

Multi Criteria Decision Making Based on Projection and Bidirectional Projection Measures of Rough Neutrosophic Sets

Surapati Pramanik^{1*}, Rumi Roy², and Tapan Kumar Roy³

¹ Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District –North 24 Parganas, Pin code-743126, West Bengal, India. E-mail: sura_pati@yahoo.co.in

² Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, West Bengal, India. E-mail: roy.rumi.r@gmail.com

³ Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah-711103, West Bengal, India. E-mail: roy_t_k@gmail.com

Corresponding author's email^{1*}: sura_pati@yahoo.co.in

ABSTRACT

In this paper, we define projection and bidirectional projection measures between rough neutrosophic sets. Then two new multi criteria decision making methods are proposed based on neutrosophic projection and bidirectional projection measures respectively. Then the proposed methods are applied for solving multiple criteria group decision making problems. Finally, two numerical examples are provided to demonstrate the applicability and effectiveness of the proposed methods.

KEYWORDS: Rough neutrosophic set; projection measure; bidirectional projection measure.

1. INTRODUCTION

The concept of fuzzy set theory made its first appearance in the literature in two nearly simultaneous publications by Zadeh (1965) and Klaua (1965). Zadeh's work caught much more attention of the researchers than Klau's pure mathematical treatment. Zadeh (1965) defined fuzzy set by introducing membership function to deal non-statistical uncertainty. Atanassov (1983, 1986) defined intuitionistic fuzzy sets by introducing non-membership function as independent component. Smarandache (1998, 1999, 2002, 2005, 2010) introduced indeterminacy membership function as independent component and defined neutrosophic set. Smarandache (1998) paved the way to define single valued neutrosophic set (SVNS) (Wang et al., 2010) to deal realistic problems. SVNSs (Wang et al., 2010) have been widely studied and applied in different fields such as medical diagnosis (Ye, 2015b), multi criteria/multi attribute decision making (Sodenkamp, 2013; Ye, 2013a, 2013b, 2014a, 2014b, 2015a,; Biswas et al. 2014a, 2014b, 2015a, 2015b, 2016a, 2016b, 2017a, 2017b; Kharal, 2014; Liu et al., 2014; Liu & Li, 2017; Liu & Wang, 2014; Sahin & Liu, 2015; Peng et al., 2016; Pramanik et al., 2015, 2016; Broumi & Smarandache, 2013; Mondal & Pramanik, 2015d, 2015e), educational problem (Mondal & Pramanik, 2014b, 2015a), conflict resolution (Pramanik & Roy 2014), social problem (Pramanik & Chakrabarti, 2013, Mondal & Pramanik, 2014a), optimization (Das & Roy, 2015; Hezam et al, 2015; Abdel-Baset et al., 2016; Pramanik, 2016a, 2016b; Sarkar et al., 2016), clustering analysis (Ye, 2014a, 2014b), image processing (Cheng & Guo, 2008; Guo & Cheng, 2009; Guo et al., 2014), etc.

Pawlak (1982) proposed the concept of rough set. Rough set is an extension of the classical set theory (Cantor, 1874). It is very useful in dealing with incompleteness.

Broumi et al. (2014a, 2014b) proposed the concept of rough neutrosophic set (RNS) by combining the concept of rough set (Pawlak, 1982) and neutrosophic set (Smarandache, 1998). Rough neutrosophic set is very useful to deal with uncertain, inconsistent and incomplete information. Yang et al. (2016) introduced single valued neutrosophic rough sets on two-universes and presented an algorithm for multi criteria decision making (MCDM). Mondal and Pramanik (2015b) presented rough multi-attribute decision making based on grey relational analysis. Pramanik and Mondal (2015a) defined cosine similarity measure of rough neutrosophic sets and presented a MCDM approach in medical diagnosis. Mondal and Pramanik (2015c) presented MADM method using rough accuracy score function. Pramanik and Mondal (2015c) proposed cotangent similarity measure under rough neutrosophic environment. Pramanik and Mondal (2015b) further proposed some similarity measures namely, Dice similarity measure and Jaccard similarity measure in rough neutrosophic environment and their applications in MADM problems. Mondal et al. (2016a) defined several trigonometric Hamming similarity measures such as cosine, sine, cotangent similarity measures and proved some of their properties. In the same study (Mondal et al., 2016a) also presented MADM models based on Hamming similarity measures. Mondal et al. (2016b) proposed rough neutrosophic variational coefficient similarity measure and presented its application in multi attribute decision making. Mondal et al. (2016c) presented rough neutrosophic TOPSIS for multi-attribute group decision making problems.

Pramanik and Mondal (2015d) studied interval neutrosophic multi-attribute decision-making method based on GRA. Mondal and Pramanik (2015f) developed MADM methods based on cosine similarity measure, Dice similarity measure and Jaccard similarity measures under interval rough neutrosophic environment.

Mondal and Pramanik (2015g) proposed tri-complex rough neutrosophic similarity measure and presented its application in multi-attribute decision making problems. Mondal, Pramanik, Smarandache (2016d) defined rough neutrosophic hyper-complex set and presented its application to multi-attribute decision making problem.

Yue & Jia (2015) proposed a method for multi attribute group decision making (MAGDM) problems based on normalized projection measure, in which the attribute values are offered by decision makers in hybrid form with crisp values and interval data. Yue (2012a) studied a new method for MAGDM based on determining the weights of decision makers using an extended projection method with interval data. Yue (2012a) Xu and Da (2004) and Xu (2005) studied projection method for decision making in uncertain environment with preference information. Yue (2012b) described a model to obtain the weights of DMs with crisp values using a projection method. Yue (2017) defined new projection measures in real number and interval settings and proposed group decision-making with hybrid decision information, including real numbers and interval data. Zheng et al. (2010) proposed an improved grey relational projection method by combining grey relational analysis (GRA) and technique for order of preference by similarity to ideal solution (TOPSIS) to select the optimum building envelope.

