



Neutrosophic C-means clustering with local information and noise distance-based kernel metric image segmentation [☆]



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ABSTRACT

The traditional FCM algorithm is developed on the basis of classical fuzzy theory, though the classical fuzzy theory has its own limitations. The lack of expressive ability of uncertain information makes it hard for FCM algorithm to handle clustered boundary pixels and outliers. This paper proposes a Neutrosophic C-means Clustering with local information and noise distance-based kernel metric for image segmentation (NKWNLICM). At first, noisy distance and fuzzy spatial information are introduced to NCM model to improve the robustness of noise image segmentation. Then, the kernel function is used to measure the distance between pixels. By mapping low-dimensional data into high-dimensional data, the classification performance is further improved. At last, the fuzzy factor is redefined based on the distance between the center pixel and its neighborhood. The new fuzzy factor can excellently reflect the influence of neighborhood pixels on central pixels and improve the classification accuracy much better. The experimental results on Berkeley Segmentation Database demonstrates the excellent performance of the proposed method for noisy image segmentation.

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1. Introduction

The traditional FCM algorithm is developed on the basis of fuzzy theory and is widely applied on computer version [1–3]. However, the fuzzy theory has certain limitations [4]. The lack of expressive ability of uncertain information makes the FCM algorithm unable to handle clustered boundary pixels and outliers when segmenting the image. The traditional FCM algorithm only considers the pixel's gray information when performing image segmentation, and ignores the spatial neighborhood information of the image, which makes the algorithm very sensitive to noise and isolated points [5]. When it is used to process images with noises, the obtained results will suffer from a lower quality [6]. Due to above drawbacks of the FCM algorithm, Dave [7] proposed a noise clustering algorithm (NC), which uses a subset of parameters to represent noise class based on the FCM algorithm. The algorithm reduces the side-effect of noises on the final clustering results to some extent. Krinidis et al. [8] proposed a fuzzy local information C-means algorithm (FLICM), which used a fuzzy local information to associate local spatial information with local gray information. The algorithm

improves the robustness in the algorithm to noise data processing. In order to improve the segmentation performance of the FLICM algorithm, Guo [9] improved the fuzzy local information by using pixel neighborhood variance information and replace the Euclidean distance with kernel distance. Fuzzy C-means clustering with local information and kernel metric for image segmentation (KWFLICM) is obtained with strong robustness and noise immunity. Guo et al. [10] improved the FCM on the basis of the Neutrosophic theory, and proposed the Neutrosophic c-means clustering algorithm (NCM). The algorithm not only includes the degree of membership, but also contains the uncertainty and opposition. The FCM algorithm makes the classification of the boundary region more obvious, and overcome the noise effects. Jian et al. [11] proposed a novel framework for underwater image saliency detection by exploiting Quaternionic Distance Based Weber Descriptor (QDWD), pattern distinctness, and local contrast. The algorithm incorporates quaternion number system and principal components analysis (PCA) simultaneously, so as to achieve superior performance. Jing et al. [12] proposed a transductive low-rank multi-view regression (TLRMVR), and it is capable of boosting the performance of micro-video popularity prediction by jointly considering the intrinsic representations of the source and target samples. Zhu et al. [13] propose a novel unsupervised visual hashing approach called semantic-assisted visual hashing (SAHV).

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Its core idea is to effectively extract the rich semantics latently embedded in auxiliary texts of images to boost the effectiveness of visual hashing without any explicit semantic labels. Zhu et al. [14] propose a novel hashing scheme, named as canonical view based discrete multimodal hashing (CV-DMH), to handle high bandwidth consumption of query transmission and the huge visual variations of query images sent from mobile devices. To solve the problem of the lack of the use of learning mechanism in feature representation, Jing et al. [15] propose a joint low-rank and sparse regression (JLRSR) framework to jointly learn a low-rank projection matrix that enables to decompose the original data into a component part and an error part and a sparse regression coefficient vector for image memorability prediction.

From above analysis, Neutrosophic C-means Clustering Algorithm (NCM) which use neutrosophic theory to improve the traditional FCM is effective to handle the boundary pixels and outliers during clustering. However, without any spatial information involved in NCM model, it cannot obtain accurate image segmentation results. Aiming to solve this problem, we use the spatial distribution of the neighborhood and noise distance-based kernel metric term to improve NCM method. At first, noisy distance and fuzzy spatial information are introduced to NCM model to improve the robustness of noise image segmentation. Then, the kernel function is used to measure the distance between pixels. By mapping low-dimensional data into high-dimensional data, the classification performance is further improved. At last, the fuzzy factor is redefined based on the distance between the center pixel and its neighborhood. The new fuzzy factor can excellently reflect the influence of neighborhood pixels on central pixels and improve the classification accuracy much better. The experimental results on Berkeley Segmentation Database demonstrates the excellent performance of the proposed method for noisy image segmentation.

