Hindawi
Journal of Mathematics
Volume 2021, Article ID 5582102, 5 pages
https://doi.org/10.1155/2021/5582102

Research Article

Neutrosophic D'Agostino Test of Normality: An Application to Water Data

Mohammed Albassam[1], **Nasrullah Khan**[2], **and Muhammad Aslam**[1]

[1]*Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21551, Saudi Arabia*
[2]*Department of Statistics, University of Veterinary and Animal Sciences, Jhang Campus, Lahore, Pakistan*

Correspondence should be addressed to Muhammad Aslam; aslam_ravian@hotmail.com

Received 25 January 2021; Revised 8 February 2021; Accepted 16 February 2021; Published 23 February 2021

Academic Editor: Parimala Mani

The D'Agostino test has been widely applied for testing the normality of the data. The existing D'Agostino test cannot be applied when the data have some indeterminate observations or observations which are obtained from the complex systems. In this paper, we present a D'Agostino test under neutrosophic statistics. We propose the D'Agostino test to test the normality of the data having indeterminate observations. The design of the proposed test is given and implemented with the help of real data. From the comparison, it is concluded that the proposed test is effective, adequate, and suitable to be applied in the presence of indeterminacy.

1. Introduction

The data obtained from various fields such as medical, physiological, education, and chemical process are assumed to follow the approximately normal distribution. Therefore, before some estimation and forecasting, the normality of the data in hand is checked first. If the data follow the normal distribution, the statistical techniques based on normal distribution are used; otherwise, the nonparametric methods are applied for the analysis of the data. Among many statistical tests, the D'Agostino test has been widely applied for testing the normality of the data. This test is used to test the null hypothesis that the data do not significantly differ from the normal distribution versus the alternative hypothesis that the data significantly differ from the normality. D'Agostino and Stephens [1] introduced statistical tests when the data follow the normal distribution. Öztuna et al. [2] studied the power of the test and type-I error rate for various tests under normality assumptions. Yap and Sim [3] discussed various statistical tests and showed that the D'Agostino test has better power. Chen and Xia [4] presented tests when data are nonnormal. Mishra et al. [5] presented the descriptive statistic for the test. More details on the statistical test for normality can be seen in [6–9].

The traditional statistical tests are applied to test the hypothesis that the data follow approximately normal distribution with exact mean and variance. In some situations, such as the measure of the water level, a lifetime of a product and melting of a material cannot be expressed in the exact form and have approximate mean and variances. In this case, the statistical test using the fuzzy logic is preferable to apply for the analysis of the data [10]. Hesamian and Akbari [11] presented the tests using fuzzy logic. Chachi and Taheri [12] worked on the optimal test using the fuzzy approach. Haktanır and Kahraman [13] discussed the role of tests in decision-making issues. For details, the reader may refer to [14–24].

The neutrosophic logic which is more efficient than the fuzzy logic and interval-based analysis was proposed by Smarandache [25]. This logic estimates the measures of truth, falsehood, and indeterminacy, while the fuzzy logic is unable to estimate the measure of indeterminacy. More applications of neutrosophic logic can be read in [26–36]. Based on the idea of neutrosophic logic, Smarandache [37] introduced the descriptive neutrosophic statistics which are applied for the analysis of the data having indeterminate observations. Kandasamy and Smarandache [38] introduced the neutrosophic numbers for the first time. Chen et al. [39] applied the neutrosophic numbers in rock measuring. Aslam [40] introduced a new branch of statistical quality control under neutrosophic statistics. Kolmogorov–Smirnov tests and Bartlett and Hartley tests using neutrosophic statistics were developed by Aslam [41, 42], respectively. More details on the application of neutrosophic statistics can be seen in [43, 44].

Although the D'Agostino test under classical statistics is available in the literature, the existing D'Agostino test cannot be applied if observations are imprecise, vague, and indeterminate. By exploring the literature and according to the best of our knowledge, there is work on the D'Agostino test. In this paper, we will propose and design the D'Agostino test under indeterminacy. The operational process of the proposed test is explained. The application of the proposed test will be given with the help of water data. We expect that the proposed test will be informative and adequate than the existing D'Agostino test under classical statistics in the indeterminate environment.

