NSS

Neutrosophic Sets and Systems, Vol. 35, 2020

University of New MeSico

Neutrosophic $\aleph$ −bi-ideals in semigroups

K. Porselvi[1], B. Elavarasan[2]*, F. Smarandache[3] and Y. B. Jun[4]

[1,2]Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore - 641 114, Tamilnadu, India.
E-mail: porselvi94@yahoo.co.in; porselvi@karunya.edu.
E-mail: belavarasan@gmail.com;elavarasan@karunya.edu.
[3] Mathematics Department, University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA.
E-mail:fsmarandache@gmail.com; smarand@unm.edu.
[4] Department of Mathematics, Gyeongsang National University, Jinju 52828, Korea.
E-mail: skywine@gmail.com
* Correspondence: belavarasan@gmail.com

Abstract: In this paper, we introduce the notion of neutrosophic $\aleph$-bi-ideal for a semigroup. We infer different semigroups using neutrosophic $\aleph$-bi-ideal structures. Moreover, for regular semigroups, neutrosophic $\aleph$-product and intersection of neutrosophic $\aleph$-ideals are identical.

Keywords: Semigroup, ideal, bi-ideal, neutrosophic $\aleph-$ ideals, neutrosophic $\aleph$ -bi-ideals, neutrosophic $\aleph$ −product.

---

1. Introduction

In 1965, Zadeh [16] introduced the idea of fuzzy sets for modeling the ambiguous theories in the globe. In 1986, Atanassov [1] generalized fuzzy set and named as intuitionistic fuzzy set, and discussed it. Also from his view point, there are two degrees for any object in the world. They are degree of membership to a vague subset and degree of non-membership to that given subset.

Smarandache generalized fuzzy and intuitionistic fuzzy set, and referred as Neutrosophic set (see [2, 3, 6, 13-15]). It is identified by a truth, a falsity and an indeterminacy membership function. These sets are applied to many branches of mathematics to overcome the complexities arising from uncertain data. Neutrosophic set can distinguish between absolute membership and relative membership. Smarandache used this in non-standard analysis such as result of sport games (winning/defeating/tie), decision making and control theory, etc. This area has been studied by several authors (see [5, 10-12]).

In [8], M. Khan et al. presented and discussed the concepts of neutrosophic $\aleph$ −subsemigroup of semigroup. In [5], Gulistan et al. have studied the idea of complex neutrosophic subsemigroups. They have introduced the notion of characteristic function of complex neutrosophic sets, direct product of complex neutrosophic sets.

In [4], B. Elavarasan et al. introduced the concepts of neutrosophic $\aleph$ −ideal of semigroup and explored its properties. Also, the conditions are given for neutrosophic $\aleph-$structure becomes neutrosophic $\aleph$ −ideal. Further, presented the notion of characteristic neutrosophic $\aleph$ −structure over semigroup.

Throughout this article, X denotes a semigroup. Recall that for any subsets A and B of X, $AB = \{uw | u \in A \text{ and } w \in B\}$, the multiplication of A and B.

For a semigroup X,

(i) $\emptyset \neq U \subseteq X$ is a subsemigroup of X if $U^2 \subseteq U$.

(ii) A subsemigroup U of X is left (resp., right) ideal if $XU \subseteq U$ (resp., $UX \subseteq U$). U is an ideal of X if U is both left and right ideal of X.

(iii) X is left (resp., right) regular if for each $s \in X$, there exists $x \in X$ such that $s = xs^2$(resp., $s = s^2x$) [7].

(iv) X is regular if for each $s \in X$, there exists $x \in X$ such that $s = sxs$ [9].

(v) X is intra-regular if for every$s \in X$, there exist $x, y \in X$ such that $s = xs^2y$ [9].

(vi) A subsemigroup Y of X is bi-ideal if $YXY \subseteq Y$. For any $r' \in X$, $B(r') = \{r', r'^2, r'Xr'\}$ is the principal bi-ideal of X generated by r'.

2. Basics of neutrosophic ℵ – structures

In this section, we present the required basic definitions of neutrosophic $\aleph-$structures of X that we need in the sequel.

The collection of functions from a set X to $[-1,0]$ is denoted by $\Im(X,[-1,0])$. Note that $f \in \Im(X,[-1,0])$ is a negative-valued function from X to $[-1,0]$ (briefly, $\aleph-$function on X). Here $\aleph-$structure means (X,f) of X.

Definition 2.1. [8] A neutrosophic $\aleph-$ structure of X is defined to be the structure:

$$X_N := \frac{X}{(T_N, I_N, F_N)} = \left\{\frac{x}{T_N(x), I_N(x), F_N(x)} \mid x \in X\right\}$$

where T_N is the negative truth membership function on X, I_N is the negative indeterminacy membership function on X and F_N is the negative falsity membership function on X.

Note that for any $x \in X$, X_N satisfies the condition $-3 \leq T_N(x) + I_N(x) + F_N(x) \leq 0$.

Definition 2.2. [8] A neutrosophic $\aleph-$structure X_N of X is called a neutrosophic $\aleph-$subsemigroup of X if the below condition is valid:

$$(\forall\, g_i, h_j \in X)\begin{pmatrix} T_N(g_ih_j) \leq T_N(g_i) \vee T_N(h_j) \\ I_N(g_ih_j) \geq I_N(g_i) \wedge I_N(h_j) \\ F_N(g_ih_j) \leq F_N(g_i) \vee F_N(h_j)\end{pmatrix}.$$

Let X_N be a neutrosophic $\aleph-$ structure of X and let $\lambda, \delta, \varepsilon \in [-1,0]$ with $-3 \leq \lambda + \delta + \varepsilon \leq 0$. Then the set $X_N(\lambda, \delta, \varepsilon) := \{x \in X \mid T_N(x) \leq \lambda,\ I_N(x) \geq \delta,\ F_N(x) \leq \varepsilon\}$ is called a *(λ, δ, ε)* – level set of X_N.

Definition 2.3. [4] A neutrosophic $\aleph-$structure X_N of X is called a neutrosophic $\aleph-$left (resp., right) ideal of X if it satisfies:

$$(\forall\, g_i, h_j \in X)\begin{pmatrix} T_N(g_ih_j) \leq T_N(h_j)\ (resp., T_N(g_ih_j) \leq T_N(g_i)) \\ I_N(g_ih_j) \geq I_N(h_j)\ (resp., I_N(g_ih_j) \geq I_N(g_i)) \\ F_N(g_ih_j) \leq F_N(h_j)\ (resp., F_N(g_ih_j) \leq F_N(g_i))\end{pmatrix}.$$

If X_N is both neutrosophic $\aleph-$left and neutrosophic $\aleph-$right ideal of X, then it is called a neutrosophic $\aleph-$ideal of X.

