]	[74,74.001]	[0.006986,0.006834]
19	[73.984,73.984]	[74.002,74.002]	[74.003,74.003]	[74.005,74.005]	[73.997,73.997]	[0.008468,0.008468]
20	[74,74]	[74.01,74.01]	[74.013,74.009]	[74.02,74.015]	[74.003,74.003]	[0.007981,0.005941]
21	[73.982,73.982]	[74.001,74.001]	[74.015,74.015]	[74.005,74.005]	[73.996,73.996]	[0.012153,0.012153]
22	[74.004,74.004]	[73.999,73.999]	[73.99,73.99]	[74.006,74.006]	[74.009,74.002]	[0.007436,0.006261]
23	[74.01,74.01]	[73.989,73.989]	[73.99,73.99]	[74.009,74.005]	[74.014,74.011]	[0.011929,0.010747]
24	[74.015,74.011]	[74.008,74.008]	[73.993,73.993]	[74,74]	[74.01,74.011]	[0.008701,0.007893]
25	[73.982,73.982]	[73.984,73.989]	[73.995,73.995]	[74.017,74.012]	[74.013,74.01]	[0.016177,0.013088]

For this real example, suppose  $n_N \in \{5, 5\}$  and  $r_{0N} = 370$ . The control limits of the proposed chart under the neutrosophic statistics are shown in Figure 3. We plotted the neutrosophic statistic  $S_N^2$  in Figure 3. From Figure 3, it can be seen that although the process is in-control state, some plotting points are near the control limits. The  $S^2$  control chart under the classical statistics is presented in Figure 4. From Figure 4, we note that the process is an in-control state with one point near the control limit. From Figure 3, we note that sample numbers 2, 7, 9, 10 and 11 are very close to  $LCL_N$ . On the other hand, Figure 4 indicates that 11th and 12th sample numbers are very near to the LCL. By comparing Figure 3 with Figure 4, it can be concluded that although the plotting statistic is an in-control state, several points near the control limits need attention.

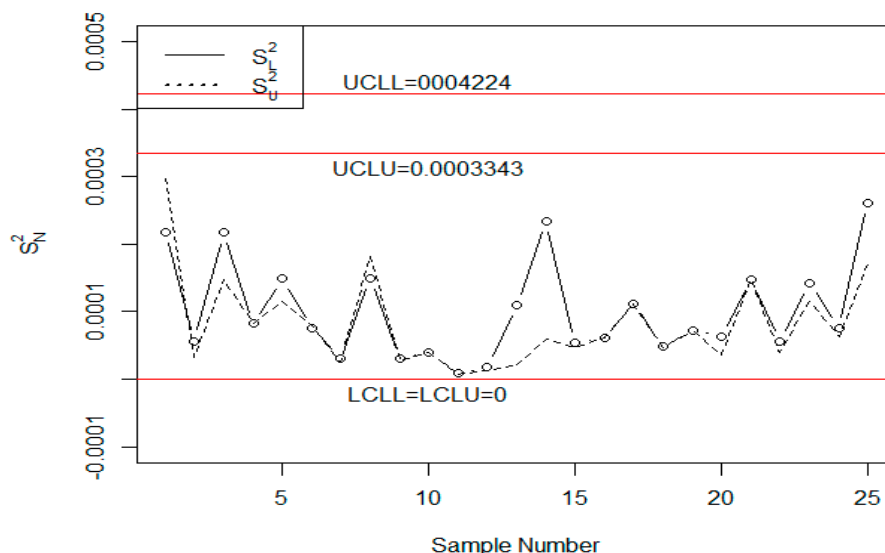


Figure 3. The proposed plan using real data.



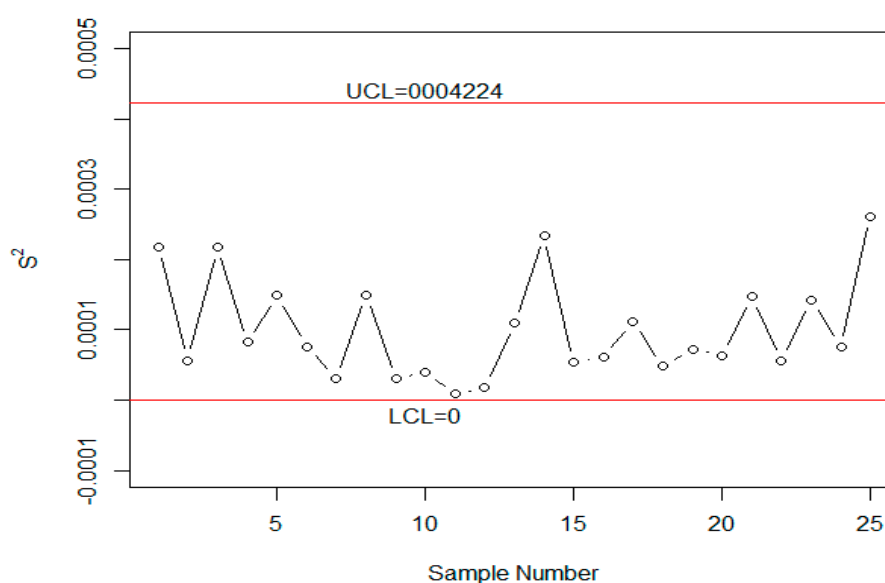


Figure 4. The existing chart for the real data.

## 5. Concluding Remarks

We presented the designing of  $S^2$  control chart under the neutrosophic interval statistical method. Some necessary measures to assess the performance of the proposed control chart are given. The advantages of the proposed control chart over the chart using the classical statistics are given. From the comparison, it is concluded that the proposed control chart is more effective and adequate under the uncertainty environment. The simulation study showed that the proposed chart has the ability to detect a shift in the process. The implementation of the proposed chart on the real data also shows its efficacies over the existing control chart. Therefore, it is recommended to apply the proposed control when the observations or the parameters are fuzzy. From the comparison and real example, it is concluded that the proposed chart under the neutrosophic statistics is quite adequate and effective in the uncertainty environment, more so than the method based on classical statistics. The proposed control chart using the exponentially weighted moving average (EWMA) will be considered as future research.

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