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NEUTROSOPHIC CRISP SUPRA BI AND TRI-TOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduced neutrosophic crisp supra topological spaces (NCSTS) and neutrosophic crisp supra bi and tri-topological spaces, new types of open and closed sets in neutrosophic crisp supra bi and tri-topological spaces, the closure and interior of neutrosophic crisp supra bi and tri-topological space, new concepts of open and closed sets, their properties are investigated.

1. Introduction

The idea of degree of membership and the concept of fuzzy set [7] was introduced by Zadeh in 1965. In 1983 generalization of fuzzy set intuitionistic fuzzy set was introduced by K. Atanassov [1] as a beyond the degree of membership and the degree of non membership of each element. Neutrosophic set is a generalization of intuitionistic fuzzy set. The idea of "neutrosophic set" was first proposed by Smarandache [6, 5]. Neutrosophic operations have been developed by Salama et al. [4, 2]. Salama and Alblowi [2] define neutrosophic topological space, established some of its properties. Salama and Smarandache [3, 6, 4] introduced the concept of neutrosophic crisp set and neutrosophic crisp operators have been investigated. In this paper we introduced neutrosophic crisp supra topological spaces and neutrosophic crisp supra bi and tri-topological spaces, new types of open and closed sets in neutrosophic crisp supra bi and tri-topological spaces, the

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closure and interior of neutrosophic crisp supra bi and tri-topological spaces and new concept of open and closed sets, their properties are investigated.

2. Preliminaries

Definition 2.1 [2]. Let X be a non-empty fixed set. A neutrosophic crisp set (NCS for short) E is an object having the form $E = \langle E_1, E_2, E_3 \rangle$, where E_1, E_2 and E_3 are sub sets of X. Then the object having the form $E = \langle E_1, E_2, E_3 \rangle$ is called a NCS if satisfying $E_1 \cap E_2 = \varphi$, $E_1 \cap E_3 = \varphi$ and $E_2 \cap E_3 = \varphi$.

Definition 2.2 [2]. Types of NCSs φ_N and X_N in X may be defined as following

1. (a) $\varphi_N = \langle \varphi, \varphi, X \rangle$ or (b) $\varphi_N = \langle \varphi, X, \varphi \rangle$ or (c) $\varphi_N = \langle \varphi, \varphi, \varphi \rangle$.

2. (a) $X_n = \langle X, \varphi, \varphi \rangle$

Definition 2.3 [2]. If F and E are two NCSs, then $E \subseteq F$ can be defined as

(a) $E \subseteq F \Leftrightarrow E_1, F_1, E_2 \subseteq F_2$ and $F_3 \subseteq E_3$.

(b) $E \subseteq F \Leftrightarrow E_1 \subseteq F_1, E_2 \subseteq F_2$ and $F_3 \subseteq E_3$.

Definition 2.4 [2]. Let X be a non-empty set, and the NCSs E and F in the form $E = \langle E_1, E_2, E_3 \rangle$, $F = \langle F_1, F_2, F_3 \rangle$ then

(1) $E \cap F$ may be defined as following

(a) $E \cap F = \langle E_1 \cap F_1, E_2 \cap F_2, E_3 \cap F_3 \rangle$ or

(b) $E \cap F = \langle E_1 \cap F_1, E_2 \cap F_2, E_3 \cap F_3 \rangle$

(2) $E \cup F$ may be defined as following

(a) $E \cup F = \langle E_1 \cup F_1, E_2 \cap F_2, E_3 \cap F_3 \rangle$

or

(b) $E \cup F = \langle E_1 \cup F_1, E_2 \cup F_2, E_3 \cap F_3 \rangle$

Definition 2.5 [2]**.** Let X be a non-empty set, and the NCS $E = \langle E_1, E_2, E_3 \rangle$. Then E^C may be defined as following

(a) $E^C = \langle E_3, E_2, E_1 \rangle$.

3. Neutrosophic Crisp Supra Topological Spaces

Definition 3.1. A neutrosophic crisp supra topology (NCST for short) on a non empty set X is a family τ^{μ} of neutrosophic crisp subset in X if satisfying the following

(a) φ_N and $X_N \in \tau^{\mu}$

(b) $\bigcup E_j \in \tau^{\mu} \forall \{E_j : j \in J\} \subseteq \tau^{\mu}$.