2. FCM algorithm

The fuzzy C-means algorithm (FCM) was proposed by Dunn and Bezdek. The core idea is to find the minimum value of the objective function by finding the appropriate membership and clustering center:

$$J_m(U, V) = \sum_{i=1}^n \sum_{j=1}^C \mu_{ij}^m d_{ij}^2(x_i, v_j) \quad (1)$$

$$\mu_{ij} = \left(\sum_{r=1}^C \left[\frac{d_{ij}}{d_{ir}} \right]^{\frac{2}{m-1}} \right)^{-1} \quad (2)$$

$$v_j = \frac{\sum_{i=1}^n \mu_{ij}^m x_i}{\sum_{i=1}^n \mu_{ij}^m} \quad (3)$$

where $J(U, V)$ denotes the square sum of the weighted distance from the pixel to the cluster center of the region, $U = (\mu_{ij})_{n \times c}$ denotes the degree of membership matrix, C is the number of clusters of the image, μ_{ij} is the value of the sample point x_i belonging to j^{th} Class. m represents a fuzzy exponent, at a typical value of 2. When $m = 1$, the fuzzy clustering degenerates to hard clustering (HCM), $V = (v_1, v_2, \dots, v_c)$ is the matrix of clustering center values, v_j Represents j^{th} Cluster Center, and $d_{ij}^2(x_i, v_j) = \|x_i - v_j\|^2$ represents the Euclidean distance between sample point x_i and the cluster center v_j .

The algorithm first determines the number of clustering and initialize the membership matrix. Then the Clustering Center and Membership matrix are updated repeatedly through formula (2) and (3). When the objective function is less than a certain

threshold, all kinds of clustering centers and membership degrees are obtained.

The algorithm has the following disadvantages: Being sensitive to the initial value and having largely dependence on the initial clustering. If the initial cluster center is far away from the global optimal Clustering center, the algorithm is easy to fall into local minima. Meanwhile, it's easy to be disturbed by noise signal, and the segmentation of noisy image is poor.

3. Noise clusters algorithm

The noise clustering algorithm (NC) [7] considers noises as an independent class. It regards the noise distance δ , representing the distance between the sample point and the center of the noise cluster, as a constant. It is a key parameter that is critical to the performance of noise clustering. Based on this argument, a simplified statistical average is used to calculate δ [7];

$$\delta^2 = \lambda \frac{\sum_{i=1}^N \sum_{k=1}^c d_{ik}^2}{Nc} \quad (4)$$

where λ is a noise multiplier used to adjust the effect of noise distance on the algorithm; N indicates the total number of sample points; c represents the number of sample clusters; d_{ik} represents the Euclidean distance between the sample x_i and the cluster center v_k

In the NC algorithm, u_{*k} is used to indicate the degree of membership of the pixel existing in the noise class. The mathematical expression is shown as follows:

$$u_{*k} = 1 - \sum_{k=1}^c u_{ik}, \quad \forall i \in \{1, 2, 3, \dots, N\} \quad (5)$$

The NC algorithm changes the membership degree constraints based on the FCM algorithm and introduces the noise distance. Its clustering target expression [9] is expressed as follows:

$$J(U, V) = \sum_{i=1}^N \sum_{k=1}^c u_{ik}^m d^2(x_i, v_k) + \sum_{i=1}^N \delta^2 \left(1 - \sum_{k=1}^c u_{ik}\right)^m \quad (6)$$

where $U = \{u_{ik}\}_{c \times N}$ denotes a fuzzy membership matrix; $V = \{v_k\}_{c \times 1}$ denotes a cluster center matrix; N denotes the total number of sample points; c denotes a number of sample clusters; $x_i (i = 1, 2, 3, \dots, N)$ denotes a sample set; u_{ik} indicates a degree of membership of the i^{th} sample x_i belonging to the k^{th} class area; $v_k (k = 1, 2, 3, \dots, c)$ denotes the k^{th} cluster center; $d(x_i, v_k)$ is the Euclidean distance between the sample x_i and the cluster center v_k ; $m \in [1, +\infty]$ is the fuzzy weighted index, which is usually specified as 2; and, δ^2 is the noise distance.

Due to the effect of noise distance in the algorithm, the noise clustering algorithm is robust and can get better results when dealing with noisy data. Therefore, the concept of noise distance can be combined with other clustering algorithms to improve the robustness of the algorithm.

4. Neutrosophic C-means cluster

4.1. Neutrosophic theory

In order to address those limitations of the classical fuzzy theory [4] and improve its capability of processing and expressing uncertain information, Smarandache [16] proposed the Neutrosophic theory, which is a generalization of other extended theories. The Neutrosophic theory can not only represent non-deterministic issues in a better way, but also work out the unsolved problems when applying the fuzzy theory.

The basic idea of the Neutrosophic theory is that any viewpoint has a degree of truth, uncertainty, and falsity. Hence, T , I , and F have been introduced as Neutrosophic Components, which represent

the authenticity, uncertainty, and absurdity of events respectively. These neutral elements are named true, indeterminate and false values.

