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NMCDA: A framework for evaluating cloud computing services

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HIGHLIGHTS

- Cloud services considered as a type of multi-criteria decision analysis problem.
- A new approach (NMCDA) for estimating the quality of cloud services is proposed.
- Ambiguous and incompatible information handling by Triangular neutrosophic numbers.
- To demonstrate the performance of the proposed model a case study is presented.

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ABSTRACT

Many organizations are currently seeking to contract services from cloud computing rather than owing the possessions to supply those services. Due to the fast expansion of cloud computing, many cloud services have been developed. Any organization that tries to achieve the best flexibility and quick response to market requests, they have the options to use cloud services. Due to the diversity of cloud service providers, it is a very significant defy for organizations to select the appropriate cloud services which can fulfill their requirements, as numerous criteria should be counted in the selection process of cloud services. Therefore, the selection process of cloud services can be considered as a type of multi-criteria decision analysis problems. In this research paper, we present how to aid a decision maker to estimate different cloud services by providing a neutrosophic multi-criteria decision analysis (NMCDA) approach for estimating the quality of cloud services. Triangular neutrosophic numbers are used to deal with ambiguous and incompatible information which exist usually in the performance estimation process. An efficacious model is evolved depending on neutrosophic analytic hierarchy process (NAHP). The aim is to solve the performance estimation problem and improve the quality of services by creating a strong competition between cloud providers. To demonstrate the pertinence of the proposed model for disbanding the multi-criteria decision analysis, a case study is presented.

1. Introduction

The most crucial "ingredients" for any organization to survive belong to the information and communication technology departments to make things happen, keep business running and generate and store lots of data from day-to-day activities. Therefore, a large budget is assigned for their confirmation and enhancement [1]. Embedding and/or outsourcing suitable technologies are tactical decisions which depend on strategies, facilities and requirements of organization [2]. The outsourcing process improves productivity and pliability in work, via minimizing management and maintenance costs of neoteric technology [3]. The cloud computing service providers have performed a significant role in latest decades. The number of cloud computing services providers have increased and became a significant sector in business environments due to increasing outsourcing process of demanded technology for cloud computing services. However, the estimation process of available cloud services providers is still a complicated problem due to the following reasons [4]:

1. Numerous and incompatible criteria: The evaluation criteria of cloud services providers are various and numerous, several researches listed about 100 criteria and grouped them to six main categories. These criteria are incompatible with each other in almost cases, such as achieving high performance and security of services providers apart from minimizing the operational costs.
2. Different interests of decision makers: To evaluate available cloud services providers, the evaluation criteria must be determined carefully. But the determination process of

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these criteria is not easy matter, due to different opinion of decision makers. Decision makers have various interests which returns to their experiences and objectives.
3. The diversity of cloud services: Cloud services is a broad category which include the trillion IT resources provided through the internet.
4. Failure to handle vague and inconsistent information which exist usually in the performance estimation process: In evaluation process of cloud services providers, the decision makers always use their own experiences and perceptions in their judgment. Therefore, it is not always reliable in reality due to inconsistent guidelines and practices which exist in the real world.
5. The altitude cognitive requests on the decision makers [5–7]: Selecting the best cloud services providers can depend on knowledge and experience of decision makers. The final selection process will affect by their own opinions.

Subsequently, determining and selecting the best services provider is not an easy decision. For solving decision problems which contains conflicting criteria, the multi-criteria decision making/analysis (MCDM/MCDA) techniques can be considered as the best choice. Since it includes several methods that aid decision makers in simplifying and solving complex problems. In our research we select the analytic hierarchy process (AHP) for estimating and choosing the appropriate services provider. The judgment of experts and decision makers can be ambiguous and inconsistent in nature. Therefore, the fuzzy set theory proposed by Zadeh [8] in 1965, has been widely used to handle the ambiguous of human's decision [9,10]. It is also able to resolve uncertainties which exist in information for MCDM/MCDA. In fuzzy MCDA, the important weights of criteria are estimated using linguistic values represented via fuzzy numbers. Since fuzzy set theory, considers only the truth-membership degree, it fails to represent reality and cannot represent vague and inconsistent information efficiently. In order to enhance performance of fuzzy set theory, Atanassove introduced intuitionistic fuzzy sets [11], which applied by several researchers in different fields [12–17]. However, it can only model incomplete information and cannot model indeterminacy and inconsistency which exists usually in real systems. The previous gapes motivated us to represent the AHP for evaluating cloud services providers for the first time in neutrosophic environment. In 1995, Smarandache introduced Neutrosophy [18]. Neutrosophic set is a popularization of classic set, fuzzy set, and intuitionistic fuzzy set, etc. To facilitate the practical side of neutrosophic sets, a single-valued neutrosophic set (SVNS) was presented [19]. It consists of three membership elements: truth, indeterminacy and falsity. In neutrosophic set indeterminacy is explicitly quantified, truth, indeterminacy and falsity membership functions are autonomous. This proposition is extremely significant in various status such as information coalition when we try to integrate the data from various sensors. The single valued neutrosophic set applied in various domains [20–22]. When decision maker gives his/her opinion about a statement, he/she may say that, this statement is 50% true, 60% false and 20% that he/she is not sure. Hence, we can consider that neutrosophic can be one of the most suitable concepts for representing real decision making process via considering truth (certain/yes), indeterminacy (unsure) and falsity (false/no) membership functions. So it can manage vague, incomplete and inconsistent information efficiently. Subsequently, the resulted information will be more precise and this will help contribute to the right decisions. It can also be used in multiple disciplines and adoption cases, although many of them are expensive applications [23–26].

To improve the existing neutrosophic research, an approach is to use the multi-criteria decision analysis MCDA (AHP) technique in neutrosophic environment. The aim is to evaluate cloud services providers and assess their drawbacks of using classical and fuzzy AHP, by considering truth, indeterminacy and falsity elements and eventually it can lead to more accurate and precise decisions. Our proposed model also aggregates different interests of decision makers into one opinion to delete confliction. In classical AHP, Saaty provided a method for checking consistency of decision makers' comparison matrices, but if the matrix is not consistent the decision makers should rebuild their judgments and it is a time consuming process. We also overcame the previous drawback by presenting the induced bias matrix in neutrosophic environment for the first time. By applying this concept, the decision makers will be able to improve the consistency rate and modify it to the desired rate. A case study is solved to explain the pertinence of the proposed model. We also estimated the benefits and applicability of the proposed criteria for evaluating cloud services providers in this research, by using IEEE Standard [27]. A comparison between proposed approach and other existing methods has been applied. We also applied the eight quality factors, which were proposed by Moody and Shanks [28] to estimate the quality of our proposed model.

This paper is structured as follows. Section 2 explains the cloud computing anatomy and various MCDA techniques. Section 3 presents the literature review about choosing cloud service providers using various MCDA methods. Section 4 illustrates the basic definitions of neutrosophic sets. Section 5 presents the proposed model of neutrosophic multi-criteria decision analysis approach depending on the analytic hierarchy process. Section 6 validates the model by solving a case study and compares the proposed model with other existing models to evaluate it. Section 7 concludes our research and determines the future directions of the work.

2. Cloud computing anatomy and various MCDA techniques

Cloud computing turned into a prevalent service due to the fast evolution of information and communication technologies [29]. Clouds are computing and data-storage systems which integrate different technologies together to interconnect and manage demanded resources on dispensed computers [30]. The conceptual view of cloud computing is presented in Fig. 1. Users can utilize data, storage and applications provided by cloud service providers to build on new data, services and work-in-progress. Similarly, users can store their own data, storage and applications on the Cloud.

Several definitions of Cloud computing have been determined in different methods from analyst corporations, academics, manufacture practitioners, and IT firms. Table 1 presents various definitions of cloud computing according to many analyst corporations.

The applications of cloud computing [35] include these four characteristics. First, it has the secure and dependable center of data storage. Second, it can share data between various equipment. Third, it can enable users to use the internet infinitely. Last, it does not require high quality equipment form user.

There exist three layers (delivery models) of cloud computing [36]:

1. Software as a Service (SaaS), which extends access service to whole applications.
2. Platform as a Service (PaaS), which extends a platform for improving other applications on its head.
3. Infrastructure as a Service (IaaS), which run, prevail and manage storages and virtual machines through the extended environment.

The delivery services on IaaS, PaaS, and (SaaS), concerned by cloud computing. By introducing interfaces on all three layers, clouds declaim various kinds of customers:

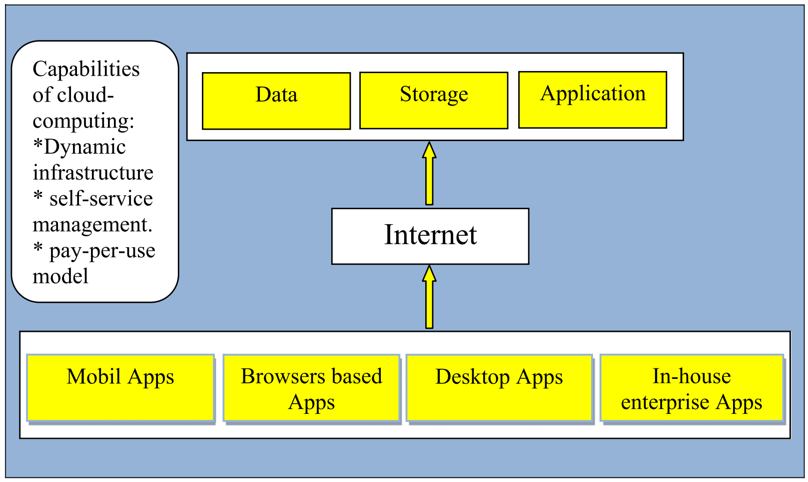

Fig. 1. Cloud computing.

Table 1
Definitions of cloud computing according to various analyst firms.