Definition 2.4. A neutrosophic $\aleph-$subsemigroup X_N of X is a neutrosophic $\aleph-$bi-ideal of X if the following condition is valid:

$$(\forall\, r,s,t \in X)\begin{pmatrix} T_N(rst) \le T_N(r) \vee T_N(t) \\ I_N(rst) \ge I_N(r) \wedge I_N(t) \\ F_N(rst) \le F_N(r) \vee F_N(t) \end{pmatrix}.$$

Clearly any neutrosophic $\aleph-$left (resp., right) ideal is neutrosophic $\aleph-$bi-ideal, but the neutrosophic $\aleph-$bi-ideal is not necessary to be a neutrosophic $\aleph-$left (resp., right) ideal.

Example 2.5. Consider the semigroup $X = \{0, a, b, c\}$ with binary operation as follows:

.	0	a	b	c
0	0	0	0	0
a	0	0	0	b
b	0	0	0	b
c	b	b	b	c

Then $X_N = \left\{\frac{0}{(-0.9,-0.1,-0.7)}, \frac{a}{(-0.8,-0.2,-0.5)}, \frac{b}{(-0.7,-0.3,-0.3)}, \frac{c}{(-0.5,-0.4,-0.1)}\right\}$ is a neutrosophic $\aleph-$bi-ideal of X, but X_N is not neutrosophic $\aleph-$left ideal as well as neutrosophic $\aleph-$right ideal of X. □

Definition 2.6. [8] For $\Phi \neq A \subseteq X$, the characteristic neutrosophic $\aleph-$structure of X is denoted by $\chi_A(X_N)$ and is defined to be neutrosophic $\aleph-$structure

$$\chi_A(X_N) = \frac{X}{(\chi_A(T)_N, \chi_A(I)_N, \chi_A(F)_N)}$$

where

$$\chi_A(T)_N : X \to [-1,0],\ x \to \begin{cases} -1 & if\ x \in A \\ 0 & otherwise, \end{cases}$$

$$\chi_A(I)_N : X \to [-1,0],\ x \to \begin{cases} 0 & if\ x \in A \\ -1 & otherwise, \end{cases}$$

$$\chi_A(F)_N : X \to [-1,0],\ x \to \begin{cases} -1 & if\ x \in A \\ 0 & otherwise. \end{cases}$$

Definition 2.7. [8] Let $X_N := \frac{X}{(T_N, I_N, F_N)}$ and $X_M := \frac{X}{(T_M, I_M, F_M)}$.

(i) X_M is called a neutrosophic $\aleph-$ substructure of X_N over X, denoted by $X_N \subseteq X_M$, if
$$T_N(t) \ge T_M(t),\ I_N(t) \le I_M(t),\ F_N(t) \ge F_M(t)\ \forall t \in X.$$
If $X_N \subseteq X_M$ and $X_M \subseteq X_N$, then we say that $X_N = X_M$.

(ii) The neutrosophic $\aleph-$ product of X_N and X_M is defined to be a neutrosophic $\aleph-$structure of X,
$$X_N \odot X_M := \frac{X}{(T_{N\circ M}, I_{N\circ M}, F_{N\circ M})} = \left\{\frac{h}{T_{N\circ M}(h), I_{N\circ M}(h), F_{N\circ M}(h)} \mid h \in X\right\},$$
where
$$(T_N \circ T_M)(h) = T_{N\circ M}(h) = \begin{cases} \bigwedge_{h=rs} \{T_N(r) \vee T_M(s)\} & if\ \exists\, r,s \in X\ such\ that\ h = rs \\ 0 & otherwise, \end{cases}$$
$$(I_N \circ I_M)(h) = I_{N\circ M}(h) = \begin{cases} \bigvee_{h=rs} \{I_N(r) \wedge I_M(s)\} & if\ \exists\, r,s \in X\ such\ that\ h = rs \\ -1 & otherwise, \end{cases}$$

$$(F_N \circ F_M)(h) = F_{N\circ M}(h) = \begin{cases} \bigwedge_{h=rs} \{F_N(r) \vee F_M(s)\} & if\ \exists\, r,s \in X\ such\ that\ h = rs \\ 0 & otherwise. \end{cases}$$

(iii) For $t \in X$, the element $\frac{t}{(T_{N\circ M}(t),\ I_{N\circ M}(t),\ F_{N\circ M}(t))}$ is simply denoted by

$$(X_N \odot X_M)(t) = (T_{N\circ M}(t),\ I_{N\circ M}(t),\ F_{N\circ M}(t)) \text{ for the sake of convenience.}$$

(iv) The union of X_N and X_M is a neutrosophic $\aleph$ −structure over X is defined as

$$X_N \cup X_M = X_{N\cup M} = (X; T_{N\cup M},\ I_{N\cup M},\ F_{N\cup M}),$$

where

$$(T_N \cup T_M)(h_i) = T_{N\cup M}(h_i) = T_N(h_i) \wedge T_M(h_i),$$
$$(I_N \cup I_M)(h_i) = I_{N\cup M}(h_i) = I_N(h_i) \vee I_M(h_i),$$
$$(F_N \cup F_M)(h_i) = F_{N\cup M}(h_i) = F_N(h_i) \wedge F_M(h_i) \ \forall h_i \in X.$$

(v) The intersection of X_N and X_M is a neutrosophic $\aleph$ −structure over X is defined as

$$X_N \cap X_M = X_{N\cap M} = (X; T_{N\cap M},\ I_{N\cap M},\ F_{N\cap M}),$$

where

$$(T_N \cap T_M)(h_i) = T_{N\cap M}(h_i) = T_N(h_i) \vee T_M(h_i),$$
$$(I_N \cap I_M)(h_i) = I_{N\cap M}(h_i) = I_N(h_i) \wedge I_M(h_i),$$
$$(F_N \cap F_M)(h_i) = F_{N\cap M}(h_i) = F_N(h_i) \vee F_M(h_i) \ \forall \ h_i \in X.$$

3. Neutrosophic $\aleph$ −bi-ideals of semigroups

In this section, we examine different properties of neutrosophic $\aleph$ −bi-ideals of X.

Theorem 3.1. For $\Phi \neq B \subseteq X$, the following assertions are equivalent:

(i) $\chi_B(X_N)$ is a neutrosophic $\aleph$ −bi-ideal of X,

(ii) B is a bi-ideal of X.