4.2. Neutrosophic C-means clustering algorithm

In cluster analysis, traditional fuzzy clustering methods can only describe the degree of every group. In fact, especial for the samples on the boundary region between different groups, it is difficult to determine which group they belong to and what partitions they join in. In order to solve these problems, Guo et al. [10] improved the FCM on the basis of the Neutrosophic theory, and proposed the Neutrosophic C-means clustering algorithm (NCM) [10]. A new unique set A has been proposed, which regards as the union of the determinant clusters and indeterminate clusters. Let $A = C_j \cup B \cup R, j = 1, 2, \dots, c$ where C_j is an indeterminate cluster, B regards the clusters in boundary regions, R is associated with noisy data and \cup is the union operation. B and R are two kinds of indeterminate clusters. T is defined as the degree to determinant clusters, I is the degree to the boundary clusters, and F is the degree belonging to the noisy data set. Considering the clustering with indeterminacy, a new objective function and membership are defined as:

$$J(T, I, F, C) = \sum_{i=1}^N \sum_{k=1}^C (w_1 T_{ik})^m \|x_i - v_k\|^2 + \sum_{i=1}^N \sum_{k=1}^{\binom{c}{2}} (w_2 I_{2ik})^m \|x_i - \overline{v_{2k}}\|^2 + \sum_{i=1}^N \sum_{k=1}^{\binom{c}{3}} (w_3 I_{3ik})^m \|x_i - \overline{v_{3k}}\|^2 + \sum_{i=1}^N \sum_{k=1}^{\binom{c}{4}} (w_4 I_{4ik})^m \|x_i - \overline{v_{4k}}\|^2 + \dots + \sum_{i=1}^N \sum_{k=1}^{\binom{c}{c}} (w_c I_{cik})^m \|x_i - \overline{v_{ck}}\|^2 + \sum_{i=1}^N (\overline{w_{c+1} F_i})^m \quad (7)$$

where w_i is the weight factor. δ is used to control the number of objects considered as outliers. When the clustering number C is greater than 3, the objective function is very complex and time consuming. After simplification, the objective function is rewritten as:

$$J(T, I, F, C) = \sum_{k=1}^C (w_1 T_{ik})^m \|x_i - v_k\|^2 + \sum_{i=1}^N (w_2 I_i)^m \|x_i - \overline{v_{imax}}\|^2 + \sum_{i=1}^N (\overline{w_{c+1} F_i})^2 \delta^2 \quad (8)$$

where, $\overline{v_{imax}} = \frac{v_{p_i} + v_{q_i}}{2}$, $p_i = \operatorname{argmax}_{k=1,2,\dots,C} (T_{ik})$, and $q_i = \operatorname{argmax}_{k \neq p_i, k=1,2,\dots,C} (T_{ik})$.

In above equations, m is a constant, and p_i and q_i are the cluster numbers with the biggest and second biggest value. When the p_i and q_i are identified, the $\overline{v_{imax}}$ is calculated and its value is a constant number for each data point i , and will not change any more. T_{ik}, I_i and F_i are the membership values belonging to the determinate clusters, boundary regions and noisy data set, $0 < T_{ik}, I_i, F_i < 1$ which satisfy with the following formula:

$$\sum_{k=1}^C T_{ik} + I_i + F_i = 1 \quad (9)$$

The partitioning is carried out through an iterative optimization of the objective function, and the membership T_{ik}, I_i, F_i and the cluster centers v_k are updated in each iteration. The $\overline{v_{imax}}$ is calculated according to indexes of the largest and second largest value of T_{ik} of each iteration. The iteration will sustain until $\max\{|T_{ik}^{(h+1)} - T_{ik}^{(h)}|\} < \varepsilon$ or $h \geq H_{max}$, in which ε a termination criterion between 0 and 1 is, k is the iteration step, and h is the number of iterations.

5. Proposed method

In NCM algorithm, since the objective function does not involve any spatial information. If we directly use it for image segmentation, the segmentation performance is not very well. In addition, using the maximum membership principle to determine pixel labels may produce some segmentation errors. Therefore, spatial neighborhood information should be added to the objective function to reduce the influence of undesired factors on the final classification result.

Aiming to solve the problems of NCM algorithm, we propose a new clustering algorithm, Neutrosophic C-means clustering with Local Information and Kernel Metric noise distance-based for Image Segmentation (NKWNLICM). The objective function is

$$J(T, I, F, C) = \sum_{i=1}^N \sum_{k=1}^c (w_1 T_{ik})^m (\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik}) + \sum_{i=1}^N \sum_{l=1}^{\binom{c}{2}} (w_2 I_{il})^m (\|\Phi(x_i) - \Phi(\overline{v_l})\|^2 + \overline{G_{il}}) + \sum_{i=1}^N (w_3 F_i)^m \delta^2 \quad (10)$$

where T_{ik} denotes the extent to which element i belongs to cluster k , I_{il} denotes the degree to which element i belongs to two cluster boundaries in cluster c , F_i denotes the degree to which element i belongs to noise; $\overline{v_l}$ is the average of any two classes value. w_i is the weighting factor, δ is the noise distance, $\overline{G_{il}}$ and G_{ik} are local information.

