Multi-attribute Group decision Making Based on Expected Value of Neutrosophic Trapezoidal Numbers

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ABSTRACT

We present an expected value based method for multiple attribute group decision making (MAGDM), where the preference values of alternatives and the importance of attributes are expressed in terms of neutrosophic trapezoidal numbers (NTrNs). First, we introduce an expected value formula for NTrNs to be used in MAGDM. Second, we determine the expected values of aggregated rating values and expected weight values of attributes, which are given by the decision makers. Third, we determine the weighted expected value of each alternative to rank the given alternatives and chose the desired alternative. Finally, we provide a numerical example to illustrate the validity and effectiveness of the proposed approach.

KEYWORDS: Trapezoidal fuzzy number, Neutrosophic trapezoidal number, Expected value of neutrosophic trapezoidal number, Multi-attribute group decision making

1 INTRODUCTION

Multi-attribute decision making (MADM) is an important part in the theory of decision making problems. In this method, we determine the best one from the set of possible alternatives after considering qualitative or quantitative assessment of finite conflicting attributes. Several methods for solving MADM such as TOPSIS (Hwang & Yoon, 2012), GRA (Deng, 1989; Li, Yamaguchi, & Nagai, 2007; Olson & Wu, 2006), AHP (Boucher & MacStravic, 1991; Saaty, 1980, 1994), VIKOR (Opricovic, 1998), ELECTREE (Roy, 1991) have been developed in crisp environment. However, decision makers cannot always evaluate the performance of alternatives with crisp numbers due to insufficient knowledge of the problem, or inability to explain directly the performance of one alternative over the others. This issue has motivated us to extend the MADM problems with imprecise environment. Fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986), interval valued fuzzy sets (Turksen, 1986), hesitant fuzzy sets (Torra, 2010) have been proved as the effective tools to model MADM in imprecise or vague environment although these sets cannot represent incomplete, inconsistent and indeterminate information that we often face in decision making problems. Neutrosophic set (Smarandache, 1998) captures all these types of information. This set represents each element of universe with three independent membership functions: truth membership function, indeterminacy membership function, and falsity membership function. Single valued neutrosophic set (SVNS) (Wang, Smarandache, Zhang, & Sunderraman, 2010), an instance of neutrosophic set, can effectively handle uncertain information existing in the real world problems.

Recently, researchers have found the potentiality of SVNS and shown an increased interest about MADM problem under neutrosophic environment. Peng, Wang, Zhang, and Chen (2014) proposed outranking method for solving multi-criteria decision making problems (MCDM) under simplified neutrosophic environment. Ye (2014b) introduced some vector similarity measures of simplified neutrosophic sets. Pramanik, Biswas, and Giri (2017) extended vector similarity measure to hybrid vector similarity measure of single valued and interval neutrosophic sets to study MADM problem. Mondal and Pramanik (2015) proposed tangent similarity measure for SVNSs and applied it to MADM. Biswas, Pramanik, and Giri (2014a) proposed entropy based grey relational analysis method for MADM with SVNSs. Biswas, Pramanik, and Giri (2014b) further studied grey relational analysis for neutrosophic MADM problems in which the weight of attribute is partially known or completely unknown. Biswas, Pramanik, and Giri (2016a) developed a TOPSIS method for neutrosophic MAGDM problem, where decision maker's weight, attribute's weight and rating values of alternatives are represented in terms of SVNSs. Biswas, Pramanik, and Giri (2017) further developed a non-linear programming based TOPSIS method for MAGDM problem under SVNS environment. Şahin and Liu (2015) put forward maximum deviation method to determine weight of attributes and then solve neutrosophic MADM. In addition, different aggregation operators of neutrosophic sets (Liu, Chu, Li, & Chen, 2014; Liu & Wang, 2014; Peng, Wang, Wang, Zhang, & Chen, 2016; Ye, 2014a) have also been developed to solve MADM.

However in MADM, the domain of single-valued neutrosophic set is discrete set. A fuzzy number (Dubois & Prade, 1987) is expressed with imprecise value rather than exact numerical values. Fuzzy numbers are considered as a connected set of possible values, where each value is characterized by membership degree, which lies between zero and one. The main advantage of fuzzy number is that it depicts the physical world more realistically than crisp numbers. Therefore to represent the physical universe with a degree of inherent uncertainty, we consider truth, indeterminacy and falsity membership functions of SVNSs with a triad of connected set of possible values rather than triad of crisp numbers. Recently neutrosophic numbers has received little attention to the researchers, and several definitions of single-valued neutrosophic numbers have been proposed. Ye (2015) proposed trapezoidal neutrosophic sets, and defined score function, accuracy functions, and two aggregation operators for trapezoidal neutrosophic sets. Biswas, Pramanik, and Giri (2015) defined cosine similarity measure and relative expected value of trapezoidal neutrosophic sets for MADM problem. Biswas, Pramanik, and Giri (2016b) introduced single-valued neutrosophic trapezoidal numbers, where each of truth, indeterminacy and falsity membership functions has been considered with trapezoidal fuzzy numbers. They (Biswas et al., 2016b) developed a value and ambiguity index based ranking method to compare neutrosophic trapezoidal numbers used in MADM problems. Deli and Şubaş (2017) introduced neutrosophic trapezoidal number (NTrNs) by assigning a set of four consecutive elements characterized by truth, indeterminacy and falsity membership degrees. and proposed a value and ambiguity index based ranking method to compare single-valued neutrosophic trapezoidal numbers.

Furthermore, the method of expected value is also used to rank fuzzy numbers and intuitionistic fuzzy numbers. Heilpern (1992) proposed the expected value for fuzzy number, and thereafter He and Wang (2009) extended expected value method to MADM with fuzzy data. Grzegrorzewski (2003) put forward the expected value and ordering method for intuitionistic fuzzy numbers. Ye (2011) extended the method of expected value for intuitionistic trapezoidal fuzzy MCDM problems. The intuitionistic trapezoidal fuzzy number (Nehi, 2010) has two parts: membership function and non-membership functions expressed by trapezoidal fuzzy numbers. Because indeterminacy is a common issue in decision making problems, extension of the Ye's method (Ye, 2011) is required to deal the issue in multi-attribute decision making problems. There is a little research about neutrosophic trapezoidal number and thus more research is needed for MADM under NTrNs.

Literature review reflects that no research has been carried out on expected value method for MADM underNTrNs. To bridge the gap, we first propose expected value of neutrosophic trapezoidal numbers to order NTrNs. Then we develop an expected value based novel method for neutrosophic trapezoidal MAGDM. We define formulas to determine the expected weight values of the attribute and weighted expected value for an alternative to determine the best alternative.

The remainder of the paper has been organized as follows. In Section 2, we review some basic notions of fuzzy set, trapezoidal fuzzy numbers, single-valued neutrosophic set, NTrN, and its some arithmetical operations. In Section 3, we introduce an expected value of NTrN and a ranking method among NTrNs. In section 4, we put forward expected value method to derive attribute weights and develop an approach to MAGDM with NTrN information.