Proof: Suppose $\chi_B(X_N)$ is a neutrosophic $\aleph$ −bi-ideal of X. Let r, t $\in B$ and $s \in X$. Then

$$\chi_B(T)_N(rst) \leq \chi_B(T)_N(r) \vee \chi_B(T)_N(t) = -1,$$
$$\chi_B(I)_N(rst) \geq \chi_B(I)_N(r) \wedge \chi_B(I)_N(t) = 0,$$
$$\chi_B(F)_N(rst) \leq \chi_B(F)_N(r) \vee \chi_B(F)_N(t) = -1.$$

Thus $rst \in B$ and hence B is a bi-ideal of X,

Conversely, assume B is a bi-ideal of X. Let $r, s, t \in X$.

If $r \in B$ and $t \in B$, then $rst \in B$. Now

$$\chi_B(T)_N(rst) = -1 = \chi_B(T)_N(r) \vee \chi_B(T)_N(t),$$
$$\chi_B(I)_N(rst) = 0 = \chi_B(I)_N(r) \wedge \chi_B(I)_N(t),$$
$$\chi_B(F)_N(rst) = -1 = \chi_B(F)_N(r) \vee \chi_B(F)_N(t).$$

If $r \notin B$ or t$\notin B$, then

$$\chi_B(T)_N(rst) \leq 0 = \chi_B(T)_N(r) \vee \chi_B(T)_N(t),$$
$$\chi_B(I)_N(rst) \geq -1 = \chi_B(I)_N(r) \wedge \chi_B(I)_N(t)$$
$$\chi_B(F)_N(rst) \leq 0 = \chi_B(F)_N(r) \vee \chi_B(F)_N(t).$$

Therefore $\chi_B(X_N)$ is a neutrosophic $\aleph$ −bi-ideal of X. □

Theorem 3.2. Let $\lambda, \delta, \varepsilon \in [-1, 0]$ be such that $-3 \leq \lambda + \delta + \varepsilon \leq 0$. If X_N is a neutrosophic $\aleph$ −bi-ideal, then $(\lambda, \delta, \varepsilon)$ −level set of X_N is a neutrosophic bi- ideal of X whenever $X_N(\lambda, \delta, \varepsilon) \neq \emptyset$.

Proof: Suppose $X_N(\lambda, \delta, \varepsilon) \neq \emptyset$ for $\lambda, \delta, \varepsilon \in [-1, 0]$ with $-3 \leq \lambda + \delta + \varepsilon \leq 0$. Let X_N be a neutrosophic $\aleph$ −bi-ideal and let $x, y, z \in X_N(\lambda, \delta, \varepsilon)$. Then

$$T_N(xyz) \leq T_N(x) \vee T_N(z) \leq \lambda,$$
$$I_N(xyz) \geq I_N(x) \wedge I_N(z) \geq \delta,$$

$$F_N(xyz) \leq F_N(x) \vee F_N(z) \leq \varepsilon$$

which imply $xyz \in X_N(\lambda, \delta, \varepsilon)$. Therefore $X_N(\lambda, \delta, \varepsilon)$ is a neutrosophic $\aleph$ −bi-ideal of X. □

Theorem 3.3. Let X_M be a neutrosophic $\aleph -$ structure of X. Then the equivalent assertions are:

(i) $X_M \odot X_M \subseteq X_M$ and $X_M \odot \chi_X(X_N) \odot X_M \subseteq X_M$ for any neutrosophic $\aleph -$ structure X_N,

(ii) X_M is a neutrosophic $\aleph$ −bi-ideal of X.

Proof: Suppose (i) holds. Then X_M is neutrosophic $\aleph -$ subsemigroup of X by Theorem 4.6 of [8]. Let $r, s, t \in X$ and let $a = rst$. Then

$$(T_M)(rst) \leq (T_M \circ \chi_X(T)_N \circ T_M)(rst) = \bigwedge_{a=rst} \{(T_M \circ \chi_X(T)_N)(rs) \vee T_M(t)\}$$

$$= \bigwedge_{a=bt} \{\bigwedge_{b=rs} \{(T_M(r) \vee \chi_X(T)_N(s)\} \vee T_M(t)\}$$

$$\leq \bigwedge \{T_M(r) \vee T_M(t)\} \leq T_M(r) \vee T_M(t),$$

$$I_M(rst) \geq (I_M \circ \chi_X(I)_N \circ I_M)(rst) = \bigvee_{a=rst} \{(I_M \circ \chi_X(I)_N)(rs) \wedge I_M(t)\}$$

$$= \bigvee_{a=bt} \{\bigvee_{b=rs} \{I_M(r) \wedge \chi_X(I)_N(s)\} \wedge I_M(t)\}$$

$$\geq \bigvee_{a=rst} \{I_M(r) \wedge I_M(t)\} \geq I_M(r) \wedge I_M(t),$$

$$(F_M)(rst) \leq (F_M \circ \chi_X(F)_N \circ F_M)(rst) = \bigwedge_{a=rst} \{(F_M \circ \chi_X(F)_N)(rs) \vee F_M(t)\}$$

$$= \bigwedge_{a=bt} \{\bigwedge_{b=rs} \{(F_M(r) \vee \chi_X(F)_N(s)\} \vee F_M(t)\}$$

$$\leq \bigwedge_{a=rst} \{F_M(r) \vee F_M(t)\} \leq F_M(r) \vee F_M(t).$$

Therefore X_M is a neutrosophic $\aleph -$ bi-ideal of X.

For converse, suppose (ii) holds. Then $X_M \odot X_M \subseteq X_M$ by Theorem 4.6 of [8].

Let $x \in X$. If $x = rb$ and r$= st$ for some r$, b, s, t \in X$, then

$$(T_M \circ \chi_X(T)_N \circ T_M)(x) = \bigwedge_{x=rb} \{(T_M \circ \chi_X(T)_N)(r) \vee T_M(b)\}$$

$$= \bigwedge_{x=rb} \{\bigwedge_{r=st} \{T_M(s) \vee \chi_X(T)_N(t)\} \vee T_M(b)\}$$

$$= \bigwedge_{x=rb} \{\bigwedge_{r=st} \{(T_M(s)\} \vee T_M(b)\}$$

$= \wedge_{x=rb}\{T_M(s_i) \vee T_M(b)\}$ for some $s_i \in X$ and r$= s_i t_i$

$$\geq \bigwedge_{x=s_i t_i b} T_M(s_i t_i b) = T_M(x),$$

$$(I_M \circ \chi_X(I)_N \circ I_M)(x) = \bigvee_{x=rb} \{(I_M \circ \chi_X(I)_N)(r) \wedge I_M(b)\}$$

$$= \bigvee_{x=rb}\{\bigvee_{r=pq}\{ I_M(s)\wedge \chi_X(I)_N(t)\}\wedge I_M(b)\}$$
$$= \bigvee_{x=rb}\{\bigvee_{r=st}\{ I_M(s)\} \wedge I_M(b)\}$$

$$= \vee_{x=ab}\{I_M(s_i)\wedge I_M(b)\}, \text{for some } s_i \in X \text{ and } r = s_i t_i$$