Analyst corporations	Definition
Gartner	"A computing technique in which a huge scalable IT linked abilities are introduced "as a service" via utilizing Internet to various outer users" [31].
IDC	"A development, deployment and delivery IT model for transmitting services, products and solutions through the Internet" [32].
The 451 Group	"A service model which integrates a generic arrangement standard for IT transmission, infrastructure ingredients, an architectural path and an economic model" [33]
Merrill Lynch	"Transferring personal and business applications from centralized servers" [34]

1. End consumers, who basically use the services of the SaaS layer via a web browsers and requisite offerings of the IaaS layer.
2. Business customers, who can access all layers: to improve the own infrastructure with extra resources on demand they access the IaaS layer, to be able to run own applications in a cloud they access the PaaS layer and they access the SaaS layer to gather benefits of available applications which presented as a service.
3. Developers and independent software vendors, who can improve and upgrade their applications through the SaaS layer of the cloud.

Different types of cloud deployment models are presented as follows [36,37]:

1. Public clouds: In these types, the service providers present their services to the public through the internet and web applications. Public clouds need a high level of security and control to manage business situations effectively.
2. Private clouds: These types are established especially for single organization. These types of clouds are characterized by a high level of security and control, because only stakeholders of organization have access to it. Private cloud is very expensive by comparing it with public clouds.
3. Community cloud: This type is established between two or more organizations that have the same requirements.
4. Hybrid clouds: A combination between at least two clouds. To keep some data in an organization, a hybrid cloud to support this feature.
5. Virtual private cloud: To overcome the drawbacks of private and public clouds, a virtual private cloud is presented.

The models and characteristics of cloud computing presented in Fig. 2.

Clouds are very useful for organizations due to the following reasons:

1. Inspire and preserve the competitive features of organizations: Using the cloud allows access to enterprise-class technology for everyone. It also enables smaller businesses to accelerate their work faster than large established competitors.
2. Better management of organizational information systems: The employees may save various versions of documents due to manage information in silos, which leads to confusion and diluted data. But using cloud computing deleted these difficulties via storing all documents in one place and in a single format.
3. Increase the productivity of the organization: The outsourcing process can improve productivity and pliability in work, via minimizing management and maintenance costs of neoteric technology.
4. Improve cooperation among organization members: Using cloud applications enhance collaboration by allowing sparse groups of people to meet virtually and facilely share information in real time.
5. Create a flexible environment in organization [38]: Cloud is the elastic facility that can be turned up, down or off depending upon circumstances.
6. Reduce cost of organization: Since organizations do not have to purchase equipment and build out and operate a data center, they do not have to disburse huge money on hardware, facilities, utilities and other aspects of operations.

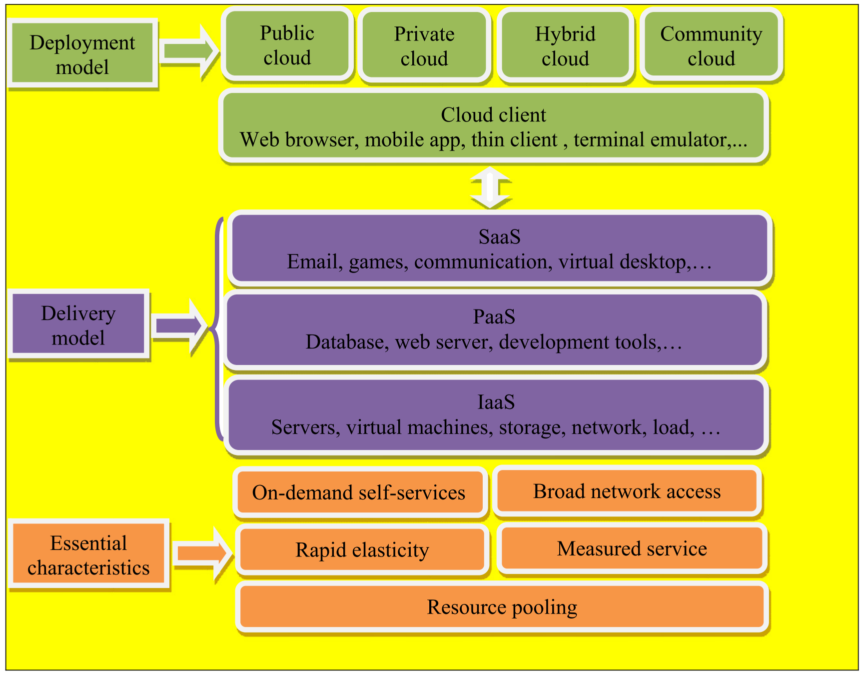

Fig. 2. Models and characteristics of cloud computing.

The well-known criteria for the estimation process of cloud services as mentioned in many researches are as follows:

1. Security: It is the ability to keep data and services protected for organization, so that a high confidence of privacy and data safety can be attained by the cloud service.
2. Performance: It is the quality of service provided by the cloud service providers.
3. Accessibility and usability: It refers to the ease of use of cloud services to support organizational requirements.
4. Scalability: It is the ability of cloud services to fit the problem and use resources effectively.
5. Adaptability: It is the adjustment process of cloud services depend on customer requests.

Once the previous criteria have been identified, the cloud services have to be estimated by a multi-criteria decision analysis approach (MCDA).

MCDA has a group of theories, methodologies and techniques for transacting with different problems. In decision making problems, the MCDA approaches choose and rank the actions efficiently. MCDA, belongs to a significant branch in operations research, seeks to plan mathematical and programming tools to select the superior alternative between various choices, according to the selected criteria. The MCDA approaches are categorized into two types in [39]:

1. Multi-Attribute Utility Theory (MAUT): It seeks to get a function which reflect the utility of a specific alternative. Each alternative assigns a marginal utility, with a real number presenting the preferability of alternative. The final utility is the sum of these marginal utilities.
2. Outranking approaches: They construct a pairwise comparison matrices to determine whether one alternative is ranked greater than another.

The MCDA approaches and its capabilities presented in Table 2.

3. Literature review

In this section a survey on various MCDA techniques which are used in selection process of cloud service.

Decision makers in many organizations face a major challenge in choosing and estimating the most suitable requirements [29,38] of cloud. Implementing cloud services in an organization and estimating their performance is a complex process due to the following reasons:

1. Incompatible and numerous criteria.
2. Different interests of decision makers.
3. Imprecision latent in the estimation process.

Due to the variety of cloud service providers, it is an extremely significant defy for organizations to choose the appropriate cloud providers which can fulfill their needs. The criteria for estimating and selecting the desirable cloud services, should be determined first. The pertinent criteria for estimating the performance of cloud services have been identified in many researches [29,36,50–63].

The estimation and selection process of cloud services has been illustrated in many researches by using various methods [36,50,51]. The analytic hierarchy process has been used by Garg et al. [36] to estimate the effectiveness of cloud services in an organization. In a multi-sourcing scenario, a mathematical decision making pattern for selecting cloud services proposed in [64]. The AHP applied to task-oriented resource allocation of cloud computing in [65]. A new AHP of cloud service selection applied to medical service cloud environment proposed in [66]. The AHP is an effective and efficient decision-making technique but subjectivity of decision makers can yield uncertainties when performing pairwise comparisons. To overcome this drawback, fuzzy AHP has been used by Safari et al. [67] for prioritizing cloud computing acceptance indicators. The fuzzy AHP used by Singla and Kaushal [68] for cloud path choosing of offloading in mobile cloud computing. Also fuzzy AHP approach has been used by Cheng [69] for cloud computing decision making problem. It is often hard and not accurate for decision makers to properly determine their opinions within [0, 1] interval. Hence, interval valued fuzzy AHP **is** proposed in [70]. The fuzzy set theory can be applied to various problems via possess a degree of uncertainty, but the obtained verdict is always comparatively vague. A novel MCDA method proposed in [71] is to assess cloud computing service using

Table 2
Capabilities and MCDA approaches.

Technique	Capabilities	Authors/year
Goal programming	Solve multiple and conflicting problems and it is an application of linear programming [40]	Charnes et al. (1955)
VIKOR	Determine the closeness degree to the ideal solution [41]	Opricovic (2004)
DEMATEL	Used to rank compromises a structural model including associations of ganglion factors [42]	Gabus and Fontela (1973)
AHP	A hierarchal structure, construct pairwise comparison of alternatives and criteria [43]	Thomas L. Saaty (1980)
ANP	A generalization of AHP, represent the interrelationships between decision levels and attributes [44]	Thomas L. Saaty (1996)
DEA	Relative to a set of similar observations it evaluate the competence of an observation [45]	Charnes (1978)
ELECTRE	Determine and exclude alternatives which are dominated by other alternatives [46]	Roy (1991)
TOPSIS	Select the alternative which is adjacent to the favorable ideal solution and the outmost from unfavorable ideal solution [47]	Hwang and Yoon (1981)
GRA	Solve problems with complex interrelationships among factors and variables [48]	Deng (1982)
PROMETHEE	As ELECTRE, but various in the pairwise comparison phase [49]	Brans and Vincke (1986)

intuitionistic fuzzy sets. However, none has proposed a MCDA technique based on AHP using neutrosophic sets like our approach does. The simple additive weighting (SAW) approach has been used by Saripalli and Pingali [50] for transacting with the cloud service estimation and selected problems. The analytic network process (ANP) is integrated with zero–one goal programming by Menzel et al. [51] to estimate the quality of cloud services. A fuzzy multi-criteria group decision making technique based on TOPSIS technique, has been used by Wibowo et al. [72] for evaluating cloud services.

A new crossbred fuzzy approach, depend on integration of fuzzy set and VIKOR techniques, was presented in [6]. A decision making model which combine interval-valued fuzzy sets and VIKOR is proposed in [73] for evaluating and selecting convenient cloud service provider.