Yang et al. (2014) develop projection method for material selection problem in fuzzy environment. Xu and Hu (2010) developed two projection based models for MADM in intuitionistic fuzzy and interval valued intuitionistic fuzzy environment. Zeng et al. (2013) provided weighted projection algorithm for intuitionistic fuzzy MADM problems and interval-valued intuitionistic fuzzy MADM problems.

Chen and Ye (2016) developed the projection based model for solving neutrosophic MADM problem and applied it to select clay-bricks in construction field.

Dey et al. (2016b) defined weighted projection measure with interval neutrosophic environment and applied it to solve MADM problems with interval valued neutrosophic information. Ye (2015c) developed a projection measure-based multiple attribute decision making method with interval neutrosophic information and credibility information.

To overcome the shortcomings of the general projection measure, Ye (2016) introduced a bidirectional projection measure between single valued neutrosophic numbers and developed MADM method for selecting problems of mechanical design schemes under a single valued neutrosophic environment. Ye (2015d) also presented the bidirectional projection method for multiple attribute group decision making with neutrosophic numbers.

Dey et al. (2016a) proposed a new approach to neutrosophic soft MADM using grey relational projection method. Yue (2012b) presented a projection method to obtain weights of the experts in a group decision making problem. Yue (2013) proposed a projection based approach for partner selection in a group decision making problem with linguistic value and intuitionistic fuzzy information.

Dey et al. (2017) defined projection, bidirectional projection and hybrid projection measures between bipolar neutrosophic sets and presented bipolar neutrosophic projection based models for multi-attribute decision making problems.

Literature review reflects that no studies have been made on multi-attribute decision making using projection and bidirectional projection measures under rough neutrosophic environment. In this paper, we propose projection and bidirectional projection measures under rough neutrosophic environment. We also present two numerical examples to show the effectiveness and applicability of the proposed measures.

Rest of the paper is organized as follows: Section 2 describes preliminaries of neutrosophic number, SVNS and rough neutrosophic set (RNS). Section 3 describes projection and bidirectional projection measures of rough neutrosophic sets. Section 4 presents projection and bidirectional projection based decision making methods for MCDM problems with rough neutrosophic information. Section 5 solves a numerical example. Finally, section 6 presents the conclusion and future scope of research.

2. PRELIMINARIES

In this Section, we provide some basic definitions regarding SVNSs, RNSs which are useful for developing the paper.

2.1 Neutrosophic set

Smarandache (1998) offered the following definition of neutrosophic set.

Definition 2.1.1. Let X be a space of points (objects) with generic element in X denoted by x . A neutrosophic set A in X is characterized by a truth-membership function T_A , an indeterminacy membership function I_A and a falsity membership function F_A . The functions T_A , I_A and F_A are real standard or non-standard subsets of $]^{-}0, 1^{+}[$ [that is $T_A: X \rightarrow]^{-}0, 1^{+}[$, $I_A: X \rightarrow]^{-}0, 1^{+}[$ and $F_A: X \rightarrow]^{-}0, 1^{+}[$. It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ i.e. $0^{-} \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$.

Definition 2.1.2: (complement) The complement of a neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = \{1^{+}\} - T_A(x)$, $I_{c(A)}(x) = \{1^{+}\} - I_A(x)$, $F_{c(A)}(x) = \{1^{+}\} - F_A(x)$.

Definition 2.1.3: (Containment) A neutrosophic set A is contained in the other neutrosophic set B, denoted by $A \subseteq B$ iff

$$\inf T_A(x) \leq \inf T_B(x), \sup T_A(x) \leq \sup T_B(x), \inf I_A(x) \geq \inf I_B(x), \sup I_A(x) \geq \sup I_B(x)$$

and $\inf F_A(x) \geq \inf F_B(x), \sup F_A(x) \geq \sup F_B(x) \forall x \in X$

Definition 2.1.4: (Single-valued neutrosophic set). Let X be a universal space of points (objects) with a generic element of X denoted by x. A single valued neutrosophic set A is characterized by a truth membership function $T_A(x)$, a falsity membership function $F_A(x)$ and indeterminacy function $I_A(x)$ with

$$T_A(x), I_A(x) \text{ and } F_A(x) \in [0,1] \forall x \text{ in } X.$$

When X is continuous, a SNVS S can be written as follows:

$$A = \int \langle T_A(x), F_A(x), I_A(x) \rangle / x \forall x \in X$$

and when X is discrete, a SVNS S can be written as follows:

$$A = \sum \langle T_A(x), F_A(x), I_A(x) \rangle / x \forall x \in X$$

For a SVNS S, $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

Definition 2.1.5: The complement of a single valued neutrosophic set A is denoted by $c(A)$ and is defined by $T_{c(A)}(x) = F_A(x)$, $I_{c(A)}(x) = 1 - I_A(x)$, $F_{c(A)}(x) = T_A(x)$.

Definition 2.1.6: A SVNS A is contained in the other SVNS B, denoted as $A \subseteq B$ iff,

$$T_A(x) \leq T_B(x), I_A(x) \geq I_B(x) \text{ and } F_A(x) \geq F_B(x), \forall x \in X.$$

2.2 Rough neutrosophic set (Broumi et al., 2014a, 2014b)

Broumi et al., (2014a, 2014b) defined hybrid intelligent structure called Rough neutrosophic set.