$$\leq \bigvee_{x=s_i t_i b} I_M(s_i t_i b) = I_M(x),$$

$$(F_M \circ \chi_X(F)_N \circ F_M)(x) = \bigwedge_{x=rb}\{(F_M \circ \chi_X(F)_N)(r) \vee F_M(b)\}$$

$$= \bigwedge_{x=rb}\{\bigwedge_{a=st}\{(F_M(s) \vee \chi_X(F)_N(t)\} \vee F_M(b)\}$$

$$= \bigwedge_{x=rb}\{\bigwedge_{r=st}\{(F_M(s)\} \vee F_M(b)\}$$
$$= \wedge_{x=rb}\{F_M(s_i) \vee F_M(b)\} \text{ for some } s_i \in X \text{ and } a = s_i t_i$$

$$\geq \bigwedge_{x=s_i t_i b} F_M(s_i t_i b) = F_M(x).$$

Otherwise $x \neq rb$ or $a \neq st$ for all r, $b, s, t \in X$. Then

$$(T_M \circ \chi_X(T)_N \circ T_M)(x) = 0 \geq T_M(x),$$
$$(I_M \circ \chi_X(I)_N \circ I_M)(x) = -1 \leq I_M(x),$$
$$(F_M \circ \chi_X(F)_N \circ F_M)(x) = 0 \geq F_M(x).$$

Therefore $X_M \odot \chi_X(X_N) \odot X_M \subseteq X_M$ for any neutrosophic $\aleph-$ structure X_N over X. □

Definition 3.4. A semigroup X is called neutrosophic $\aleph-$left (resp., right) duo if every neutrosophic $\aleph-$left (resp., right) ideal is neutrosophic $\aleph-$ideal of X.

If X is both neutrosophic $\aleph-$left duo and neutrosophic $\aleph-$right duo, then X is called neutrosophic $\aleph-$duo

Theorem 3.5. If X is regular left duo (resp., duo, right duo), then the equivalent assertions are:

(i) X_M in X is neutrosophic $\aleph-$bi- ideal,

(ii) X_M in X is neutrosophic $\aleph-$right ideal (resp., ideal, left ideal).

Proof: $(\boldsymbol{i}) \Longrightarrow (\boldsymbol{ii})$ Suppose X_M is a neutrosophic $\aleph-$bi- ideal and $g, h \in X$. As X is regular, we get $g = gtg \in gX \cap Xg$ for some $t \in X$ which gives $gh \in (gX \cap Xg)X \subseteq gX \cap Xg$ as X is left duo. So $gh = gs$ and $gh = s'g$ for some $s, s' \in X$. As X is regular, $\exists r \in X:\ gh = ghrgh = gsrs'g = g(srs')g$. Since X_M is neutrosophic $\aleph-$bi- ideal, we have

$$T_M(gh) = T_M(g(srs')g) \leq T_M(g) \vee T_M(g) = T_M(g),$$

$$I_M(gh) = I_M(g(srs')g) \geq I_M(g) \wedge I_M(g) = I_M(g),$$

$$F_M(gh) = F_M(g(srs')g) \leq F_M(g) \vee F_M(g) = F_M(g).$$

Therefore X_M is neutrosophic $\aleph-$right ideal.

$(\boldsymbol{ii}) \Longrightarrow (\boldsymbol{i})$ Suppose X_M is neutrosophic $\aleph-$right ideal and let $x, y, z \in X$. Then

$$T_M(xyz) \leq T_M(x) \leq T_M(x) \vee T_M(z),$$

$$I_M(xyz) \geq I_M(x) \geq I_M(x) \wedge I_M(z),$$

$$F_M(xyz) \leq F_M(x) \leq F_M(x) \vee F_M(z).$$

Therefore X_M is a neutrosophic $\aleph$ −bi-ideal. □

Theorem 3.6. If X is regular, then the equivalent assertions are:
(i) X is left duo (resp., right duo, duo),
(ii) X is neutrosophic $\aleph$ −left duo (resp., right duo, duo).

Proof: $(\boldsymbol{i}) \Longrightarrow (\boldsymbol{ii})$ Let r, s $\in X$, we have $rs \in (rXr)s \subseteq r(Xr)X \subseteq Xr$ as Xr is left ideal. Since X is regular, we have $rs = tr$ for some $t \in X$.

If X_M is neutrosophic $\aleph$ −left ideal, then $T_M(rs) = T_M(tr) \leq T_M(r), I_M(rs) = I_M(tr) \geq I_M(r)$ and $F_M(rs) = F_M(tr) \leq F_M(r)$. Thus X_M is neutrosophic $\aleph$ −right ideal and therefore X is neutrosophic $\aleph$ −left duo.

$(\boldsymbol{ii}) \Longrightarrow (\boldsymbol{i})$ Let A be a left ideal of X. Then $\chi_A(X_M)$ is a neutrosophic $\aleph$ −left ideal by Theorem 3.5 of [4]. By assumption, $\chi_A(X_M)$ is neutrosophic $\aleph$ −ideal. Thus A is a right ideal of X. □

Theorem 3.7. If X is regular, then the equivalent assertions are:
(i) Every neutrosophic $\aleph$ −bi-ideal is a neutrosophic $\aleph$ −right (resp., left ideal, ideal) ideal,
(ii) Every bi-ideal of X is a right ideal (resp., left ideal, ideal).

Proof: $(\boldsymbol{i}) \Longrightarrow (\boldsymbol{ii})$ Let A be a bi-ideal of X. Then *by* Theorem 3.1 $\chi_A(X_M)$ is neutrosophic $\aleph$ −bi-ideal for a neutrosophic $\aleph$ −structure X_M. Now by assumption, $\chi_A(X_M)$ is neutrosophic $\aleph$ −right ideal. So by Theorem 3.5 of [4], A is right ideal.

$(\boldsymbol{ii}) \Longrightarrow (\boldsymbol{i})$ Let X_M be a neutrosophic $\aleph$ −bi-ideal and let $r, s \in X$. Then we get rXr is a bi-ideal of X. By hypothesis, we can have rXr is right ideal. Since X is regular, we can get r$\in rXr$. So $rs \in (rXr)X \subseteq rXr$ implies $rs = rxr$ for some $x \in X$. Now,

$$T_M(rs) = T_M(rxr) \leq T_M(r) \vee T_M(r) = T_M(r),$$
$$I_M(rs) = I_M(rxr) \geq I_M(r) \wedge I_M(r) = I_M(r)$$
$$F_M(rs) = F_M(rxr) \leq F_M(r) \vee F_M(r) = F_M(r).$$

Thus X_M is a neutrosophic $\aleph$ −right ideal of X. □

Theorem 3.8. For any X, the equivalent conditions are:

(i) X is regular,

(ii) $X_M \cap X_N = X_M \odot X_N \odot X_M$ for every neutrosophic $\aleph$ − bi-ideal X_M and neutrosophic $\aleph$ − ideal X_N of X.