4. Concepts and definitions of neutrosophic set

The requisite definitions of neutrosophic sets, triangular neutrosophic numbers and its operations presented in this section [18,19].

Definition 1. Any neutrosophic set A in X has a truth $T_A(x)$, indeterminacy $I_A(x)$ and falsity $F_A(x)$ membership functions. Where X is a set of points, $x \in X$, $T_A(x)$:$X \to\ ]-0, 1+[$, $I_A(x)$:$X \to\ ]-0, 1+[$ and $F_A(x)$: $X \to\ ]-0, 1+[$. The sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ has no restriction .

Definition 2. A single valued neutrosophic set A over X is an object having the form $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$, where $T_A(x) : X \to [0, 1]$, $I_A(x) : X \to [0, 1]$ and $F_A(x) : X \to [0, 1]$ with $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ for all $x \in X$. For convenience, a SVN number is represented by $A = (a, b, c)$, where $a, b, c \in [0, 1]$ and $a + b + c \le 3$.

Definition 3. Suppose that $\alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \in [0, 1]$ and $a_1, a_2, a_3 \in R$ where $a_1 \le a_2 \le a_3$. Then a single valued triangular neutrosophic number, $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ is a neutrosophic set whose truth, indeterminacy and falsity membership functions are as follows:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\dfrac{x-a_1}{a_2-a_1}\right) & (a_1 \le x \le a_2) \\ \alpha_{\tilde{a}} & (x = a_2) \\ \alpha_{\tilde{a}}\left(\dfrac{a_3-x}{a_3-a_2}\right) & (a_2 < x \le a_3) \\ 0 & \text{otherwise}, \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \theta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \le x \le a_2) \\ \theta_{\tilde{a}} & (x = a_2) \\ \dfrac{(x - a_2 + \theta_{\tilde{a}}(a_3-x))}{(a_3-a_2)} & (a_2 < x \le a_3) \\ 1 & \text{otherwise}, \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2 - x + \beta_{\tilde{a}}(x-a_1))}{(a_2-a_1)} & (a_1 \le x \le a_2) \\ \beta_{\tilde{a}} & (x = a_2) \\ \dfrac{(x - a_2 + \beta_{\tilde{a}}(a_3-x))}{(a_3-a_2)} & (a_2 < x \le a_3) \\ 1 & \text{otherwise}. \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \theta_{\tilde{a}}$ and $\beta_{\tilde{a}}$, represent the greatest degree of truth membership, least degree of indeterminacy and falsity memberships respectively.

Definition 4. Let $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \theta_{\tilde{b}}, \beta_{\tilde{b}} \rangle$ be two single valued triangular neutrosophic numbers and $\gamma \ne 0$ be any real number. Then,

1. Addition of two triangular neutrosophic numbers

$$\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3) ; \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$$

2. Subtraction of two triangular neutrosophic numbers

$$\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1) ; \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle$$

3. Inverse of a triangular neutrosophic number

$$\tilde{a}^{-1} = \left\langle \left(\frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1}\right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle, \text{ Where } (\tilde{a} \ne 0)$$

4. Multiplication of triangular neutrosophic number by constant value

$$\gamma\tilde{a} = \begin{cases} \langle (\gamma a_1, \gamma a_2, \gamma a_3)\,; \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle \; if (>0) \\ \langle (\gamma a_3, \gamma a_2, \gamma a_1)\,; \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \rangle \; if\ (\gamma < 0) \end{cases}$$

5. Division of triangular neutrosophic number by constant value

$$\frac{\tilde{a}}{\gamma} = \begin{cases} \left\langle \left(\frac{a_1}{\gamma}, \frac{a_2}{\gamma}, \frac{a_3}{\gamma} \right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle if (>0) \\ \left\langle \left(\frac{a_3}{\gamma}, \frac{a_2}{\gamma}, \frac{a_1}{\gamma} \right); \alpha_{\tilde{a}}, \theta_{\tilde{a}}, \beta_{\tilde{a}} \right\rangle if\ (\gamma < 0) \end{cases}$$

6. Division of two triangular neutrosophic numbers

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \left\langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle \\ \quad if\ (a_3 > 0, b_3 > 0) \\ \left\langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle \\ \quad if\ (a_3 < 0, b_3 > 0) \\ \left\langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3} \right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \right\rangle \\ \quad if\ (a_3 < 0, b_3 < 0) \end{cases}$$

7. Multiplication of two triangular neutrosophic numbers

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3)\,; \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle \\ \quad if\ (a_3 > 0, b_3 > 0) \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1)\,; \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle \\ \quad if\ (a_3 < 0, b_3 > 0) \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1)\,; \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \theta_{\tilde{a}} \vee \theta_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}} \rangle \\ \quad if\ (a_3 < 0, b_3 < 0) \end{cases}$$

5. A neutrosophic multi-criteria decision analysis approach

In this section, we present our proposed model based on the neutrosophic analytic hierarchy process.

5.1. Neutrosophic analytic hierarchy process

The analytic hierarchy process is a vastly used method in diverse multi-criteria decision analysis problems. The hierarchy of AHP consists of the following:

1. The objectives of the problem were defined in the first level.
2. The criteria and sub-criteria are presented in the second and third levels respectively.
3. The alternatives are presented in the last level.

Since classical AHP does not deal with vague information, the fuzzy theory was embedded to the classical AHP. Because the back stone of fuzzy programming is the membership function and decision makers assumed it according to their experience, then it fails to deal with indeterminacy and falsity membership functions which exist usually in real life situations and affect the quality of decision. The scale used in Fuzzy AHP cannot reflect the perceptions of the decision maker accurately. Also in classical AHP, if the pair wise comparison matrix is not consistent, Saaty did not provide any method to make it consistent. To overcome the previous drawbacks, this research introduces AHP in neutrosophic surroundings.

The steps of the proposed approach are as follows:

Step 1: Draw the hierarchy of the problem at various levels, which is called the decomposition process.

Step 2: Let decision makers compare criteria and alternatives through the linguistic terms, which shown in Table 3 and represented according to Abdel-Basset opinion.

If the decision maker illustrates "criterion 1 is absolutely significant than criterion 2", then it takes the triangular neutrosophic scale as $\langle (9, 9, 9); 1.00, 0.00, 0.00 \rangle$. Conversely, the comparison of criterion 2 to criterion 1 will take the triangular neutrosophic scale as $\left\langle (\frac{1}{9}, \frac{1}{9}, \frac{1}{9}); 1.00, 0.00, 0.00 \right\rangle$.

The pair-wise comparison matrices will have the following form as shown in Eq. (4).

$$\tilde{A}^k = \begin{bmatrix} \tilde{r}^k_{11} & \tilde{r}^k_{12} \cdots & \tilde{r}^k_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{r}^k_{n1} & \tilde{r}^k_{n2} \cdots & \tilde{r}^k_{mn} \end{bmatrix} \tag{4}$$

Where $\tilde{r}^k_{ij}$ is the preference relation of ith criterion over jth criterion according to kth decision maker. The "tilde" symbolize the triangular neutrosophic numbers, which have the following form $\tilde{r}^k_{ij} = \left\langle (l^k_{ij}, m^k_{ij}, u^k_{ij}); T^k_{ij}, \text{I}^k_{ij}, F^k_{ij} \right\rangle$, where $l^k_{ij}, m^k_{ij}, u^k_{ij}$ are the lower, median and upper bound of neutrosophic number,$T^k_{ij}, \text{I}^k_{ij}, F^k_{ij}$ are the truth-membership, indeterminacy and falsity membership functions respectively of triangular neutrosophic number. For example $\tilde{r}^1_{12}$ is the preference relation of first criterion via second criterion, with respect to the first decision makers and equal to $\tilde{r}^1_{12} = \langle (2, 3, 4); 0.30, 0.75, 0.70 \rangle$. Here in this research the truth, indeterminacy and falsity membership functions quantified for each triangular neutrosophic number according to decision maker opinion.

Step 3: By having more than on decision maker in the estimation process then, the aggregated $\tilde{r}_{ij}$ of all decision makers calculated as in Eq. (5) for obtaining the final comprehensive preference values via taking average values of all decision makers preferences.

$$\tilde{r}_{ij} = \frac{\sum_{k=1}^{K} \left\langle (l^k_{ij}, m^k_{ij}, u^k_{ij}); T^k_{ij}, \text{I}^k_{ij}, F^k_{ij} \right\rangle}{K} \tag{5}$$

The aggregated pair-wise comparison matrix according to the averaged preference values has the following form:

$$\tilde{A} = \begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} \cdots & \tilde{r}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{r}_{n1} & \tilde{r}_{n2} \cdots & \tilde{r}_{nn} \end{bmatrix} \tag{6}$$

Step 4: from the previous matrix we can calculate weight and creating a ranking of priorities, as follows:

1. Take the totality row averages:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n} \tag{7}$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using the following score equation:

$$S(\tilde{r}_{ij}) = \left| {}^{(l_{\tilde{r}_{ij}} + m_{\tilde{r}_{ij}} + u_{\tilde{r}_{ij}})}/_{3} + (T_{\tilde{r}_{ij}} - I_{\tilde{r}_{ij}} - F_{\tilde{r}_{ij}}) \right| \tag{8}$$

$$S(\tilde{w}_i) = \left| {}^{(l_{w_i} + m_{w_i} + u_{w_i})}/_{3} + (T_{w_i} - I_{w_i} - F_{w_i}) \right| \tag{9}$$

The previous score function apply to each triangular neutrosophic number for converting it to its crisp numerical value via taking the mean value of triangular number and added it to confirmation degree which equal $(T - I - F)$ of triangular number.

After de-neutrosophic of $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using the following equation:

$$w^N_i = \frac{w_i}{\sum_{i=1}^{n} w_i} \tag{10}$$

Step 5: Check consistency of judgments.