Then the pair (X, τ^{μ}) is called neutrosophic crisp supra topological space (NCSTS) in X. The elements in τ^{μ} are said to be neutrosophic crisp supra open sets (NCSOS). The complement of τ^{μ} are called neutrosophic crisp supra closed sets (NCSCS).

Example 3.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau^{\mu} = \{\varphi_N, X_N, L, M, N\}$, where $L = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$, $M = \langle \{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\} \rangle$, $N = \{\eta_2\}, \varphi, \{\delta_1, \psi_3\} \rangle$. Then (X, τ^{μ}) is a NCSTS.

4. Neutrosophic Crisp Supra Bi-Topological Spaces

Definition 4.1. Let $\tau_1^{\mu}, \tau_2^{\mu}$ be any two neutrosophic crisp supra topology (NCST) on a non empty set X then $(X, \tau_1^{\mu}, \tau_2^{\mu})$ is a neutrosophic crisp supra bi-topological space (NCS-bi-TS).

Example 4.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$ where $L = \langle \{\delta_1, \eta_2\}, \varphi, \{\psi_3\} \rangle$, $M = \langle \{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\} \rangle$, $N = \langle \{\eta_2\}, \varphi, \{\delta_1, \psi_3\} \rangle$, $O = \langle \{\delta_1\}, \varphi, \{\psi_3\} \rangle$, $P = \langle \{\delta_1\}, \varphi, \{\sigma_4, \psi_3\} \rangle$, $Q = \langle \{\delta_1\}, \varphi, \{\eta_2, \psi_3\} \rangle$. Then $(X, \tau_1^{\mu}, \tau_2^{\mu})$ is NCS-bi-TS.

Definition 4.3. Let $(X, \tau_1^{\mu}, \tau_2^{\mu})$ be a NCS-bi-TS. Then $\tau_1^{\mu} \cup \tau_2^{\mu}$ is said to be neutrosophic crisp supra bi-open sets (NCS-bi-OSs for short) and the complement of NCS-bi-OSs are called neutrosophic crisp supra bi-closed sets (NCS-bi-CSs for short).

Note 4.4. (a) The family of all NCS-bi-OSs can be denoted as NCS-bi-OS(X).

(b) The family of all NCS-bi-CSs can denoted as NCS-bi-CS(X).

Example 4.5. (a) From example 4.2 NCS-bi-OSs are NCS-bi-OS) $(X) = \tau_1^{\mu} \cup \tau_2^{\mu} = \{\varphi_N, X_N, L, M, N, O, P, Q\}$.

(b) From example 4.2 NCS-bi-CSs are NCS-bi-CS $(X) = \tau_1^{\mu} \cup \tau_2^{\mu} = \{\varphi_N, X_N, L^C, M^C, N^C, O^C, P^C, Q^C\}$. where $L^C = \langle\{\psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$, $M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$, $N^C = \langle\{\delta_1, \psi_3\}, \varphi, \{\eta_2\}\rangle$, $O^C = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1\}\rangle$, $Q^C = \langle\{\eta_2, \psi_3\}, \varphi, \{\delta_1\}\rangle$.

Remark 4.6. (a) Every NCSOS (NCSCS) in (X, τ_1^{μ}) or (X, τ_2^{μ}) is a NCS-bi-OS (NCS-bi-CS).

Remark 4.7. Every NCS-bi-TS $(X, \tau_1^{\mu}, \tau_2^{\mu})$ induces two NCSTSs as (X, τ_1^{μ}), (X, τ_2^{μ}).

Theorem 4.8. *Let $(X, \tau_1^{\mu}, \tau_2^{\mu})$ be a NCS-bi-TS. Then the union of two NCS-bi-OS (NCS-bi-CS) need not a NCS-bi-OS (NCS-bi-CS).*

Proof. The proof of this theorem follows from the example 4.9.

Example 4.9. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$ where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $Q = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^{μ}), (X, τ_2^{μ}) are NCSTS. Therefore $(X, \tau_1^{\mu}, \tau_2^{\mu})$ is NCS-bi-TS.

N, O are NCS-bi-OSs but $N \cup O = \langle\{\eta_2, \psi_3\}, \varphi, \{\delta_1\}\rangle$ is not a NCS-bi-OS.

$N^C = \langle\{\delta_1, \psi_3\}, \varphi, \{\eta_2\}\rangle$ and $O^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-bi-CSs but $N^C \cup O^C = \langle\{\delta_1, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-bi-CS.