Proof: $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Suppose $X$is regular, X_M is a neutrosophic $\aleph$ − bi-ideal and X_N is a neutrosophic $\aleph$ − ideal of X. Then by Theorem 3.3, we have $X_M \odot X_N \odot X_M \subseteq X_M$ and $X_M \odot X_N \odot X_M \subseteq X_N$. So $X_M \odot X_N \odot X_M \subseteq X_M \cap X_N$.

Let $r' \in X$. As X is regular, there is $p \in X$ such that $r' = r'pr' = r'pr'pr'$. Now

$$T_{M \circ N \circ M}(r') = \bigwedge_{r'=de} \{T_M(d) \vee T_{N \circ M}(e)\}$$
$$= \bigwedge_{r'=r'e} \{T_M(r') \vee \{ \bigwedge_{v=pr'pr'} \{T_N(pr'p) \vee T_M(r')\}\}$$
$$\leq \bigwedge_{r'=r'e} \{T_M(r') \vee T_N(r')\} \leq T_M(r') \vee T_N(r') = T_{M \cap N}(r'),$$

$$I_{M \circ N \circ M}(r') = \vee_{r'=de}\{I_M(d) \wedge I_{N \circ M}(e)\}$$

$$= \bigvee_{r'=r'e} \{I_M(r') \wedge \{ \bigvee_{v=pr'pr'} \{ I_N(pr'p) \wedge I_M(r')\}\}$$

$$\geq \bigvee_{r'=r'e} \{I_M(r') \wedge I_N(r')\} \geq I_M(r') \wedge I_N(r') = I_{M\cap N}(r'),$$

$$F_{M\circ N\circ M}(r') = \bigwedge_{r'=de} \{F_M(d) \vee F_{N\circ M}(e)\}$$

$$= \bigwedge_{r'=r'e} \{F_M(r') \vee \{ \bigwedge_{v=pr'pr'} \{F_N(pr'p) \vee F_M(r')\}\}$$

$$\leq \bigwedge_{r'=r'e} \{F_M(r') \vee F_N(r')\} \leq F_M(r') \vee F_N(r') = F_{M\cap N}(r').$$

Thus $X_{M\cap N} \subseteq X_M \odot X_N \odot X_M$ and hence $X_{M\cap N} = X_M \odot X_N \odot X_M$.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Suppose (ii) holds. Then $X_M \cap \chi_X(X_N) = X_M \odot \chi_X(X_N) \odot X_M$. But $X_M \cap \chi_X(X_N) = X_M$, so $X_M = X_M \odot \chi_X(X_N) \odot X_M$ for every neutrosophic $\aleph-$ bi-ideal X_M of X.

Let $u' \in X$. Then $\chi_{B(u')}(X_M)$ is neutrosophic $\aleph-$ bi-ideal by Theorem 3.1.

By assumption, we have

$$\chi_{B(u')}(T)_M = \chi_{B(u')}(T)_M \circ \chi_X(T)_N \circ \chi_{B(u')}(T)_M = \chi_{B(u')XB(u')}(T)_M,$$
$$\chi_{B(u')}(I)_M = \chi_{B(u')}(I)_M \circ \chi_X(I)_N \circ \chi_{B(u')}(I)_M = \chi_{B(u')XB(u')}(I)_M,$$
$$\chi_{B(u')}(F)_M = \chi_{B(u')}(F)_M \circ \chi_X(F)_N \circ \chi_{B(u')}(F)_M = \chi_{B(u')XB(u')}(F)_M.$$

Since $u' \in B(u')$, we have

$$\chi_{B(u')XB(u')}(T)_M(u') = \chi_{B(u')}(T)_M(u') = -1,$$
$$\chi_{B(u')XB(u')}(I)_M(u') = \chi_{B(u')}(I)_M(u') = 0,$$
$$\chi_{B(u')XB(u')}(F)_M(u') = \chi_{B(u')}(F)_M(u') = -1$$

Thus $u' \in B(u')XB(u')$ and hence X is regular. □

Theorem 3.9. For any X, the below statements are equivalent:

(i) X is regular,

(ii) $X_M \cap X_N = X_M \odot X_N$ for every neutrosophic $\aleph-$ bi-ideal X_M and neutrosophic $\aleph-$ left ideal X_N of X.

Proof:$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Let X_M and X_N be neutrosophic $\aleph-$ bi-ideal and neutrosophic $\aleph-$left ideal of X respectively. Let $r \in X$. Then $\exists x \in X$: $r = rxr$. Now

$$T_{M\circ N}(r) = \bigwedge_{r=uv} \{T_M(u) \vee T_N(v)\} \leq T_M(r) \vee T_N(xr) \leq T_M(r) \vee T_N(r) = T_{M\cap N}(r),$$

$$I_{M\circ N}(r) = \bigvee_{r=uv} \{I_M(u) \wedge I_N(v)\} \geq I_M(r) \wedge I_N(xr) \geq I_M(r) \wedge I_N(r) = I_{M\cap N}(r),$$

$$F_{M\circ N}(r) = \bigwedge_{r=uv} \{F_M(u) \vee F_N(v)\} \leq F_M(r) \vee F_N(xr) \leq F_M(r) \vee F_N(r) = F_{M\cap N}(r).$$

Therefore $X_{M\cap N} \subseteq X_M \odot X_N$.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Suppose (ii) holds, and let X_M and X_N be neutrosophic $\aleph-$ right ideal and neutrosophic $\aleph-$ left ideal of X respectively. Since every neutrosophic $\aleph-$ right ideal is neutrosophic $\aleph-$ bi-ideal, X_M is neutrosophic $\aleph-$ bi-ideal. Then by assumption, $X_{M\cap N} \subseteq X_M \odot X_N$. By Theorem 3.8 and Theorem 3.9 of [4], we can get $X_M \odot X_N \subseteq X_N$ and $X_M \odot X_N \subseteq X_M$ and so $X_M \odot X_N \subseteq X_M \cap X_N = X_{M\cap N}$. Therefore $X_M \odot X_N = X_{M\cap N}$.