Table 3
Linguistic terms and the identical triangular neutrosophic numbers.

Saaty scale	Explanation	Neutrosophic Triangular Scale
1	Equally significant	$\tilde{1} = \langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$
3	Slightly significant	$\tilde{3} = \langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$
5	Strongly significant	$\tilde{5} = \langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$
7	Very strongly significant	$\tilde{7} = \langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$
9	Absolutely significant	$\tilde{9} = \langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$
2	Sporadic values between two close scales	$\tilde{2} = \langle(1, 2, 3); 0.40, 0.65, 0.60\rangle$
4		$\tilde{4} = \langle(3, 4, 5); 0.60, 0.35, 0.40\rangle$
6		$\tilde{6} = \langle(5, 6, 7); 0.70, 0.25, 0.30\rangle$
8		$\tilde{8} = \langle(7, 8, 9); 0.85, 0.10, 0.15\rangle$

Table 4
Upper bound for pair-wise comparison matrix to be convenient.

N	3×3	4×4	$n > 4$
CR≤	0.58	0.90	1.12

Table 5
Saaty table for random consistency index (RI) per different number of criteria.

0	2	3	4	5	6	7	8	9	10
0.0	0.0	0.58	0.90	1.12	1.24	1.32	1.4	1.45	1.49

To ensure decision quality, we have to consider the consistency of the pair-wise comparison matrix during the evaluation process. If the pair-wise comparison matrix has a transitive relation i.e. $a_{ik} = a_{ij}a_{jk}$ for all i, j and k, then the comparison matrix is consistent. But this method does not calculate the degree of consistency or inconsistency (i.e. the greater or lesser degree of consistency and inconsistency). So in this research we use the transitive relation (i.e. $(l_{ik}, m_{ik}, u_{ik}) = \left(l_{ij}, m_{ij}, u_{ij}\right) . \left(l_{jk}, m_{jk}, u_{jk}\right)$) to determine the consistency and calculate consistency degree according to Saaty. Not only this, but we also enhance the degree of consistency for the pair-wise comparison matrix and make it consistent by developing the concept in [74]. The value of the consistency of the pair-wise comparison matrix depend on n (i.e. the number of items being compared), and the consistency rate (CR) have to be calculated. The consistency rate is the ratio between the consistency index (CI) and a random consistency index(RI). The value of (CR) should not exceed 0.1 for comparison matrix which is smaller than or equal to 4×4 . The pair-wise comparison matrix is convenient, if the upper-bound of the consistency rate like what is shown in Table 4 [75,76].

For calculating CI and CR, the following steps should be executed:

1. All values in the first column of the pair-wise comparison matrix should be multiplied by the priority of the first item; this process continues for all columns of the comparison matrix. Sum the values across the rows to obtain a vector of values labeled "weighted sum".
2. The elements of the weighted sum vector should be divided by the corresponding priority for each criterion.
3. Compute the mean of the values found in the previous step; this mean is denoted $\tilde{\lambda}_{max}$.
   Since $\tilde{\lambda}_{max}$ still neutrosophic number, then we need to de-neutrosophic it (λ_{max}) by using in Eq. (8) as follows:
   $$S(\tilde{\lambda}_{max}) = \left|{}^{(l_{\tilde{\lambda}_{max}} + m_{\tilde{\lambda}_{max}} + u_{\tilde{\lambda}_{max}})}/_{3} + (T_{\tilde{\lambda}_{max}} - I_{\tilde{\lambda}_{max}} - F_{\tilde{\lambda}_{max}})\right| \quad (11)$$
4. Compute the consistency index (CI) as follows:
   $$\text{CI} = \frac{\lambda_{max} - n}{n - 1}, \quad (12)$$
   where n is the number of criteria being compared.
5. Compute the consistency ratio, which is defined as:
   $$CR = \frac{CI}{RI}, \quad (13)$$
   Where RI is the consistency index of a randomly generated pair-wise comparison matrix and shown in Table 5.

Step 6: If the pair-wise comparison matrix is not consistent, the decision maker does not repeat the exercise as in classical AHP but he/she can repair the matrix and make it consistent and he/she can also enhance the degree of consistency by using the following steps:

To identify the inconsistent elements in a pair-wise comparison matrix or enhance the consistency degree, Ergu et al. [74] proposed the induced bias matrix. The theorem and corollaries are as follows:

Theorem 1. *If the comparison matrix $\tilde{A}$ is a consistent matrix then, neutrosophic induced matrix*

$$\tilde{I} = \tilde{A} \times \tilde{A} - n \times \tilde{A}. \quad (14)$$

Corollary 1. *If the comparison matrix $\tilde{A}$ is approximately consistent, then the neutrosophic induced matrix $\tilde{I}$ should be close to zero.*

Corollary 2. *If the pair-wise matrix is inconsistent, then there must exist some inconsistent element.*

Ergu et al. [74] proposed major steps to identify inconsistency, and we developed these steps to deal with neutrosophic theory and enhance consistency degree of pair-wise matrix as follows:

1. Construct the neutrosophic induced matrix $\tilde{I} = \tilde{A} \times \tilde{A} - n \times \tilde{A}$.
2. Determine the largest preference relation $\tilde{r}_{ij}$, which has the largest lower, median and upper-bound of triangular number.
3. Determine the ith row and jth column which contain the inconsistent triangular neutrosophic number and calculate the dot product of row vector $\widetilde{Ro}_i = (\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in})$ and column vector $\widetilde{Co}_j^T = (\tilde{r}_{1j}, \tilde{r}_{2j}, \ldots, \tilde{r}_{nj})^T$, where $\widetilde{Co}_i^T$ is the transpose vector of $\widetilde{co}_j$.
4. The dot product
   $$\tilde{P} = \widetilde{Ro}_i \cdot \widetilde{Co}_j^T = (\tilde{r}_{i1}\tilde{r}_{1j}, \tilde{r}_{i2}\tilde{r}_{2j}, \ldots \tilde{r}_{in}\tilde{r}_{nj}) \quad (15)$$
5. Calculate elements which are distant from $\tilde{r}_{ij}$ in vector $\tilde{P}$ by the following formula:$\tilde{b} = \tilde{P}$-$\tilde{r}_{ij}$(16), where $\tilde{P}$ is the prejudice vector.
6. Determine the elements in the original pair-wise comparison matrix $\tilde{A}$ that cause inconsistency, by using the prejudice vector.
7. These elements are with the largest lower, median and upper bounds and far from scratch in the prejudice vector.
8. Try to modify these elements for enhancing the consistency of the judgments.

Step 7: Calculate the normalized weight of alternatives as in criteria weight calculation process.

Step 8: To calculate the scores of alternatives, multiply the weight of each alternative with the related criteria.

Step 9: Rank alternatives according to the largest score value.

The overall description of the proposed model presented in Fig. 3.

6. Case study: ranking of cloud computing based on proposed model

The proposed neutrosophic multi-criteria decision analysis approach submitted in Section 4 is utilized for estimating cloud computing services quality for the next case study.

A big e-learning service provider company in Egypt has more than 100 employees. The company has major activities such as e-learning content expansion and business delivery over marketing. The top agenda of the company includes safeness and piracy of the contents. If the contents of the company are pirated on the internet, this will cause a wasteful loss, and for this reason the company is looking for the appropriate cloud service. The board of directors of the company nominated three cloud service alternatives, according to five of the most important criteria in the estimation process. The cloud computing service alternatives are (1) Dropbox, (2) Google Drive and (3) Microsoft Sky Drive. The five criteria in the cloud service estimation process are (1) Security, (2) Performance, (3) Accessibility, (4) Scalability and (5) Adaptability.

Step 1: Draw the hierarchy of cloud service estimation process as in Fig. 4.

The decision makers in our case study are presented in Table 6 and the information about them are gathered from interviewees as illustrated in appendix. Three interviewees from the Egyptian company were interviewed. To ensure consistency of our proposal is independent of countries and other non-technical factors, three additional interviews of the decision-makers from the UK and China were undertaken. All these six interviewers are the main decision-makers of their institutions and have used the cloud service providers extensively. They were chosen due to their expertise, number of years in service and the frequent users of Dropbox, Google Drive and Microsoft Sky Drive. Their feedback was used to validate our proposed model.

Step 2: For allowing decision makers to compare criteria and alternatives through the linguistic terms shown in Table 3, a meeting was executed with the directors of the company and the averaged preference relation of the criteria is presented in Table 7, where $C_1, \ldots, C_5$ are the criteria's names as listed in the main example respectively.

Step3: Calculating weights of criteria as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (3.2, 3.8, 4.4)\, ; 0.50, 0.50, 0.50 \rangle\,,$$
$$\tilde{w}_2 = \langle (3.6, 4.2, 4.8); 0.50, 0.50, 0.50 \rangle\,,$$
$$\tilde{w}_3 = \langle (0.7, 0.95, 1.2); 0.30, 0.75, 0.70 \rangle\,,$$
$$\tilde{w}_4 = \langle (0.68, 0.89, 1); 0.30, 0.75, 0.70 \rangle\,,$$
$$\tilde{w}_5 = \langle (1.1, 1.3, 1.6); 0.30, 0.75, 0.70 \rangle\,.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$$w_1 = 3.3,\ w_2 = 3.7,\ w_3 = 0.2,\ w_4 = 0.29,\ w_5 = 0.18.$$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10).
Then, the normalized weight value of criteria are as follows:

$$w_1 = 0.4,\ w_2 = 0.5,\ w_3 = 0.03,\ w_4 = 0.04,\ w_5 = 0.02.$$

It is obvious that $\sum w_i = 1$.
The priorities of criteria are as follows:
C_2, C_1, C_4, C_3 and C_5 respectively.

Step 4: Check consistency of judgment.