When the clustering number C is greater than 3, the objective function in Eq. (10) is very complex and time consuming. In this situation, if we only consider the two closest determinate clusters which have the top two largest membership values, the objective function will be simplified. Meanwhile, computation cost will be reduced without decreasing the clustering accuracy greatly.

$$\delta^2 = \lambda \left[\frac{\sum_{i=1}^N \sum_{k=1}^c \|\Phi(x_i) - \Phi(v_k)\|^2}{Nc} \right] \quad (11)$$

$$G_{ik} = \sum_{j \in N_i, i \neq j} w_{ij} (1 - T_{jk})^m \|\Phi(x_i) - \Phi(v_k)\|^2 \quad (12)$$

where T_{ik}, I_i and F_i are the membership values belonging to the determinate clusters, boundary regions and noisy data set, $0 < T_{ik}, I_i, F_i < 1$ which satisfy with the following formula:

$$\sum_{k=1}^c T_{ik} + \sum_{l=1}^{\binom{c}{2}} I_{il} + F_i = 1. \quad (13)$$

According to the above formula, the Lagrange objective function is constructed as

$$L(T, I, F, C, \lambda) = \sum_{i=1}^N \sum_{k=1}^c (w_1 T_{ik})^m (\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik}) + \sum_{i=1}^N \sum_{l=1}^{\binom{c}{2}} (w_2 I_{il})^m (\|\Phi(x_i) - \Phi(\overline{v_l})\|^2 + \overline{G_{il}}) + \sum_{i=1}^N (w_3 F_i)^m \delta^2 + \sum_{i=1}^N \lambda_i \left(\sum_{k=1}^c T_{ik} + \sum_{l=1}^{\binom{c}{2}} I_{il} + F_i - 1 \right) = 0 \quad (14)$$

To minimize the Lagrange objective function, we use the following operations:

$$\frac{\partial L}{\partial T_{ik}} = m(w_1 T_{ik})^{m-1} (\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik}) - \lambda_i \quad (15)$$

$$\frac{\partial L}{\partial I_{il}} = m(w_2 I_{il})^{m-1} (\|\Phi(x_i) - \Phi(\overline{v_l})\|^2 + \overline{G_{il}}) - \lambda_i \quad (16)$$

$$\frac{\partial L}{\partial F_i} = m(w_3 F_i)^{m-1} \delta^2 - \lambda_i \tag{17}$$

$$\frac{\partial L}{\partial v_k} = -2 \sum_{i=1}^N (w_1 T_{ik})^2 (\Phi(x_i) - \Phi(v_k)) \tag{18}$$

Using Euclidean universal numbers, let $\frac{\partial L}{\partial T_{ik}} = 0, \frac{\partial L}{\partial I_i} = 0, \frac{\partial L}{\partial F_i} = 0, \frac{\partial L}{\partial v_k} = 0$, then

$$T_{ij} = \frac{1}{w_1} \left(\frac{\lambda_i}{m} \right)^{\frac{1}{m-1}} \left(\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik} \right)^{-\frac{1}{m-1}} \tag{19}$$

$$I_{il} = \frac{1}{w_2} \left(\frac{\lambda_i}{m} \right)^{\frac{1}{m-1}} \left(\|\Phi(x_i) - \Phi(\bar{v}_l)\|^2 + \bar{G}_{il} \right)^{-\frac{1}{m-1}} \tag{20}$$

$$F_i = \frac{1}{w_3} \left(\frac{\lambda_i}{m} \right)^{\frac{1}{m-1}} \delta^{-\frac{2}{m-1}} \tag{21}$$

$$v_k = \frac{\sum_{i=1}^N (w_1 T_{ik})^m T_{ik} x_i}{\sum_{i=1}^N (w_1 T_{ik})^m T_{ik}} \tag{22}$$

Let $\left(\frac{\lambda_i}{m}\right)^{\frac{1}{m-1}} = K_i$, then we obtain:

$$1 = \sum_{j=1}^c \frac{K_i}{w_1} \left(\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik} \right)^{-\frac{1}{m-1}} + \sum_{l=1}^c \frac{K_i}{w_2} \left(\|\Phi(x_i) - \Phi(\bar{v}_l)\|^2 + \bar{G}_{il} \right)^{-\frac{1}{m-1}} + \frac{K_i}{w_3} \delta^{-\frac{2}{m-1}} \tag{23}$$

$$K_i = \left[\frac{1}{w_1} \sum_{k=1}^c \left(\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik} \right)^{-\frac{1}{m-1}} + \sum_{l=1}^c \frac{1}{w_2} \left(\|\Phi(x_i) - \Phi(\bar{v}_l)\|^2 + \bar{G}_{il} \right)^{-\frac{1}{m-1}} + \frac{1}{w_3} \delta^{-\frac{2}{m-1}} \right]^{-1} \tag{24}$$

Therefore,

$$T_{ik} = \frac{K_i}{w_1} \left(\|\Phi(x_i) - \Phi(v_k)\|^2 + G_{ik} \right)^{-\frac{1}{m-1}} \tag{25}$$

$$I_{il} = \frac{K_i}{w_2} \left(\|\Phi(x_i) - \Phi(\bar{v}_l)\|^2 + \bar{G}_{il} \right)^{-\frac{1}{m-1}} \tag{26}$$

$$F_i = \frac{K_i}{w_3} \delta^{-\frac{2}{m-1}} \tag{27}$$

$$\bar{v}_l = \frac{1}{2} (v_q + v_p), p, q \in \{1, 2, \dots, c\}, p \neq q \tag{28}$$

The partitioning is carried out through an iterative optimization of the objective function, and the membership T_{ik}, I_i, F_i and the cluster centers v_k are updated in each iteration. The $v_{i\max}$ is calculated according to indexes of the top two largest value of T_{ik} in each iteration. The iteration will continuous until $\max\{|T_{ik}^{(h+1)} - T_{ik}^{(h)}|\} < \varepsilon$ or $h \geq H_{\max}$, where ε a termination criterion between 0 and 1, k is the iteration step, and h is the number of iterations.