Definition 2.2.1: Let Y be a non-null set and R be an equivalence relation on Y. Let P be a neutrosophic set in Y with the membership function T_P , indeterminacy membership function I_P and falsity membership function F_P . The lower and the upper approximations of P in the approximation space (Y, R) denoted by $\underline{N(P)}$ and $\overline{N(P)}$ are respectively defined as:

$$\underline{N(P)} = \left\langle \langle x, T_{\underline{N(P)}}(x), I_{\underline{N(P)}}(x), F_{\underline{N(P)}}(x) \rangle / y \in [x]_R, x \in Y \right\rangle$$

and

$$\overline{N(P)} = \left\langle \langle x, T_{\overline{N(P)}}(x), I_{\overline{N(P)}}(x), F_{\overline{N(P)}}(x) \rangle / y \in [x]_R, x \in Y \right\rangle$$

where,

$$T_{\underline{N(P)}}(x) = \wedge z \in [x]_R T_P(Y), I_{\underline{N(P)}}(x) = \wedge z \in [x]_R I_P(Y), F_{\underline{N(P)}}(x) = \wedge z \in [x]_R F_P(Y) \text{ and}$$

$$T_{\overline{N(P)}}(x) = \vee z \in [x]_R T_P(Y), I_{\overline{N(P)}}(x) = \vee z \in [x]_R I_P(Y), F_{\overline{N(P)}}(x) = \vee z \in [x]_R F_P(Y).$$

So,

$$0 \leq T_{\underline{N(P)}}(x) + I_{\underline{N(P)}}(x) + F_{\underline{N(P)}}(x) \leq 3$$

and

$$0 \leq T_{\overline{N(P)}}(x) + I_{\overline{N(P)}}(x) + F_{\overline{N(P)}}(x) \leq 3.$$

Here \vee and \wedge denote “max” and “min” operators respectively. $T_P(y)$, $I_P(y)$ and $F_P(y)$ are the membership, indeterminacy and non-membership of Y with respect to P .

Thus NS mappings \underline{N} , $\overline{N}: N(Y) \rightarrow N(Y)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P), \overline{N}(P))$ is called the rough neutrosophic set in (Y, R) .

Definition 2.2.2 If $N(P) = (\underline{N}(P), \overline{N}(P))$ is a rough neutrosophic set in (Y, R) , the rough complement of $N(P)$ is the rough neutrosophic set denoted by $\sim(N(P))$ and defined as: $\sim(N(P)) = ((\underline{N}(P))^c, (\overline{N}(P))^c)$, where $(\underline{N}(P))^c$ and $(\overline{N}(P))^c$ are the complements of neutrosophic sets $\underline{N}(P)$ and $\overline{N}(P)$ respectively.

3. PROJECTION AND BIDIRECTIONAL PROJECTION MEASURE OF ROUGH NEUTROSOPHIC SETS

Existing projection and bidirectional projection measure are not capable of dealing with MCDM problems in rough neutrosophic environment. Therefore, new projection and bidirectional projection measures between RNSs are proposed.

Assume that M and N are two RNSs represented by

$$M = \{ \langle (\underline{T}_M(x_i), \underline{I}_M(x_i), \underline{F}_M(x_i)), (\overline{T}_M(x_i), \overline{I}_M(x_i), \overline{F}_M(x_i))) \rangle : i = 1, 2, \dots, n \}$$

and

$$N = \{ \langle (\underline{T}_N(x_i), \underline{I}_N(x_i), \underline{F}_N(x_i)), (\overline{T}_N(x_i), \overline{I}_N(x_i), \overline{F}_N(x_i))) \rangle : i = 1, 2, \dots, n \}.$$

Then, the inner product of M and N denoted by $M.N$ can be defined as:

$$M.N = \sum_{i=1}^n [\underline{T}_1(x_i) \cdot \underline{T}_2(x_i) + \underline{I}_1(x_i) \cdot \underline{I}_2(x_i) + \underline{F}_1(x_i) \cdot \underline{F}_2(x_i) + \overline{T}_1(x_i) \cdot \overline{T}_2(x_i) + \overline{I}_1(x_i) \cdot \overline{I}_2(x_i) + \overline{F}_1(x_i) \cdot \overline{F}_2(x_i)].$$

The modulus of M can be defined as

$$\|M\| = \sqrt{\sum_{i=1}^n [\underline{T}_1(x_i)^2 + \underline{I}_1(x_i)^2 + \underline{F}_1(x_i)^2 + \overline{T}_1(x_i)^2 + \overline{I}_1(x_i)^2 + \overline{F}_1(x_i)^2]}$$

and the modulus of N can be defined as

$$\|N\| = \sqrt{\sum_{i=1}^n [\underline{T}_2(x_i)^2 + \underline{I}_2(x_i)^2 + \underline{F}_2(x_i)^2 + \overline{T}_2(x_i)^2 + \overline{I}_2(x_i)^2 + \overline{F}_2(x_i)^2]}.$$

Definition 4.1. The projection of M on N can be defined as:

$$\text{Pr oj}(M)_N = \frac{1}{\|N\|} M.N.$$

Definition 4.2. The bidirectional projection measure between the RNSs M and N is defined as:

$$\text{B Pr oj}(M, N) = \frac{1}{1 + \left| \frac{\|M\| - \|N\|}{\|M.N\|} \right|} = \frac{\|M\| \|N\|}{\|M\| \|N\| + \left| \frac{\|M\| - \|N\|}{\|M.N\|} \right|}.$$

Here also the bidirectional projection measure satisfies the following properties:

- (1) $\text{BProj}(M, N) = \text{BProj}(N, M)$;
- (2) $0 \leq \text{BProj}(M, N) \leq 1$;
- (3) $\text{BProj}(M, N) = 1$, if $M = N$.