Theorem 4.10. *Let* $(X, \tau_1^\mu, \tau_2^\mu)$ *be a NCS-bi-TS. Then the intersection of two NCS-bi-OS (NCS-bi-CS) need not a NCS-bi-OS (NCS-bi-CS).*

Proof. The proof of this theorem follows from the example 4.11.

Example 4.11. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^\mu = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^\mu = \{\varphi_N, X_N, O, P, Q\}$ where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $Q = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^μ), (X, τ_2^μ) are NCSTS. Therefore $(X, \tau_1^\mu, \tau_2^\mu)$ is NCS-bi-TS.

L, O are two NCS-bi-OSs but $L \cap O = \langle\phi, \phi, \{\psi_3, \delta_1\}\rangle$ is not a NCS-bi-OS. $L^C = \langle\{\psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $O^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-bi-CSs but $L^C \cap O^C = \langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is not a NCS-bi-CS.

5. The Closure and Interior via NCS-bi-OS and NCS-bi-CS

Definition 5.1. Let $(X, \tau_1^\mu, \tau_2^\mu)$ be a NCS-bi-TS and A is NCS. Then neutrosophic crisp supra bi-interior of A can be defined as NCS-bi-Int $(A) = \cup\{\lambda : \lambda \subseteq A, \lambda$ is a NCS-bi-OS$\}$.

Definition 5.2. Let $(X, \tau_1^\mu, \tau_2^\mu)$ be a NCS-bi-TS and A is NCS. Then neutrosophic crisp supra bi-closure of A can be defined as NCS-bi-Cl $(A) = \cap\{\lambda^* : A \subseteq \lambda^*; \lambda^*$ is a NCS-bi-CS$\}$.

Theorem 5.3. *Let* $(X, \tau_1^\mu, \tau_2^\mu)$ *be NCS-bi-TS, A is a NCS then*

(a) *NCS-bi-int* $(A) \subseteq A$.

(b) *NCS-bi-int* (A) *is not a NCS-bi-OS.*

Proof. (a) it is clear from definition 5.1.

(b) follows from Theorem 4.8.

Theorem 5.4. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu})$ *be NCS-bi-TS, A is a NCS then*

(a) $A \subseteq NCS\text{-}bi\text{-}cl\,(A)$.

(b) *NCS-bi-cl (A) is not a NCS-bi-CS.*

Proof. (a) it is clear from definition 5.2.

(b) follows from theorem 4.10.

6. Neutrosophic Crisp Supra S-Open Sets (NCS-SOS) and Neutrosophic Crisp Supra S-closed Set (NCS-SCS)

The concepts of open and closed sets in NCS-bi-TS were introduced in this section.

Definition 6.1. Let $(X, \tau_1^{\mu}, \tau_2^{\mu})$ be a NCS-bi-TS. Then a subset A of space X is said to be neutrosophic crisp supra S-open set (NCS-SOS) if $A \in \tau_1^{\mu}$ and $A \notin \tau_2^{\mu}$ or $A \in \tau_2^{\mu}$ and $A \notin \tau_1^{\mu}$ and the complement of NCS-SOS is said to be neutrosophic crisp supra S-closed set (NCS-SCS).

Example 6.2. From example 4.9 L and O are any two NCS-SOSS.

Theorem 6.3. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu})$ *be a NCS-bi-TS then*

(a) *Every NCS-SOS is NCS-bi-OS.*

(b) *Every NCS-SCS is NCS-bi-CS.*

Proof. (a) Let A be a NCS-SOS, then $A \in \tau_1^{\mu}$ and $A \notin \tau_2^{\mu}$ or $A \in \tau_2^{\mu}$ and $A \notin \tau_1^{\mu}$, therefore A is NCS-bi-OS.

(b) Let A be a NCS-SCS, then A^C is NCS-SOS therefore $A^C \in \tau_1^{\mu}$ and $A^C \notin \tau_2^{\mu}$ or $A^C \in \tau_2^{\mu}$ and $A^C \notin \tau_1^{\mu}$ hence A^C is NCS-bi-OS therefore A is NCS-bi-CS.

Remark 6.4. The converse of 6.2 is not true as seen from the following 6.5.

Example 6.5. From any NCS-bi-TS, φ_N, X_N are two NCS-bi-OS but not NCS-SOS and also φ_N, X_N are two NCS-bi-CS but not NCS-SCS

Theorem 6.6. *Let* $(X, \tau_1^\mu, \tau_2^\mu)$ *be a NCS-bi-TS, then the union of two NCS-SOS (NCS-SCS) is not a NCS-SOS (NCS-SCS).*

Proof. Proof follows from the following example.