The above equations allow the formulation of NKWNLICM algorithm. It can be summarized in the following steps:

- Step 1:** Initialize the $m, \varepsilon, w_1, w_2, w_3, H_{\max}$ and w_{ij} ;
- Step 2:** Initialize $T^{(0)}, I^{(0)}$ and $F^{(0)}$, let $h = 0$;
- Step 3:** Calculate the centers vectors $v^{(h)}$ and cluster boundary \bar{v}_{il} at h step using Eq. (22) and Eq. (28);
- Step 4:** Calculate noise distance using Eq. (11);
- Step 5:** Update $T^{(h)}$ to $T^{(h+1)}, I^{(h)}$ to $I^{(h+1)}$, and $F^{(h)}$ to $F^{(h+1)}$ using Eqs. (25)–(27);

Table 1
The notation table of main symbols.

J	Weighted distance	σ	Noise distance
U	Membership	λ	Noise multiplier
C	Number of clusters	N	Total number of sample points
m	Fuzzy exponent	d	Euclidean distance
V	Clustering center values	w	Weight factor
ε	Termination criterion	T	Membership values belonging to determinate clusters
G	Local information	I	Membership values belonging to boundary regions
h	Number of iterations	F	Membership values belonging to noisy data set

Step 6: If $|T^{(h+1)} - T^{(h)}| < \varepsilon$ or $h \geq H_{\max}$ then stop; otherwise return to **Step 3**, let $h = h + 1$;

Step 7: Assign each data into the class with the largest $TM = [T, I, F]$ value: $x(i) \in h^{th}$ class if $h = \underset{j=1, \dots, c+2}{\operatorname{argmax}} (TM_{ij})$

To easily understand the proposed NKWNLICM, the main symbols are summarized in **Table 1**.

The effects of each component in the proposed method are given as follows:

- a) Neutrosophic C-means clustering: In addition of the membership of pixels in each class, Neutrosophic theory estimates the uncertainty of clustering. Thus, the pixels located in edge region can be accurately categorized.
- b) Kernel function: Kernel function is used to measure the distance between pixels, thus mapping the complex non-linear problems in the original low-dimensional space to the high-dimensional space.
- c) Weighed fuzzy factor: The trade-off weighted fuzzy factor contains both the spatial distance information and gray information of neighborhood pixels, it can better reflect the influence of neighborhood pixels on the central pixel, thus further improving the clustering performance.
- d) Noise distance term: Introducing noise distance term in classification criteria, the segmentation of the proposed method is robust to noisy image.
- e) Fuzzy local information: By considering the fuzzy spatial distribution in the local region of image, the proposed method is able to distinguish between image boundary pixels and noise during classification and further improve the image segmentation performance.

6. Evaluation criteria

The evaluation criteria used in this paper are given as follows:

An entropy-based evaluation function (E), which combines both the layout entropy (H_r) and the expected region entropy (H_l), is often used in measuring the effectiveness of a segmentation method [17].

The entropy for region R_j is defined as:

$$H(R_j) = - \sum_{m \in V_j} \frac{L_j(m)}{S_j} \log \frac{L_j(m)}{S_j} \tag{29}$$

The expected region entropy of segmentation I is defined as:

$$H_r(I) = \sum_{j=1}^c \left(\frac{S_j}{S_I} \right) H(R_j) \tag{30}$$

The layout entropy is defined as:

$$H_l(I) = - \sum_{j=1}^c \left(\frac{S_j}{S_I} \right) \log \frac{S_j}{S_I} \tag{31}$$

The entropy-based evaluation function E is defined as:

$$E = H_l(I) + H_r(I) \tag{32}$$

where $L_j(m)$ denotes to the number of pixels in region j that have a value of m for feature (e.g. luminance) in the original image. $S_j = |R_j|$ denotes to the area of region R_j . Maximizing the consistency of pixels in each segmentation region and minimizing the consistency of different regions, the best segmentation result is obtained when E achieved to the minimum value.

Partition coefficient V_{pc} and partition entropy V_{pe} are proposed by Bezdek et al. [18], which is mainly used to measure the pixel membership of the segmented images. The definition is as follows:

$$V_{pc} = \frac{1}{n} \sum_{i=1}^c \sum_{j=1}^n u_{ij}^2 \tag{33}$$

$$V_{pe} = -\frac{1}{n} \sum_{i=1}^c \sum_{j=1}^n u_{ij} \log u_{ij} \tag{34}$$

During image classification, the higher compactness of the object indicates the lower the separation between the objects, which means that the segmentation effect is much better. Therefore, the better the clustering effect means the larger the value of V_{pc} and the smaller the value of V_{pe} .