Let K and L be right and left ideals of X respectively, and $r \in K \cap L$. Then $\chi_K(X_M) \odot \chi_L(X_M) = \chi_K(X_M) \cap \chi_L(X_M)$ which implies $\chi_{KL}(X_M) = \chi_{K\cap L}(X_M)$. Since $r \in K \cap L$, we have

$\chi_{K\cap L}(T)_M(r) = -1 = \chi_{KL}(T)_M(r), \chi_{K\cap L}(I)_M(r) = 0 = \chi_{KL}(I)_M(r)$ and $\chi_{K\cap L}(F)_M(r) = -1 = \chi_{KL}(F)_M(r)$ which imply r $\in KL$.Thus $K \cap L \subseteq KL \subseteq K \cap L$. So $K \cap L = KL$. Thus X is regular. □

Theorem 3.10. For any X, the equivalent conditions are:

(i) X is regular,

(ii) $X_M \cap X_N \subseteq X_M \odot X_N$ for every neutrosophic $\aleph -$ right ideal X_N and neutrosophic $\aleph -$ bi-ideal X_M of X.

Proof: It is same as Theorem 3.9. □

Theorem 3.11. For any X, the equivalent assertions are:

(i) X is regular,

(ii) $X_L \cap X_M \cap X_N \subseteq X_L \odot X_M \odot X_N$ for every neutrosophic$\aleph -$ right ideal X_L, neutrosophic $\aleph -$ bi-ideal X_M and neutrosophic $\aleph -$ left ideal X_N of X.

Proof: $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Suppose X is regular, and let X_L, X_M, X_N be neutrosophic $\aleph -$ right, bi-ideal, left ideals of X respectively. Let r $\in X$. Then there is $x \in X$ with r= $rxr = rxrxr$. Now

$$T_{L\circ M\circ N}(r) = \bigwedge_{r=uv} \{T_L(u) \vee T_{M\circ N}(v)\} \leq T_L(rx) \vee T_{M\circ N}(rxr) \leq T_L(r) \vee \{T_M(r) \vee T_N(xr)\}$$
$$\leq T_L(r) \vee T_M(r) \vee T_N(r) = T_{L\cap M\cap N}(r),$$
$$I_{L\circ M\circ N}(r) = \bigvee_{r=uv} \{I_L(u) \wedge I_{M\circ N}(v)\} \geq I_L(rx) \wedge I_{M\circ N}(rxr) \geq I_L(r) \wedge \{I_M(r) \wedge I_N(xr)\}$$
$$\geq I_L(r) \wedge I_M(r) \wedge I_N(r) = I_{L\cap M\cap N}(r),$$
$$F_{L\circ M\circ N}(r) = \bigwedge_{r=uv} \{F_L(u) \vee F_{M\circ N}(v)\} \leq F_L(rx) \vee F_{M\circ N}(rxr) \leq F_L(r) \vee F_M(r) \vee F_N(xr)$$
$$\leq F_L(r) \vee F_M(r) \vee F_N(r) = F_{L\cap M\cap N}(r).$$

Therefore $X_{L\cap M\cap N} \subseteq X_L \odot X_M \odot X_N$.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Suppose (ii) holds, and let X_L and X_N be neutrosophic $\aleph -$ right and neutrosophic $\aleph -$ left ideal of X respectively, and X_M a neutrosophic $\aleph -$bi-ideal of X. Then $\chi_X(X_M)$ is a neutrosophic $\aleph -$ bi-ideal by Theorem 3.1. Now $X_L \cap X_N = X_L \cap \chi_X(X_M) \cap X_N \subseteq X_L \odot \chi_X(X_M) \odot X_N \subseteq X_L \odot X_N$. Again by Theorem 3.8 and Theorem 3.9 of [4], we can get $X_L \odot X_N \subseteq X_L \cap X_N$and so $X_L \odot X_N = X_L \cap X_N$.

Let K and L be right and left ideals of X respectively. Then $\chi_K(X_M) \odot \chi_L(X_M) = \chi_K(X_M) \cap \chi_L(X_M)$. By Theorem 3.6 of [4], we have $\chi_{KL}(X_M) = \chi_{K\cap L}(X_M)$. Let r $\in K \cap L$. Then

$$\chi_{KL}(T)_M(r) = \chi_{K\cap L}(T)_M(r) = -1,$$
$$\chi_{KL}(I)_M(r) = \chi_{K\cap L}(I)_M(r) = 0,$$
$$\chi_{KL}(F)_M(r) = \chi_{K\cap L}(F)_M(r) = -1.$$

So r $\in KL$. Thus $K \cap L \subseteq KL \subseteq K \cap L$. Hence $K \cap L = KL$. Therefore X is regular. □

Theorem 3.12. For any X, the equivalent conditions are:

(i) X is regular and intra- regular,

(ii) $X_M \cap X_N \subseteq X_M \odot X_N$ for every neutrosophic $\aleph -$ bi-ideals X_M, X_N of X.

Proof: $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Let X_M and X_N be neutrosophic $\aleph -$ bi-ideals. Let $h \in X$. Then by regularity of X, h $= hxh = hxhxh$ for some x $\in X$. Since X is intra-regular, $\exists y, z \in X$: h $= yh^2z$. Then $h = hxyhhzxh$. Now

$$T_{M\circ N}(h) = \bigwedge_{h=rt}\{T_M(r) \vee T_N(t)\} \leq T_M(hxyh) \vee T_N(hzxh) \leq T_M(h) \vee T_N(h) \quad = T_{M\cap N}(h),$$
$$I_{M\circ N}(h) = \bigvee_{h=rt}\{I_M(r) \wedge I_N(t)\} \geq I_M(hxyh) \wedge I_N(hzxh) \geq I_M(h) \wedge I_N(h) = I_{M\cap N}(h),$$
$$F_{M\circ N}(h) = \bigwedge_{h=rt}\{F_M(r) \vee F_N(t)\} \leq F_M(hxyh) \vee F_N(hzxh) \leq F_M(h) \vee F_N(h) = F_{M\cap N}(h).$$

Therefore $X_M \cap X_N \subseteq X_M \odot X_N$ for every neutrosophic $\aleph-$ bi-ideals X_M and X_N.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Suppose (ii) holds, and let X_M and X_N be neutrosophic $\aleph-$ right and left ideal of X respectively. Then X_M and X_N are neutrosophic $\aleph-$ bi-ideals. By assumption, $X_{M\cap N} \subseteq X_M \odot X_N$. By Theorem 3.8 and Theorem 3.9 of [4], we can get $X_M \odot X_N \subseteq X_N$ and $X_M \odot X_N \subseteq X_M$ and so $X_M \odot X_N \subseteq X_M \cap X_N = X_{M\cap N}$. Therefore $X_M \odot X_N = X_{M\cap N}$.

Let K, L be right, left ideals of X respectively. Then $\chi_K(X_M) \odot \chi_L(X_M) = \chi_K(X_M) \cap \chi_L(X_M)$.