Proof:

$$(i) \text{B Pr oj}(M, N) = \frac{1}{1 + \left| \frac{\|M\| - \|N\|}{\|M.N\|} \right|} = \frac{1}{1 + \left| \frac{\|N\| - \|M\|}{\|N.M\|} \right|} = \text{B Pr oj}(N, M)$$

(ii) As $\frac{1}{1 + \left| \|M\| - \|N\| \right| \|M.N\|} \geq 0$ and $\frac{1}{1 + \left| \|M\| - \|N\| \right| \|M.N\|} \leq 1$ so, $0 \leq BProj(M, N) \leq 1$

(iii) If $M = N$ then $BProj(M, N) = BProj(M, M) = \frac{1}{1 + \left| \|M\| - \|M\| \right| \|M.M\|} = 1$

4. PROJECTION AND BIDIRECTIONAL PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION

In this section, we develop projection and bidirectional projection based MCDM models to solve MCDM problems with rough neutrosophic information. Consider $E = \{E_1, \dots, E_n\}$ be a set of alternatives and $A = \{A_1, \dots, A_m\}$ be a set of attributes. Now we present two algorithms for MCDM problems involving rough neutrosophic information.

4.1 PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION

Algorithm 1.

Step 1. The value of alternative $E_i (i = 1, 2, \dots, n)$ for the attribute $A_j (j = 1, 2, \dots, m)$ is evaluated by the decision maker in terms of RNSs and the rough neutrosophic decision matrix is constructed as:

$$Z = \langle Z_{ij} \rangle_{n \times m} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2m} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & Z_{nm} \end{bmatrix} \text{ where } Z_{ij} = \langle (\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), (\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}) \rangle \text{ with}$$

$$0 \leq \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \leq 3 \text{ and } 0 \leq \overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij} \leq 3.$$

Step 2. Determine the ideal solution $S^* = \{S_1, S_2, \dots, S_m\}$.

If A_i is benefit type attribute then $S_i = \{(\min_j \underline{T}_{ji}, \max_j \underline{I}_{ji}, \max_j \underline{F}_{ji}), (\max_j \overline{T}_{ji}, \min_j \overline{I}_{ji}, \min_j \overline{F}_{ji})\}$.

If A_i is cost type attribute then $S_i = \{(\max_j \underline{T}_{ji}, \min_j \underline{I}_{ji}, \min_j \underline{F}_{ji}), (\min_j \overline{T}_{ji}, \max_j \overline{I}_{ji}, \max_j \overline{F}_{ji})\}$.

Step 3. Compute the projection measure between S^* and $Z_i = \langle Z_{ij} \rangle_{n \times m}$ for all $i = 1, \dots, n$ and $j = 1, \dots, m$. According to the descending order of projection measure $Proj(Z_i)_{S^*}$ for $i = 1, \dots, n$ alternatives are ranked and highest value of $Proj(Z_i)_{S^*}$ reflects the best option.

4.2. BIDIRECTIONAL PROJECTION BASED DECISION MAKING METHODS FOR MCDM PROBLEMS WITH ROUGH NEUTROSOPHIC INFORMATION

Algorithm 2.

Step 1. The value of alternative $E_i (i = 1, 2, \dots, n)$ for the attribute $A_j (j = 1, 2, \dots, m)$ is evaluated by the decision maker in terms of RNSs and the rough neutrosophic decision matrix is constructed as:

$$Z = \langle Z_{ij} \rangle_{n \times m} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1m} \\ Z_{21} & Z_{22} & \dots & Z_{2m} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Z_{n1} & Z_{n2} & \dots & Z_{nm} \end{bmatrix} \text{ where } Z_{ij} = \langle (\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}), (\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}) \rangle \text{ with}$$

$$0 \leq \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \leq 3 \text{ and } 0 \leq \overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij} \leq 3.$$

Step 2. Determine the ideal solution $S^* = \{S_1, S_2, \dots, S_m\}$.

If A_i is benefit type attribute then $S_i = \{(\min_j \underline{T}_{ji}, \max_j \underline{I}_{ji}, \max_j \underline{F}_{ji}), (\max_j \overline{T}_{ji}, \min_j \overline{I}_{ji}, \min_j \overline{F}_{ji})\}$.

If A_i is cost type attribute then $S_i = \{(\max_j \underline{T}_{ji}, \min_j \underline{I}_{ji}, \min_j \underline{F}_{ji}), (\min_j \overline{T}_{ji}, \max_j \overline{I}_{ji}, \max_j \overline{F}_{ji})\}$.

Step 3. Compute the bidirectional projection measure between S^* and $Z_i = \langle Z_{ij} \rangle_{n \times m}$ for all $i = 1, \dots, n$ and $j = 1, 2, \dots, m$. According to the descending order of bidirectional projection measure $BProj(Z_i, S^*)$ for $i = 1, 2, \dots, n$ alternatives are ranked and highest value of $BProj(Z_i, S^*)$ reflects the best option.

Section 5. NUMERICAL EXAMPLES

Example 1: Assume that a decision maker intends to select the most suitable smartphone from the three initially chosen smartphones (S_1, S_2, S_3) by considering four attributes namely: feature A_1 , price A_2 , customer care A_3 , and risk factor A_4 .