Section 5 provides a numerical example to illustrate the developed approach, and finally, in Section 6, we conclude the paper with future direction of research.

2 PRELIMINARIES

In this section, we recall some basic notions of fuzzy sets, trapezoidal fuzzy numbers, single-valued neutrosophic sets, and single-valued neutrosophic trapezoidal numbers.

Definition 1. (Zadeh, 1965) A fuzzy set A in a universe of discourse X is defined by $A = \{\langle x, \mu_A(x)\rangle \, | x \in X\}$, where, $\mu_A(x)$: $X \to [0,1]$ is called the membership function of A and the value of $\mu_A(x)$ is called the degree of membership for $x \in X$.

Definition 2. (Dubois & Prade, 1987; Kauffman & Gupta, 1991) A fuzzy number A is called a trapezoidal fuzzy number(TrFN) if its membership function is defined by

$$\mu_A(x) = \begin{cases} \mu_A^L(x) = \dfrac{x - a_1}{a_2 - a_1}, & a_1 \leq x \leq a_2 \\ 1, & a_2 \leq x \leq a_3 \\ \mu_A^U(x) = \dfrac{a_4 - x}{a_4 - a_3}, & a_3 \leq x \leq a_4 \\ 0, & otherwise. \end{cases}$$

The TrFN A is denoted by the quadruplet A=(a_1, a_2, a_3, a_4), where a_1, a_2, a_3, a_4 are the real numbers and $a_1 \leq a_2 \leq a_3 \leq a_4$.

Definition 3. (Heilpern, 1992) Let $A = (a_1, a_2, a_3, a_4)$ be a trapezoidal fuzzy number in the set of real number $\mathbb{R}$. Then the expected interval and expected value of $\tilde{A}$ are respectively

$$EI(A) = [E(A^L), E(A^U)] \qquad \text{and} \qquad EI(A) = (E(A^L) + E(A^U))/2 \tag{1}$$

where, $E(A^L) = a_2 - \int_{a_1}^{a_2} \mu_{\tilde{A}}^L(x)\,\mathrm{dx}$ and $E(A^U) = a_4 + \int_{a_3}^{a_4} \mu_{\tilde{A}}^U(x)\,\mathrm{dx}$

Definition 4. (Wang et al., 2010) A single valued neutrosophic set $\tilde{A}$ in a universe of discourse X is given by

$$\tilde{A} = \Big\{\langle x, T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x)\rangle | x \in X\Big\},$$

where, $T_{\tilde{A}} : X \to [0,1]$, $I_{\tilde{A}} : X \to [0,1]$ and $F_{\tilde{A}} : X \to [0,1]$, with the condition

$$0 \leq T_{\tilde{A}}(x) + I_{\tilde{A}}(x) + F_{\tilde{A}}(x) \leq 3, \text{ for all } x \in X.$$

The numbers $T_{\tilde{A}}(x)$, $I_{\tilde{A}}(x)$ and $F_{\tilde{A}}(x)$ respectively represent the truth membership, indeterminacy membership and falsity membership degree of the element x to the set $\tilde{A}$. For convenience, we take the single valued neutrosophic set $A = \big\langle T_A(x), I_A(x), F_A(x)\big\rangle$.

Definition 5. (Biswas et al., 2016b) Let $\tilde{A}$ be a neutrosophic trapezoidal number in the set of real numbers $\mathbb{R}$, then its truth membership function, indeterminacy membership function and falsity membership function are defined as

$$T_{\tilde{A}}(x)=\begin{cases} T^L_{\tilde{A}}(x), & a_{11}\le x\le a_{21},\\ 1, & a_{21}\le x\le a_{31},\\ T^U_{\tilde{A}}(x), & a_{31}\le x\le a_{41},\\ 0, & otherwise.\end{cases} \quad I_{\tilde{A}}(x)=\begin{cases} I^L_{\tilde{A}}(x), & b_{11}\le x\le b_{21},\\ 0, & b_{21}\le x\le b_{31},\\ I^U_{\tilde{A}}(x), & b_{31}\le x\le b_{41},\\ 1, & otherwise,\end{cases}$$

$$F_{\tilde{A}}(x)=\begin{cases} F^L_{\tilde{A}}(x), & c_{11}\le x\le c_{21},\\ 0, & c_{21}\le x\le c_{31},\\ F^U_{\tilde{A}}(x), & c_{31}\le x\le c_{41},\\ 1, & otherwise.\end{cases}$$

The sum of three independent membership degrees of a single-valued neutrosophic set $\tilde{A}$ lie between the interval $[0,3]$ and a_{11}, a_{21}, a_{31}, a_{41}, b_{11}, b_{21}, b_{31}, b_{41}, c_{11}, c_{21}, c_{31}, and c_{41} belong to $\mathbb{R}$ such that $a_{11}\le a_{21}\le a_{31}\le a_{41}$, $b_{11}\le b_{21}\le b_{31}\le b_{41}$, and $c_{11}\le c_{21}\le c_{31}\le c_{41}$. The functions $T^L_{\tilde{A}}$, $I^L_{\tilde{A}}$, and $F^L_{\tilde{A}}$ are non-decreasing continuous functions and $T^U_{\tilde{A}}$, $I^U_{\tilde{A}}$, and $F^U_{\tilde{A}}$ are non-increasing continuous functions.

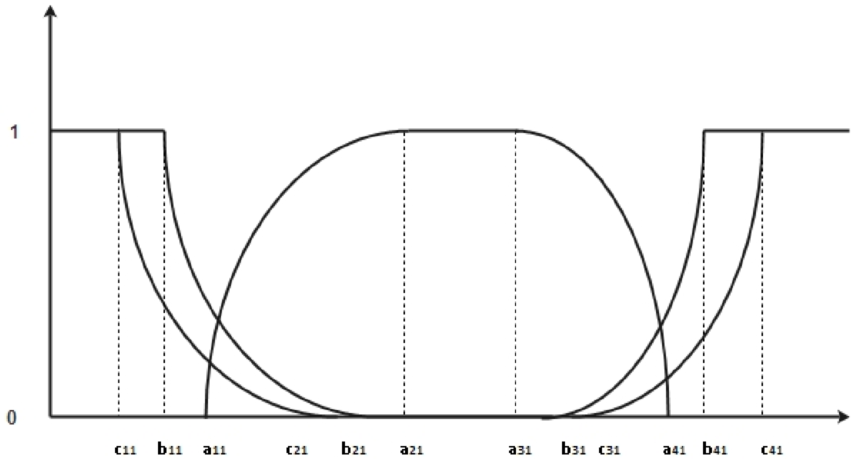

Figure 1: Neutrosophic number

Definition 6. (Biswas et al., 2016b) A neutrosophic trapezoidal number (NTrN) $\tilde{A}$ is a set of twelve parameters satisfying the inequality $c_{11}\le b_{11}\le a_{11}\le c_{21}\le b_{21}\le a_{21}\le a_{31}\le b_{31}\le c_{31}\le a_{41}\le b_{41}\le c_{41}$ and is denoted by $\tilde{A}=\big\langle(a_{11},a_{21},a_{31},a_{41}),\ (b_{11},b_{21},b_{31},b_{41}),\ (c_{11},c_{21},c_{31},c_{41})\big\rangle$ in the set of real numbers $\mathbb{R}$. Then the truth membership , the indeterminacy membership and the falsity membership degree of $\tilde{A}$ are defined as