Example 6.7. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^\mu = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^\mu = \{\varphi_N, X_N, O, P, Q\}$ where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $Q = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence $(X, \tau_1^\mu), (X, \tau_2^\mu)$ are NCSTS. Therefore $(X, \tau_1^\mu, \tau_2^\mu)$ is NCS-bi-TS.

N, O are two NCS-SOSs but $N \cup O = \langle\{\eta_2, \psi_3\}, \varphi, \{\delta_1\}\rangle$ is not a NCS-SOS. $N^C = \langle\{\delta_1, \psi_3\}, \varphi, \{\eta_2\}\rangle$ and $O^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-bi-CSs but $N^C \cup O^C = \langle\{\delta_1, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-SCS.

Theorem 6.8. *Let* $(X, \tau_1^\mu, \tau_2^\mu)$ *be a NCS-bi-TS, then the intersection of two NCS-SOS (NCS-SCS) is not a NCS-SOS (NCS-SCS).*

Proof. Proof follows from the following example.

Example 6.9. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^\mu = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^\mu = \{\varphi_N, X_N, O, P, Q\}$ where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $Q = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence $(X, \tau_1^\mu), (X, \tau_2^\mu)$ are NCSTS. Therefore $(X, \tau_1^\mu, \tau_2^\mu)$ is NCS-bi-TS.

L, O are two NCS-SOS but $L \cap O = \langle\phi, \phi, \{\psi_3, \delta_1\}\rangle$ is not a NCS-SOS. $L^C = \langle\{\psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $O^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-SCSs but $L^C \cap O^C = \langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is not a NCS-SCS.

7. Neutrosophic Crisp Supra Tri-Topological Spaces

Definition 7.1. Let τ_1^{μ}, τ_2^{μ} and τ_3^{μ} are three NCSTs on a non-empty set X then $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ is called neutrosophic crisp supra tri-topological spaces (NCS-tri-TS for short).

Example 7.2. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$, $\tau_3^{\mu} = \{\varphi_N, X_N, R, S, T\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $P = \langle\{\delta_1\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$, $R = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $S = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $T = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^{μ}), (X, τ_2^{μ}) and (X, τ_2^{μ}) are NCSTS. Therefore $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ is NCS-tri-TS.

Definition 7.3. Let $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ be a (NCS-tri-TS) then $\tau_1^{\mu} \cup \tau_2^{\mu} \cup \tau_3^{\mu}$ are called neutrosophic crisp supra tri-open sets (NCS-tri-OSs for short) and the complement of NCStri-OSs are called neutrosophic crisp supra bi-closed sets (NCS-tri-CSs for short).

Note 7.4. (a) The family of all NCS-tri-OSs can be written as NCS-tri-OS(X).

(b) The family of all NCS-tri-CSs can be written as NCS-tri-CS(X).

Example 7.5. From Example 7.2 NCS-tri-OSs are NCS-tri-OS $(X) = \tau_1^{\mu} \cup \tau_2^{\mu} \cup \tau_3^{\mu} = \{\varphi_N, X_N, L, M, N, O, P, Q, R, S, T\}$ and NCS-tri-CSs are NCS-tri-CS $(X) = \tau_1^{\mu} \cup \tau_2^{\mu} \cup \tau_3^{\mu} = \{\varphi_N, X_N, L^C, M^C, N^C, O^C, P^C, Q^C, R^C, S^C, T^C\}$, where $L^C = \langle\{\psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$, $M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$, $N^C = \langle\{\delta_1, \psi_3\}, \varphi, \{\eta_2\}\rangle$, $O^C = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $P^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1\}\rangle$, $Q^C = \langle\{\eta_2, \psi_3\}, \varphi, \{\delta_1\}\rangle$, $R^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $S^C = \langle\{\sigma_4, \delta_1\}, \varphi, \{\psi_3\}\rangle$, $T^C = \langle\{\eta_2, \delta_1\}, \varphi, \{\psi_3\}\rangle$.

Remark 7.6. (a) Every NCSOS in (X, τ_1^{μ}) or (X, τ_2^{μ}) or (X, τ_3^{μ}) is NCS-tri-OS.