2. Preliminary

Suppose that a_i and $b_i I_N; I_N \varepsilon [I_L, I_U]$ are determinate and indeterminate parts of neutrosophic random variable $z_N = a_i + b_i I_N; I_N \epsilon [I_L, I_U]$, $i = 1, 2, \ldots, n_N$, where n_N denotes the neutrosophic sample size. The values of z_N reduce to a_i when $I_N = 0$. Based on this information, compute the neutrosophic average for variable $z_N \epsilon [z_L, z_U]$ as follows:

$$\overline{z}_N = \overline{a} + \overline{b} I_N,\ I_N \epsilon [I_L, I_U], \tag{1}$$

where $\overline{a} = (1/n_N) \sum_{i=1}^{n_N} a_i$ and $\overline{b} = (1/n_N) \sum_{i=1}^{n_N} b_i$.

The neutrosophic sum of squares (NSS) by following [39] is computed as follows:

$$\sum_{i=1}^{n_N} (z_i - \overline{z}_{iN})^2 = \sum_{i=1}^{n_N} \left[\begin{matrix} \min\left(\begin{matrix} (a_i + b_i I_L)(\overline{a} + \overline{b} I_L), (a_i + b_i I_L)(\overline{a} + \overline{b} I_U) \\ (a_i + b_i I_U)(\overline{a} + \overline{b} I_L), (a_i + b_i I_U)(\overline{a} + \overline{b} I_U) \end{matrix} \right) \\ \max\left(\begin{matrix} (a_i + b_i I_L)(\overline{a} + \overline{b} I_L), (a_i + b_i I_L)(\overline{a} + \overline{b} I_U) \\ (a_i + b_i I_U)(\overline{a} + \overline{b} I_L), (a_i + b_i I_U)(\overline{a} + \overline{b} I_U) \end{matrix} \right) \end{matrix} \right], \quad I_N \epsilon [I_L, I_U]. \tag{2}$$

3. Design of the Proposed D'Agostino Test under Neutrosophic Statistics

The main objective is to design D'Agostino test under neutrosophic statistics for testing the null hypothesis H_{0N} that the neutrosophic data follow the neutrosophic normal distribution versus the alternative hypothesis H_{1N} that the data do not belong to the neutrosophic normal distribution. The acceptance of the null hypothesis means that the data are not significantly away from the normal distribution. The operational procedure of the proposed test is stated as follows.

Step 1: Compute the neutrosophic averages of lower values $a_i (i = 1, 2, \ldots, n_L)$ and upper values $b_i (i = 1, 2, \ldots, n_U)$ as follows: $\overline{a} = (1/n_N) \sum_{i=1}^{n_N} a_i$ and. $\overline{b} = (1/n_N) \sum_{i=1}^{n_N} b_i$.

Step 2: Find neutrosophic average as follows:

$$\overline{z}_N = \overline{a} + \overline{b} I_N, I_N \epsilon [I_L, I_U]. \tag{3}$$

Step 3: The neutrosophic sum of squares (NSS) by following [39] is calculated using the following expression:

$$\sum_{i=1}^{n_N} (z_i - \overline{z}_{iN})^2 = \sum_{i=1}^{n_N} \left[\begin{matrix} \min\left((a_i - \overline{a})^2, \left((a_i - \overline{a}) \left((a_i - \overline{a}) + 1 \times \left(b_i - \overline{b} \right) \right), (a_i - \overline{a}) + 1 \times \left(b_i - \overline{b} \right)^2 \right) \right) \\ \max\left((a_i - \overline{a})^2, \left((a_i - \overline{a}) \left((a_i - \overline{a}) + 1 \times \left(b_i - \overline{b} \right) \right), (a_i - \overline{a}) + 1 \times \left(b_i - \overline{b} \right)^2 \right) \right) \end{matrix} \right]. \tag{4}$$

Step 4: Compute the neutrosophic numerator $T_N \epsilon [T_L, T_U]$ of the proposed test as follows:

$$T_N = \sum \left(i_N - \left(\frac{n_N + 1}{2} \right) \right) X_{iN} T_N \epsilon [T_L, T_U], \tag{5}$$

where i_N denotes the rank of neutrosophic observations X_{iN} for $a_i (i = 1, 2, \ldots, n_L)$ and $b_i (i = 1, 2, \ldots, n_U)$.

Step 5: Compute the neutrosophic test statistic $D_N \epsilon [D_L, D_U]$ of the proposed test as follows:

$$D_N = \frac{T_N}{\sqrt{n_N^3 \left(\sum_{i=1}^{n_N} (z_i - \overline{z}_{iN})^2 \right)}}, T_N \epsilon [T_L, T_U], D_N \epsilon [D_L, D_U]. \tag{6}$$

Step 6: Decide the level of significance α and select the critical values from the D'Agostino table. The null hypothesis will be accepted if $D_N \epsilon [D_L, D_U]$ lies within the range of the tabulated values.