$$T_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_{11}}{a_{21}-a_{11}}, & a_{11}\le x\le a_{21},\\ 1, & a_{21}\le x\le a_{31},\\ \dfrac{a_{41}-x}{a_{41}-a_{31}}, & a_{31}\le x\le a_{41},\\ 0, & otherwise.\end{cases}\quad I_{\tilde{A}}(x)=\begin{cases}\dfrac{x-b_{21}}{b_{21}-b_{11}}, & b_{11}\le x\le b_{21},\\ 0, & b_{21}\le x\le b_{31},\\ \dfrac{x-b_{31}}{b_{41}-b_{31}}, & b_{31}\le x\le b_{41},\\ 1, & otherwise.\end{cases}$$

$$F_{\tilde{A}}(x)=\begin{cases}\dfrac{x-c_{21}}{c_{21}-c_{11}}, & c_{11}\le x\le c_{21},\\ 0, & c_{21}\le x\le c_{31},\\ \dfrac{x-c_{31}}{c_{41}-c_{31}}, & c_{31}\le x\le c_{41},\\ 1, & otherwise.\end{cases}$$

For a_{21}=a_{31}, b_{21}=b_{31} , and c_{21}=c_{31} in a NTrN $\tilde{A}$, we get a new type of neutrosophic number and call it neutrosophic triangular number.

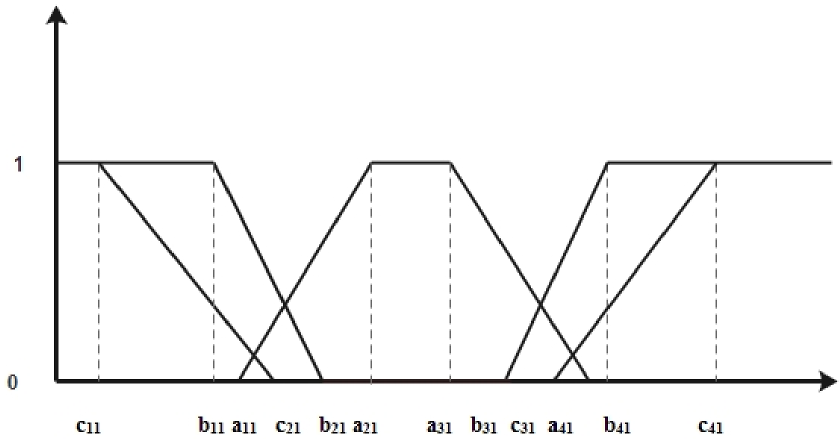

Figure 2: Neutrosophic trapezoidal number

Definition 7. (Biswas et al., 2016b) Let $\tilde{A}=\langle(a_{11},a_{21},a_{31},a_{41}),(b_{11},b_{21},b_{31},b_{41}),(c_{11},c_{21},c_{31},c_{41})\rangle$ and $\tilde{B}=\langle(a_{12},a_{22},a_{32},a_{42}),(b_{12},b_{22},b_{32},b_{42}),(c_{12},c_{22},c_{32},c_{42})\rangle$ be two NTrNs in the set of real numbers $\mathbb{R}$, then the following operations are valid:

1. $\tilde{A}\oplus\tilde{B}=\left\langle\begin{array}{c}(a_{11}+a_{12},a_{21}+a_{22},a_{31}+a_{32},a_{41}+a_{42}),\\ (b_{11}+b_{12},b_{21}+b_{22},b_{31}+b_{32},b_{41}+b_{42}),\\ (c_{11}+c_{12},c_{21}+c_{22},c_{31}+c_{32},c_{41}+c_{42})\end{array}\right\rangle,$

2. $\tilde{A}\otimes\tilde{B}=\left\langle\begin{array}{c}(a_{11}a_{12},a_{21}a_{22},a_{31}a_{32},a_{41}a_{42}),\\ (b_{11}b_{12},b_{21}b_{22},b_{31}b_{32},b_{41}b_{42}),\\ (c_{11}c_{12},c_{21}c_{22},c_{31}c_{32},c_{41}c_{42})\end{array}\right\rangle,$

3. $\lambda\tilde{A} = \left\langle \begin{array}{c} (\lambda a_{11}, \lambda a_{21}, \lambda a_{31}, \lambda a_{41}), (\lambda b_{11}, \lambda b_{21}, \lambda b_{31}, \lambda b_{41}), \\ (\lambda c_{11}, \lambda c_{21}, \lambda c_{31}, \lambda c_{41}) \end{array} \right\rangle \text{for} \lambda > 0,$

4. $\tilde{A}^{\lambda} = \left\langle \begin{array}{c} (a_{11}^{\lambda}, a_{21}^{\lambda}, a_{31}^{\lambda}, a_{41}^{\lambda}), (b_{11}^{\lambda}, b_{21}^{\lambda}, b_{31}^{\lambda}, b_{41}^{\lambda}), \\ (c_{11}^{\lambda}, c_{21}^{\lambda}, c_{31}^{\lambda}, c_{41}^{\lambda}) \end{array} \right\rangle \text{for} \lambda > 0.$

3 EXPECTED VALUE OF NEUTROSOPHIC TRAPEZOIDAL NUMBER

For a NTrN $\tilde{A} = \langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \rangle$, we assume that $T_{\tilde{A}}^{L}(x) = \dfrac{x - a_{11}}{a_{21} - a_{11}}$ and $T_{\tilde{A}}^{U}(x) = \dfrac{x - a_{41}}{a_{31} - a_{41}}$ are the two sides of trapezoidal fuzzy number $T_{\tilde{A}}(x) = (a_{11}, a_{21}, a_{31}, a_{41})$ in $\tilde{A}$. Similarly, $I_{\tilde{A}}^{L}(x) = \dfrac{x - b_{21}}{b_{11} - b_{21}}$ and $I_{\tilde{A}}^{U}(x) = \dfrac{x - a_{31}}{a_{41} - a_{31}}$ are the two sides of trapezoidal fuzzy number $I_{\tilde{A}}(x) = (b_{11}, b_{21}, b_{31}, b_{41})$ and $F_{\tilde{A}}^{L}(x) = \dfrac{x - c_{21}}{c_{11} - c_{21}}$ and $F_{\tilde{A}}^{U}(x) = \dfrac{x - c_{31}}{c_{41} - c_{31}}$ are the two sides of trapezoidal fuzzy number $F_{\tilde{A}}(x) = (c_{11}, c_{21}, c_{31}, c_{41})$.