The classification accuracy rate SA [19] is defined as follows:

$$SA = \sum_{i=1}^c \frac{card(A_i \cap C_i)}{\sum_{j=1}^c card(C_j)} \tag{35}$$

where A_i denotes the set of pixels of the i^{th} cluster in the segmentation result, C_i denotes the set of pixels of the i^{th} cluster in the standard segmentation image, *card* is used to calculate the number of elements in the set. The higher the accuracy indicates the better segmentation result.

Reconstruction error rate V_{RE} proposed by Pedrycz et al. [20] is mainly used to measure the difference between the segmented image and the original image. Its definition is given as follows:

$$V_{RE} = \frac{1}{N} \sum_{k=1}^N \|x_k - \bar{x}_k\|^2 \tag{36}$$

where x_i represents the gray value of the i^{th} pixel of the original image and \bar{x}_i represents the gray value of the i^{th} pixel of the reconstructed image, which is defined as follows:

$$\bar{x}_k = \frac{\sum_{i=1}^c u_{ik}^m v_k}{\sum_{i=1}^c u_{ik}^m} \tag{37}$$

Among them, u_{ik}^m denotes the membership degree of cluster k , and v_k denotes the clustering center value of cluster k . The segmented image is reconstructed according to formula (37). The smaller reconstruction error indicates better segmentation result.

7. Experimental results

The testing environment of this experiment in this paper is CPU core 2.5 GHz, memory 4 GB, and programmed by Matlab 2016a. In this section, in order to validate the effectiveness of this method in image segmentation, a comparative experiment is conducted with images in Berkeley Segmentation Database. The public benchmark based on Berkeley Segmentation Database consists 200 images with hand-labeled segmentation. We use 4 images (camera image and no. 388016, 15088 and 176039 from Berkeley Segmentation Database) to compare the proposed method with other approaches named FCM, FGFCM, FLICM, NCM and KWFLICM to demonstrate the performance in clustering.

In the experiments, the parameter m has the same meaning to the fuzzification constant in the fuzzy clustering algorithm, and its value usually selected as 2. We selected $N_R = 8$, $\lambda_s = 3$, and $\lambda_g = 0.5$. The noise type is 20% salt and pepper noise. We take different

Table 2
The E values of the segmentation results obtained by different parameters.

	m				T				λ			
	2	3	4	5	0.8	0.6	0.4	0.2	0.1	0.2	0.5	1.0
Hr	1.66	1.68	1.72	1.78	1.66	1.68	1.72	1.7	1.66	1.71	1.68	1.72
Hl	0.36	0.39	0.38	0.37	0.36	0.36	0.36	0.37	0.36	0.36	0.37	0.36
E	2.02	2.08	2.13	2.15	2.02	2.05	2.06	2.05	2.02	2.02	2.14	2.10

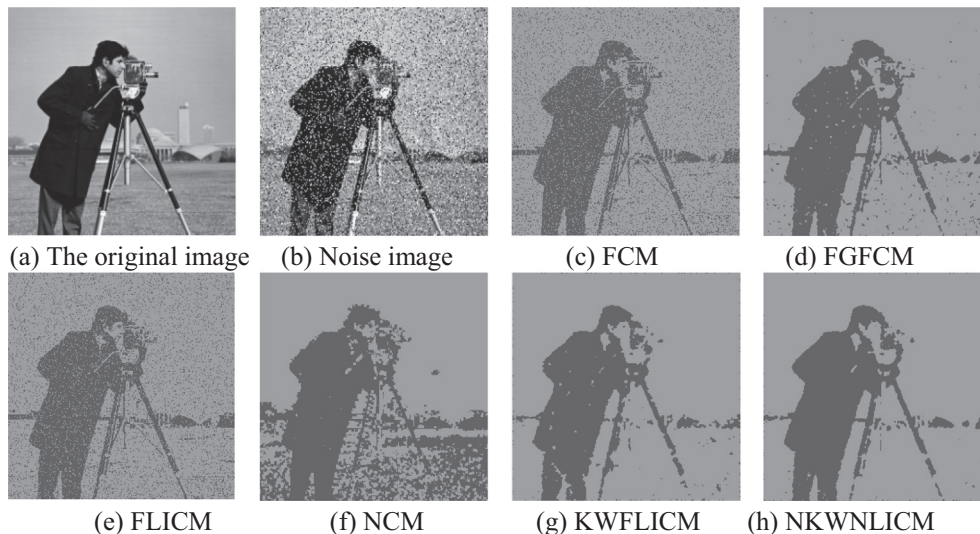


Fig. 1. Comparison of camera image segmentation results.

values for each parameter to select the best segmentation parameter settings. It can be seen from Table 2 that when m takes 2, T takes 0.8, and λ takes 0.1, it works best.

We compare the proposed method with other approaches named FCM, FGFCM, FLICM, NCM and KWFLICM to demonstrate their segmentation performance. The original images and segmentation results are shown from Fig. 1 to Fig. 4. Figs. 1(a), 2(a), 3(a), 4(a) are original images without noise. Figs. 1(b), 2(b), 3(b), 4(b) are images with 20% salt and pepper noise. Figs. 1(c), 2(c), 3(c), 4(c) are segmentation results of FCM method. Figs. 1(d), 2(d), 3(d), 4(d) are

segmentation results of FGFCM method. Figs. 1(e), 2(e), 3(e), 4(e) are segmentation results of FLICM method. Figs. 1(f), 2(f), 3(f), 4(f) are segmentation results of NCM method. Figs. 1(g), 2(g), 3(g), 4(g) are segmentation results of KWFLICM method. Figs. 1(h), 2(h), 3(h), 4(h) are segmentation results of NKWNLICM method. By comparison, it can be clearly seen that the NKWNLICM method has the best segmentation effect, and noise is significantly less than other methods.