Table 1: The PMW data.

Portuguese mineral	n. 1		n. 2		n. 3		n. 4		n. 5	
	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i	a_i	b_i
HCO_3^-	21	41	113	119	2.2	4.2	8	11.6	4.6	5
CI^-	7	9	16.5	17.5	3.6	4	4.1	4.7	6.6	7.4
N_a^+	10	16	10.3	10.7	2.8	3.8	2.8	3.6	5.4	5.6
C_a^{2+}	3	4	15	21	0.01	1.01	1.9	2.9	0.72	0.84
SiO_2	23	29	13.7	14.9	1.01	7.8	5.8	6.8	16.7	18.3
pH	6.1	6.5	6.7	7.1	5.71	5.81	5.9	6	5.4	5.8

Table 2: Neutrosophic means of five different types of water.

Water	$\bar{a}_N$	$\bar{b}_N$	$\bar{z}_N$
n. 1	11.68	17.58	[11.68, 29.26]
n. 2	29.2	31.7	[29.2, 60.9]
n. 3	2.55	4.43	[2.55, 6.98]
n. 4	4.75	5.93	[4.75, 10.68]
n. 5	5.57	7.15	[5.57, 12.72]

4. Application for Portuguese Mineral Water

In this section, we will give the application of the proposed test using the Portuguese mineral water (PMW) data. D'Urso and Giordani [45] used the same data and analyzed them using classical statistics. D'Urso and Giordani [45] conidered six mineral concentrations such as six mineral concentrations of HCO_3^-, CI^-, N_a^+, C_a^{2+}, SiO_2, and pH. The PMW data are reported in Table 1. Table 1 clearly indicates that the data are reported in intervals. Before any prediction or estimation is given for the data, it is necessary to see that the data do not significantly differ from the normal distribution. Therefore, we will apply the proposed test on these data to test whether the six variables are from the neutrosophic normal distribution or not.

The necessary computations for PMW data are given in the following steps.

Step 1: The neutrosophic averages of lower values $a_i\ (i = 1, 2, \ldots, n_L)$ and upper values $b_i\ (i = 1, 2, \ldots, n_U)$ of PMW data of five different types of water are given in Table 2.

Step 2: The neutrosophic averages $\bar{z}_N$; $I_N \epsilon [0, 1]$for the water data are also shown in Table 2.

Step 3: The values of NSS are given in Table 3 by following [39]:

$$\sum_{i=1}^{n_N} \left(z_i - \bar{z}_{iN}\right)^2 = \sum_{i=1}^{n_N} \begin{bmatrix} \min\left(\left(a_i - \bar{a}\right)^2, \left(\left(a_i - \bar{a}\right)\left(\left(a_i - \bar{a}\right) + 1 \times \left(b_i - \bar{b}\right)\right), \left(a_i - \bar{a}\right) + 1 \times \left(b_i - \bar{b}\right)^2\right)\right) \\ \max\left(\left(a_i - \bar{a}\right)^2, \left(\left(a_i - \bar{a}\right)\left(\left(a_i - \bar{a}\right) + 1 \times \left(b_i - \bar{b}\right)\right), \left(a_i - \bar{a}\right) + 1 \times \left(b_i - \bar{b}\right)^2\right)\right) \end{bmatrix}. \quad (7)$$

Step 4: The values $T_N \epsilon [T_L, T_U]$ and $D_N \epsilon [D_L, D_U]$ are also shown in Table 3.

Step 5: Let $\alpha = 0.05$; the range of the tabulated values is 0.2513, 0.2849. The null hypothesis that the data follow the normal distribution is accepted if $D_N \epsilon [D_L, D_U]$ is within the range of the tabulated values. The acceptance or rejection of H_{0N} is shown in Table 3. From Table 3, it is clear that the PMW data for all waters do not follow the neutrosophic normal distribution.

5. Comparative Study and Discussion

The proposed D'Agostino test under neutrosophic statistics is the extension of the D'Agostino test under classical statistics. The proposed test reduces to D'Agostino test under classical statistics when $D_N = D_L = 0$. We compare the proposed test with the existing D'Agostino test using the PMW data of five types of water with the same values of α. The values of statistic D for the existing test and the proposed test along with the measure of indeterminacy are shown in Table 4. From Table 4, it can be seen that the proposed test statistic $D_N \epsilon [D_L, D_U]$ has the results in the neutrosophic form with the probability of the indeterminacy. On the contrary, the existing test provides only the determined values of statistic D. For example, when $\alpha = 0.05$ and $n.1$, the null hypothesis H_{0N} will accepted the probability of 0.95, the chance to do not accept H_{0N} is 0.05, and the probability of indeterminacy is 0.0621. From the proposed test, it can be seen that $0.95 + 0.05 + 0.062 > 1$ which shows the case of paraconsistent neutrosophic probability, see [37].