Definition 8. (**Expected interval of a neutrosophic number**)
The expected interval of a NTrN $\tilde{A} = \langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \rangle$ is defined by

$$EI(\tilde{A}) = [E(\tilde{A}^{L}), E(\tilde{A}^{U})]. \tag{2}$$

Here, the lower limit of expected interval for the functions $F_{\tilde{A}}^{L}(x)$, $I_{\tilde{A}}^{L}(x)$ and $T_{\tilde{A}}^{L}(x)$ is

$$\begin{aligned} E(\tilde{A}^{L}) &= \frac{1}{3}\left[\left(c_{11} - \int_{c_{21}}^{c_{11}} F_{\tilde{A}}^{L}(x)\,dx\right) + \left(b_{11} - \int_{b_{21}}^{b_{11}} I_{\tilde{A}}^{L}(x)\,dx\right) + \left(a_{21} - \int_{a_{11}}^{a_{21}} T_{\tilde{A}}^{L}(x)\,dx\right)\right] \\ &= \frac{c_{11} + b_{11} + a_{21}}{3} + \frac{1}{3}\int_{c_{11}}^{c_{21}} F_{\tilde{A}}^{L}(x)\,dx + \frac{1}{3}\int_{b_{11}}^{b_{21}} I_{\tilde{A}}^{L}(x)\,dx - \frac{1}{3}\int_{a_{11}}^{a_{21}} T_{\tilde{A}}^{L}(x)\,dx; \end{aligned} \tag{3}$$

and the upper limit of expected interval for the functions $F_{\tilde{A}}^{U}(x)$, $I_{\tilde{A}}^{U}(x)$ and $T_{\tilde{A}}^{U}(x)$ is

$$\begin{aligned} E(\tilde{A}^{U}) &= \frac{1}{3}\left[\left(c_{41} + \int_{c_{41}}^{c_{31}} F_{\tilde{A}}^{L}(x)\,dx\right) + \left(b_{41} + \int_{b_{41}}^{b_{31}} I_{\tilde{A}}^{L}(x)\,dx\right) + \left(a_{31} - \int_{a_{11}}^{a_{21}} T_{\tilde{A}}^{L}(x)\,dx\right)\right] \\ &= \frac{a_{31} + b_{41} + c_{41}}{3} + \frac{1}{3}\int_{a_{31}}^{a_{41}} T_{\tilde{A}}^{U}(x)\,dx - \frac{1}{3}\int_{b_{31}}^{b_{41}} I_{\tilde{A}}^{U}(x)\,dx - \frac{1}{3}\int_{c_{31}}^{c_{41}} F_{\tilde{A}}^{U}(x)\,dx \end{aligned} \tag{4}$$

Definition 9. Let $\tilde{A} = \langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \rangle$ be a neutrosophic number in the set of real numbers $\mathbb{R}$. Then the expected value of $\tilde{A}$ is determined by taking the mid values of expected interval of $\tilde{A}$ and is defined by

$$EV(\tilde{A}) = \frac{E(\tilde{A}^L) + E(\tilde{A}^U)}{2} \tag{5}$$

Therefore the expected value of a neutrosophic trapezoidal number can be determined by the expected interval of neutrosophic numbers with the following theorem.

Theorem 3.1. *Let* $\tilde{A}=\langle(a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41})\rangle$ *be a NTrN in the set of real numbers* $\mathbb{R}$ *satisfying the relation* $c_{11} \leq b_{11} \leq a_{11} \leq c_{21} \leq b_{21} \leq a_{21} \leq a_{31} \leq b_{31} \leq c_{31} \leq a_{41} \leq b_{41} \leq c_{41}$. *Then for* $T^L_{\tilde{A}}(x)=\frac{x-a_{11}}{a_{21}-a_{11}}$, $T^U_{\tilde{A}}(x)=\frac{x-a_{41}}{a_{31}-a_{41}}$; $I^L_{\tilde{A}}(x)=\frac{x-b_{21}}{b_{11}-b_{21}}$, $I^U_{\tilde{A}}(x)=\frac{x-a_{31}}{a_{41}-a_{31}}$, $F^L_{\tilde{A}}(x)=\frac{x-c_{21}}{c_{11}-c_{21}}$ *and* $F^U_{\tilde{A}}(x)=\frac{x-c_{31}}{c_{41}-c_{31}}$, *the expected value of* $\tilde{A}$ *is obtained by*

$$EV(\tilde{A}) = \frac{\sum_{i=1}^{4} a_{i1} + \sum_{i=1}^{4} b_{i1} + \sum_{i=1}^{4} c_{i1}}{12}. \tag{6}$$

Proof. Putting the values of $T^L_{\tilde{A}}(x)$, $I^L_{\tilde{A}}(x)$, and $F^L_{\tilde{A}}(x)$ in Eq.(3), we get

$$\begin{aligned} E(\tilde{A}^L) &= \frac{c_{11}+b_{11}+a_{21}}{3} + \frac{1}{3}\int_{c_{11}}^{c_{21}} \frac{x-c_{21}}{c_{11}-c_{21}}\,dx + \frac{1}{3}\int_{b_{11}}^{b_{21}} \frac{x-b_{21}}{b_{11}-c_{21}}\,dx \\ &\quad - \frac{1}{3}\int_{a_{11}}^{a_{21}} \frac{x-a_{11}}{a_{21}-a_{11}}\,dx \\ &= \frac{c_{11}+b_{11}+a_{21}}{3} + \frac{c_{21}-c_{11}}{6} + \frac{b_{21}-b_{11}}{6} + \frac{a_{11}-a_{21}}{6} \\ &= \frac{c_{11}+b_{11}+a_{21}+c_{21}+b_{21}+a_{11}}{6}. \end{aligned} \tag{7}$$

Similarly, putting the values of $T^U_{\tilde{A}}(x)$, $I^U_{\tilde{A}}(x)$, and $F^U_{\tilde{A}}(x)$ in Eq.(4), we obtain

$$\begin{aligned} E(\tilde{A}^U) &= \frac{a_{31}+b_{41}+c_{41}}{3} + \frac{1}{3}\int_{a_{31}}^{a_{41}} \frac{x-a_{41}}{a_{31}-a_{41}}\,dx - \frac{1}{3}\int_{b_{31}}^{b_{41}} \frac{x-b_{31}}{b_{41}-b_{31}}\,dx \\ &\quad - \frac{1}{3}\int_{c_{31}}^{c_{41}} \frac{x-c_{31}}{a_{41}-a_{31}}\,dx \\ &= \frac{a_{31}+b_{41}+c_{41}}{3} + \frac{a_{41}-a_{31}}{6} + \frac{b_{31}-b_{41}}{6} + \frac{c_{31}-c_{41}}{6} \\ &= \frac{c_{11}+b_{11}+a_{21}+c_{21}+b_{21}+a_{11}}{6}. \end{aligned} \tag{8}$$