By Theorem 3.6 of [4], $\chi_{KL}(X_M) = \chi_{K\cap L}(X_M)$. Let $r \in K \cap L$. Then $\chi_{K\cap L}(T)_M(r) = -1 = \chi_{KL}(T)_M(r), \chi_{K\cap L}(I)_M(r) = 0 = \chi_{KL}(I)_M(r)$ and $\chi_{K\cap L}(F)_M(r) = -1 = \chi_{KL}(F)_M(r)$ which imply $r \in KL$. Thus $K \cap L \subseteq KL \subseteq K \cap L$ and hence $K \cap L = KL$. Therefore X is regular.

Also, for $r \in X, \chi_{B(r)}(X_M) \cap \chi_{B(r)}(X_M) = \chi_{B(r)}(X_M) \odot \chi_{B(r)}(X_M)$. By Theorem 3.8 and Theorem 3.9 of [4], we get $\chi_{B(r)}(X_M) = \chi_{B(r)B(r)}(X_M)$.since$\chi_{B(r)}(T)_M(r) = -1 = \chi_{B(r)}(F)_M(r)$and $\chi_{B(r)}(I)_M(r) = 0$, we get $\chi_{B(r)B(r)}(T)_M(r) = -1 = \chi_{B(r)B(r)}(F)_M(r)$ and $\chi_{B(r)B(r)}(I)_M(r) = 0$ which imply $r \in B(r)B(r)$. Thus X is intra-regular. □

Theorem 3.13. For any X, the equivalent conditions are:

(i) X is intra-regular and regular,

(ii) $X_M \cap X_N \subseteq (X_M \odot X_N) \cap (X_N \odot X_M)$ for every neutrosophic $\aleph-$ bi-ideals X_M and X_N of X.

Proof:$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Suppose X is regular and intra- regular, and let X_M and X_N be neutrosophic $\aleph-$ bi-ideals of X. Then by Theorem 3.12, $X_M \odot X_N \supseteq X_M \cap X_N$. Similarly we can prove that $X_N \odot X_M \supseteq X_N \cap X_M$.Therefore $(X_M \odot X_N) \cap (X_N \odot X_M) \supseteq X_M \cap X_N$ for every neutrosophic $\aleph-$ bi-ideals X_M and X_N of X.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Let X_M and X_N be neutrosophic $\aleph-$ bi-ideals of X. Then $X_M \cap X_N \subseteq X_M \odot X_N$ gives X is intra-regular and regular by Theorem 3.12. □

Theorem 3.14. For any X, the equivalent assertions are:

(i) X is intra-regular and regular,

(ii) $X_M \cap X_N \subseteq X_M \odot X_N \odot X_M$ for every neutrosophic $\aleph-$ bi-ideals X_M and X_N of X.

Proof:$(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Let X_M and X_N be neutrosophic $\aleph-$ bi-ideals, and $a \in X$. As X is regular, $a = axa = axaxaxa$ for some $x \in X$. Since X is intra-regular, $a = ya^2z$ for some $y, z \in X$. Then $a = (axya)(azxya)(azxa)$. Now

$$\begin{aligned}
T_{M\circ N\circ M}(a) &= \bigwedge_{a=km}\{T_M(k) \vee T_{N\circ M}(m)\} \\
&= \bigwedge_{a=(axya)v}\{T_M(axya) \vee \{\bigwedge_{v=rt}\{T_N(r) \vee T_M(t)\}\} \\
&\leq T_M(axya) \vee T_N(azxya) \vee T_M(azxa) \\
&\leq T_M(a) \vee T_N(a) \vee T_M(a) \quad = T_{M\cap N}(a),
\end{aligned}$$

$$\begin{aligned}
I_{M\circ N\circ M}(a) &= \bigvee_{a=km}\{I_M(k) \wedge I_{N\circ M}(m)\} \\
&= \bigvee_{a=(axya)v}\{I_M(axya) \wedge \{\bigvee_{v=rt}\{I_N(r) \wedge I_M(t)\}\}
\end{aligned}$$

$$\geq I_M(axya) \wedge I_N(azxya) \wedge I_M(azxa)$$
$$\geq I_M(a) \wedge I_N(a) \wedge I_M(a) = I_{M\cap N}(a),$$

and

$$F_{M\circ N\circ M}(a) = \bigwedge_{a=km}\{F_M(k) \vee F_{N\circ M}(m)\}$$
$$= \bigwedge_{a=(axya)v}\{F_M(axya) \vee \{\bigwedge_{v=rt}\{F_N(r) \vee F_M(t)\}\}$$
$$\leq F_M(axya) \vee F_N(azxya) \vee F_M(azxa)$$
$$\leq F_M(a) \vee F_N(a) \vee F_M(a) = F_{M\cap N}(a).$$

Therefore $X_M \cap X_N \subseteq X_M \odot X_N \odot X_M$ for every neutrosophic $\aleph-$ bi-ideals X_M and X_N of X.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Let $h_j \in X$. Then

$$\chi_{B(h_j)}(X_M) \subseteq \chi_{B(h_j)}(X_M) \cap \chi_{B(h_j)}(X_M) \subseteq \chi_{B(h_j)}(X_M) \odot \chi_{B(h_j)}(X_M) \odot \chi_{B(h_j)}(X_M).$$

So

$$\chi_{B(h_j)}(T)_M(h_j) \geq \chi_{B(h_j)B(h_j)B(h_j)}(T)_M(h_j),$$
$$\chi_{B(h_j)}(I)_M(h_j) \leq \chi_{B(h_j)B(h_j)B(h_j)}(I)_M(h_j),$$
$$\chi_{B(h_j)}(F)_M(h_j) \geq \chi_{B(h_j)B(h_j)B(h_j)}(F)_M(h_j).$$

Since $\chi_{B(h_j)}(T)_M(h_j) = -1 = \chi_{B(h_j)}(F)_M(h_j)$ and $\chi_{B(h_j)}(I)_M(h_j) = 0$, we get $\chi_{B(h_j)B(h_j)B(h_j)}(T)_M(h_j) = -1 = \chi_{B(h_j)B(h_j)B(h_j)}(F)_M(h_j)$ and $\chi_{B(h_j)B(h_j)B(h_j)}(I)_M(h_j) = 0$ which imply $h_j \in B(h_j)B(h_j)B(h_j)$. Therefore X is intra-regular and regular. □

Theorem 3.15. For any X, the equivalent assertions are:

(i) X is intra-regular,

(ii) For each neutrosophic $\aleph-$ideal X_M of X, $X_M(a) = X_M(a^2)$ $\forall a \in X$.