Table 3: The values of NSS of five waters.

Water	NSS	$T_N \epsilon [T_L, T_U]$	$D_N \epsilon [D_L, D_U]$	Decision
n. 1	[811.01, 4915.23]	[73.85, 117.75]	[0.1764, 0.1142]	Do not accept H_{0N}
n. 2	[7858.36, 33176.61]	[274.2, 296.5]	[0.2104, 0.1107]	Do not accept H_{0N}
n. 3	[54.55, 268.79]	[18.43, 20.09]	[0.7847, 0.6489]	Do not accept H_{0N}
n. 4	[84.73, 521.31]	[20.75, 27.2]	[0.1533, 0.0810]	Do not accept H_{0N}
n. 5	[385.16, 1686.90]	[42.95, 47.35]	[0.1489, 0.0784]	Do not accept H_{0N}

Table 4: The comparison of two tests.

Water	The proposed test		The existing test
	$D_N \epsilon [D_L, D_U]$	Measure of indeterminacy $I_N \epsilon [I_L, L_U]$	D
n. 1	[0.1764, 0.1142]	0.1764–0.1142 I_N; [0, 0.0621]	0.1764
n. 2	[0.2104, 0.1107]	0.2104–0.1107 I_N; [0, 0.90]	0.2104
n. 3	[0.7847, 0.6489]	0.7847–0.6489 I_N; [0, 0.21]	0.7847
n. 4	[0.1533, 0.0810]	0.1533–0.0810 I_N; [0, 0.892]	0.1533
n. 5	[0.1489, 0.0784]	0.1489–0.0784 I_N; [0, 0.0.899]	0.1489

On the contrary, the existing test provides only the determined value which is not adequate when the data have interval, uncertain, and indeterminate values or the data are obtained from the complex system. From this comparison, it is concluded that the proposed test provides the values of statistic in the indeterminate interval, and this theory is the same as in [39]. Therefore, the use of the proposed test is adequate under an indeterminate environment.

6. Concluding Remarks

In this paper, we presented a D'Agostino test under neutrosophic statistics. We proposed the D'Agostino test to test the normality of the data having indeterminate observations. The design of the proposed test was given and implemented with the help of real data. The proposed test was the extension of an existing D'Agostino test under classical statistics. From the comparison, it was concluded that the proposed test is effective, adequate, and suitable to be applied in the presence of indeterminacy. The development of software for the proposed test will be a fruitful area of research. The application of the proposed test for big datasets such as testing the normality of ocean data, Facebook user data, and rail data can be considered as future research.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This article was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah (Grant no. D-103-130–1441). The authors, therefore, gratefully acknowledge DSR for the technical and financial support.

References

[1] R. D'Agostino and M. Stephens, *Tests for Normal Distribution in Goodness-Of-Fit Techniques, Statistics: Textbooks and Monographs*, R. B. D'Agostino and Stephens, Eds., Dekker, New York, NY, USA, 1986.

[2] D. Öztuna, A. H. Elhan, and E. Tüccar, "Investigation of four different normality tests in terms of type 1 error rate and power under different distributions," *Turkish Journal of Medical Sciences*, vol. 36, no. 3, pp. 171–176, 2006.

[3] B. W. Yap and C. H. Sim, "Comparisons of various types of normality tests," *Journal of Statistical Computation and Simulation*, vol. 81, no. 12, pp. 2141–2155, 2011.

[4] H. Chen and Y. Xia, "A nonparametric normality test for high-dimensional data," 2019, https://arxiv.org/pdf/1904.05289.pdf.

[5] P. Mishra, C. M. Pandey, U. Singh, A. Gupta, C. Sahu, and A. Keshri, "Descriptive statistics and normality tests for statistical data," *Annals of Cardiac Anaesthesia*, vol. 22, no. 1, p. 67, 2019.

[6] E. S. Pearson, R. B. D'"Agostino, and K. O. Bowman, "Tests for departure from normality: comparison of powers," *Biometrika*, vol. 64, no. 2, pp. 231–246, 1977.