Following Eq.(5), we obtain the required expected value of $\tilde{A}$

$$EV(\tilde{A}) = \frac{\sum_{i=1}^{4} a_{i1} + \sum_{i=1}^{4} b_{i1} + \sum_{i=1}^{4} c_{i1}}{12}.$$

This completes the proof. □

Proposition 3.2. *Let* $\tilde{A}_1 = \big\langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \big\rangle$ *and* $\tilde{A}_2 = \big\langle (a_{12}, a_{22}, a_{32}, a_{42}),\ (b_{12}, b_{22}, b_{32}, b_{42}),\ (c_{12}, c_{22}, c_{32}, c_{42}) \big\rangle$ *be two NTrNs in the set of real numbers* $\mathbb{R}$*. Then the following relations are satisfied:*

1. $EV(\tilde{A}_1 + \tilde{A}_2) = EV(\tilde{A}_1) + EV(\tilde{A}_2)$;
2. $EV(\lambda \tilde{A}_1) = \lambda EV(\tilde{A}_1)$.

Proof. Following the Eq.(6) about expected value and addition of NTrNs, we have

$$\begin{aligned} EV(\tilde{A}_1 + \tilde{A}_2) &= \frac{\sum_{i=1}^{4} (a_{i1} + a_{i2}) + \sum_{i=1}^{4} (b_{i1} + b_{i2}) + \sum_{i=1}^{4} (c_{i1} + c_{i2})}{12} \\ &= \frac{\Big(\sum_{i=1}^{4} a_{i1} + \sum_{i=1}^{4} b_{i1} + \sum_{i=1}^{4} c_{i1} \Big) + \Big(\sum_{i=1}^{4} a_{i2} + \sum_{i=1}^{4} b_{i2} + \sum_{i=1}^{4} c_{i2} \Big)}{12} \\ &= EV(\tilde{A}_1) + EV(\tilde{A}_2) \end{aligned}$$

Similarly,

$$\begin{aligned} EV(\lambda \tilde{A}_1) &= \frac{\sum_{i=1}^{4} (\lambda a_{i1}) + \sum_{i=1}^{4} (\lambda b_{i1}) + \sum_{i=1}^{4} (\lambda c_{i1})}{12} \\ &= \lambda \Big[\frac{\sum_{i=1}^{4} a_{i1} + \sum_{i=1}^{4} b_{i1} + \sum_{i=1}^{4} c_{i1}}{12} \Big] \\ &= \lambda EV(\tilde{A}_1). \end{aligned}$$

This completes the proof. □

Definition 10. Let $\tilde{A}_1 = \big\langle (a_{11}, a_{21}, a_{31}, a_{41}), (b_{11}, b_{21}, b_{31}, b_{41}), (c_{11}, c_{21}, c_{31}, c_{41}) \big\rangle$ and $\tilde{A}_2 = \big\langle (a_{12}, a_{22}, a_{32}, a_{42}), (b_{12}, b_{22}, b_{32}, b_{42}), (c_{12}, c_{22}, c_{32}, c_{42}) \big\rangle$ be two NTrNs. Then the following relations are satisfied:

1. $\tilde{A}_1 \prec_{EV} \tilde{A}_2 \Leftrightarrow EV(\tilde{A}_1) < EV(\tilde{A}_2)$;
2. $\tilde{A}_1 \succ_{EV} \tilde{A}_2 \Leftrightarrow EV(\tilde{A}_1) > EV(\tilde{A}_2)$;

3. $\tilde{A}_1 \sim_{EV} \tilde{A}_2 \Leftrightarrow EV(\tilde{A}_1) = EV(\tilde{A}_2)$.

Example 11. Let $\tilde{A}_1$=$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9), (0.4, 0.5, 0.8, 0.9)\rangle$ and $\tilde{A}_2$ =$\langle(0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 1.0), (0.7, 0.8, 0.9, 0.1)\rangle$ be two NTrNs, then by Definition 3.2 we can calculate

$$\tilde{A}_1 + \tilde{A}_2 = \langle(1.2, 1.4, 1.6, 1.8), (1.1, 1.4, 1.6, 1.9), (1.1, 1.3, 1.7, 1.9)\rangle;$$
$$5\tilde{A}_1 = \langle(2.5, 3.0, 3.5, 4.0), (2.0, 2.4, 3.5, 4.5), (2.0, 2.5, 4.0, 4.5)\rangle.$$

Following Eq. (6), we obtain the results: $EV(\tilde{A}_1) = 0.65$, $EV(\tilde{A}_2) = 0.85$, $EV(\tilde{A}_1 + \tilde{A}_2)$, and $EV(5\tilde{A}_1) = 3.25$. It follows that $EV(\tilde{A}_1 + \tilde{A}_2) = EV(\tilde{A}_1) + EV(\tilde{A}_2) = 1.5$ and $EV(5\tilde{A}_1) = 5EV(\tilde{A}_1) = 3.25$.
Because $EV(\tilde{A}_2) > EV(\tilde{A}_1)$, we can consider that $\tilde{A}_2 \succ_{EV} \tilde{A}_1$ i.e. $\tilde{A}_2$ is greater than $\tilde{A}_1$.

4 MADM USING EXPECTED VALUE OF NEUTROSOPHIC TRAPEZOIDAL FUZZY NUMBER

In this section we develop multi-attribute group decision making with neutrosophic trapezoidal number.
Assume that A= $\{A_1, A_2, \ldots, A_m\}$ be the set of m alternatives, C= $\{C_1, C_2, \ldots, C_n\}$ be the set of n attributes, D= $\{D_1, D_2, \ldots, D_K\}$ be the set of k decision makers (experts). We also consider that $\lambda^k = \{\tilde{\lambda}_j^1, \tilde{\lambda}_j^2, \ldots, \tilde{\lambda}_j^k\}$ be the k-th decision maker's weight vector of jth attribute for $j = 1, 2, \ldots n$, where $\tilde{\lambda}_j^2$ takes the form on NTrN $\tilde{\lambda}_j^k = \langle(u_{j1}^k, u_{j2}^k, u_{j3}^k, u_{j4}^k)$, $(v_{j1}^k, v_{j2}^k, v_{j3}^k, v_{j4}^k)$, $(w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k)\rangle$. The rating values of kth decision maker of the alternatives A_i for i=$1, 2, \ldots m$ with respect to attributes C_j for j=$1, 2, \ldots n$ can be concisely expressed in matrix format. Then we obtain the decision matrix $(d_{ij}^k)_{m\times n}$ for the kth decision maker as

$$(d_{ij}^k)_{m\times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & d_{11}^k & d_{12}^k & \cdots & d_{13}^k \\ A_2 & d_{21}^k & d_{22}^k & \cdots & d_{2n}^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & d_{m1}^k & d_{m2}^k & \cdots & d_{mn}^k \end{array} \tag{9}$$