Proof: $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Let $a \in X$. Then $a = ya^2z$ for some $y, z \in X$. For a neutrosophic $\aleph-$ideal X_M, we have

$$T_M(a) = T_M(ya^2z) \leq T_M(a^2z) \leq T_M(a^2) \leq T_M(a),$$
$$I_M(a) = I_M(ya^2z) \geq I_M(a^2z) \geq I_M(a^2) \geq I_M(a),$$
$$F_M(a) = F_M(ya^2z) \leq F_M(a^2z) \leq F_M(a^2) \leq F_M(a),$$

so $T_M(a) = T_M(a^2)$; $I_M(a) = I_M(a^2)$ and $F_M(a) = F_M(a^2)$ for all $a \in X$. Therefore $X_M(a) = X_M(a^2)$

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Let $a \in X$. Then $I(a^2)$ is an ideal of X. Thus $\chi_{I(a^2)}(X_M)$ is neutrosophic $\aleph-$ideal by Theorem 3.5 of [4]. By assumption, $\chi_{I(a^2)}(X_M)(a) = \chi_{I(a^2)}(X_M)(a^2)$. Since $\chi_{I(a^2)}(T)_M(a^2) = -1 = \chi_{I(a^2)}(F)_M(a^2)$ and $\chi_{I(a^2)}(I)_M(a^2) = 0$, we get $\chi_{I(a^2)}(T)_M(a) = -1 = \chi_{I(a^2)}(F)_M(a)$ and $\chi_{I(a^2)}(I)_M(a) = 0$ imply $a \in I(a^2)$. Thus X is intra-regular. □

Theorem 3.16. For any X, the equivalent assertions are:

(i) X is left (resp., right) regular,

(ii) For each neutrosophic $\aleph-$left (resp., right) ideal X_M of X, $X_M(a) = X_M(a^2)$ $\forall a \in X$.

Proof: $(\boldsymbol{i}) \Rightarrow (\boldsymbol{ii})$ Suppose X is left regular. Then $a = ya^2$ for some $y \in X$ Let X_M be neutrosophic $\aleph-$ left ideal. Then $T_M(a) = T_M(ya^2) \leq T_M(a^2)$ and so $T_M(a) = T_M(a^2), I_M(a) =$

$I_M(ya^2) \geq I_M(a)$ and so $I_M(a) = I_M(a^2)$, and $F_M(a) = F_M(ya^2) \leq F_M(a)$ and so $F_M(a) = F_M(a^2)$. Therefore $X_M(a) = X_M(a^2)$ for all $a \in X$.

$(\boldsymbol{ii}) \Rightarrow (\boldsymbol{i})$ Let X_M be neutrosophic $\aleph -$left ideal. Then for any $a \in X$, we have $\chi_{L(a^2)}(T)_M(a) = \chi_{L(a^2)}(T)_M(a^2) = -1, \chi_{L(a^2)}(I)_M(a) = \chi_{L(a^2)}(I)_M(a^2) = 0$ and $\chi_{L(a^2)}(F)_M(a) = \chi_{L(a^2)}(F)_M(a^2) = -1$ imply $a \in L(a^2)$. Thus X is left regular. □

Corollary 3.17. Let X be a regular right duo (resp., left duo). Then the equivalent conditions are:

(i) X is left regular,

(ii) For each neutrosophic $\aleph -$bi- ideal X_M of X, we have $X_M(a) = X_M(a^2)$ for all $a \in X$.

Proof: It is evident from Theorem 3.5 and Theorem 3.16. □

Conclusions

In this paper, we have presented the concept of neutrosophic $\aleph -$ bi $-$ideals of semigroups and explored their properties, and characterized regular semigroups, intra-regular semigroups and semigroups using neutrosophic ℵ-bi-ideal structures. We have also shown that the neutrosophic ℵ-product of ideals and the intersection of neutrosophic ℵ-ideals are identical for a regular semigroup. In future, we will focus on the idea of neutrosophic $\aleph -$prime ideals of semigroups and its properties.

Acknowledgments: The authors express their gratitude to the referees for valuable comments and suggestions which improve the article a lot.

Reference

1. Atanassov, K. T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **1986**, 20, 87-96.
2. Abdel-Baset, M.; Chang, V.; Gamal, A.; Smarandache, F. An integrated neutrosophic ANP and VIKOR method for achieving sustainable supplier selection: A case study in importing field. *Computers in Industry* **2019**, 106, 94-110.
3. Abdel-Basset, M.; Saleh, M.; Gamal, A.; Smarandache, F. An approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number. *Applied Soft Computing* **2019**, 77, 438-452.
4. Elavarasan, B.; Smarandache, F.; Jun, Y. B. Neutrosophic $\aleph -$ ideals in semigroups. *Neutrosophic Sets and Systems* **2019**, 28, 273-280.
5. Gulistan, M.; Khan, A.; Abdullah, A.; Yaqoob, N. Complex Neutrosophic subsemigroups and ideals. *International J. Analysis and Applications* **2018**, 16, 97-116.
6. Jun, Y. B.; Lee, K. J.; Song, S. Z. $\aleph -$Ideals of BCK/BCI-algebras. *J. Chungcheong Math. Soc.* **2009**, 22, 417-437.
7. Kehayopulu, N. A note on strongly regular ordered semigroups. *Sci. Math.* **1998**, 1, 33-36.
8. Khan, M. S.; Anis; Smarandache, F.; Jun, Y. B. Neutrosophic $\aleph -$structures and their applications in semigroups. *Annals of Fuzzy Mathematics and Informatics*, reprint.
9. Mordeson, J.N.; Malik, D. S.; Kuroki. N. Regular semigroups. *Fuzzy Smigroups* **2003**, 59-100.
10. Muhiuddin, G.; Ahmad, N.; Al-Kenani; Roh, E. H.; Jun, Y. B. Implicative neutrosophic quadruple BCK-algebras and ideals, *Symmetry* **2019**, 11, 277.

11. Muhiuddin, G.; Bordbar, H.; Smarandache, F.; Jun, Y. B. Further results on (2; 2)-neutrosophic subalgebras and ideals in BCK/BCI- algebras, *Neutrosophic Sets and Systems* **2018**, Vol. 20, 36-43.
12. Muhiuddin, G.; Kim, S. J.; Jun, Y. B. Implicative N-ideals of BCK-algebras based on neutrosophic N-structures, *Discrete Mathematics, Algorithms and Applications* **2019**, Vol. 11, No. 01, 1950011.
13. Muhiuddin, G.; Smarandache, F.; Jun, Y. B. Neutrosophic quadruple ideals in neutrosophic quadruple BCI-algebras, *Neutrosophic Sets and Systems* **2019**, 25, 161-173 (2019).
14. Smarandache, F. A. Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. *American Research Press* 1999, Rehoboth, NM.
15. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set. *Int. J. Pure Appl. Math.* **2005,** 24(3), 287-297.
16. Zadeh, L. A. Fuzzy sets. *Information and Control* **1965**, 8, 338 - 353.

Received: Apr 28, 2020. Accepted: July 17, 2020