If the pair-wise comparison matrix has a transitive relation i.e. $a_{ik} = a_{ij}a_{jk}$ for all i, j and k, then the comparison matrix is consistent. By focusing only on the lower, median and upper values of triangular neutrosophic number of the comparison matrix of criteria it is obvious that the matrix is consistent. To measure the degree of consistency do the following steps:

1. Calculate the "weighted sum" for each row.
The weighted sum of the first row = $\langle (1.34, 1.43, 1.52); 0.50, 0.50, 0.50 \rangle$,
The weighted sum of the second row = $\langle (1.38, 1.47, 1.56); 0.50, 0.50, 0.50 \rangle$,
The weighted sum of the third row = $\langle (0.23, 0.28, 0.35); 0.30, 0.75, 0.70 \rangle$,
The weighted sum of the fourth row = $\langle (0.21, 0.26, 0.30); 0.30, 0.75, 0.70 \rangle$,
The weighted sum of the fifth row = $\langle (0.32, 0.34, 0.46); 0.30, 0.75, 0.70 \rangle$.
2. By dividing the values of the weighted sum vector by the corresponding priority for each criterion we obtain the following :

$$\frac{\langle (1.34, 1.43, 1.52)\,; 0.50, 0.50, 0.50\rangle}{0.4}, \frac{\langle (1.38, 1.47, 1.56)\,; 0.50, 0.50, 0.50\rangle}{0.5}, \frac{\langle (0.23, 0.28, 0.35)\,; 0.30, 0.75, 0.70\rangle}{0.03}, \frac{\langle (0.21, 0.26, 0.30)\,; 0.30, 0.75, 0.70\rangle}{0.04}, \frac{\langle (0.32, 0.34, 0.46)\,; 0.30, 0.75, 0.70\rangle}{0.02}.$$

3. Compute the mean of the values found in the previous step; this mean is denoted $\tilde{\lambda}_{max}$, then

$$\tilde{\lambda}_{max} = \text{Average}\{\frac{\langle (1.34, 1.43, 1.52)\,; 0.50, 0.50, 0.50\rangle}{0.4}, \frac{\langle (1.38, 1.47, 1.56)\,; 0.50, 0.50, 0.50\rangle}{0.5}, \frac{\langle (0.23, 0.28, 0.35)\,; 0.30, 0.75, 0.70\rangle}{0.03}, \frac{\langle (0.21, 0.26, 0.30)\,; 0.30, 0.75, 0.70\rangle}{0.04}, \frac{\langle (0.32, 0.34, 0.46)\,; 0.30, 0.75, 0.70\rangle}{0.02}\} = \langle (7, 8, 9)\,; 0.30, 0.75, 0.70\rangle\,.$$

Since $\tilde{\lambda}_{max}$ still neutrosophic number, then we need to de-neutrosophic it as follows:

$$S(\tilde{\lambda}_{max}) = \left| {}^{(l_{\tilde{\lambda}max} + m_{\tilde{\lambda}max} + u_{\tilde{\lambda}max})}/{}_{3} + (T_{\tilde{\lambda}_{max}} - I_{\tilde{\lambda}_{max}} - F_{\tilde{\lambda}_{max}}) \right|,$$

Then $\lambda_{max} = 6.85$.

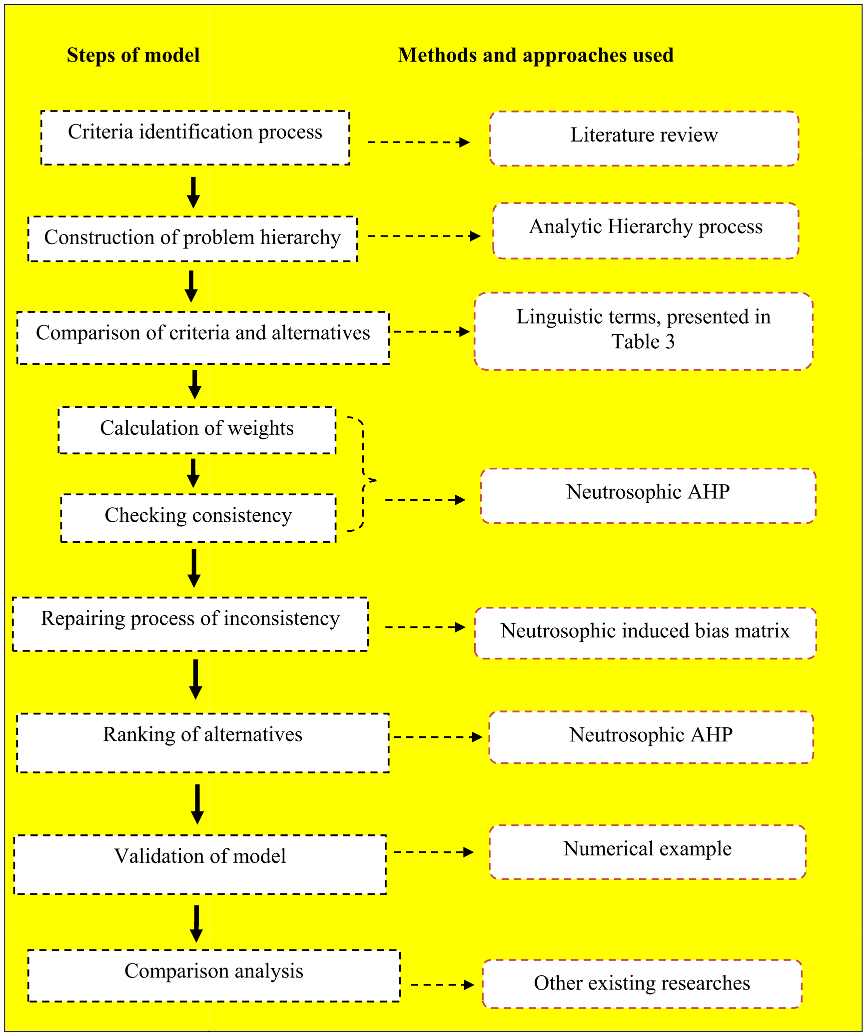

Fig. 3. Model description.

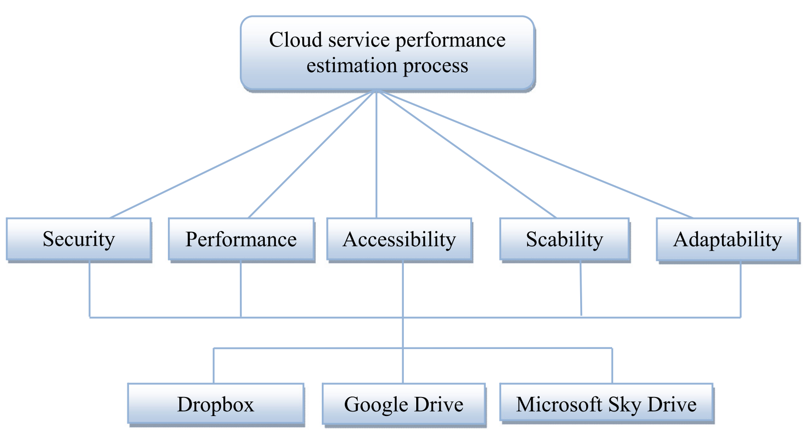

Fig. 4. The hierarchical structure of criteria and alternatives.

4. Compute the consistency index (CI) as follows:

$$\mathrm{CI} = \frac{\lambda_{max} - n}{n - 1} = \frac{6.85 - 5}{4} = 0.46.$$

, where n is the number of criteria being compared.

5. Compute the consistency ratio, which is defined as:

$$CR = \frac{CI}{RI} = \frac{0.46}{1.12} = 0.4$$

Because the comparison matrix is 5×5 (i.e. $n = 5$), the CR should be ≤ 1.12 as presented in Table 4, so it is a very compatible ratio of the comparison matrix, but we can also improve this ratio

Table 6
The record for interviewees.

Interviewers	Job	Sector	Years of experience	Organization to which they belong
First interviewer	IT manager	Information and communication	>5 years	Egyptian organization
Second interviewer	CXO	Sales and service	>4 years	Egyptian organization
Third interviewer	IS manager	Engineering	>7years	Egyptian organization
Fourth interviewer	Director	R&D	>20 years	UK organization
Fifth interviewer	Managing Director	Information and communication	>12 years	UK organization
Sixth interviewer	Director	Information and communication	>30 years	Chinese organization

Table 7
Comparison matrix of criteria.

Criteria	C_1	C_2	C_3	C_4	C_5
C_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$
C_2	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$
C_3	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}); 0.30, 0.75, 0.70\rangle$	$\langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$
C_4	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$
C_5	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}); 0.30, 0.75, 0.70\rangle$	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

and make it very near to 0.1 to increase degree of consistency during the following steps:

1. Construct the neutrosophic induced matrix $\tilde{I} = \tilde{A} \cdot \tilde{A} - n \cdot \tilde{A}$ (see the equation in Box I).
2. The largest preference relation $\tilde{r}_{ij}$, which has the largest lower, median and upper-bound of triangular number is $\tilde{r}_{24}$.
3. The dot product (see the equation in Box II).
4. Calculate elements which are distant from $\tilde{r}_{ij}$ in vector $\tilde{P}$ by the formula in Box III.
5. All elements with zero or negative values for each lower, median and upper bound of triangular number in $\tilde{b}$ is consistent element and we should enhance other elements in $\tilde{b}$.

To improve consistency of original pair-wise comparison matrix, we try to modify $\tilde{r}_{24}$ and $\tilde{r}_{25}$ as Table 8.

The normalized weight values of the previous matrix will be as follows:

$$w_1 = 0.5, w_2 = 0.4, w_3 = 0.03, w_4 = 0.04, w_5 = 0.03$$

The priorities of criteria are presented in Fig. 5 as follows:

C_1, $C_{2,}C_4$, C_3 and C_5 respectively and this means that, security and performance are the most important criteria according to company's directors.