Step1: The decision maker forms the following decision matrix:

	A_1	A_2	A_3	A_4
S_1	$\langle (.6, .3, .3), (.8, .1, .1) \rangle$	$\langle (.6, .4, .4), (.8, .2, .2) \rangle$	$\langle (.6, .4, .4), (.8, .2, .2) \rangle$	$\langle (.7, .4, .4), (.9, .2, .2) \rangle$
S_2	$\langle (.7, .3, .3), (.9, .1, .3) \rangle$	$\langle (.6, .3, .3), (.8, .3, .3) \rangle$	$\langle (.6, .2, .2), (.8, .4, .2) \rangle$	$\langle (.7, .3, .3), (.9, .3, .3) \rangle$
S_3	$\langle (.6, .2, .2), (.8, .0, .2) \rangle$	$\langle (.7, .3, .3), (.9, .1, .1) \rangle$	$\langle (.7, .4, .6), (.9, .2, .4) \rangle$	$\langle (.6, .3, .2), (.8, .1, .2) \rangle$

Step2: Here A_2 and A_4 are the cost type attributes.

So, the ideal solution is:

$S^* = [\langle (.6, .3, .3), (.9, .0, .1) \rangle, \langle (.7, .3, .3), (.8, .3, .3) \rangle, \langle (.6, .4, .6), (.9, .2, .2) \rangle, \langle (.7, .3, .2), (.8, .3, .3) \rangle]$.

Step3: Determination of the projection and bidirectional projection measure:

$$\|S^*\| = 6.06, \|S_1\| = 2.387467, \|S_2\| = 2.424871, \|S_3\| = 2.412468$$

$$S_1.S^* = 5.78, S_2.S^* = 5.80, S_3.S^* = 5.82.$$

$$\text{Pr oj}(S_1)_{S^*} = 0.953795, \text{Pr oj}(S_2)_{S^*} = 0.957095, \text{Pr oj}(S_3)_{S^*} = 0.960396.$$

$$\Rightarrow \text{Pr oj}(S_3)_{S^*} > \text{Pr oj}(S_2)_{S^*} > \text{Pr oj}(S_1)_{S^*}$$

$$\Rightarrow S_3 > S_2 > S_1.$$

$$\text{BPr oj}(S_1, S^*) = 0.405320, \text{BPr oj}(S_2, S^*) = 0.410714, \text{BPr oj}(S_3, S^*) = 0.407818.$$

$$\Rightarrow \text{BPr oj}(S_2, S^*) > \text{BPr oj}(S_3, S^*) > \text{BPr oj}(S_1, S^*)$$

$$\Rightarrow S_2 > S_3 > S_1.$$

Here S_3 is the best alternative according to projection measure and S_2 is the best alternative according to bidirectional projection measure. As bidirectional projection measure is better than projection measure so the decision maker selects the smartphone S_2 .

Example 2: Assume that a decision maker intends to select the most suitable location of modern logistic centre from the three initially chosen locations (K_1, K_2, K_3) by considering six attributes namely: cost L_1 , distance to suppliers L_2 , distance to customers L_3 , conformance to government and law L_4 , quality of service L_5 , environmental impact L_6 .

Step1: The decision maker forms the following decision matrix:

	L_1	L_2	L_3	L_4	L_5	L_6
K_1	$\langle (.85,.05, .05), (.95,.15,.15) \rangle$	$\langle (.75,.15, .10), (.85,.25,.20) \rangle$	$\langle (.75,.15, 5,.10), (.85,.25, .20) \rangle$	$\langle (.75,.15, 5,.10), (.85,.25, .20) \rangle$	$\langle (.75,.15,.10), (.85,.25,.20) \rangle$	$\langle (.85,.05,.05), (.95,.15,.15) \rangle$
K_2	$\langle (.45,.45, .35), (.55,.55,.55) \rangle$	$\langle (.75,.15, .10), (.85,.25,.20) \rangle$	$\langle (.45,.45, 5,.35), (.55,.55, .55) \rangle$	$\langle (.75,.15, 5,.10), (.85,.25, .20) \rangle$	$\langle (.75,.15,.10), (.85,.25,.20) \rangle$	$\langle (.45,.45,.35), (.55,.55,.55) \rangle$
K_3	$\langle (.45,.45, .35), (.55,.55,.55) \rangle$	$\langle (.85,.05, .05), (.95,.15,.15) \rangle$	$\langle (.75,.15, 5,.10), (.85,.25, .20) \rangle$	$\langle (.75,.15, 5,.10), (.85,.25, .20) \rangle$	$\langle (.85,.05,.05), (.95,.15,.15) \rangle$	$\langle (.45,.45,.35), (.55,.55,.55) \rangle$

Step2: Here L_1, L_2, L_3 are cost type attributes

So, the ideal solution is:

$$S^* = [\langle (.85,.05,.05), (.55,.55,.55) \rangle, \langle (.85,.05,.05), (.85,.25,.20) \rangle, \langle (.75,.15,.10), (.55,.55,.55) \rangle, \langle (.55,.30,.25), (.85,.25,.20) \rangle, \langle (.75,.15,.10), (.95,.15,.15) \rangle, \langle (.45,.45,.35), (.95,.15,.15) \rangle].$$

Step3: Determination of the projection and bidirectional projection measure:

$$\|S^*\| = 2.997916$$

$$\|K_1\| = 3.004995, \|K_2\| = 2.926602, \|K_3\| = 2.966479$$

$$K_1.S^* = 8.3475, K_2.S^* = 8.1450, K_3.S^* = 8.2325.$$

$$\text{Pr oj}(K_1)_{S^*} = 2.784434, \text{Pr oj}(K_2)_{S^*} = 2.716897, \text{Pr oj}(K_3)_{S^*} = 2.746074.$$

$$\Rightarrow \text{Pr oj}(K_1)_{S^*} > \text{Pr oj}(K_3)_{S^*} > \text{Pr oj}(K_2)_{S^*}$$

$$\Rightarrow K_1 > K_3 > K_2.$$

$$\text{BPr oj}(K_1, S^*) = 0.993481, \text{BPr oj}(K_2, S^*) = 0.937908, \text{BPr oj}(K_3, S^*) = 0.971721.$$

$$\Rightarrow \text{BPr oj}(K_1, S^*) > \text{BPr oj}(K_3, S^*) > \text{BPr oj}(K_2, S^*)$$

$$\Rightarrow K_1 > K_3 > K_2.$$

Hence, K_1 is the best alternative.