where, $\tilde{d}_{ij}^k$= $\langle T_{ij}^k, I_{ij}^k, F_{ij}^k\rangle$ is the neutrosophic rating of alternative A_i with respect to attribute C_j. In the rating $\tilde{d}_{ij}^k$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, the component T_{ij}^k=$(a_{ij1}^k, a_{ij2}^k, a_{ij3}^k, a_{ij4}^k)$ represents the truth membership function , I_{ij}^k=$(b_{ij1}^k, b_{ij2}^k, b_{ij3}^k, b_{ij4}^k)$ represents the indeterminacy membership function, and F_{ij}^k=$(c_{ij1}^k, c_{ij2}^k, c_{ij3}^k, c_{ij4}^k)$ represents the falsity membership function. Hence we can consider the NTrN $\tilde{d}_{ij}^k = \langle(a_{ij1}^k, a_{ij2}^k, a_{ij3}^k, a_{ij4}^k)$,
$(b_{ij1}^k, b_{ij2}^k, b_{ij3}^k, b_{ij4}^k)$,
$(c_{ij1}^k, c_{ij2}^k, c_{ij3}^k, c_{ij4}^k)\rangle$ as the neutrosophic rating of the decision matrix.

Step 1. Aggregate the rating values of alternatives

In the decision making process, experts provide their different ratings for each alternative. Therefore, the method of average value can be used to aggregate the neutrosophic ratings $\langle T_{ij}^k, I_{ij}^k, F_{ij}^k \rangle$ of K decision makers.

Thus the aggregated neutrosophic rating $\tilde{d}_{ij}$ $(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$ of the alternatives are calculated as $\tilde{d}_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$ where,

$$\begin{aligned} T_{ij} =& \Big(\frac{\sum_{k=1}^K a_{ij1}^k}{K}, \frac{\sum_{k=1}^K a_{ij2}^k}{K}, \frac{\sum_{k=1}^K a_{ij3}^k}{K}, \frac{\sum_{k=1}^K a_{ij4}^k}{K} \Big) \\ =& (a_{ij1}, a_{ij2}, a_{ij3}, a_{ij4}) \end{aligned} \tag{10}$$

$$\begin{aligned} I_{ij} =& \Big(\frac{\sum_{k=1}^K b_{ij1}^k}{K}, \frac{\sum_{k=1}^K b_{ij2}^k}{K}, \frac{\sum_{k=1}^K b_{ij3}^k}{K}, \frac{\sum_{k=1}^K b_{ij4}^k}{K} \Big) \\ =& (b_{ij1}, b_{ij2}, b_{ij3}, b_{ij4}) \end{aligned} \tag{11}$$

$$\begin{aligned} F_{ij} =& \Big(\frac{\sum_{k=1}^K c_{ij1}^k}{K}, \frac{\sum_{k=1}^K c_{ij2}^k}{K}, \frac{\sum_{k=1}^K c_{ij3}^k}{K}, \frac{\sum_{k=1}^K c_{ij4}^k}{K} \Big) \\ =& (c_{ij1}, c_{ij2}, c_{ij3}, c_{ij4}) \end{aligned} \tag{12}$$

Then the aggregated group decision matrix $\tilde{D}$ can be obtained as

$$(\tilde{d}_{ij})_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \tilde{d}_{11} & \tilde{d}_{12} & \cdots & \tilde{d}_{13} \\ A_2 & \tilde{d}_{21} & \tilde{d}_{22} & \cdots & \tilde{d}_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & \tilde{d}_{m1} & \tilde{d}_{m2} & \cdots & \tilde{d}_{mn} \end{array} \tag{13}$$

and the corresponding expected value based decision matrix of $\tilde{D}$ can be obtained by Eq.(6) as

$$(E(\tilde{d}_{ij}))_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & EV(\tilde{d}_{11}) & EV(\tilde{d}_{12}) & \cdots & EV(\tilde{d}_{1n}) \\ A_2 & EV(\tilde{d}_{21}) & EV(\tilde{d}_{22}) & \cdots & EV(\tilde{d}_{2n}) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_m & EV(\tilde{d}_{m1}) & EV(\tilde{d}_{m2}) & \cdots & EV(\tilde{d}_{mn}) \end{array} \tag{14}$$

Step 2. Aggregate of the weight of attributes

Similarly, using the method of average value, the aggregated neutrosophic weight $\tilde{\lambda}_j =$

$\langle u_j, v_j, w_j \rangle$ of $C_j (j = 1, 2, \ldots, n)$ can be calculated as follows:

$$\begin{aligned} u_j =& \Big(\frac{\sum_{k=1}^{K} u_{j1}^k}{K}, \frac{\sum_{k=1}^{K} u_{j2}^k}{K}, \frac{\sum_{k=1}^{K} u_{j3}^k}{K}, \frac{\sum_{k=1}^{K} u_{j4}^k}{K} \Big) \\ =& (u_{j1}, u_{j2}, u_{j3}, u_{j4}) \end{aligned} \tag{15}$$

$$\begin{aligned} v_j =& \Big(\frac{\sum_{k=1}^{K} v_{j1}^k}{K}, \frac{\sum_{k=1}^{K} v_{j2}^k}{K}, \frac{\sum_{k=1}^{K} v_{j3}^k}{K}, \frac{\sum_{k=1}^{K} v_{j4}^k}{K} \Big) \\ =& (v_{j1}, v_{j2}, v_{j3}, v_{j4}) \end{aligned} \tag{16}$$

$$\begin{aligned} w_j =& \Big(\frac{\sum_{k=1}^{K} w_{j1}^k}{K}, \frac{\sum_{k=1}^{K} w_{j2}^k}{K}, \frac{\sum_{k=1}^{K} w_{j3}^k}{K}, \frac{\sum_{k=1}^{K} w_{j4}^k}{K} \Big) \\ =& (w_{j1}, w_{j2}, w_{j3}, w_{j4}). \end{aligned} \tag{17}$$

Then, the aggregated attribute weight $\tilde{W}$ can be taken as

$$W \quad = [\tilde{\lambda}_1, \tilde{\lambda}_2, \ldots, \tilde{\lambda}_n]. \tag{18}$$

where, $\tilde{\lambda}_j = \langle u_j, v_j, w_j \rangle$ for $j = 1, 2, \ldots, n$. Now by Eq.(6), we determine the expected value of weight $\tilde{\lambda}_j$ $(j = 1, 2, \ldots, n)$ for an attribute C_j and obtain the normalized expected weight vector

$$W^N \quad = [\lambda_1^N, \lambda_2^N, \ldots, \lambda_n^N] \tag{19}$$

where,

$$\lambda_j^N = \frac{EV\left(\tilde{\lambda}_j\right)}{\sum\limits_{j=1}^{n} EV\left(\tilde{\lambda}_j\right)} \quad j = 1, 2, \ldots, n. \tag{20}$$

Step 3. Determine the weighted expected value of alternative

We now determine the weighted expected value of the alternative A_i for $i = 1, 2, \ldots, m$ by summing up the multiplicative values of normalized expected weight and expected value of aggregated rating value for an attribute $C_j (j = 1, 2, \ldots, n)$ in the decision matrix $(E(\tilde{d}_{ij}))_{m \times n}$ shown in Eq.(14). Therefore, the weighted expected value of alternative $A_i (i = 1, 2, \ldots, m)$ is

$$EV_w(A_i) = \sum_{j=1}^{n} \lambda_j^N EV(\tilde{d}_{ij}). \tag{21}$$

Step 4. Rank the alternatives

Largest value of the weighted expected value $EV_w(A_i)$ of an alternative $A_i (i = 1, 2, \ldots, m)$ determines the best alternative.