(b) Every NCSCS in (X, τ_1^{μ}) or (X, τ_2^{μ}) or (X, τ_3^{μ}) is NCS-tri-CS.

Theorem 7.7. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ *be a NCS-tri-TS then the union of two NCS-tri-OS (NCS-tri-CS) need not a NCS-tri-OS (NCS-tri-CS).*

Proof. Proof follows 7.8.

Example 7.8. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$, $\tau_3^{\mu} = \{\varphi_N, X_N, R, S, T\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $P = \langle\{\delta_1\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$, $R = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $S = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $T = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^{μ}), (X, τ_2^{μ}) and (X, τ_2^{μ}) are NCSTS. Therefore $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ is NCS-tri-TS.

M, *R* are NCS-tri-OSs but $M \cup R = \langle\{\delta_1, \eta_2, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-tri-OS. $M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $R^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-tri-CSs but $M^C \cup R^C = \langle\{\delta_1, \sigma_4, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-tri-CS.

Theorem 7.9. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ *be a NCS-tri-TS then the intersection of two NCS-tri-OS (NCS-tri-CS) need not a NCS-tri-OS (NCS-tri-CS).*

Proof. Follows from the following example.

Example 7.10. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$, $\tau_3^{\mu} = \{\varphi_N, X_N, R, S, T\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $P = \langle\{\delta_1\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$, $R = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $S = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $T = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^{μ}), (X, τ_2^{μ}) and (X, τ_2^{μ}) are NCSTS. Therefore $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ is NCS-tri-TS.

M, *R* are two NCS-tri-OSs but $M \cap R = \langle\phi, \phi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is not a NCS-tri-OS. $M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $R^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-tri-CSs but $M^C \cap R^C = \langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is not a NCS-tri-CS.

8. The Closure and Interior via NCS-tri-OS and NCS-tri-CS

Definition 8.1. Let $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ be a NCS-tri-TS and A is NCS. Then neutrosophic crisp supra tri-interior of A can be defined as NCS-tri-Int $(A) = \bigcup\{\lambda : \lambda \subseteq A;\ \lambda$ is a NCS-tri-OS$\}$.

Definition 8.2. Let $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ be a NCS-tri-TS and A is a NCS. Then neutrosophic crisp supra tri-closure of A can be defined as NCS-tri-Cl $(A) = \bigcap\{\lambda^* : A \subseteq \lambda^*;\ \lambda^*$ is a NCS-tri-CS$\}$.

Theorem 8.3. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu})$ *be NCS-bi-TS*, *A is NCS then*

(a) *NCS-tri-int* $(A) \subseteq A$.

(b) *NCS-tri-int* (*A*) *is not a NCS-tri-OS*.

Proof. (a) It is clear from Definition 8.1.

(b) follows from Theorem 7.7.

Theorem 8.4. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu})$ *be NCS-tri-TS*, *A is a NCS then*

(a) $A \subseteq$ *NCS-tri-cl* (*A*).

(b) *NCS-tri-cl* (*A*) *is not a NCS-tri-CS*.

Proof. (a) It is clear from Definition 8.2.

(b) follows from Theorem 7.9.

9. The Neutrosophic Crisp Supra Tri *S*-Open Sets (NCS-tri-SOS) and Neutrosophic Crisp Supra Tri *S*-Closed Set (NCS-tri-SCS)

In this section we introduced new concept of open and closed sets in NCS-tri-TS. Also we introduced the basic properties of this new concept.

Definition 9.1. Let $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ be a NCS-tri-TS. Then the neutrosophic crisp supra open set only in one of the three neutrosophic crisp supra topological spaces $(X, \tau_1^{\mu}), (X, \tau_2^{\mu})$ and (X, τ_3^{μ}) are called the neutrosophic crisp tri-S-open set (NCS-tri-SOS).

The complement of NCS-tri-SOS is called neutrosophic crisp supra S-closed set (NCS-tri-SCS).

Example 9.2. From 3.8. A, C are any two NCS-tri-SOS.

Theorem 9.3. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ *be a NCS-tri-TS. Then*

(a) *Every NCS-tri-SOS is NCS-tri-OS.*

(b) *Every NCS-tri-SCS is NCS-tri-CS.*

Proof. (a) Let A be a NCS-tri-SOS, then it is one of the NCSOS in three neutrosophic crisp supra topological space $(X, \tau_1^{\mu}), (X, \tau_2^{\mu})$ and (X, τ_3^{μ}) Therefore A is NCS-tri-OS.