6. CONCLUSION

This paper defines projection measure and bidirectional projection measure between rough neutrosophic sets. Two new multi criteria decision making methods have been proposed based on the proposed neutrosophic projection and bidirectional projection measures respectively. Finally, two numerical examples are provided to demonstrate the applicability and effectiveness of the proposed methods. The proposed methods can be extended for solving multi criteria decision making in interval neutrosophic rough environments.

REFERENCES

- Abdel-Baset, M., Hezam, I. M., & Smarandache. F. (2016). Neutrosophic goal programming. *Neutrosophic Sets and Systems*, 11, 112-118.
- Atanassov, K. (1983). Intuitionistic fuzzy sets. in: *Proceedings of the VII ITKR's Session*, Sofia (Deposited in Central Sci.-Techn. Library of Bulgaria Academy of Science), 1677-1684.
- Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.
- Biswas, P., Pramanik, S., & Giri, B. C. (2014a). Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. *Neutrosophic Sets and Systems*, 2, 102-110.
- Biswas, P., Pramanik, S., & Giri, B. C. (2014b). A new methodology for neutrosophic multi-attribute decision making with unknown weight information. *Neutrosophic Sets and Systems*, 3, 42-52.
- Biswas, P., Pramanik, S., & Giri, B. C. (2015a). Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. *Neutrosophic Sets and Systems*, 8, 47-57.
- Biswas, P., Pramanik, S., & Giri, B. C. (2015b). TOPSIS method for multi-attribute group decision-making under single valued neutrosophic environment. *Neural Computing and Applications*, 27(3), 727-737. doi: 10.1007/s00521-015-1891-2.
- Biswas, P., Pramanik, S., & Giri, B. C. (2016a). Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. *Neutrosophic Sets and Systems*, 12, 20-40.
- Biswas, P., Pramanik, S., & Giri, B. C. (2016b). Value and ambiguity index based ranking method of single-valued trapezoidal neutrosophic numbers and its application to multi-attribute decision making. *Neutrosophic Sets and Systems*, 12, 127-138.
- Biswas, P., Pramanik, S., & Giri, B. C. (2017a). Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. In F. Smarandache, & S. Pramanik (Eds), *New Trends in Neutrosophic Theory and Applications, Vol. II*. Brussels:Pons Editions. In Press.

- Biswas, P., Pramanik, S., & Giri, B., C. (2017b). Non-linear programming approach for single-valued neutrosophic TOPSIS method, *New Mathematics and Natural Computation*, In Press.
- Broumi, S., & Smarandache, F. (2013). Several similarity measures of neutrosophic sets. *Neutrosophic Sets and Systems*, 1, 54–62.
- Broumi, S., Smarandache, F., & Dhar, M. (2014a). Rough neutrosophic sets. *Italian Journal of Pure and Applied Mathematics*, 32, 493–502.
- Broumi, S., Smarandache, F., & Dhar, M. (2014b). Rough neutrosophic sets. *Neutrosophic Sets and Systems*, 3, 60–66.
- Cantor, G. (1874). Ueber eine Eigenschaft des Inbegriffes aller reellen algebraischen Zahlen. *Journal für die reine und angewandte Mathematik*, 77, 258–262. doi:10.1515/crll.1874.77.258.
- Chen, J., & Ye, J. (2016). A projection model of neutrosophic numbers for multiple attribute decision making of clay-brick selection. *Neutrosophic Sets and Systems*, 12, 139–142.
- Cheng, H.D.; Guo, Y. (2008). A new neutrosophic approach to image thresholding. *New Mathematics and Natural Computation*, 4, 291–308.
- Das, P., & Roy, T., K. (2015). Multi-objective non-linear programming problem based on neutrosophic optimization technique and its application in riser design problem. *Neutrosophic Sets and Systems*, 9, 88–95.
- Dey, P. P., Pramanik, S., & Giri, B. C. (2016a). Neutrosophic soft multi-attribute decision making based on grey relational projection method. *Neutrosophic Sets and Systems*, 11, 98–106.
- Dey, P. P., Pramanik, S., & Giri, B. C. (2016b). Extended projection based models for solving multiple attribute decision making problems with interval valued neutrosophic information. In F. Smarandache, & S. Pramanik (Eds), *New Trends in Neutrosophic Theory and Applications* (pp. 127–140). Brussels: Pons Editions.
- Guo, Y., Cheng, H.D. (2009). New neutrosophic approach to image segmentation. *Pattern Recognition*, 42, 587–595.
- Guo, Y., Sengur, A., Ye, J. (2014). A novel image thresholding algorithm based on neutrosophic similarity score. *Measurement*, 58, 175–186.
- Hezam, I. M., Abdel-Baset, M., & Smarandache, F. (2015). Taylor series approximation to solve neutrosophic multiobjective programming problem. *Neutrosophic Sets and Systems*, 10, 39–45.
- Kharal, A. (2014). A neutrosophic multi-criteria decision making method. *New Mathematics and Natural Computing*, 10, 143–162. doi: 10.1142/S1793005714500070.
- Klaua, D. (1965). Über einen Ansatz zur mehrwertigen Mengenlehre. *Monatsber. Deut. Akad. Wiss. Berlin*, 7, 859–876.
- Liu, P., Chu, Y., Li, Y., & Chen, Y. (2014). Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making, *International Journal of Fuzzy Systems*, 16(2)242–255.
- Liu, P. D. & Li, H. G. (2017). Multiple attribute decision-making method based on some normal neutrosophic Bonferroni mean operators. *Neural Computing and Applications*, 28, 179–194.
- Liu, P., & Wang, Y. (2014). Multiple attribute decision-making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Computing and Applications*, 25(7)2001–2010.
- Mondal, K., & Pramanik, S. (2014a). A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. *Neutrosophic Sets and Systems*, 5, 21–26.
- Mondal, K., & Pramanik, S., (2014b). Multi-criteria group decision making approach for teacher recruitment in higher education under simplified Neutrosophic environment. *Neutrosophic Sets and Systems*, 6, 28–34.
- Mondal, K., & Pramanik, S., (2015a). Neutrosophic decision making model of school choice. *Neutrosophic Sets and Systems*, 7, 62–68.
- Mondal, K., & Pramanik, S., (2015b). Rough neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 7, 8–17.