5 ILLUSTRATIVE EXAMPLE

To illustrate the proposed approach, we provide an illustrative example. Assume that an organization desires to purchase some cars. After initial choice, four models (i.e. alternatives) A_1, A_2, A_3 and A_4 are considered for further evaluation. A committee of four experts D_1, D_2, D_3 and D_4 is set up to select the most appropriate alternative car. Six attributes are considered which include:

1. Performance (C_1),
2. Style (C_2),
3. Comfort (C_3),
4. Safety (C_4),
5. Specifications (C_5),
6. Customer service (C_6).

Linguistic variables are generally presented with linguistic terms Zadeh (1975). These terms play an important role to present uncertain information that are either too complex or too ill-defined to be described properly with conventional quantitative expressions. For example, the ratings of alternatives over the qualitative attributes could be expressed with linguistic variables such as very poor, poor, medium poor, fair, medium good, good, very good, etc. These linguistic terms can also be represented by NTrNs such as the term "fair" can be considered with $\big\langle(0.3, 0.4, 0.5, 0.6),\ (0.2, 0.4, 0.5, 0.7),\ (0.2, 0.3, 0.6, 0.7)\big\rangle$. We now define the following linguistic scales characterizing NTrNs.

Table 1: Linguistic variables for the importance of attributes

Linguistic variables	Corresponding NTrNs
Very low(VL)	$\langle(0.0, 0.0, 0.0, 0.0),\ (0.0, 0.0, 0.0, 0.0),\ (0.0, 0.0, 0.0, 0.0)\rangle$
Low(L)	$\langle(0.0, 0.1, 0.2, 0.3),\ (0.0, 0.1, 0.2, 0.3),\ (0.0, 0.1, 0.2, 0.3)\rangle$
Medium(M)	$\langle(0.1, 0.2, 0.3, 0.4),\ (0.0, 0.2, 0.3, 0.5),\ (0.0, 0.1, 0.4, 0.5)\rangle$
High(H)	$\langle(0.3, 0.4, 0.5, 0.6),\ (0.2, 0.4, 0.5, 0.7),\ (0.2, 0.3, 0.6, 0.7)\rangle$
Very High(VH)	$\langle(0.5, 0.6, 0.7, 0.8),\ (0.4, 0.6, 0.7, 0.9),\ (0.4, 0.5, 0.8, 0.9)\rangle$

We consider that the four experts describe the importance of the attribute and the rating of alternatives by linguistic variables such as very good, good, fair, poor, very poor, etc. The linguistic ratings of four alternatives under the pre-assigned attributes and the weights of the attributes for $k(k = 1, 2, \ldots, K)$ are shown in Table 1. We first convert the assessed rating values of alternative and weights of each attribute with the help of pre-defined linguistic variables in the form of NTrNs defined in Table 2. The proposed method is applied to solve the problem and its computational procedure is summarized as follows:

Table 2: Linguistic variables for the ratings of alternatives

Linguistic variables	Corresponding NTrNs
Very Poor(VP)	$\langle(0.0, 0.0, 0.0, 0.0),\ (0.0, 0.0, 0.0, 0.0),\ (0.0, 0.0, 0.0, 0.0)\rangle$
Poor(P)	$\langle(0.0, 0.1, 0.2, 0.3),\ (0.0, 0.1, 0.2, 0.3),\ (0.0, 0.1, 0.2, 0.3)\rangle$
Medium Poor(MP)	$\langle(0.1, 0.2, 0.3, 0.4),\ (0.0, 0.2, 0.3, 0.5),\ (0.0, 0.1, 0.4, 0.5)\rangle$
Fair(F)	$\langle(0.3, 0.4, 0.5, 0.6),\ (0.2, 0.4, 0.5, 0.7),\ (0.2, 0.3, 0.6, 0.7)\rangle$
Good(G)	$\langle(0.5, 0.6, 0.7, 0.8), (0.4, 0.6, 0.7, 0.9), (0.4, 0.5, 0.8, 0.9)\rangle$
Medium Good(MG)	$\langle(0.7, 0.8, 0.9, 1.0),\ (0.7, 0.8, 0.9, 1.0),\ (0.7, 0.8, 0.9, 1.0)\rangle$
Very Good(VG)	$\langle(1.0, 1.0, 1.0, 1.0),\ (1.0, 1.0, 1.0, 1.0),\ (1.0, 1.0, 1.0, 1.0)\rangle$

Table 3: Rating of alternatives and weight of attributes

Alternatives (A_i)	Decision Makers	C_1	C_2	C_3	C_4	C_5	C_6
A_1	DM-1	VG	G	G	G	G	VG
	DM-2	VG	VG	G	G	G	VG
	DM-3	G	VG	G	G	VG	G
	DM-4	G	G	G	G	G	G
A_2	DM-1	F	G	F	G	G	F
	DM-2	G	MG	G	MG	G	G
	DM-3	G	F	G	F	VG	F
	DM-4	F	G	F	F	G	F
A_3	DM-1	VG	VG	G	G	VG	VG
	DM-2	G	VG	VG	G	G	VG
	DM-3	VG	G	G	MG	G	MG
	DM-4	G	G	G	MG	G	G
A_4	DM-1	F	VG	G	G	VG	F
	DM-2	F	F	G	G	F	G
	DM-3	G	MG	G	MG	MG	G
	DM-4	G	G	F	G	G	G
Weights	DM-1	VH	VH	H	M	H	H
	DM-2	H	VH	H	H	M	M
	DM-3	M	H	M	M	H	M
	DM-4	M	H	M	H	VH	H

Step 1. Determine the aggregated rating values of alternatives

Using Eqs.(10),(11), and (12), we aggregate each of four decision makers' opinion into a group opinion (see Table 4). Then employing expected value of neutrosophic trapezoidal number defined in Eq.(6), we construct the following expected value matrix:

$$(\tilde{d}_{ij})_{m\times n} = \begin{array}{c|cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ \hline A_1 & 0.9250 & 0.9250 & 0.8500 & 0.8500 & 0.8875 & 0.9250 \\ A_2 & 0.6500 & 0.7000 & 0.6500 & 0.6000 & 0.8875 & 0.5500 \\ A_3 & 0.9250 & 0.9250 & 0.8875 & 0.7500 & 0.8875 & 0.8750 \\ A_4 & 0.6500 & 0.7375 & 0.7500 & 0.8000 & 0.7375 & 0.7500 \end{array} \quad (22)$$