By calculating λ_{max} as we illustrated previously with details, we found that $\lambda_{max} = 5.85$.

Compute the consistency index (CI) as follows:

$$\text{CI} = \frac{\lambda_{max} - n}{n - 1} = \frac{5.8 - 5}{4} = 0.2$$

$$CR = \frac{CI}{RI} = \frac{0.2}{1.12} = 0.17$$

It is obvious that CR became close to 0.1, and we reduced the consistency rate CR from 0.4 to 0.17 and it is an efficient ratio by comparing it with 1.12 as in Table 4 of Saaty.

To estimate the benefit and applicability of the previous proposed criteria in this research, we focus on the four criteria which are determined from IEEE Standard [27]:

This standard is a method to establish quality requirements and identify, implement, analyze, and validate any process or product. Then by applying this standard on proposed criteria, we ensure that the determined criteria are valid and establish quality requirements. As a result, the selected product (cloud service provider) can be possibly the best and a high quality product. The four criteria which are determined from IEEE Standard are as follows:

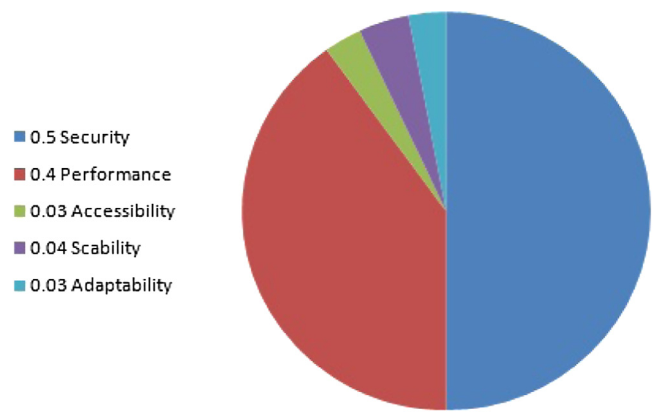

Fig. 5. The weight obtained for the evaluation criteria.

- Correlation: in order to show the interdependency between criteria, we will use the correlation coefficient of Spearman through the following formula:

$$\rho = 1 - \frac{6 * \sum D_i^2}{n * (n^2 - 1)} \tag{16}$$

Where ρ is the correlation coefficient, $D_i = x_i - y_i$ and it is the difference between ranked criteria values and n is the number of criteria.

$\rho_{\text{security-performance}} = 0.99$ and this means that the correlation between security and performance is very high. Also $\rho_{\text{security-accessibility}} = 0.77$, $\rho_{\text{security-scalability}} = 0.78$,

$\rho_{\text{security-adaptability}} = 0.77$,

$\rho_{\text{performance-accessibility}} = 0.86$,

$\rho_{\text{performance-scalability}} = 0.87$,

$\rho_{\text{performance-adaptability=0.86}}$,

$\rho_{\text{accessibility-scalability}} = 0.99$,

$\rho_{\text{accessibility-adaptability}} = 1$,

$\rho_{\text{scalability-adaptability}} = 0.99$. It is obvious that the correlation between criteria is very high.
- Consistency of criteria: as we illustrated in the previous example by calculating consistency ratio of the criteria. It is noted that the criteria are consistent.
- Computability and practicability of the criteria: it is obvious that the proposed criteria are easy to compute and practical.

$$\tilde{I} = \begin{bmatrix}
\langle(-1,0,1);0.50,0.50,0.50\rangle & \langle(-1,0,1);0.50,0.50,0.50\rangle & \langle(5,13,28);0.30,0.75,0.70\rangle & \langle(5,13,23);0.30,0.75,0.70\rangle & \langle(3,8,16);0.30,0.75,0.70\rangle \\
\langle(-1,0,1);0.50,0.50,0.50\rangle & \langle(-1,0,1);0.50,0.50,0.50\rangle & \langle(-16,22,33);0.30,0.75,0.70\rangle & \langle(13,23,35);0.30,0.75,0.70\rangle & \langle(-5,0,8);0.30,0.75,0.70\rangle \\
\left\langle(-\frac{1}{4},0,\frac{1}{6});0.50,0.50,0.50\right\rangle & \left\langle(-\frac{1}{4},0,\frac{1}{6});0.50,0.50,0.50\right\rangle & \langle(-2,0,3);0.30,0.75,0.70\rangle & \left\langle(\frac{53}{2},\frac{49}{3},\frac{35}{4});0.30,0.75,0.70\right\rangle & \langle(-4,7,8);0.30,0.75,0.70\rangle \\
\left\langle(\frac{7}{6},\frac{2}{7},\frac{3}{8});0.50,0.50,0.50\right\rangle & \left\langle(\frac{1}{6},\frac{2}{7},\frac{3}{8});0.50,0.50,0.50\right\rangle & \langle(-10,-8,-5);0.30,0.75,0.70\rangle & \langle(-1,0,2);0.30,0.75,0.70\rangle & \left\langle(\frac{31}{6},10,\frac{7}{4});0.30,0.75,0.70\right\rangle \\
\left\langle(0,\frac{1}{6},\frac{3}{4});0.50,0.5,0.5\right\rangle & \left\langle(\frac{7}{6},\frac{2}{7},\frac{3}{8});0.50,0.50,0.50\right\rangle & \left\langle(\frac{49}{2},\frac{46}{3},\frac{35}{4});0.30,0.75,0.70\right\rangle & \langle(-13,-10,14);0.30,0.75,0.70\rangle & \langle(-1,0,2);0.30,0.75,0.70\rangle
\end{bmatrix}$$

Box I.

$$\tilde{P} = \tilde{Ro}_2 \cdot \tilde{Co}_4^T = \left\{ \begin{array}{c} \langle (-5, 0, 23) ; 0.30, 0.75, 0.70 \rangle \ \langle (-13, 0, 35) ; 0.30, 0.75, 0.70 \rangle \ \langle (-424, 192, 539) ; 0.30, 0.75, 0.70 \rangle \\ \langle (-13, 0, 70) ; 0.30, 0.75, 0.70 \rangle \ \langle (50, 0, 112) ; 0.30, 0.75, 0.70 \rangle \end{array} \right\}$$

Box II.

$$\tilde{b} = \tilde{P} - \tilde{r}_{24} = \left\{ \begin{array}{c} \langle (-18, -23, -12) ; 0.30, 0.75, 0.70 \rangle \ \langle (-26, -23, 0) ; 0.30, 0.75, 0.70 \rangle \ \langle (-459, 179, 516) ; 0.30, 0.75, 0.70 \rangle \\ \langle (-26, -23, 35) ; 0.30, 0.75, 0.70 \rangle \ \langle (-23, 37, 77) ; 0.30, 0.75, 0.70 \rangle \end{array} \right\}$$

Box III.

Table 8
The modified comparison matrix of criteria.

Criteria	C_1	C_2	C_3	C_4	C_5
C_1	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	⟨(4, 5, 6); 0.80, 0.15, 0.20⟩	⟨(6, 7, 8); 0.90, 0.10, 0.10⟩	⟨(4, 5, 6); 0.80, 0.15, 0.20⟩
C_2	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	⟨(4, 5, 6); 0.80, 0.15, 0.20⟩	⟨(4, 5, 6); 0.90, 0.10, 0.10⟩	⟨(4, 5, 6); 0.90, 0.10, 0.10⟩
C_3	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	$\langle(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}); 0.30, 0.75, 0.70\rangle$	⟨(2, 3, 4); 0.30, 0.75, 0.70⟩
C_4	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.90, 0.10, 0.10\rangle$	⟨(2, 3, 4); 0.30, 0.75, 0.70⟩	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$
C_5	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.80, 0.15, 0.20\rangle$	$\langle(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}); 0.30, 0.75, 0.70\rangle$	⟨(4, 5, 6); 0.80, 0.15, 0.20⟩	⟨(1, 1, 1); 0.50, 0.50, 0.50⟩

- Power of discriminative: the proposed criteria can handle various cloud providers.

Step 5: Determine weights of alternatives according to each criterion.

The pair-wise comparison matrix of alternatives according to security criterion is presented in Table 9.

A_1, A_2 and A_3 are Dropbox, Google Drive and Microsoft Sky Drive respectively.

Similar to weight calculation methodology of criteria, we will also calculate the normalized weight of alternatives as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (0.4, 0.44, 0.45) ; 0.50, 0.50, 0.50 \rangle,$$
$$\tilde{w}_2 = \langle (1.75, 2.1, 2.5) ; 0.30, 0.75, 0.70 \rangle,$$
$$\tilde{w}_3 = \langle (4, 4.3, 4.7); 0.30, 0.75, 0.70 \rangle.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$w_1 = 0.07, w_2 = 0.96, w_3 = 3.2.$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10). then,

$w_1 = 0.4, w_2 = 0.5, w_3 = 0.03$

According to the security criterion, the weight of Dropbox is 0.4, the weight of Google Drive is 0.5 and the weight of Microsoft Sky Drive is 0.03. These values are basically dependent on decision makers' comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to performance criterion is presented in Table 10.

The normalized weight of alternatives according to performance criterion is as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (0.43, 0.44, 0.47) ; 0.50, 0.50, 0.50 \rangle,$$
$$\tilde{w}_2 = \langle (1.7, 2, 2.5) ; 0.50, 0.50, 0.50 \rangle,$$
$$\tilde{w}_3 = \langle (3.7, 4.3, 5); 0.50, 0.50, 0.50 \rangle.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$w_1 = 0.05, w_2 = 1.5, w_3 = 3.8.$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10), Then

$w_1 = 0.01, w_2 = 0.3, w_3 = 0.7.$

According to the performance criterion, the weights of Dropbox, Google Drive and Microsoft Sky Drive are 0.01, 0.3 and 0.7 respectively. These values are basically depend on decision makers' comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to accessibility criterion is presented in Table 11.