- Mondal, K., & Pramanik, S., (2015c). Rough neutrosophic multi-attribute decision-making based on rough accuracy score function. *Neutrosophic Sets and Systems*, 8, 14-21.
- Mondal, K., & Pramanik, S., (2015d). Neutrosophic tangent similarity measure and its application to multiple attribute decision making. *Neutrosophic Sets and Systems*, 9, 80-87.
- Mondal, K., & Pramanik, S., (2015e). Neutrosophic refined similarity measure based on tangent function and its application to multi attribute decision making. *Journal of New Theory*, 8, 41-50.
- Mondal, K., Pramanik, S., (2015f). Decision making based on some similarity measures under interval rough neutrosophic environment. *Neutrosophic Sets and Systems*, 10, 46-57.
- Mondal, K., Pramanik, S., (2015g). Tri-complex rough neutrosophic similarity measure and its application in multi-attribute decision making. *Critical Review*, 11, 26-40.
- Mondal, K., Pramanik, S., & Smarandache, F. (2016a). Several trigonometric Hamming similarity measures of rough neutrosophic sets and their applications in decision making. In F. Smarandache, & S. Pramanik (Eds), *New Trends in Neutrosophic Theory and Application* (pp 93-103). Brussels, Belgium: Pons Editions.
- Mondal, K., Pramanik, S., & Smarandache, F. (2016b). Multi-attribute decision making based on rough neutrosophic variational coefficient similarity measure. *Neutrosophic Sets and Systems*, 13, 3-17.
- Mondal, K., Pramanik, S., & Smarandache, F. (2016c). Rough neutrosophic TOPSIS for multi-attribute group decision making. *Neutrosophic Sets and Systems*, 13, 105-117.
- Mondal, K., Pramanik, S., & Smarandache, F. (2016d). Rough neutrosophic hyper-complex set and its application to multi-attribute decision making. *Critical Review*, 13, 111-126.
- Pawlak, Z. (1982). Rough sets. *International Journal of Information and Computer Sciences*, 11(5), 341-356.
- Peng, J., J., Wang, J., Q., Wang, J., Zhang, H., Y., & Chen, X. H. (2016). Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International Journal of Systems Science*, 47 (10), 2342-2358.
- Pramanik, S. (2016a). Neutrosophic linear goal programming. *Global Journal of Engineering Science and Research Management*, 3(7), 01-11.
- Pramanik, S. (2016b). Neutrosophic multi-objective linear programming. *Global Journal of Engineering Science and Research Management*, 3(8), 36-46.
- Pramanik, S., Banerjee, D., & Giri, B. C. (2016). Multi – criteria group decision making model in neutrosophic refined set and its application. *Global Journal of Engineering Science and Research Management*, 3(6), 12-18.
- Pramanik, S., Biswas, P., & Giri, B. C. (2015). Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. *Neural Computing and Applications*, doi:10.1007/s00521015-2125-3.
- Pramanik, S., & Chakrabarti, S. (2013). A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. *International Journal of Innovative Research in Science, Engineering and Technology*, 2(11), 6387-6394.
- Pramanik, S., Dalapati, S., & Roy, T. K. (2016). *Logistics center location selection approach based on neutrosophic multi-criteria decision making*. In F. Smarandache, & S. Pramanik (Eds), *New Trends in Neutrosophic Theories and Applications*, (pp 161-174). Brussels, Belgium: Pons Editions.
- Pramanik, S., Dey, P. P., Giri, B.C. & Smarandache, F. (2017). Bipolar neutrosophic projection based models for multi-attribute decision making problems. *Neutrosophic Sets and Systems*, 15, 74-83.
- Pramanik, S., & Mondal, K. (2015a). Cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. *Global Journal of Advanced Research*, 2(1), 212-220.
- Pramanik, S., & Mondal, K. (2015b). Some rough neutrosophic similarity measure and their application to multi attribute decision making. *Global Journal of Engineering Science and Research Management*, 2(7) (2015) 61-74.
- Pramanik, S., & Mondal, K. (2015c). Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis. *Journal of New Theory*, 4, 90-102.