[7] R. B. D'agostino, A. Belanger, and R. B. D'Agostino Jr, "A suggestion for using powerful and informative tests of normality," *The American Statistician*, vol. 44, no. 4, pp. 316–321, 1990.

[8] L. Baringhaus and N. Henze, "A test for uniformity with unknown limits based on d'agostino's D," *Statistics & Probability Letters*, vol. 9, no. 4, pp. 299–304, 1990.

[9] S. Kallithraka, I. S. Arvanitoyannis, P. Kefalas, A. El-Zajouli, E. Soufleros, and E. Psarra, "Instrumental and sensory analysis of Greek wines; implementation of principal component analysis (PCA) for classification according to geographical origin," *Food Chemistry*, vol. 73, no. 4, pp. 501–514, 2001.

[10] N. Watanabe and T. Imaizumi, "A fuzzy statistical test of fuzzy hypotheses," *Fuzzy Sets and Systems*, vol. 53, no. 2, pp. 167–178, 1993.

[11] G. Hesamian and M. G. Akbari, "Statistical test based on intuitionistic fuzzy hypotheses," *Communications in Statistics-Theory and Methods*, vol. 46, no. 18, pp. 9324–9334, 2017.

[12] J. Chachi and S. M. Taheri, "Optimal statistical tests based on fuzzy random variables," *Iranian Journal of Fuzzy Systems*, vol. 15, no. 5, pp. 27–45, 2018.

[13] E. Haktanır and C. Kahraman, "Fuzzy hypothesis testing in statistical decision making," *Journal of Intelligent & Fuzzy Systems (Preprint)*, vol. 37, no. 5, pp. 6545–6555, 2019.

[14] B. Van Cutsem and I. Gath, "Detection of outliers and robust estimation using fuzzy clustering," *Computational Statistics & Data Analysis*, vol. 15, no. 1, pp. 47–61, 1993.

[15] M. Montenegro, M. A. R. Casals, M. A. A. Lubiano, and M. A. A. Gil, "Two-sample hypothesis tests of means of a fuzzy random variable," *Information Sciences*, vol. 133, no. 1-2, pp. 89–100, 2001.

[16] V. Mohanty and P. AnnanNaidu, "Fraud detection using outlier analysis: a survey," *International Journal of Engineering Sciences and Research Technology*, vol. 2, no. 6, 2013.

[17] Y. M. Moradnezhadi, "Determination of a some simple methods for outlier detection in maximum daily rainfall (case study: baliglichay Watershed Basin–Ardebil Province–Iran)," *Bulletin of Environment, Pharmacology and Life Sciences*, vol. 3, no. 3, pp. 110–117, 2014.

[18] T.-Y. Ling, C.-L. Soo, J.-J. Liew, L. Nyanti, S.-F. Sim, and J. Grinang, "Application of multivariate statistical analysis in evaluation of surface river water quality of a tropical river," *Journal of Chemistry*, vol. 2017, 2017.

[19] G. Shahzadi, M. Akram, and A. B. Saeid, "An application of single-valued neutrosophic sets in medical diagnosis," *Neutrosophic Sets and Systems*, vol. 18, pp. 80–88, 2017.

[20] A. K. Shrestha and N. Basnet, "The Correlation and regression analysis of physicochemical parameters of River water for the evaluation of percentage contribution to electrical conductivity," *Journal of Chemistry*, vol. 2018, 2018.

[21] Y. Choi, H. Lee, and Z. Irani, "Big data-driven fuzzy cognitive map for prioritising IT service procurement in the public sector," *Annals of Operations Research*, vol. 270, no. 1-2, pp. 75–104, 2018.

[22] S. Habib and M. Akram, "Diagnostic methods and risk analysis based on fuzzy soft information," *International Journal of Biomathematics*, vol. 11, no. 08, Article ID 1850096, 2018.

[23] S. Habib and M. Akram, "Medical decision support systems based on Fuzzy Cognitive Maps," *International Journal of Biomathematics*, vol. 12, no. 06, Article ID 1950069, 2019.

[24] S. Habib, M. A. Butt, M. Akram, and F. Smarandache, "A neutrosophic clinical decision-making system for cardiovascular diseases risk analysis," *Journal of Intelligent & Fuzzy Systems (Preprint)*, vol. 39, no. 5, pp. 7807–7829, 2020.

[25] F. Smarandache, *Neutrosophy. Neutrosophic Probability, Set, and Logic*, vol. 105, pp. 118–123, ProQuest Information & Learning, Ann Arbor, MI, USA, 1998.