The normalized weight of alternatives with according to accessibility criterion is as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (2.4, 2.7, 3) ; 0.50, 0.50, 0.50 \rangle,$$
$$\tilde{w}_2 = \langle (3.4, 3.4, 3.4) ; 0.50, 0.50, 0.50 \rangle,$$
$$\tilde{w}_3 = \langle (1.7, 2, 2.4); 0.50, 0.50, 0.50 \rangle.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$w_1 = 2.2, w_2 = 2.9, w_3 = 1.5.$

Table 9
The pair-wise comparison matrix of alternatives according to security.

Alternatives	A_1	A_2	A_3
A_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\left\langle\left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}\right); 0.80, 0.15, 0.20\right\rangle$	$\left\langle\left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}\right); 1.00, 0.00, 0.00\right\rangle$
A_2	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\left\langle\left(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}\right); 0.30, 0.75, 0.70\right\rangle$
A_3	$\langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$	$\langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

Table 10
The pair-wise comparison matrix of alternatives according to performance.

Alternatives	A_1	A_2	A_3
A_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\left\langle\left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}\right); 0.80, 0.15, 0.20\right\rangle$	$\left\langle\left(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}\right); 0.90, 0.10, 0.10\right\rangle$
A_2	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\left\langle\left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}\right); 0.80, 0.15, 0.20\right\rangle$
A_3	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

Table 11
The pair-wise comparison matrix of alternatives according to accessibility.

Alternatives	A_1	A_2	A_3
A_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$	$\left\langle\left(\frac{1}{6}, \frac{1}{5}, \frac{1}{4}\right); 0.80, 0.15, 0.2\right\rangle$
A_2	$\left\langle\left(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}\right); 0.90, 0.10, 0.10\right\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$
A_3	$\langle(4, 5, 6); 0.80, 0.15, 0.20\rangle$	$\left\langle\left(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}\right); 1.00, 0.00, 0.00\right\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10). Then,

$$w_1 = 0.33, w_2 = 0.44, w_3 = 0.23$$

According to the accessibility criterion, the weights of Dropbox, Google Drive and Microsoft Sky Drive are 0.33, 0.44 and 0.23 respectively. These values are basically dependent on decision makers' comparison matrix, according to their opinions and requirements of e-learning company.

The pair-wise comparison matrix of alternatives according to scalability criterion is presented in Table 12.

The normalized weight of alternatives with according to scalability criterion is as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (5.3, 5.7, 6)\,; 0.50, 0.50, 0.50\rangle\,,$$

$$\tilde{w}_2 = \langle (3.4, 3.4, 3.4)\,; 0.50, 0.50, 0.50\rangle\,,$$

$$\tilde{w}_3 = \langle (0.4, 0.42, 0.4); 0.50, 0.50, 0.50\rangle\,.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$$w_1 = 5.2, w_2 = 2.9, w_3 = 0.09.$$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10). Then,

$$w_1 = 0.63, w_2 = 0.35, w_3 = 0.01$$

According to the scalability criterion, the weights of Dropbox, Google Drive and Microsoft Sky Drive are 0.63, 0.35 and 0.01 respectively. The pair-wise comparison matrix of alternatives according to adaptability criterion is presented in Table 13.

The normalized weight of alternatives with respect to adaptability criterion is as follows:

1. Take the totality row averages using the following equation:

$$\tilde{w}_i = \frac{\sum_{j=1}^{n} \left\langle (l_{ij}, m_{ij}, u_{ij}); T_{ij}, I_{ij}, F_{ij} \right\rangle}{n},$$

Then,

$$\tilde{w}_1 = \langle (0.4, 0.42, 0.42)\,; 0.50, 0.50, 0.50\rangle\,,$$
$$\tilde{w}_2 = \langle (3, 3.6, 4.3)\,; 0.30, 0.75, 0.7\rangle\,,$$
$$\tilde{w}_3 = \langle (3.4, 3.4, 3.5); 0.30, 0.75, 0.7\rangle\,.$$

2. Since $\tilde{w}_i$ are still triangular neutrosophic numbers, so we need to de-neutrosophic it using Eq. (9).

$$w_1 = 0.09, w_2 = 2.5, w_3 = 2.3.$$

3. After de-neutrosophic $\tilde{w}_i$, we obtain w_i, which is a crisp value and we need to normalize it using Eq. (10), then:

$$w_1 = 0.02, w_2 = 0.51, w_3 = 0.47$$

According to each criterion, the weights of three clouds computing service providers are presented in Fig. 6.

Step 6: Calculate the scores of alternatives by multiplying the weight of each alternative with the related criteria.

Then the relative scores for each alternative as follows:

$$\begin{bmatrix} 0.4 & 0.01 & 0.33 & 0.63 & 0.02 \\ 0.5 & 0.3 & 0.44 & 0.35 & 0.51 \\ 0.03 & 0.7 & 0.23 & 0.01 & 0.47 \end{bmatrix} \times \begin{bmatrix} 0.5 \\ 0.4 \\ 0.03 \\ 0.04 \\ 0.03 \end{bmatrix} = \begin{bmatrix} 0.24 \\ 0.41 \\ 0.32 \end{bmatrix}.$$

Step 7: Rank alternatives according to the largest score value.

According to proposed criteria and e-learning company requirements and comparisons from decision makers, the weights of Dropbox, Google Drive and Microsoft Sky Drive are 0.24, 0.41 and 0.32 respectively. Subsequently, the rank of alternatives is as follows: Google Drive followed by Microsoft Sky Drive and Dropbox as shown in Fig. 7. Since Google Drive has the highest weight comparing to two other drives with respect to predefined criteria and e-learning company requirements. Hence, we recommended to the e-learning company the selection of Google Drive, because it is the better choice taking into account all the determined criteria and the preference of decision makers' judgment.

7. Related work and model evaluation

In this section, the neutrosophic multi-criteria decision analysis methodology depends on the analytic hierarchy process for

Table 12
The pair-wise comparison matrix of alternatives according to scalability.

Alternatives	A_1	A_2	A_3
A_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$
A_2	$\langle(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}); 1.00, 0.00, 0.00\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$
A_3	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}); 1.00, 0.00, 0.00\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

Table 13
The pair-wise comparison matrix of alternatives according to adaptability.

Alternatives	A_1	A_2	A_3
A_1	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(\frac{1}{8}, \frac{1}{7}, \frac{1}{6}); 0.90, 0.10, 0.10\rangle$	$\langle(\frac{1}{9}, \frac{1}{9}, \frac{1}{9}); 1.00, 0.00, 0.00\rangle$
A_2	$\langle(6, 7, 8); 0.90, 0.10, 0.10\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$	$\langle(2, 3, 4); 0.30, 0.75, 0.70\rangle$
A_3	$\langle(9, 9, 9); 1.00, 0.00, 0.00\rangle$	$\langle(\frac{1}{4}, \frac{1}{3}, \frac{1}{2}); 0.30, 0.75, 0.70\rangle$	$\langle(1, 1, 1); 0.50, 0.50, 0.50\rangle$

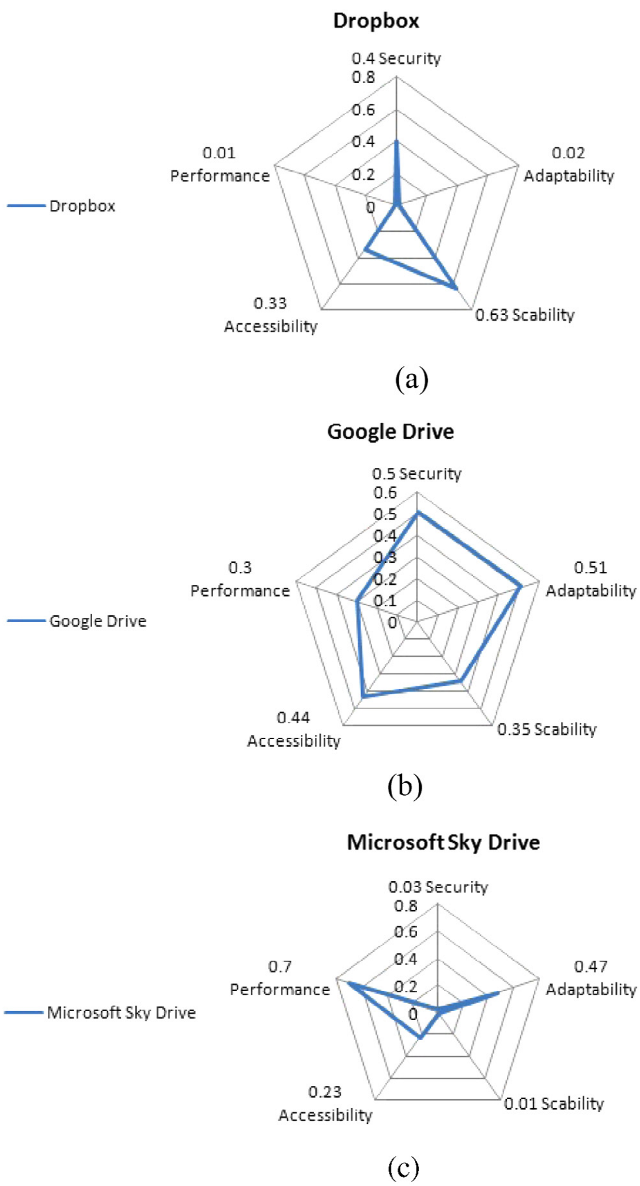

Fig. 6. Comparison of three clouds computing according to different criteria.