- Pramanik, S., & Mondal, K. (2015d). Interval neutrosophic multi-attribute decision-making based on grey relational analysis. *Neutrosophic Sets and Systems*, 9, 13-22.
- Pramanik, S., & Roy, T. K. (2014). Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. *Neutrosophic Sets and Systems*, 2, 82-101.
- Sahin, R., & Liu, P. (2015). Maximizing deviation method for neutrosophic multiple attribute decision making with incomplete weight information. *Neural Computing and Applications*. doi: 10.1007/s00521-015-1995-8.
- Sarkar, M., Dey, S., & Roy, T. K. (2016). Truss design optimization using neutrosophic optimization technique., *Neutrosophic Sets and Systems*, 13, 62-69.
- Smarandache, F. (1998). *A unifying field in logics: neutrosophic logic. Neutrosophy, neutrosophic set, neutrosophic probability, and neutrosophic statistics*. Rehoboth: American Research Press.
- Smarandache, F. (1999). Linguistic paradoxes and tautologies, *Libertas Mathematica, University of Texas at Arlington*, IX, 143-154.
- Smarandache, F., (2002). A unifying field in logics: neutrosophic logics. *Multiple valued logic*, 8(3), 385-438.
- Smarandache, F. (2005). Neutrosophic set – a generalization of intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24(3), 287-297.
- Smarandache, F. (2010). Neutrosophic set – a generalization of intuitionistic fuzzy set. *Journal of Defense Resources Management*, 1(1), 107-116.
- Sodenkamp, M. (2013). *Models, methods and applications of group multiple-criteria decision analysis in complex and uncertain systems*. Dissertation, University of Paderborn, Germany.
- Wang, H. Smarandache, F. Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410–413.
- Xu, Z. S., & Da, Q. L. (2004). Projection method for uncertain multi-attribute decision making with preference information on alternatives. *International Journal of Information Technology & Decision Making*, 3, 429-434.
- Xu, Z. (2005). On methods for uncertain multiple attribute group decision making with uncertain multiplicative preference information on alternatives. *Fuzzy Optimization and Decision Making*, 4, 131-139.
- Xu, Z., & Hu, H. (2010). Projection models for intuitionistic fuzzy multiple attribute decision making. *International Journal of Information Technology & Decision Making*, 9(2), 267-280.
- Yang, L., Xiao, S., & Yuan, S. (2014). Projection method for material selection problem with triangular fuzzy numbers. *Advanced Materials Research*, 1046, 375-379.
- Yang, H. L., Zhang, C. L., Guo, Z. L., Liu, Y. L., & Liao, X. (2016). A hybrid model of single valued neutrosophic sets and rough sets: single valued neutrosophic rough set model. *Soft Computing*, 1-15. doi:10.1007/s00500-016-2356-y.
- Ye, J. (2013a). Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment. *International Journal of General Systems*, 42, 386–394.
- Ye, J. (2013b). Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Applied Mathematical Modelling*, 38 (3),1170–1175.
- Ye, J. (2014a). A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *Journal of Intelligent and Fuzzy Systems*, 26, 2459–2466.
- Ye, J. (2014b). Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. *International Journal of Fuzzy Systems*, 16, 204–215.
- Ye, J. (2015a). Trapezoidal neutrosophic set and its application to multiple attribute decision-making. *Neural Computing and Applications*, 26,1157–1166.
- Ye, J. (2015b). Improved cosine similarity measures of simplified neutrosophic sets for medical diagnoses. *Artificial Intelligence in Medicine*, 63, 171–179.
- Ye, J. (2015c). Interval neutrosophic multiple attribute decision-making method with credibility information. *International Journal of Fuzzy Systems*. doi 10.1007/s40815-015-0122-4.
- Ye, J. (2015d). Bidirectional projection method for multiple attribute group decision making with neutrosophic number. *Neural Computing and Applications*. doi: 10.1007/s00521-015-2123-5.

- Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, 10(4), 106.
- Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. *Measurement*. Volume 124, August 2018, Pages 47-55
- Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.
- Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.
- Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, 86, 12-29.
- Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*, 33(6), 4055-4066.
- Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. *Symmetry* 2018, 10, 116.
- Ye, J. (2016). Projection and bidirectional projection measures of single valued neutrosophic sets and their decision – making method for mechanical design scheme. *Journal of Experimental & Theoretical Artificial Intelligence*. doi:10.1080/0952813X.2016.1259263.
- Ye, J. (2014a). Single valued neutrosophic minimum spanning tree and its clustering method. *Journal of Intelligent Systems*, 23, 311–324.
- Ye, J. (2014b). Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. *Journal of Intelligent Systems*, 23, 379–389.
- Yue, C. (2017). Normalized projection approach to group decision-making with hybrid decision information. *International Journal of Machine Learning and Cybernetics*. doi:10.1007/s13042-017-0650-3.
- Yue, Z. L. (2012a). Application of the projection method to determine weights of decision makers for group decision making. *Scientia Iranica*, 19(3), 872-878.
- Yue, Z. L. (2012b). Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling*, 36(7), 2900-2910.
- Yue, Z. L. (2013). An intuitionistic fuzzy projection-based approach for partner selection. *Applied Mathematical Modelling*, 37, 9538-9551.
- Yue, Z., L., & Jia, Y. (2015). A direct projection-based group decision-making methodology with crisp values and interval data. *Soft Computing*. doi: 10.1007/s00500-015-1953-5
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338-353.
- Zeng, S., Balezentis, T., Chen, J., & Luo, G. (2013). A projection method for multiple attribute group decision making with intuitionistic fuzzy information. *Informatica*, 24(3), 485-503.
- Zheng, G., Jing, Y., Huang, H., & Gao, Y. (2010). Application of improvement grey relational projection method to evaluate sustainable building envelope performance. *Applied Energy*, 87(2), 710-720.