Step 2. Aggregate of the weight of attributes

Similarly, we aggregate the weights of attributes by Eqs.(15), (16), and (17). Then the aggregated weight vector W is

$$W = \begin{bmatrix} \langle (0.60, 0.70, 0.80, 0.90), & \langle (0.60, 0.70, 0.80, 0.90), \\ (0.58, 0.70, 0.80, 0.92), & (0.58, 0.70, 0.80, 0.92), \\ (0.58, 0.67, 0.82, 0.92)\rangle, & (0.58, 0.67, 0.82, 0.92)\rangle, \\ \langle (0.60, 0.70, 0.80, 0.90), & \langle (0.60, 0.70, 0.80, 0.90), \\ (0.58, 0.70, 0.80, 0.92), & (0.58, 0.70, 0.80, 0.92), \\ (0.58, 0.67, 0.82, 0.92)\rangle, & (0.58, 0.67, 0.82, 0.92)\rangle, \\ \langle (0.60, 0.70, 0.80, 0.90), & \langle (0.60, 0.70, 0.80, 0.90), \\ (0.58, 0.70, 0.80, 0.92), & (0.58, 0.70, 0.80, 0.92), \\ (0.58, 0.67, 0.82, 0.92)\rangle, & (0.58, 0.67, 0.82, 0.92)\rangle \end{bmatrix}. \quad (23)$$

Using Eq.(6), we calculate the expected value of each element of the weight vector W:

$$EV(W) = (0.40, 0.55, 0.35, 0.35, 0.45, 0.35)^T. \quad (24)$$

Following Eq.(20), we determine the normalized weight vector

$$W^N = (0.1633, 0.2246, 0.1428, 0.1428, 0.1428, 0.1837)^T. \quad (25)$$

Step 3. Determine the weighted expected value of alternative

By Eq.(21), we determine the following weighted expected value of alternative A_i for $(i = 1, 2, 3, 4)$: $EV(A_1) = 0.8967$, $EV(A_2) = 0.6834$, $EV(A_1) = 0.8806$, $EV(A_1) = 0.7357$.

Table 4: Aggregated rating values of alternatives with NTrNs

(A_i)	C_1	C_2	C_3
A_1	$\langle(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),\rangle$	$\langle(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),\rangle$	$\langle(0.70,0.80,0.90,1.00),$ $(0.70,0.80,0.90,1.00),$ $(0.70,0.80,0.90,1.00),\rangle$
A_2	$\langle(0.50,0.60,0.70,0.80),$ $(0.45,0.60,0.70,0.85),$ $(0.45,0.55,0.75,0.85),\rangle$	$\langle(0.55,0.65,0.75,0.85),$ $(0.50,0.65,0.75,0.90),$ $(0.55,0.60,0.80,0.90),\rangle$	$\langle(0.50,0.60,0.70,0.80),$ $(0.45,0.60,0.70,0.85),$ $(0.45,0.55,0.75,0.85),\rangle$
A_3	$\langle(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),\rangle$	$\langle(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),\rangle$	$\langle(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),\rangle$
A_4	$\langle(0.50,0.60,0.70,0.80),$ $(0.45,0.60,0.70,0.85),$ $(0.45,0.55,0.75,0.85),\rangle$	$\langle(0.63,0.70,0.77,0.85),$ $(0.58,0.70,0.77,0.90),$ $(0.58,0.65,0.82,0.90),\rangle$	$\langle(0.60,0.70,0.80,0.90),$ $(0.58,0.70,0.80,0.92),$ $(0.58,0.68,0.82,0.92),\rangle$

(A_i)	C_4	C_5	C_6
A_1	$\langle(0.70,0.80,0.90,1.00),$ $(0.70,0.80,0.90,1.00),$ $(0.70,0.80,0.90,1.00),\rangle$	$\langle(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),\rangle$	$\langle(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),$ $(0.85,0.90,0.95,1.00),\rangle$
A_2	$\langle(0.45,0.55,0.65,0.75),$ $(0.38,0.55,0.65,0.82),$ $(0.38,0.48,0.72,0.82),\rangle$	$\langle(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),\rangle$	$\langle(0.40,0.50,0.60,0.70),$ $(0.33,0.50,0.60,0.77),$ $(0.33,0.43,0.67,0.77),\rangle$
A_3	$\langle(0.60,0.70,0.80,0.90),$ $(0.55,0.70,0.80,0.95),$ $(0.55,0.65,0.85,0.95),\rangle$	$\langle(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),$ $(0.78,0.85,0.92,1.00),\rangle$	$\langle(0.80,0.85,0.90,0.95),$ $(0.78,0.85,0.90,0.97),$ $(0.78,0.83,0.92,0.97),\rangle$
A_4	$\langle(0.65,0.75,0.85,0.95),$ $(0.63,0.75,0.85,0.97),$ $(0.63,0.73,0.87,0.97),\rangle$	$\langle(0.63,0.70,0.77,0.85),$ $(0.58,0.70,0.77,0.90),$ $(0.58,0.65,0.82,0.90),\rangle$	$\langle(0.60,0.70,0.80,0.90),$ $(0.58,0.70,0.80,0.92),$ $(0.58,0.67,0.82,0.92),\rangle$

Step 4. Rank the alternatives

We set the following ranking order according to the weighted expected value of alternative $A_i (i = 1, 2, \dots, m)$ as

$$EV(A_1) > EV(A_3) > EV(A_4) > EV(A_4).$$

Thus the ranking order $A_1 \succ A_3 \succ A_4 \succ A_2$ of alternatives reflects that A_1 is the best car for purchasing.

6 CONCLUSIONS

In MAGDM problems, the rating values provided by decision makers are often evaluated qualitatively and quantitatively due to uncertainty of real world problems. Neutrosophic trapezoidal number (NTrN) is an alternative tool that can represent incomplete and inconsistent information. In this paper, we have taken decision maker's qualitative opinion in-terms of linguistic variables represented by predefined NTrNs. We have developed an exact formula of expected value for NTrN. Then we have determined the expected values of aggregated rating values and expected weight values of attributes. Furthermore, we have calculated the weighted expected values of alternatives to get the ranking order of alternatives. Finally, we have provided a numerical example about MAGDM with NTrNs to illustrate the proposed method. The developed method is straightforward and effective. We hope that the proposed method has a great chance of success for dealing with uncertainty in MAGDM problems such as personal selection, supplier selection, project evaluation, and manufacturing systems. This method can be extended to MAGDM problems under interval neutrosophic trapezoidal number information.

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