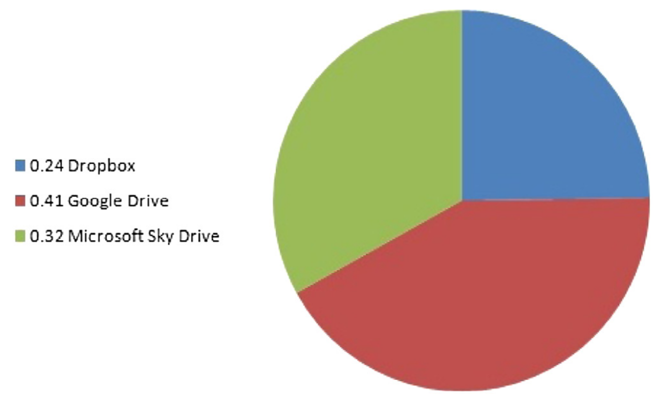

Fig. 7. The priorities of alternatives.

estimating the quality of cloud services has been evaluated and compared with other existent methods:

The analytic hierarchy process has been used by Garg et al. [36] to estimate the effectiveness of cloud services in an enterprise and this method has some drawbacks such as: The failure to handle vague, inconsistent information and the altitude cognitive request on the decision makers. They also did not provide any method to repairs the inconsistency of judgments.

The fuzzy AHP has been used by Safari et al. [67] for prioritizing cloud computing acceptance indicators, by Singla and Kaushal to allow users to select an optimal cloud service [68], by Cheng [69] for cloud computing decision making problem and a fuzzy multi-criteria group decision making method based on TOPSIS technique, has been used by Wibowo et al. [72] for evaluating cloud services. But these methods have some drawbacks such as: It should not represent real life situation efficiently, because it considers only the membership function and did not take into account the indeterminacy and falsity function. Also, the scale used in Fuzzy HP cannot reflect the perceptions of the decision maker accurately. They did not provide any technique to repair the judgments and make it consistent. There are two frameworks developed and led by Chang et al. [73,74]. The first one was focused on the security implementation and service delivery for cloud security. The second one was focused on the guidelines, recommendation and best practices. Each framework is either too practical or too theoretical. Our current proposal blends both theories into practices and ensures other institutions can adopt them according to their needs and decision-makers' criteria.

We overcame the previous drawbacks by proposing a simple model to evaluate the effectiveness of cloud services and select the optimal choice. Our proposed model provides the user with a richer structure framework than the classical and fuzzy AHP. Our model can handle vagueness and uncertainty over classical AHP and fuzzy AHP because it considers three different grades "membership degree, indeterminacy degree and non-membership degree". We also pointed out how to repair inconsistent judgments by developing the induced bias matrix and applying it in neutrosophic surroundings.

By applying the eight quality factors proposed by Moody and Shanks [28] to estimate the quality of our proposed model we found that:

1. The main criteria to estimate cloud services are presented, so our proposal is complete.
2. The decision makers can add or remove criteria for adjusting the proposed model to their organization, so our proposal is flexible.
3. The proposal is easy to understand.
4. The proposal is valid and correct.
5. Our proposal can be applied easily, since as we have verified with one real example.
6. Our proposal has the integrity.
7. Our proposal has an implement-ability feature.
8. Our proposal can help organization to make the most suitable choice because its consistent with problem.

8. Conclusions and future work

Clouds are made up of computing, data-storage, virtualization and networking systems to interconnect different systems, manage resources and provide services based on demands. Due to the fast development of cloud computing, many cloud services have appeared and demands for using more and better cloud services have increased. To meet the growing needs of examining some cloud service providers, this paper has introduced a neutrosophic multi-criteria decision analysis method depending on AHP, for estimating the performance of cloud services. A group of decision makers were consulted to compare alternatives according to various criteria. The priorities of decision makers are aggregated to achieve consensus of decision makers. The consistency degree of pairwise comparison matrix calculated in this research not only this, but we also improved it via representing induced bias matrix in neutrosophic environment. The estimation process of cloud services is modeled by using triangular neutrosophic numbers represented by linguistic variables in comparison matrices. A score function is introduced to transform triangular neutrosophic number to its equivalent crisp value. Real examples of organizations in Egypt, the UK and China were provided. All of these organizations have used and checked the applicability of our proposed technique, particularly the e-learning company in Egypt. The suggested neutrosophic multi-criteria decision analysis approach has achieved many benefits for transacting with ambiguous and inconsistent information which exist usually in cloud services estimation process. In the near future, we will estimate the performance of cloud computing services by using different multi-criteria decision analysis techniques and compare between their results.

Limitation of Proposed Research: More involvements from more companies will make our research better.

Competing Interests

The authors announce that there is no discrepancy of interests concerning the publication of this research.

Appendix A

The ranking process of cloud services providers varies regarding to decision makers opinion's and organization's requirements. So three decision makers were interviewed and we asked them to utilize the nine point scale which presented previously in Table 3. We gathered information about them for determining their cloud experiences as illustrated in Table 6.

The interviews were either face to face or online. Three services providers were nominated by the e-learning company managers which are (1) Dropbox, (2) Google Drive and (3) Microsoft Sky Drive. Six interviewees were consulted to determine the most important criteria for evaluating available providers and determine the relative significance of each pair of criteria from equally significant to absolutely significant.

The questions which asked to the interviewers were as follows:

1. What is your current job position?
2. Which industry sector you belong to it?
3. How many years of experience do you have in could computing field?
4. What is the organization to which you belong?
5. What are the most important criteria for evaluating cloud services providers?
6. In your opinion, what are the top cloud services providers **and** why?

After gathering the previous answers from interviewers and recording it, we selected only five criteria that decision makers agreed on its important for evaluating cloud services providers. These criteria are as follows: (1) Security, (2) Performance, (3) Accessibility, (4) Scalability and (5) Adaptability. The selection process of five criteria depended also on e-learning company requirements.

We again asked reviewers the following questiones:

7. Please utilize the nine point scale to determine the relative significance of each pair of criteria in the following table:

Criteria	Security	Performance	Accessibility	Scability	Adaptability
Security					
Performance					
Accessibility					
Scability					
Adaptability					

8. What is the best cloud services provider with respect to security criteria:
   - Dropbox
   - Google Drive
   - Microsoft Sky Drive
9. What is the best cloud services provider with respect to Performance criteria:
   - Dropbox
   - Google Drive
   - Microsoft Sky Drive
10. What is the best cloud services provider with respect to Accessibility criteria:
    - Dropbox
    - Google Drive
    - Microsoft Sky Drive
11. What is the best cloud services provider with respect to Scalability criteria:
    - Dropbox
    - Google Drive
    - Microsoft Sky Drive
12. What is the best cloud services provider with respect to Adaptability criteria:

- Dropbox
- Google Drive
- Microsoft Sky Drive

13. Please sir utilize again the nine point scale to determine the relative significance of three providers with respect to each criteria:

Security	Dropbox	Google Drive	Microsoft Sky Drive
Dropbox			
Google Drive			
Microsoft Sky Drive			

Performance	Dropbox	Google Drive	Microsoft Sky Drive
Dropbox			
Google Drive			
Microsoft Sky Drive			

Accessibility	Dropbox	Google Drive	Microsoft Sky Drive
Dropbox			
Google Drive			
Microsoft Sky Drive			

Scability	Dropbox	Google Drive	Microsoft Sky Drive
Dropbox			
Google Drive			
Microsoft Sky Drive			

Adaptability	Dropbox	Google Drive	Microsoft Sky Drive
Dropbox			
Google Drive			
Microsoft Sky Drive			

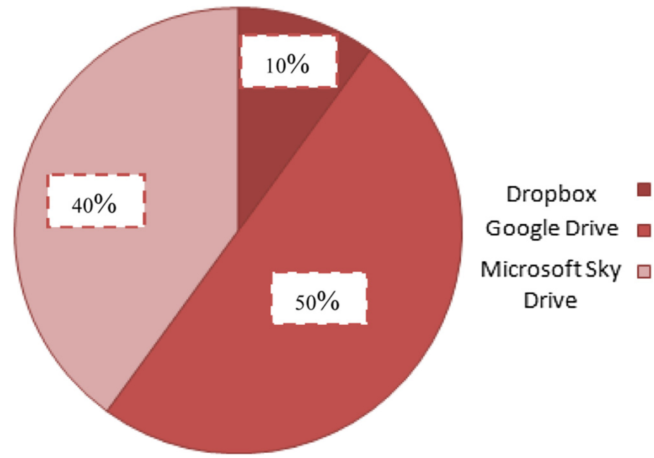

Fig. 8. The percentage of user's opinions on three services providers.

Appendix B

For evaluating results of our model and obtaining the right feedback, we asked 200 users from various organization to estimate the presented services providers in our case study. All users belong to organizations with experience in the use of cloud computing. The users were 150 men and 50 women. The users were asked to evaluate the performance of three cloud providers with respect to presented criteria (Security, Performance, Accessibility, Scalability and Adaptability) and ranking them according to their own opinions. To do this we were prepared a questionnaire as follows:

1. Current job position.........?
2. Industry sector ....?
3. Years of experience in cloud domain.........?
4. Rank the following criteria for evaluating cloud services providers according to your opinion: Security, Performance, Accessibility, Scalability and Adaptability.
    a. ....
    b. ....
    c. ....
    d. ....
    e. ....
5. Select the best services provider according to your opinion:
    - Dropbox
    - Google Drive
    - Microsoft Sky Drive
6. Depending on your previous choice, give the reasons for selecting this provider?

After collecting the answers to the previous questions we have obtained that only 10% from users agreed on Dropbox as the best services providers, 40% agreed on Microsoft Sky Drive and 50% agreed on Google Drive as the best services providers as in Fig. 8.

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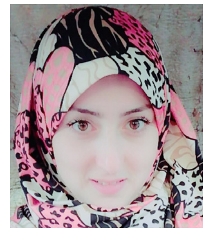

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