For quantitative comparison, the evaluation criteria V_{pc} , V_{pe} , V_{RE} , SA and E are calculated for the segmentation results obtained

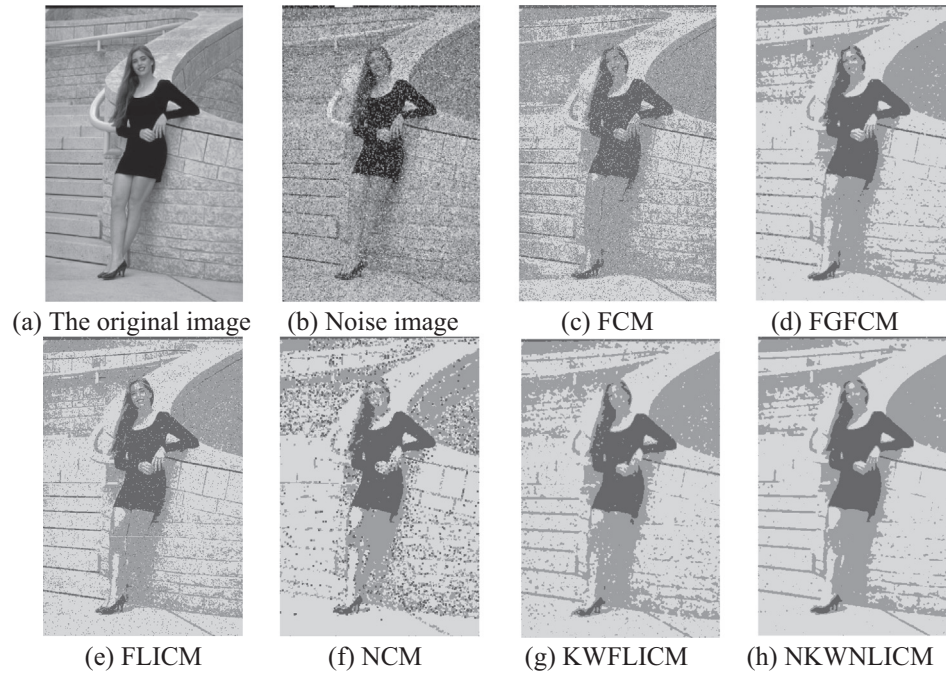


Fig. 2. Berkeley University image library 388,016 image segmentation results.

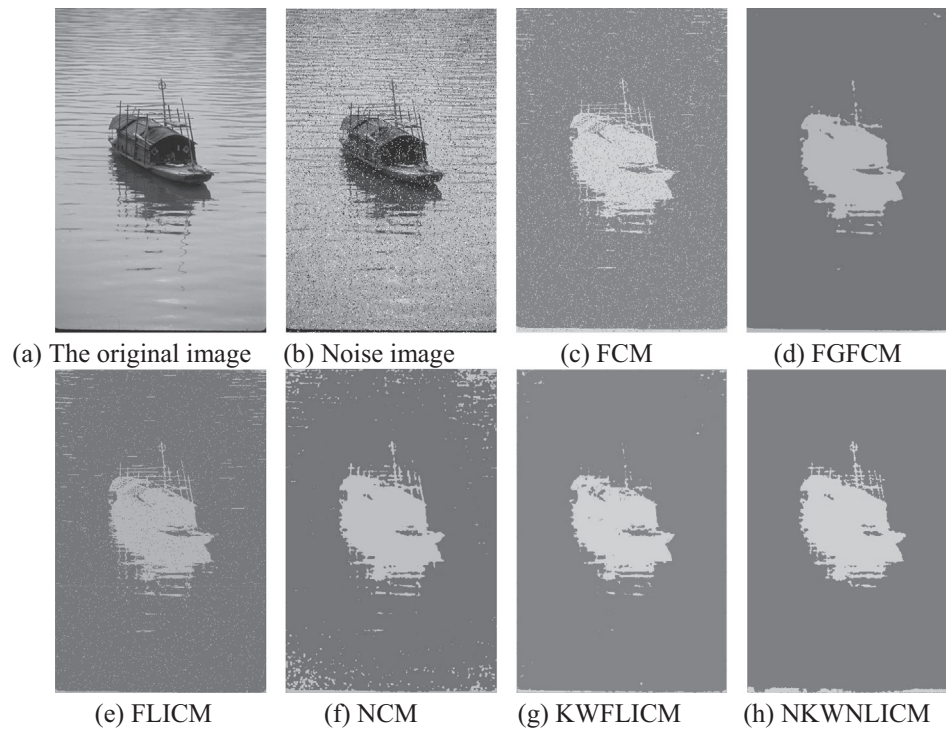


Fig. 3. Berkeley University image library 15,088 image segmentation results.