[26] H. Wang, F. Smarandache, R. Sunderraman, and Y.-Q. Zhang, "Interval neutrosophic sets and logic: theory and applications in computing: theory and applications in computing," *Infinite Study*, vol. 5, 2005.

[27] I. Hanafy, A. Salama, and K. Mahfouz, "Neutrosophic classical events and its probability," *International Journal of Mathematics and Computer Applications Research (IJMCAR)*, vol. 3, no. 1, pp. 171–178, 2013.

[28] Y. Guo and A. Sengur, "NECM: neutrosophic evidential c-means clustering algorithm," *Neural Computing and Applications*, vol. 26, no. 3, pp. 561–571, 2015.

[29] M. Abdel-Basset, G. Manogaran, A. Gamal, and F. Smarandache, "A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria," *Design Automation for Embedded Systems*, vol. 22, no. 3, pp. 257–278, 2018.

[30] R. Alhabib, M. M. Ranna, H. Farah, and A. Salama, "Some neutrosophic probability distributions," *Neutrosophic Sets and Systems*, vol. 30, 2018.

[31] S. Broumi, A. Bakali, M. Talea, and F. Smarandache, "Bipolar neutrosophic minimum spanning tree," *Infinite Study*, vol. 127, 2018.

[32] X. Peng and J. Dai, "Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function," *Neural Computing and Applications*, vol. 29, no. 10, pp. 939–954, 2018.

[33] A. I. Shahin, K. M. Amin, A. A. Sharawi, and Y. Guo, "A novel enhancement technique for pathological microscopic image using neutrosophic similarity score scaling," *Optik*, vol. 161, pp. 84–97, 2018.

[34] M. Abdel-Basset, M. Mohamed, M. Elhoseny, L. H. Son, F. Chiclana, and A. E.-N. H. Zaied, "Cosine similarity measures of bipolar neutrosophic set for diagnosis of bipolar disorder diseases," *Artificial Intelligence in Medicine*, vol. 101, Article ID 101735, 2019.

[35] C. Jana and M. Pal, "A robust single-valued neutrosophic soft aggregation operators in multi-criteria decision making," *Symmetry*, vol. 11, no. 1, Article ID 110, 2019.

[36] N. A. Nabeeh, M. Abdel-Basset, H. A. El-Ghareeb, and A. Aboelfetouh, "Neutrosophic multi-criteria decision making approach for iot-based enterprises," *Institute of Electrical and Electronics Engineers Access*, vol. 7, pp. 59559–59574, 2019.

[37] F. Smarandache, "Introduction to neutrosophic statistics," *Infinite Study*, https://arxiv.org/abs/1406.2000, 2014.

[38] W. V. Kandasamy and F. Smarandache, "Fuzzy cognitive maps and neutrosophic cognitive maps," *Infinite Study*, https://arxiv.org/abs/math/0311063, 2003.

[39] J. Chen, J. Ye, and S. Du, "Scale effect and anisotropy analyzed for neutrosophic numbers of rock joint roughness coefficient based on neutrosophic statistics," *Symmetry*, vol. 9, no. 10, p. 208, 2017.

[40] M. Aslam, "A new sampling plan using neutrosophic process loss consideration," *Symmetry*, vol. 10, no. 5, p. 132, 2018.

[41] M. Aslam, "Introducing Kolmogorov–smirnov tests under uncertainty: an application to radioactive data," *ACS Omega*, vol. 5, no. 1, pp. 914–917, 2019.

[42] M. Aslam, "Design of the Bartlett and Hartley tests for homogeneity of variances under indeterminacy environment," *Journal of Taibah University for Science*, vol. 14, no. 1, pp. 6–10, 2020.

[43] M. Aslam, "Neutrosophic analysis of variance: application to university students," *Complex & Intelligent Systems*, vol. 5, no. 4, pp. 403–407, 2019.

[44] M. Aslam and M. Albassam, "Application of neutrosophic logic to evaluate correlation between prostate cancer mortality and dietary fat assumption," *Symmetry*, vol. 11, no. 3, p. 330, 2019.

[45] P. D'Urso and P. Giordani, "A least squares approach to principal component analysis for interval valued data," *Chemometrics and Intelligent Laboratory Systems*, vol. 70, no. 2, pp. 179–192, 2004.

[46] J. Chen, J. Ye, S. Du, and R. Yong, "Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers," *Symmetry*, vol. 9, no. 7, p. 123, 2017.