(b) Let A be a NCS-tri-SCS, then it is one of the NCSCS in one of the three neutrosophic crisp supra topological space $(X, \tau_1^{\mu}), (X, \tau_2^{\mu})$ and (X, τ_3^{μ}) Therefore A is NCS-tri-OS.

Remark 9.4. The converse 9.3 is not true as seen from 9.5.

Example 9.5. From any NCS-tri-TS, φ_N, X_N are two NCS-tri-OS but not NCS-tri-SOS and also φ_N, X_N are two NCS-tri-CS but not NCS-tri-SCS.

Theorem 9.6. *Let* $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ *be a NCS-tri-TS, then the union of two NCS-tri-SOS (NCS-tri-SCS) is not a NCS-tri-SOS (NCS-tri-SCS).*

Proof. Proof follows from the following example.

Example 9.7. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^{\mu} = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^{\mu} = \{\varphi_N, X_N, O, P, Q\}$, $\tau_3^{\mu} = \{\varphi_N, X_N, R, S, T\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $P = \langle\{\delta_1\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$, $R = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $S = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $T = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence $(X, \tau_1^{\mu}), (X, \tau_2^{\mu})$ and (X, τ_2^{μ}) are NCSTS. Therefore $(X, \tau_1^{\mu}, \tau_2^{\mu}, \tau_3^{\mu})$ is NCS-tri-TS.

M, *R* are NCS-tri-SOSs but $M \cup R = \langle\{\delta_1, \eta_2, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-tri-SOS.

$M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $R^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are NCS-tri-SCSs but $M^C \cup R^C = \langle\{\delta_1, \sigma_4, \psi_3\}, \varphi, \varphi\rangle$ is not a NCS-tri-SCS.

Theorem 9.8. *Let* $(X, \tau_1^\mu, \tau_2^\mu)$ *be a NCS-tri-TS, then the intersection of two NCS-tri-SOS (NCS-tri-SCS) is not a NCS-tri-SOS (NCS-tri-SCS).*

Proof. Proof follows from the following example.

Example 9.9. Let $X = \{\delta_1, \eta_2, \psi_3, \sigma_4\}$, $\tau_1^\mu = \{\varphi_N, X_N, L, M, N\}$, $\tau_2^\mu = \{\varphi_N, X_N, O, P, Q\}$, $\tau_3^\mu = \{\varphi_N, X_N, R, S, T\}$, where $L = \langle\{\delta_1, \eta_2\}, \varphi, \{\psi_3\}\rangle$, $M = \langle\{\delta_1, \eta_2\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $N = \langle\{\eta_2\}, \varphi, \{\delta_1, \psi_3\}\rangle$, $O = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$, $P = \langle\{\delta_1\}, \varphi, \{\sigma_4, \psi_3\}\rangle$, $Q = \langle\{\delta_1\}, \varphi, \{\eta_2, \psi_3\}\rangle$, $R = \langle\{\psi_3\}, \varphi, \{\delta_1\}\rangle$, $S = \langle\{\psi_3\}, \varphi, \{\sigma_4, \delta_1\}\rangle$, $T = \langle\{\psi_3\}, \varphi, \{\eta_2, \delta_1\}\rangle$. Hence (X, τ_1^μ), (X, τ_2^μ) and (X, τ_2^μ) are NCSTS. Therefore $(X, \tau_1^\mu, \tau_2^\mu, \tau_3^\mu)$ is NCS-tri-TS.

M, R are two NCS-tri-SOSs but $M \cap R = \langle\phi, \phi, \{\delta_1, \psi_3, \sigma_4\}\rangle$ is not a NCS-tri-SOS. $M^C = \langle\{\sigma_4, \psi_3\}, \varphi, \{\delta_1, \eta_2\}\rangle$ and $R^C = \langle\{\delta_1\}, \varphi, \{\psi_3\}\rangle$ are two NCS-tri-SCSs but $M^C \cap R^C = \langle\varphi, \varphi, \{\delta_1, \eta_2, \psi_3\}\rangle$ is not a NCS-tri-SCS.

10. Conclusion

In this paper we introduced NCSTS, NCS-bi-TS, NCS-tri-TS, NCS-bi-OS, NCS-bi-CS, NCS-SOS, NCS-SCS, NCS-tri-SCS and NCS-tri-SCS also we investigated some of its basic properties. Finally these concepts going to pave the way for new types of open and closed sets in NCST.

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