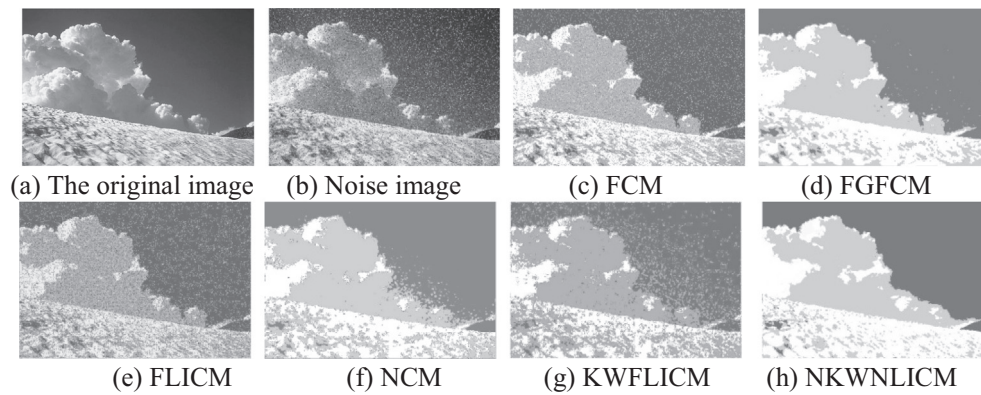


Fig. 4. Berkeley University image library 176,039 image segmentation results.

Table 3

The V_{pc} , V_{pe} , V_{RE} and SA values of the segmentation result obtained by different methods on Berkeley Segmentation Database.

	FCM	FGFCM	FLICM	NCM	KWFLICM	NKWNLICM
V_{pc}	0.9220	0.9255	0.9303	0.9289	0.9362	0.9962
V_{pe}	0.1426	0.1269	0.1453	0.1343	0.0998	0.0084
V_{RE}	153.48	152.06	152.71	150.63	122.71	102.49
SA	90.8362	95.2290	93.1943	93.8031	96.1574	97.1244

Table 4

The E of the segmentation result obtained by different methods on each image.

Image	Metric	FCM	FGFCM	FLICM	NCM	KWFLICM	NKWNLICM
Fig. 1	H_r	1.7306	1.7076	1.7248	1.7249	1.7072	1.7026
	H_l	0.4637	0.4576	0.4625	0.4536	0.4566	0.4548
	E	2.1943	2.1652	2.1873	2.1785	2.1638	2.1574
Fig. 2	H_r	1.8981	1.8813	1.8799	1.8932	1.8792	1.8698
	H_l	0.5176	0.5057	0.5064	0.5184	0.5062	0.4985
	E	2.4157	2.3870	2.3863	2.4116	2.3854	2.3683
Fig. 3	H_r	1.9482	1.7453	1.6471	1.8759	1.7291	1.6663
	H_l	0.3407	0.3650	0.3525	0.3589	0.3651	0.3619
	E	2.2890	2.1103	2.0996	2.2349	2.0943	2.0282
Fig. 4	H_r	2.5946	2.4186	2.4405	2.1537	2.1319	2.1141
	H_l	0.3484	0.3086	0.3315	0.3393	0.3082	0.3236
	E	2.8430	2.7272	2.7721	2.4931	2.4402	2.4377

by all methods. Table 3 gives the partition coefficients, partition entropy, reconstruction error rate and segmentation accuracy rate obtained by different methods running on Berkeley Segmentation Database. The maximum of partition coefficient, minimum of partition entropy, minimum of reconstruction error rate and maximum of segmentation accuracy rate are all marked in bold type. It is obvious that the value obtained by this method is much better than other methods.

Table 4 shows the evaluation indicators based on entropy obtained by running the various methods on different images, where the minimum values are marked in bold. It can be seen from the table that the evaluation index value corresponding to the segmentation result obtained by the NKWNLICM method is the smallest, and these results fully show that the NKWNLICM method has a superior effect on noise images.

Both of visual results and quantitative comparison demonstrate the excellent segmentation performance obtained by the proposed NKWNLICM method. This result indicates that Neutrosophic C-means Clustering with local information and noise distance-based kernel metric is able to overcome the effect of noise on segmentation results and is good at maintaining image structure information.

8. Conclusions

In this paper, we proposed a new clustering algorithm, Neutrosophic C-means clustering with Local Information and noise distance-based Kernel Metric for Image segmentation (NKWNLICM), and applied it to the segmentation study of noise images. The objective function of NKWNLICM is convex to each variables, and the value of the objective function tends to be stable with the increase of iteration number. So we think it is convergence empirically. By introducing the concept of local fuzzy information and noise distance in the NCM algorithm, the algorithm does not need parameters setting for different noises when segmenting images, which makes the algorithm overcome the noise effect. The efficiency of the proposed method is evaluated on grayscale image segmentation applications. Experimental results show that the algorithm has better segmentation results for noisy images. In addition, we plan to apply the method to the more complex data in our future works. In the proposed method, the local information of image, distance of noise and kernel function are integrated for image segmentation modeling. Thus, its segmentation efficiency has little advantage over the other methods. In future, we will use parallel computing to improve its efficiency.

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